IDENTIFICATION OF PULSE STREAMS OF UNKNOWN SHAPE FROM TIME ENCODING MACHINE SAMPLES

Meghna Kalra[⋆] *Yoram Bresler*[†] *Kiryung Lee*[⋆]

[⋆]Ohio State University, USA [†]University of Illinois at Urbana-Champaign, USA

ABSTRACT

We present an algorithm for the resolution of delayed and overlapping pulses of a common unknown shape from multichannel measurements. We show that just a few Fourier samples acquired by a *Time Encoding Machine* (TEM) suffice to solve this challenging problem. This acquisition scheme is desired for ultra-low power applications in wearables, such as EMG skin sensor tattoo. Numerical experiments demonstrate exact recovery of the time delays and Fourier series coefficient of the pulse shape in the noiseless case as predicted by the theory, with acceptable error in the presence of noise.

***Index Terms*—** Blind deconvolution, superresolution, time encoding machine, subspace method

1. INTRODUCTION

We consider the problem of blind identification of the locations of delayed and scaled (and possibly overlapping) versions of a pulse of unknown shape from multichannel recordings illustrated in Fig. 1, using an ultra-low power and robust acquisition scheme.

This problem arises in surface electromyography (EMG), where the EMG signal received on the skin electrodes consist of impulses convolved with the shape of the action potential of the active muscle unit. It is desired to localize the pulses accurately and thus infer the impulse train, but this is challenging because of the unknown pulse shape and their overlap, which requires super-resolution.

An additional challenge arises in our motivating application in a wearable EMG device [1], where the power supply available for the electrode sensors is limited. This results in a trade off between higher resolution and higher power consumption.

We address the problem by using *Time Encoding Machine* (TEM) [2–5] signal-adaptive acquisition, and leveraging the algorithm by Bresler and Delaney [6] to solve the blind super-resolution and identification problem from the multichannel Fourier samples produced by TEM. Bresler and Delaney [6] considered the acquisition of Fourier measurements from conventional time samples taken at the Nyquist rate with respect to bandwidth of the pulse shape. Their goal was to recover the time delays and estimate the pulse shape simultaneously. A high sampling rate was required for the latter task. In this paper, we consider a modified goal, that is, to recover the time delays (comparatively from less information) and construct a Fourier sketch of the pulse shape. This modification lowers the required sample rate significantly, and together with the use of TEM enables a lower-power signal-adaptive acquisition scheme.

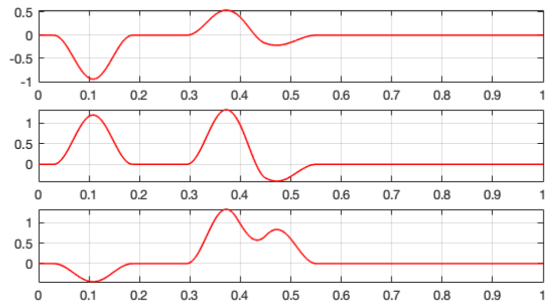

Fig. 1: Illustration of (overlapping) pulse streams with a shared delay pattern and varying amplitudes.

TEM is an alternative method to conventional sampling which has no global clock (asynchronous) and helps in reducing power consumption and electromagnetic interference [2]. It works on the principle of selective sampling. The time instants are recorded from the analog signal whenever a certain threshold is met. One of the popular TEM method is IF-TEM (integrate and fire TEM) [3–5]. It is inspired by the *integrate-and-fire* mechanism of neural system in the human brain, where the neurons use TEM samples to represent the neural information via their action potentials. Once firing instants are recorded using this method, the Fourier coefficients of the input signal is then reconstructed from the recordings.

The contributions of this paper are two folds. We propose a robust method for the multichannel blind identification of pulse parameters without requiring prior knowledge on the pulse shape. This is possible thanks to the redundant observations from a set of pulse streams sharing a common pulse shape and delay profile. We also present a sufficient condition for the exact recovery of the parameters by the proposed method. This work differs from the literature in the following aspects. First, unlike state-of-the-art methods in the sur-

KL was support by NSF CAREER Award CCF-1943201.

face EMG literature [7], which formulate the problem over a discretized signal model, our approach is in a continuous-time setting and provides infinite resolution in the absence of noise. Compared to prior art in recovering a similar model from TEM data [4], by invoking a mild condition on the pulse shape, we do not require knowledge of the pulse shape. We also quantify the deviation from this condition in terms of the temporal support size of the pulse shape.

