Subgraph Query Generation with Fairness and Diversity Constraints

Hanchao Ma, Sheng Guan, Mengying Wang, Yen-shuo Chang, Yinghui Wu
Case Western Reserve University
Email: {hxm382,sxg967,mxw767,yxc1425,yxw1650}@case.edu

***Abstract*—This paper studies the problem of subgraph query generation with guarantees on both diversity and group fairness. Given a query template (with parameterized search predicates) and a set of node groups in a graph, it is to compute a set of subgraph queries that instantiate the query template, and each query ensures diversified answers that meanwhile covers each group with a desired number of nodes. Such need is evident in web and social search with fairness constraints, query optimization, and query benchmarking. We formalize a bi-criteria optimization problem that aims to find a Pareto optimal set of query instances in terms of diversity and fairness measures. We show the problem is in Δ_2^P and verify its hardness (NP-hard and fixed-parameter tractable). We provide (1) two efficient algorithms that can approximate Pareto optimal sets with ϵ-dominance relations that yield representative query instances with a bounded size, and (2) an online algorithm that progressively generates and maintains fixed-size ϵ-Pareto set with small delay time. We experimentally verify that our algorithms can efficiently generate queries with desired diversity and coverage properties for targeted groups.**

***Index Terms*—attributed graph, query suggestion, fairness**

I. Introduction

Subgraph queries have been routinely used in *e.g.,* social search [31] and knowledge search [41]. Given a graph G, a subgraph query $Q(u_o)$ with a designated output node u_o computes a set of nodes (matches) of u_o in G. A number of algorithms [8] have been developed to process subgraph queries in terms of subgraph isomorphism and variants [41].

The emerging need for data systems that requires both results diversity and fairness [7], [3], [34], [15] poses new challenges to graph querying. In such scenarios, queries are expected to return diversified matches that meanwhile ensure a required coverage of designated groups (node sets) of interests from G. Such groups may refer to the population of vulnerable social groups that are characterized in terms of sensitive attributes (*e.g.,* gender, race, professions) [19], relevant articles yet with diversified labels for Web exploration [2] and recommendation engines [21], or designated columns for query benchmark [5]. Consider the following real-world scenarios.

Example 1: Talent search. A talent search over a professional network G [21] finds strong candidates with desired skills. Each node in G denotes a user with attributes such as *title*, *skill*, *profession*, or an organization with attributes such as the number of *employees*. Each edge indicates the affiliation (worksAt) of a user or recommendation (recommend) between users. A recruiter issues a graph search query q_1 (illustrated in Fig. 1) to find directors u_o who have expertise in managing IT business, and moreover, recommended by at least two users from large companies. In our test (Section V), this query returns a set of qualified candidates $q(G)$, yet with a skewed distribution of 375 male users and 173 female users.

The recruiter pursues the desired gender distribution and diversity of the candidates, and wonders how to revise the search such that (1) the new answer can properly cover $q(G)$ with an equal number of male and female candidates (*e.g.,* both with 200 candidates; a case of "Equal opportunity" [21]); and (2) the candidates are also more diversified in their majors. A more desirable query q_2 can be suggested, which finds 202 male and 198 female candidates that span 10 majors. The query q_2 suggests that a relaxed condition on recommendation (removing the edge from u_1 to u_3) and a relaxation that also recommends candidates from smaller businesses (reducing '1000' employees to '500' employees) help to achieve the desired answer with proper coverage of the gender groups. □

It is desirable that such queries can be automatically suggested, which ensures diversified answers that can meanwhile cover designated node groups with desired cardinalities. On the other hand, there may exist multiple queries with "better" answers in terms of either diversity or equal opportunity.

Example 2: Continuing the talent search, two more queries q_3 and q_4 (Fig. 1) can be found by "perturbing" the ranges of the years of experience of recommenders and the number of employees they work at. (1) Compared with q_1, both queries can identify more diversified candidates with a less skewed distribution of genders. (2) Compared with q_2, both find a more skewed result over gender groups, yet each finds more diversified candidates with more than 30 majors. While all three queries provide more "desired" answers in terms of diversity or promoting equal opportunity, it remains a daunting task for users to inspect all these queries. □

This calls for efficient algorithms that can suggest a set of representative subgraph queries with desirable guarantees on both diversity and group coverage, over specified groups of interests. Such need is evident in social search and recommendation with group fairness [21], workload generation for query benchmark [5], and query optimization in large graphs [27].