2. PROBLEM STATEMENT

Consider a collection of M real-valued channels of pulse streams, each given as linear combinations of d delayed versions of an unknown real-valued pulse shape $g(t)$ with unknown weights:

$$h_m(t) = \sum_{l=1}^{d} \gamma_{l,m} g(t - \tau_l), \quad m \in [M], \tag{1}$$

where $[M] = \{1, 2, \ldots, M\}$. We suppose that $g(t)$ is supported within $[-R/2, R/2)$ and the lth pulse in each stream is delayed by delay $\tau_l \in [R/2, T - R/2)$, which is shared across streams.

The problem is to recover the pulse parameters from ultra low power and robust measurements derived from the M channels over the observation interval $[0, T)$.

2.1. Approach and main results

The Fourier transform of $h_m(t)$ is written as

$$H_m(\omega) = \sum_{l=1}^{d} \gamma_{l,m} G(\omega) e^{-\mathrm{j}2\pi\tau_l}. \tag{2}$$

Bresler and Delaney [6] have shown that the redundant observation across jointly supported pulse streams enables the blind recovery of $\tau_1, \ldots, \tau_d$ from a few Fourier samples of the pulse streams under a mild assumption on the unknown pulse shape $g(t)$. They assumed that the Fourier transform of $g(t)$ satisfies the *pairwise constant condition* (PCC) given by

$$G(2k\omega_0) = G((2k-1)\omega_0), \quad k \in [K] \tag{3}$$

for some $K \in \mathbb{N}$, where $\omega_0 = \frac{2\pi}{T}$.

Although Condition (3) does not hold for pulse shapes in applications, we now show that it is satisfied up to a small error if $g(t)$ is supported on a narrow interval. This is quantitatively justified by Bernstein's inequality (cf. [8, Theorem 6.7.1]), that is

$$\left| \frac{dG(\omega)}{d\omega} \right| \le 2\pi R \sup_{\omega \in \mathbb{R}} |G(\omega)|$$

if $g(t)$ is supported within the interval $[-R/2, R/2)$. This implies that the relative gradient of $G(\omega)$ normalized by the peak value is upper-bounded by the width of pulse shape. Therefore, the difference between both sides of (3) is at most R/T up to a constant. In other words, Condition (3) holds to a good approximation when the pulse width R is small compared to the length T of the observation interval.

The original problem statement in [6] addressed the simultaneous recovery of $(\tau_l)_{l=1}^d$ and $g(t)$. To this end, they set K to a large number so that $K\omega_0$ corresponds to an approximate bandwidth of $g(t)$. However, if one is interested in the recovery of only $(\tau_l)_{l=1}^d$, then it suffices to satisfy $2K > d$.[1] We see a disparity between the sufficient conditions for the success of two recovery problems. In this work we consider a modified goal as follows.

The problem is to estimate only the $(\tau_l)_{l=1}^d$ (up to a global shift) from $\{H_m(k\omega_0)\}_{m=1}^M$ for $k \in \mathcal{K}$, where $\mathcal{K}$ is a minimal set sufficient for the recovery of $(\tau_l)_{l=1}^d$. In the course of this recovery, we also estimate $G(k\omega_0)$ for $k \in \mathcal{K}$. While the latter may be insufficient for a high-fidelity recovery of the pulse shape $g(t)$, they may suffice for e.g. a classification task. Importantly, for the acquisition of the Fourier measurements of the pulse streams, we consider here the recently developed IF-TEM acquisition scheme.

The proposed algorithm consists of the following two steps: First we recover Fourier coefficients for all channels by using the total least squares (TLS). Then we recover the pulse parameters by ESPRIT leveraging the structure due to the pairwise constant assumption. These steps are described in the next sections.

The following theorem presents a sufficient condition for the parameter recovery in the noiseless case.

Theorem 1. *Suppose the following conditions: i) The Fourier transform $G(\omega)$ of $g(t)$ satisfies* (3)*; ii) $G(2k\omega_0) \neq 0$ for all $k \in [K]$; iii) $\mathbf{\Gamma} \in \mathbb{C}^{d \times M}$ with $(\mathbf{\Gamma})_{l,m} = \gamma_{l,m}$ has full rank. If the total number of TEM samples N_m per pulse stream is larger than $2d+1$ for all $m \in [M]$ with $M \geq d$, then $(\tau_l)_{l=1}^d$ is recovered by the proposed algorithm upto a global shift within $[-R/2, R/2)$. Furthermore, the Fourier coefficients $G(k\omega_0)$ for $k \in \mathcal{K}$ are recovered up to a global scaling ambiguity.*

Theorem 1 is obtained by combining [4, Theorem 1] and a sufficient condition for the success of ESPRIT [6], as applied to the data derived from the measurements.

3. ACQUISITION OF FOURIER MEASUREMENTS FROM IF-TEM SAMPLES

IF-TEM recordings of the analog signal $h_m(t)$ for $m \in [M]$ are obtained by using the acquisition scheme by Namman et al. [4]. Each pulse stream $h_m(t)$ is first convolved with a

[1] Since $g(t)$ is real-valued, $G(-\omega)$ is determined by $G^*(\omega)$. Therefore, the total number of Fourier measurements is $4K$.

sum-of-sinc (SoS) kernel [9] given by

$$f(t) = \mathrm{rect}\left(\frac{t}{T}\right) \sum_{k\in\mathcal{K}} e^{\mathrm{j}k\omega_0 t}, \tag{4}$$

where $\mathcal{K} = \{-2K, \dots, -1\} \cup \{1, \dots, 2K\}$. Then the filtered signal is written as

$$y_m(t) = \int_{-\infty}^{\infty} \sum_{i=-r}^{r} \bar{f}(\lambda - t + iT) h_m(\lambda) d\lambda, \tag{5}$$

where $r = \lceil \frac{R+3T}{2T} \rceil - 1$ and $\bar{f}(t)$ denotes the complex conjugate of $f(t)$. It has been shown [4, 9] that (5) is equivalently rewritten as

$$\begin{aligned} y_m(t) &= \int_{-\infty}^{\infty} \bar{f}(\lambda - t) \sum_{i\in\mathbb{Z}} h_m(\lambda - iT) d\lambda \\ &= \sum_{k\in\mathcal{K}} H_m(k\omega_0) e^{\mathrm{j}k\omega_0 t}. \end{aligned} \tag{6}$$

Therefore, $y_m(t)$ is fully determined by $H_m(k\omega_0)$ for $k \in \mathcal{K}$.

Next a bias term b is added to the filtered pulse stream $y_m(t)$ ensure that $y_m(t) + b \geq 0$. Then the resulting signal is integrated and compared to a given threshold δ. Once the threshold is met, the time instant $t_{j,n}$ is recorded and the integrator is reset to zero. IF-TEM recordings satisfy

$$\int_{t_{m,n}}^{t_{m,n+1}} (y_m(t) + b) dt = \delta, \quad \forall n. \tag{7}$$

Recovery of the Fourier coefficients $H_m(k\omega_0)$ from the IF-TEM recordings is cast as a simple linear inverse problem [4]. Given the N_m time encoding samples $t_{m,1}, t_{m,2}, \dots, t_{m,N_m}$, one obtains linear measurements given by

$$y_{m,n} = \int_{t_{m,n}}^{t_{m,n+1}} y_j(t) dt = -b(t_{m,n+1} - t_{m,n}) + \delta, \tag{8}$$

which, by (6), is expressed as

$$y_{m,n} = \sum_{k\in\mathcal{K}} \left(\frac{e^{\mathrm{j}k\omega_0 t_{m,n+1}} - e^{\mathrm{j}k\omega_0 t_{m,n}}}{\mathrm{j}k\omega_0} \right) H_m(k\omega_0). \tag{9}$$