This paper studies a novel problem called *subgraph query generation with diversity and fairness constraints* (FairSQG), which has a general form below:

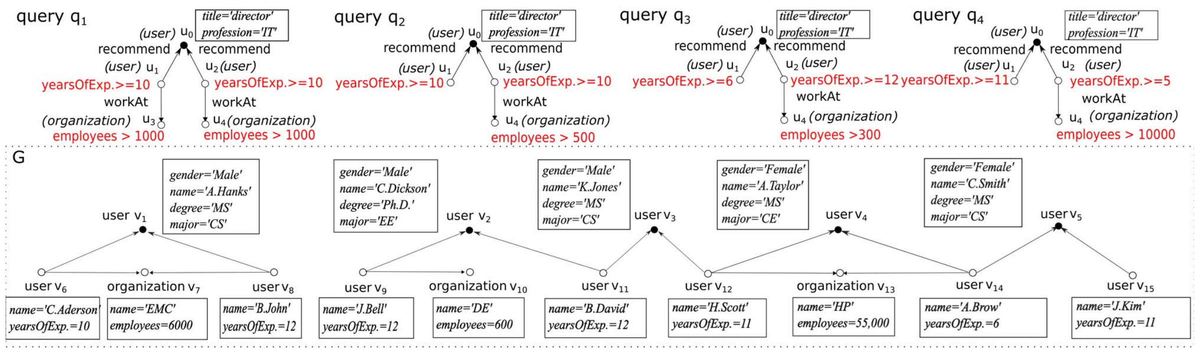


Fig. 1. Subgraph queries with Diversity and Group Fairness: Talent Search

- **Input:** graph G, an initial query (template) $Q(u_o)$, and a set of groups $\mathcal{P}$, where each group $P_i \in \mathcal{P}$ is associated with a coverage constraint c_i ($c_i \leq |P_i|$);
- **Output:** a set of subgraph queries $\mathcal{Q}$ obtained by revising $Q(u_o)$; each query can retrieve a set of diversified matches ("*Diversity*") from G that also cover each group P_i with desired cardinality c_i ("*Group fairness*").

Several methods have been developed to generate graph queries that lead to desired answers. Notable examples include query suggestion with diversified answers [33], [24], coverage of similar counterparts of "examples" [39], [35], or cardinality constraints on output sizes [28], [4]. These approaches are designed to revise queries towards specific properties rather than ensuring both group fairness and answer diversity, thus cannot be directly applied to our problem.

Contributions & Organization. We formally analyze the subgraph query generation problem with group fairness constraints. We propose both feasible approximation schemes as well as practical exact algorithms for large graphs. We refer to the subgraph query as "query" in the rest of the paper.

A Bi-objective formalization. We provide a practical formalization of the FairSQG problem (Section III). (1) We introduce a class of *query templates* (denoted as $Q(u_o)$). A template carries variables defined on search predicates and edges yet to be instantiated at processing time. Query generation yields value binding to the variables to produce a set of query instances. (2) We introduce diversity and fairness measures to measure the quality of an instance. Despite the need of optimizing both, they may come in conflict: a single optimal instance may not exist. On the other hand, a complete Pareto set is of substantial size for users to inspect. Instead, we introduce a bi-objective optimization problem to compute an ϵ-Pareto set of instances based on a query dominance relation. ϵ-Pareto set is a desirable subset approximation of the Pareto optimal set that strikes a balance between dominance relation and the number of instances to be returned.

We show FairSQG is solvable in Δ_2^P (a class of P^{NP} problems with an NP oracle) for templates with fixed variables, and show its hardness varies from PTIME to NP-hard if $Q(u_o)$ has fixed size and variable sizes, and for templates without range variables. These results verify useful upper and lower bound results for query generation scenarios in practice.

Query generation with quality guarantees (Section IV). Generating a Pareto set of substantial size is often not desirable. We first introduce algorithms that can approximate the exact Pareto set $\mathcal{Q}$ with a quality guarantee controlled by an error bound ϵ. The algorithm ensures to find a subset of $\mathcal{Q}$, denoted as $\mathcal{Q}_\epsilon$, such that for each possible instance of $Q(u_o)$, there is an instance in $\mathcal{Q}_\epsilon$ that approximately dominates it on both diversity and coverage within a constant factor $\epsilon > 0$. Better still, the algorithm ensures to return a representative query set with a bounded size. We also introduce optimization strategies to reduce the cost of query generation.

Online maintenance of fixed-sized set (Section IV-C). Our analysis shows that one often needs to sacrifice query instance quality (ϵ) in trade for a smaller, representative set to inspect. We follow up with an online algorithm, which progressively constructs and incrementally maintains an ϵ-Pareto set with k instances and an ϵ as small as possible. The online algorithm uses a sliding window strategy to dynamically swap or replace queries and incrementally update ϵ-Pareto set only when necessary, and incurs a small delay time.

Real-world evaluation and case analysis (Section V). Using real-life graphs, we verify the effectiveness and efficiency of our algorithms (Section V). We show that our algorithms can generate subgraph queries with both desired diversity and small errors in covering designated groups. These algorithms are also feasible. For example, it takes 78 seconds to produce instances with desired coverage in real-life graphs with 30 million nodes and edges. We also illustrate that our algorithms can generate favorable queries for different user preferences.

Related Work. We categorize the related work as follows.

Graph query suggestion. Subgraph query suggestion has been studied to discover queries with different desirable properties [24], [33], [39], [35]. Graph query by example [24] computes subgraph queries with answers that are close to a set of user-specified (triple) examples in knowledge graphs. Given an initial query, diversified query suggestion [33] extends an initial query with additional edges to derive a set of queries with diversified answers. Answering Why-questions [39], [35] suggest queries with query rewriting operators to approach answers by including or excluding specified nodes. Query suggestion in terms of both diversity and group coverage constraints are not addressed by these methods.

Subset selection with group fairness. Another relevant work is subset selection with group fairness. Given a universal set, and a set of groups (subsets), it is to select a set of nodes that can cover each group with a desirable number of nodes. [32] proposed a fairness metric that considers a fair group coverage of the outputs of decision making models. [20] computes a size-k seed set such that it maximizes the coverage of one group and covers "t-fraction" of the other. Diversified subset selection with group fairness has been studied [40], [34]. Approximation algorithms have been studied to generate subsets for max-sum and max-min diversification [34] as well as for online selection [40]. These methods study set coverage properties and cannot be directly used to suggest graph queries and coverage in terms of graph search.

Query workload generation. Query generation with output size constraints and distribution constraints have been investigated for graph benchmarking. [4] generates regular path queries from a given schema that can output answers with required cardinalities when projected to pre-defined attributes. [27] generates SPARQL queries that can cover the answer of given queries with cheaper plans for query optimization. Workload generation with group coverage [30] aims to generate a set of subgraph queries, where the union of their answers cover a desired fraction of each group.

Our work differs from prior work as follows. (1) We study query generation under fairness constraints for arbitrary groups with guaranteed coverage requirements. (2) We consider queries defined in subgraph isomorphism with search predicates, a more involved query class compared with regular path queries [4]. (3) Unlike [30], we study a different problem which aims to approximate the Pareto set of subgraph queries in diversity and coverage. The Pareto-optimality is not studied in these works. On the other hand, our algorithms can be readily applied to generate queries for benchmark needs.

Skyline search. Multiobjective search such as skyline queries [11], [12] has been extensively studied. Existing multi-objective optimization algorithms are studied to compute Pareto optimal sets [23], [10] or their approximate variants [36], [26] over data points. [23] transforms the multi objectives to a single objective by linear summation of all objectives with importance weights. Constraint based method (CBM) [10] initializes a set of anchor points that optimize each single objective function. It then bisects the straight lines between pairs of anchor points with a fixed vertical separation distance. This transforms bi-objective optimization into a set of single objective optimization problems. Then, it solves each to get a set of anchor nodes to approximate the Pareto frontier. ϵ-Pareto set [36], [26] has been studied as a desirable approximation for Pareto optimal set.

Our problem can be considered as computing a representative bi-objective skyline front in subgraph query space with diversity and coverage preferences. It differs from prior work as follows. (1) Our problem solves the multi-objective optimization problem defined on query instances. It needs to compute diversity and coverage for query instances via subgraph isomorphism. Nevertheless, traditional skyline search problems are defined on data points with given feature vectors. (2) It is also not desirable to return a large Pareto set [11]

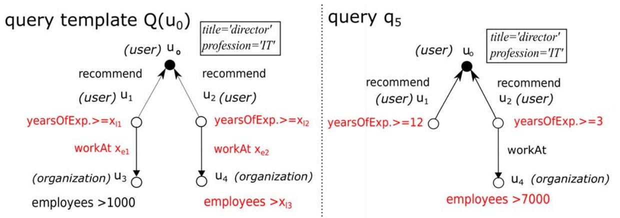


Fig. 2. Query Template and Query Instance

for practical query generation scenarios. We advocate feasible algorithms that can (a) efficiently generate and maintain ϵ-Pareto instance sets for large graphs, and (b) strike a balance between the solution quality and size. (3) Our approach aims to balance between efficiency and provable group coverage. Moreover, our method generates size bounded query sets for the user to inspect. This can not be guaranteed by [10].