The relationship between the desired Fourier coefficients and the measurements is compactly written in matrix form by

$$\boldsymbol{y}_m = \boldsymbol{B}_m \boldsymbol{h}_m \tag{10}$$

for some matrix $\boldsymbol{B}_m \in \mathbb{C}^{N_m \times |\mathcal{K}|}$, where $\boldsymbol{y}_m = (y_{m,n})_{\in[N_m]}$ and $\boldsymbol{h}_m = (H_m(k\omega_0))_{k\in\mathcal{K}}$. It has been shown that $\boldsymbol{B}_m$ has full column rank when $N_m \geq |\mathcal{K}|$ [4]. Therefore, $\boldsymbol{h}_m$ is determined from $\boldsymbol{y}_m$ via (10) when there is no noise. In the presence of noise, we propose to estimate $\boldsymbol{h}_m$ by total least squares, to account for the dependence of $\boldsymbol{B}_m$ on the IF-TEM recordings, leading to perturbation in both $\boldsymbol{y}_m$ and $\boldsymbol{B}_m$.

4. IDENTIFICATION OF PULSE STREAMS BY TLS-ESPRIT

Bresler and Delaney [6] showed that the pairwise constant assumption in (3) enables the retrieval of the pulse parameters. We summarize their method in our notation for the sake of completeness.

The vector of Fourier coefficients $\boldsymbol{h}_m \triangleq (H_m(k\omega_0))_{k=1}^{2K} \in \mathbb{C}^{2K}$ obtained from TEM from the mth channel signal $h_m(t)$ can be written in the form:

$$\boldsymbol{h}_m = \mathrm{diag}(\boldsymbol{g}) \boldsymbol{V} \boldsymbol{\alpha}_m,$$

where $\boldsymbol{g} = (G(k\omega_0))_{k\in[2K]} \in \mathbb{C}^{2K}$, $\boldsymbol{V} \in \mathbb{C}^{k\times d}$ is a Vandermonde matrix such that $(\boldsymbol{V})_{k,l} = \phi_l^k$ with $\phi_l = e^{-\mathrm{j}\omega_0 \tau_l}$, and $\boldsymbol{\gamma}_m = (\gamma_{l,m})_{l\in[d]}$ denotes the mth column of $\boldsymbol{\Gamma}$.

Let $\boldsymbol{\Pi} \in \mathbb{R}^{2K\times 2K}$ denote the symmetric permutation (flipping) matrix. By the conjugate symmetry of a real-valued signal, the Fourier coefficients of all pulse streams are rearranged in a matrix form as

$$\begin{bmatrix} \boldsymbol{\Pi}\bar{\boldsymbol{H}} \\ \boldsymbol{H} \end{bmatrix} = \begin{bmatrix} \mathrm{diag}(\boldsymbol{\Pi}\bar{\boldsymbol{g}}) & \boldsymbol{0} \\ \boldsymbol{0} & \mathrm{diag}(\boldsymbol{g}) \end{bmatrix} \begin{bmatrix} \boldsymbol{\Pi}\bar{\boldsymbol{V}} \\ \boldsymbol{V} \end{bmatrix} \boldsymbol{\Gamma}, \tag{11}$$

where $\bar{\boldsymbol{H}}$ denotes the element-wise complex conjugate of $\boldsymbol{H}$. Then the data matrix in (11) provides the rotation-invariance, which enables applying ESPRIT. Let $\boldsymbol{J}_1, \boldsymbol{J}_2 \in \mathbb{R}^{2K\times 4K}$ denote selection matrices selecting the odd rows and the even rows, respectively. Then we have

$$\boldsymbol{J}_2 \begin{bmatrix} \boldsymbol{\Pi}\bar{\boldsymbol{H}} \\ \boldsymbol{H} \end{bmatrix} = \boldsymbol{J}_2 \begin{bmatrix} \mathrm{diag}(\boldsymbol{\Pi}\bar{\boldsymbol{g}}) & \boldsymbol{0} \\ \boldsymbol{0} & \mathrm{diag}(\boldsymbol{g}) \end{bmatrix} \boldsymbol{J}_2^\top \boldsymbol{J}_2 \begin{bmatrix} \boldsymbol{\Pi}\bar{\boldsymbol{V}} \\ \boldsymbol{V} \end{bmatrix} \boldsymbol{\Gamma} \tag{12}$$