II. Graph, Query Templates and Instances

Graphs. We consider directed graphs $G = (V, E, L, T)$, where (1) V is a finite set of nodes, (2) $E \subseteq V \times V$ is a set of edges, (3) each node $v \in V$ (resp. edge $e \in E$) carries a label $L(v)$ (resp. $L(e)$); and (4) each node v carries a tuple $T(v)$ = $<(A_1, a_1), \ldots, (A_n, a_n)>$, where each A_i $(i \in [1, n])$ is a distinct node attribute with a value a_i.

We denote the finite set of all the node attributes in G as $\mathcal{A}$. The *active domain* $\mathsf{adom}(A)$ of an attribute $A \in \mathcal{A}$ refers to the set of values of $v.A$ as the node v ranges over V.

Query Template. A *query template* (or simply "template") $Q(u_o)$ is a connected graph (V_Q, E_Q, L_Q, T_Q), where V_Q (resp. $E_Q \subseteq V_Q \times V_Q$) is a set of query nodes (resp. query edges). Each query node $u \in V_Q$ (resp. query edge $e \in V_E$) has a label $L_Q(u)$ (resp. $L_Q(e)$). Specifically, there is a designated *output node* $u_o \in V_Q$.

Variables. A template allows "placeholders" in search predicates that can be bound to specific values when executed. It extends parameterized queries [9] for graph query generation. We consider two types of variables in a template $Q(u_o)$. (a) For each node $u \in V_Q$, $T_Q(u)$ is a set of literals. A literal l is in the form of $u.A$ op x_l, where op is from $\{>, >=, =, <=, <\}$, and x_l is a *range variable* that can be assigned to a constant. (b) For each edge $e \in E_Q$, $T_Q(e)$ is a Boolean *edge variable* x_e (either '0' or '1'). The set of all the variables in $Q(u_o)$ is denoted as $X = X_L \cup X_E$.

Query Instances. Given a template $Q(u_o)$, an *instantiation* of $Q(u_o)$ is a function I, such that for each variable $x \in X$, $I(x)$ is either a constant or a wildcard '_'. A *query instance* (or simply "instance") $q(u_o)$ of $Q(u_o)$ induced by an instantiation I is a connected graph (V_Q, E_q, L_Q, T_q) with the same V_Q, output node u_o and L_Q, and moreover,

- for each literal $l \in T_Q(u)$ in $Q(u_o)$, if $I(x_l)$ is a constant, then there is a literal $l = u.A$ op $I(x_l)$ in $T_q(u)$; and
- there is an edge $e \in E_Q$ if and only if (a) $I(x_e)$ = '1', and (b) e is in the same connected component of u_o.

An instance q of $Q(u_o)$ contains no variables but only literals and the edges in the connected component where u_o resides,

Notation	Description
G=(V,E,L,T)	attributed graph G
$Q(u_o)$=(V_Q,E_Q,L_Q,T_Q)	query template $Q(u_o)$; u_o: output node
x_l; x_e	range variable; boolean edge variable
$q(u_o)$	an instantiation of $Q(u_o)$
$q(u,G)$	match set of a query node u of $q(u_o)$ in G
$q(G)$	match set $q(u_o,G)$
$V(u_o)$	set $\{v \mid L(v)=L(u_o), v\in V\}$
$\mathcal{P}$	m disjoint node groups in G
C	cardinality constraints of groups $\mathcal{P}$
$\mathcal{I}(Q)$	all the query instances of Q
$\mathcal{Q}^* \subseteq \mathcal{I}(Q)$	Pareto instance set of $\mathcal{I}(Q)$
$\mathcal{Q}^*_\epsilon \subseteq \mathcal{I}(Q)$	ϵ-Pareto instance set of $\mathcal{I}(Q)$

TABLE I
SUMMARY OF NOTATION.

induced by the constant binding from I. We denote the set of all the possible instances of $Q(u_o)$ as $\mathcal{I}(Q)$.

Matches. Given an instance $q(u_o)$ and a graph G, a matching from $q(u_o)$ to G is a function $h \subseteq V_q \times V$, where (1) for each node $u \in V_Q$, $L_Q(u) = L(h(u))$, and for each literal $u.A$ op c in L_q, $h(u).A$ op c; and (2) for each edge $e = (u,u')$ in $q(u_o)$, $h(e) = (h(u), h(u'))$ is an edge in G, and $L_Q(e) = L(h(e))$.

The *matches* of a query node u of $q(u_o)$ in G, denoted as $q(u,G)$, refers to the set of all the nodes in G that can match node u via a matching $h(u)$ from q to G. The *result* of q in G, denoted as $q(G)$, refers to the match set $q(u_o,G)$. We summarize the notations in Table I.