By the pairwise constant assumption (3), we have

$$\boldsymbol{J}_2 \begin{bmatrix} \mathrm{diag}(\boldsymbol{\Pi}\bar{\boldsymbol{g}}) & \boldsymbol{0} \\ \boldsymbol{0} & \mathrm{diag}(\boldsymbol{g}) \end{bmatrix} \boldsymbol{J}_2^\top = \boldsymbol{J}_1 \begin{bmatrix} \mathrm{diag}(\boldsymbol{\Pi}\bar{\boldsymbol{g}}) & \boldsymbol{0} \\ \boldsymbol{0} & \mathrm{diag}(\boldsymbol{g}) \end{bmatrix} \boldsymbol{J}_1^\top.$$

Furthermore, since $\boldsymbol{V}$ is a Vandermonde matrix, we also have

$$\boldsymbol{J}_2 \begin{bmatrix} \boldsymbol{\Pi}\bar{\boldsymbol{V}} \\ \boldsymbol{V} \end{bmatrix} = \boldsymbol{J}_1 \begin{bmatrix} \boldsymbol{\Pi}\bar{\boldsymbol{V}} \\ \boldsymbol{V} \end{bmatrix} \boldsymbol{\Phi},$$

where $\boldsymbol{\Phi} \in \mathbb{C}^{d\times d}$ is a diagonal matrix with $(\Phi)_{l,l} = \phi_l$. Then (12) is rewritten as

$$\boldsymbol{J}_2 \begin{bmatrix} \boldsymbol{\Pi}\bar{\boldsymbol{H}} \\ \boldsymbol{H} \end{bmatrix} = \underbrace{\boldsymbol{J}_1 \begin{bmatrix} \mathrm{diag}(\boldsymbol{\Pi}\bar{\boldsymbol{g}}) & \boldsymbol{0} \\ \boldsymbol{0} & \mathrm{diag}(\boldsymbol{g}) \end{bmatrix} \boldsymbol{J}_1^\top}_{\boldsymbol{\Lambda}} \underbrace{\boldsymbol{J}_1 \begin{bmatrix} \boldsymbol{\Pi}\bar{\boldsymbol{V}} \\ \boldsymbol{V} \end{bmatrix}}_{\boldsymbol{A}} \boldsymbol{\Phi}\,\boldsymbol{\Gamma},$$

whereas the submatrix with odd rows satisfies

$$\boldsymbol{J}_1 \begin{bmatrix} \boldsymbol{\Pi}\bar{\boldsymbol{H}} \\ \boldsymbol{H} \end{bmatrix} = \boldsymbol{\Lambda}\boldsymbol{A}\boldsymbol{\Gamma}.$$

In other words, the data matrix is divided into two submatrices with odd rows and even rows, which are related to each other with a diagonal displacement matrix. This structure enables the recovery of $\mathbf{\Phi}$ by TLS-ESPRIT [10]. Then $\boldsymbol{A}$ is identified from the estimate of $(\tau_l)_{l=1}^d$, which are obtained from $\mathbf{\Phi}$.

Bresler and Delaney [6] proposed further recovery of $\boldsymbol{g}$ by the following procedure. Let $\boldsymbol{E_z} = [\boldsymbol{E}_1; \boldsymbol{E}_2]$ with $\boldsymbol{E}_1, \boldsymbol{E}_2 \in \mathbb{C}^{2K \times d}$ denote the matrix with columns consisting of the eigenvectors of the permuted data matrix given by

$$\boldsymbol{Z} = \begin{bmatrix} \boldsymbol{J}_1 \\ \boldsymbol{J}_2 \end{bmatrix} \begin{bmatrix} \mathbf{\Pi} \bar{\boldsymbol{H}} \\ \boldsymbol{H} \end{bmatrix}.$$

Let $\boldsymbol{E_\Psi}$ be the matrix with columns consisting of the eigenvectors of $\mathbf{\Psi}$, which is the TLS solution to $\boldsymbol{E}_1 \mathbf{\Psi} = \boldsymbol{E}_2$. Then we have

$$\begin{aligned} \boldsymbol{E_z} \boldsymbol{E_\Psi} &= \boldsymbol{J} \begin{bmatrix} \operatorname{diag}(\mathbf{\Pi} \bar{\boldsymbol{g}}) & \mathbf{0} \\ \mathbf{0} & \operatorname{diag}(\boldsymbol{g}) \end{bmatrix} \boldsymbol{J}^\top \begin{bmatrix} \boldsymbol{A} \\ \boldsymbol{A} \mathbf{\Phi} \end{bmatrix} \operatorname{diag}(\boldsymbol{q}) \\ &= \boldsymbol{J} \begin{bmatrix} \mathbf{\Pi} \bar{\boldsymbol{g}} \\ \boldsymbol{g} \end{bmatrix} \boldsymbol{q}^\top \odot \begin{bmatrix} \boldsymbol{A} \\ \boldsymbol{A} \mathbf{\Phi} \end{bmatrix} \end{aligned}$$

for some $\boldsymbol{q} \in \mathbb{C}^d$. Therefore, we obtain

$$\begin{bmatrix} \mathbf{\Pi} \bar{\boldsymbol{g}} \\ \boldsymbol{g} \end{bmatrix} \boldsymbol{q}^\top = \boldsymbol{J}^\top \left(\boldsymbol{E_z} \boldsymbol{E_\Psi} \oslash \begin{bmatrix} \boldsymbol{A} \\ \boldsymbol{A} \mathbf{\Phi} \end{bmatrix} \right), \qquad (13)$$

where $\oslash$ denotes the Hadamard division operator. This recovers the Fourier coefficients $G(k\omega_0)$ for $k \in [2K]$ up to a global scale ambiguity. In the noisy case, it is approximated from the best rank-1 approximation of the matrix in (13).

5. NUMERICAL RESULTS

We present the results of numerical experiments illustrating the proposed method.The pulse shape that is used for the simulation is $g(t) = \operatorname{rect}(20t/\pi) \cos^2(20t)$.

Note that the squared cosine waveform $g(t)$ above does not satisfy PCC in (3). Hence we first approximated $g(t)$ to the nearest signal among those satisfying PCC. When TEM recordings were obtained from pulse streams with this projected pulse shape with PCC, we verified exact recovery up to numerical precision.[2] This result is consistent with Theorem 1. Here the parameters were set to minimal numbers as follows: $d = 3$, $K = 2d + 2$, $M = d + 1$. The firing rate was set by the choice of $\delta = 0.07$ in (7).

Next we show that, although we only presented a sufficient condition for exact recovery without any model error or noise, the method degrades gracefully in their presence. To this end we consider pulses streams generated with the original $g(t)$, which does not satisfy PCC. Recall that the recovery of Fourier series coefficients from TEM measurements does not rely on any structure of the pulse streams except they are supported within $[0, T)$. Therefore, recovery of Fourier coefficients from IF-TEM recordings does not suffer. However, since the following step with TLS-ESPRIT relies on PCC, the model error propagates to the final parameter recovery. To mitigate the artifact, we used more channels than the minimal required for exact recovery without any artifact. We set $M = 100$. This helps obtain an accurate subspace estimate in ESPRIT. Furthermore, we also oversample the number of Fourier coefficients for ESPRIT so that $K = 4d$. To obtain the increased number of Fourier coefficients from IF-TEM recordings, we used a higher firing rate setting $\delta = 0.02$.

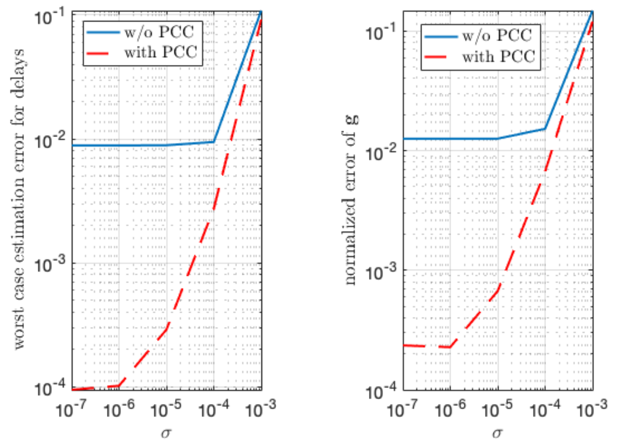

Fig. 2: Estimation error for delays (left) and Fourier coefficients (right).