Example 3: Fig. 1 illustrates a template $Q(u_o)$ that searches for directors in G [21]. (1) $Q(u_o)$ has five variables $\bar{X}$ = $\{x_{l1}, x_{l2}, x_{l3}, x_{e1}, x_{e2}\}$, with three range variables in literals u_1.yearsOfExp. $\geq x_{l1}$, u_2.yearsOfExp. $\geq x_{l2}$, and u_4.employees $\geq x_{l3}$ respectively, and edge variables x_{e1}, x_{e2}. (2) An instantiation {10, 10, 1000, '1', '1'} (resp. {10, 10, 500, '0', '1'}, {6, 12, 300, '0', '1'}, {11, 5, 1000, '0', '1'}) of $\bar{X}$ induces an instance q_1 (resp. q_2 and q_3 and q_4) of $Q(u_o)$. (3) Given G, $q_1(G) = \{v_1\}$, $q_2(G) = \{v_1, v_2, v_3\}$, $q_3(G) = \{v_1, v_2\}$, and $q_4(G) = \{v_3, v_4, v_5\}$. □

Remarks. The instances are well-defined for a "partial" instantiation in which some variables are assigned a wildcard '_' ("don't care"). For such a case, q is induced by removing corresponding parameterized predicates or edges to ensure valid $q(G)$. A user-defined "initial" query (*e.g.,* q_1 in Example 1) can be captured by a template with a partial instantiation.

III. Query Generation Problem

Given a template $Q(u_o)$, graph G and m disjoint node groups $\mathcal{P}$ in G, where each group $P_i \in \mathcal{P}$ has a cardinality constraint $c_i \in [0, |P_i|]$, the query generation problem aims to compute a set of instances $\mathcal{Q} \subseteq \mathcal{I}(Q)$ of $Q(u_o)$ with maximized diversity and required coverage properties.

A. Quality Measures

We consider two functions to quantify the "goodness" of instances in terms of diversity and fairness.

Diversification. We consider Max-sum diversity as a natural objective for result diversification [22]. Given an instance q and G, the diversity of q is defined as:

$$\delta(q,G) = (1-\lambda)\sum_{v\in q(G)} r(u_o,v) + \frac{2\lambda}{|V_{u_o}|-1}\sum_{v,v'\in q(G)} d(v,v')$$

where (1) $\lambda \in [0,1]$ is a constant to balance relevance and diversity; (2) the function $r(u_o,v) \in [0,1]$ (resp. $d(v,v') \in [0,1]$) computes a relevance score between u_o and a match v (resp. difference between two matches v and v'). In practice, $d(v,v')$ can be the edit distance between tuples $T(v)$ and $T(v')$ [25], and $r(u_o,v)$ can be an entity linkage score or impact of v in social networks [16].

The set V_{u_o} refers to the nodes in G with the same label of u_o. Given G, the pairwise dissimilarity is normalized with a constant $\frac{|V_{u_o}|-1}{2}$, as there are at most $\frac{|V_{u_o}|(|V_{u_o}|-1)}{2}$ pairs but $|V_{u_o}|$ relevance numbers. That is, $\delta(q,G) \in [0, |V_{u_o}|]$.

Coverage. Ideally, an instance should satisfy the coverage requirement c_i posed on each group $P_i \in \mathcal{P}$, and cover exactly c_i nodes in each $P_i \in \mathcal{P}$. Given $\mathcal{P}$, G, and template $Q(u_o)$, an instance is *feasible*, if for each $P_i \in \mathcal{P}$, $|q(G) \cap P_i| \geq c_i$.

We next introduce a function to quantify the quality in terms of desired coverage as:

$$f(q,\mathcal{P}) = C - \sum_{P_i\in\mathcal{P}} (|q(G)\cap P_i| - c_i)$$

where the constant $C = \sum_1^{|\mathcal{P}|} c_i$. The function penalizes the total accumulated errors between desired coverage and actual counterpart by $q(G)$ over each group in $\mathcal{P}$. The larger $f(q,\mathcal{P})$ is, the better ($f(q,\mathcal{P}) \in [0,C]$).

Example 4: Continue with the queries as illustrated in Fig. 1. Suppose we want to cover exactly 2 male users and 2 female users over the qualified candidates, one may verify the following: $\delta(q_1,G) = 0$ and $f(q_1,\mathcal{P}) = 1$; $\delta(q_2,G) = 1$ and $f(q_2,\mathcal{P}) = 1$; and $\delta(q_3,G) = 0.75$ and $f(q_3,\mathcal{P}) = 2$, and $\delta(q_4,G) = 0.5$ and $f(q_3,\mathcal{P}) = 3$. □

In the rest of the paper, we only consider feasible instances. We shall denote $\delta(q,G)$ and $f(q,\mathcal{P})$ simply as $\delta(q)$ and $f(q)$, respectively, when G and $\mathcal{P}$ are specified in the context.