To model the effects of noise, we adopted the jitter model for the IF-TEM introduced in [5]. A random bias, uniformly distributed over $(-\sigma/2, \sigma/2)$, is added to each recording time, while σ varies over $\{10^{-7}, 10^{-6}, 10^{-5}, 10^{-4}, 10^{-3}\}$.

In this setting, we carried out a Monte Carlo simulation to observe the empirical performance of parameter recovery. The error in recovering delays is measured as the worst case match given by $\max_l \min_{l'} d(\hat{\tau}_{l'}, \tau_l)$, where the distance is modulo $[0, T)$. The error for $\boldsymbol{g}$ is measured by the normalized projection error, that ie, $\|\hat{\boldsymbol{g}} - \boldsymbol{g}\boldsymbol{g}^{\mathsf{H}} \hat{\boldsymbol{g}} / \|\boldsymbol{g}\|_2^2\|_2 / \|\hat{\boldsymbol{g}}\|_2$. The median value of 50 trials for each of these metrics is plotted for each setting of parameters.

Fig. 2 illustrate the experimental results. In the absence of model error (plotted with the dotted line), the error scales gracefully with the increasing jitter. With the model error due to the violation of PCC, it dominates the effect of jitter for $\sigma \leq 10^{-4}$. Our claim here is that our method can tolerate the model error and jitter up to a certain level and still provides robust recovery of parameters.

6. CONCLUSION

The proposed method requires just a few Fourier samples obtained by an energy-efficient TEM multichannel signal acquisition scheme to solve the problem of blind identification of delayed and overlapped unknown pulse shape. The recovery is obtained for a single common pulse shape, but this method can be extended for multiple shapes as well.

[2] We also simulated IF-TEM through Riemannian approximation with a small step size. It adds extra distortion to TEM recordings.

7. REFERENCES

[1] Sourav Chandra, Jinghua Li, Babak Afsharipour, Andres F Cardona, Nina L Suresh, Limei Tian, Yujun Deng, Yishan Zhong, Zhaoqian Xie, and Haixu Shen, "Performance evaluation of a wearable tattoo electrode suitable for high-resolution surface electromyogram recording," *IEEE Transactions on Biomedical Engineering*, vol. 68, no. 4, pp. 1389–1398, 2020.

[2] Aurel A Lazar and László T Tóth, "Perfect recovery and sensitivity analysis of time encoded bandlimited signals," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 51, no. 10, pp. 2060–2073, 2004.

[3] Roxana Alexandru and Pier Luigi Dragotti, "Reconstructing classes of non-bandlimited signals from time encoded information," *IEEE Transactions on Signal Processing*, vol. 68, pp. 747–763, 2019.

[4] Hila Namman, Satish Mulleti, and Yonina C Eldar, "FRI-TEM: Time encoding sampling of finite-rate-of-innovation signals," *arXiv preprint arXiv:2106.05564*, 2021.

[5] Abijith Jagannath Kamath, Sunil Rudresh, and Chandra Sekhar Seelamantula, "Time encoding of finite-rate-of-innovation signals," *arXiv preprint arXiv:2107.03344*, 2021.

[6] Yoram Bresler and Alexander H Delaney, "Resolution of overlapping echoes of unknown shape," in *International Conference on Acoustics, Speech, and Signal Processing,*. IEEE, 1989, pp. 2657–2660.

[7] Ales Holobar and Damjan Zazula, "Multichannel blind source separation using convolution kernel compensation," *IEEE Transactions on Signal Processing*, vol. 55, no. 9, pp. 4487–4496, 2007.

[8] Amos Lapidoth, *A foundation in digital communication*, Cambridge University Press, 2017.

[9] Ronen Tur, Yonina C Eldar, and Zvi Friedman, "Innovation rate sampling of pulse streams with application to ultrasound imaging," *IEEE Transactions on Signal Processing*, vol. 59, no. 4, pp. 1827–1842, 2011.

[10] Richard Roy and Thomas Kailath, "Esprit-estimation of signal parameters via rotational invariance techniques," *IEEE Transactions on acoustics, speech, and signal processing*, vol. 37, no. 7, pp. 984–995, 1989.