B. Query Generation Problem

Pareto Optimality. Given G, $\mathcal{P}$ and a template $Q(u_o)$, an "optimal" instance q^* should maximize both diversity and relative coverage, (*i.e.,* a Pareto optimal instance):

$$q^* = \arg\max_{q\in\mathcal{I}(Q)} \delta(q); \qquad q^* = \arg\max_{q\in\mathcal{I}(Q)} f(q)$$

While desirable, such a solution may not always exist, as diversity (which favors instances with diversified matches) and group fairness (which requires desired coverage) may be in conflict. A proper option is to compute a Pareto set. Given two instances q and q' in $\mathcal{I}(Q)$, we say q *dominates* q', if either (1) $\delta(q) \geq \delta(q')$ and $f(q) > f(q')$, or (2) $\delta(q) > \delta(q')$ and $f(q) \geq f(q')$. A set $\mathcal{Q}^* \subseteq \mathcal{I}(Q)$ is a *Pareto instance set* if (1) there is no pair of instances (q,q') from $\mathcal{Q}^*$, such that q dominates q', and (2) for any instance $q'' \in \mathcal{I}(Q)$, there exists an instance $q \in \mathcal{Q}^*$ that dominates q''.

One may wonder if there exist multiple Pareto instance sets. The result below verifies the uniqueness of the Pareto instance set for a given template and groups.

Lemma 1: *Given a template $Q(u_o)$, graph G and groups $\mathcal{P}$, there exists a unique, finite and maximum Pareto set $\mathcal{Q}^*$.* □

Lemma 1 justifies that computing a Pareto instance set $\mathcal{Q}^*$ is desirable as a unique optimal solution for query suggestion. Nevertheless, the exact set $\mathcal{Q}^*$ is often of substantial size in practice. For a small template of 3 edges and 3 variables, a complete Pareto set may already contain 100 instances (see Section V), which remains a daunting task for users to inspect. Alternatively, we consider a smaller, representative set of instances to approximate the unique optimal Pareto set $\mathcal{Q}^*$. To this end, we introduce a notion of *ϵ-Pareto instance set*, which approximates $\mathcal{Q}^*$ with a polynomially bounded size.

ϵ-Pareto instance set. Given $Q(u_o)$, G, and $\mathcal{P}$, we say an instance q *ϵ-dominates* q' for some $\epsilon > 0$, denoted as $q \succ_\epsilon q'$, if and only if $(1+\epsilon)\delta(q) \geq \delta(q')$, and $(1+\epsilon)f(q) \geq f(q')$.

A set of instances $\mathcal{Q}^*_\epsilon \subseteq \mathcal{I}(Q)$ is an *ϵ-Pareto instance set*, if (a) $\mathcal{Q}^*_\epsilon \subseteq \mathcal{Q}^*$, and (b) for any instance $q' \in \mathcal{I}(Q)$, there exists an instance $q \in \mathcal{Q}^*_\epsilon$, such that $q \succeq_\epsilon q'$.

An ϵ-Pareto instance set is desirable: it not only ϵ-dominates all the instances in $\mathcal{I}(Q)$, with a configurable quality controlled by ϵ, but also contains instances from the Pareto instance set $\mathcal{Q}^*$. That is, it approximates $\mathcal{Q}^*$ with a subset of representative but less number of instances. On the other hand, there exist multiple such ϵ-Pareto instance sets.

Problem statement. A *configuration* of query generation is a tuple $\mathcal{C} = (G, Q(u_o), \mathcal{P}, \epsilon)$, which contains a graph G, template $Q(u_o)$, disjoint groups $\mathcal{P}$ with coverage constraints, and a constant $\epsilon > 0$. Given a configuration $(G, Q(u_o), \mathcal{P}, \epsilon)$, the FairSQG problem is to compute an ϵ-Pareto instance set $\mathcal{Q}_\epsilon$.

Here u_o specifies the common output nodes of the instances over which the diversity and fairness constraints are consistently enforced. We characterize fairness with group coverage, which readily expresses a few practical measures, including (1) Equal Opportunity [21], by assigning the same coverage bound (c) to social groups; (2) Disparate fairness [18] such as "80% rules", which advocates that the ratio of the size of a minor group (e.g. female employees) to a majority counterpart (e.g. male ones) be at least 0.8.

Example 5: Given the $\mathcal{I}(Q)$ that contains q_1, q_2, q_3 and q_4, the Pareto set of $I(Q)$ is $\{q_2, q_3, q_4\}$ since q_1, q_2 and q_3 all dominate $q1$. Here, we set $\epsilon = 0.3$. Given the δ and f values of these query instances (See Example 4). We can compute the"boxing" coordinates of $\mathcal{I}(Q)$. From q_2 to q_4, "boxing" coordinates are $\{2.0, 2.0)(2.0, 4.0)(1.0, 5.0)\}$, respectively. Then, the ϵ-Pareto instance set of $\mathcal{I}(Q)$ is $\{q_2, q_3, q_4\}$. We can see that $q3$ and $q4$ can not dominate each other, however, q_3 dominates q_2 with the "boxing" coordinates. Thus, q_2 will be removed from the Pareto Set of $\mathcal{I}(Q)$ to form the ϵ-Pareto set which is $\{q_3, q_4\}$. □

Although desirable, FairSQG remains nontrivial. We provide the following upper and lower bound analysis.

Theorem 1: *Given a configuration $\mathcal{C} = (G, Q(u_o), \mathcal{P}, \epsilon)$, the* FairSQG *problem (1) is in Δ_2^P when $Q(u_o)$ has a fixed number of variables $|X|$, (2) remains* NP*-hard when $Q(u_o)$ has no range variables, and (3) is fixed-parameter tractable, for $Q(u_o)$ with fixed size (number of edges) and fixed $|X|$.* □

Proof sketch: The decision problem of FairSQG is to determine whether there is a non-empty ϵ-Pareto instance set $\mathcal{Q}_\epsilon$. (1) FairSQG is solvable in Δ_2^P for fixed template $Q(u_o)$. Here Δ_2^P is the class of problems in P^{NP}. A Δ_2^P algorithm first enumerates $\mathcal{I}(Q)$. For each instance, it consults an NP oracle to verify $f(q)$ and $\delta(q)$, and computes an ϵ-Pareto set with a pairwise comparison (*e.g.,* nested loop). As $|\mathcal{I}(Q)| \leq 2^{|X_E|}|\mathsf{adom}_m|^{|X_L|}$ ($|X_E|$ and $|X_L|$ are constants), the verification is in PTIME. Here adom_m refers to the largest active domain in G. (2) The NP-hardness can be verified from the hardness of deciding subgraph isomorphism, even when X_E is fixed (thus in total $2^{|X_E|}$ instances).

To see Theorem 1 (3), we observe that the Δ_2^P algorithm in Theorem 1 (1) takes polynomial time for fixed $|X|$ and $|Q(u_o)|$, given that it is in PTIME to verify the coverage and diversity for $\mathcal{I}(Q)$ with polynomially bounded size. □

The above analysis provides a naive algorithm (denoted as EnumQGen): enumerates up to $2^{|X_E|}|\mathsf{adom}_m|^{|X_L|}$ instances, verifies each instance to find feasible ones, and invokes a nested loop comparison to generate ϵ-Pareto instance set. This is infeasible when G is large.

We next show that an ϵ-Pareto instance set $\mathcal{Q}^*_\epsilon$ with a *bounded size* can be efficiently computed (Section IV) and dynamically maintained (Section IV-C), where the size bound is only determined by ϵ and the range of diversity and coverage. This enables flexible query generation that strikes a balance between quality and instance sizes. We present detailed proofs of lemmas and Theorem 1 in [1].

IV. Approximating Pareto Instance Sets

We start with a generic query generation algorithm, denoted as QGen, for FairSQG without enumeration.

Auxiliary structures. To characterize the search space, we start with a notion of *refinement* relation defined on $\mathcal{I}(Q)$.

Refinement. Given a template $Q(u_o)$, and instantiations I and I' of $Q(u_o)$, I' *refines* I at a variable x (denoted as $I' \succeq_x I$) if it binds a constant to x that makes the predicate parameterized by x no less selective than the counterpart from I. Specifically, (1) for a literal l in the form of $u.A > x_l$ or $u.A \geq x_l$ (resp. $u.A < x_l$ or $u.A \leq x_l$), $I'(x_l)$ refines $I(x_l)$ if $I'(x_l) \geq I(x_l)$ (resp. $I'(x_l) \leq I(x_l)$) ("refines a selection condition"); (2) for edge variable x_e, $I'(x_e)$ refines $I(x_e)$ if $I'(x_e) = 1$ and $I(x_e) = 0$ ("adds a query edge"); (3) $I' \succeq_x I$ if $I(x) = '\_'$.

We say I' *refines* I (denoted as $I' \succeq I$) if for every variable x in $Q(u_o)$, $I' \succeq_x I'$. Given two instances q' and q induced by I' and I respectively, q' refines q (denoted as $q' \succeq_\mathcal{I} q$) if $I' \succeq I$. We observe the following result.

Lemma 2: *Given a configuration $\mathcal{C} = (G, Q(u_o), \mathcal{P}, \epsilon)$, (1) the refinement relation is a preorder; and (2) for any instances q and q' in $\mathcal{I}(Q)$, if $q' \succeq_\mathcal{I} q$, then $\delta(q) \geq \delta(q')$; and $f(q) \leq f(q')$ when both q and q' are feasible.* □

Proof sketch: The above results can be verified by observing that (a) refinement relation is reflexive and transitive, and (b) $q'(G) \subseteq q(G)$ if $q' \succeq_\mathcal{I} q$, *i.e.,* any match of u_o in q' remains to be a match of u_o in q if q' refines q at some variables. □

These results justify the following. Lemma 2 (1) provides a convenient lattice encoding of the search space induced by the refinement preorder. Lemma 2 (2) verifies useful monotonicity properties on diversity and coverage measures that shall be exploited for effective pruning.

Instance Lattice. Following Lemma 2, the algorithm QGen maintains a lattice encoding of the instance space Ł = $(\mathcal{I}(Q), \prec_{\mathcal{I}})$ induced by refinement. (1) It initializes $(\mathcal{I}(Q), \prec_{\mathcal{I}}$) with a single root q_r (upper bound) induced by the "most relaxed" instantiation, and a single lower bound q_b induced by the "most refined" instantiation. (2) Each node q in $(\mathcal{I}(Q), \prec_{\mathcal{I}})$ is an instance. For each node q, QGen maintains

- (a) two Boolean flags: 'verified' to record if q is verified, and 'feasible' to indicate if q is a feasible instance;
- (b) $q(G)$, the (estimated) query answer; and
- (c) a coordinate $(\delta(q), f(q))$, and a "boxing" coordinate Box(q) = $(\delta_\epsilon(q)), f_\epsilon(q)$. The coordinate value $\delta_\epsilon(q)$ (resp. $f_\epsilon(q)$) is defined as $\frac{\log(1+\delta(q))}{\log(1+\epsilon)}$ (resp. $\frac{\log(1+f(q))}{\log(1+\epsilon)}$). Box(q) specifies a box region in the bi-objective (2-dimensional) space of instance q to decide the ϵ-dominance relation (see "Updater" below).

(3) There is an edge (q, q') with a label x if (a) $q' \succeq q$, and (b) q' differs from q in the value of only one variable x, and q' refines q by modifying the value of x to its closest counterpart in the corresponding active domain (e.g., changing 0 to 1 if x is an edge variable). Intuitively, an edge indicates a stepwise refinement action of q by adjusting the value of x only.

Example 6: A fraction of the lattice structure Ł that contains $\{q_r, q_1, \ldots q_4, q_5\}$ is shown on the left-hand side of Fig. 4. Algorithm QGen maintains the auxiliary information of *e.g.,* q_3 once it is verified, including the coordinates $\delta(q_3)$, $f(q_3)$, and the boxing coordinates $Box(q_3)$. As q_4 refines q_3 at variable x_l1 at the node u_1, (q_3, q_4) is a direct edge in Ł. □

Generic Algorithm. Given a configuration $\mathcal{C}$ = $(G, Q(u_o), \mathcal{P}, \epsilon)$, algorithm QGen maintains an instance set $\mathcal{Q}$ and iteratively refines $\mathcal{Q}$ towards an ϵ-Pareto instance set of $\mathcal{I}(Q)$. At each iteration i, it refines the solution $\mathcal{Q}_i$ from the last iteration by interacting two procedures.

(1) *Spawner*. A spawner (Spawn) constructs new instances to be verified that may contribute to the current instance set $\mathcal{Q}_i$ with new ϵ-dominance relation in diversity and coverage. In each iteration, Spawn (a) refines the current configuration given the quality of $\mathcal{Q}_i$ to reduce unnecessary generation, (b) constructs a *front set* of instances $\mathcal{Q}_F$ (thus a fraction of lattice $(\mathcal{I}(Q), \prec_{\mathcal{I}})$) on-the-fly, and (c) prunes unpromising instances that are already ϵ-dominated by $\mathcal{Q}_i$ whenever possible. The spawner performs no actual query processing and verification.

(2) *Updater*. An updater (Update) refines $\mathcal{Q}_i$ with the front set $\mathcal{Q}_F$ from Spawn towards a better solution $\mathcal{Q}_{i+1}$. Our idea extends [26] to maintain "boxes" of instances that discretize the bi-objective (2-dimensional) space of answer diversity and coverage of groups. Each box is represented by a single instance q and specified by its boxing coordinates $(\delta_\epsilon(q), f_\epsilon(q))$. To verify ϵ-dominance, it then suffices to verify the dominance of boxing coordinates at both box level and instance level.

```
Algorithm RfQGen
Input: configuration $\mathcal{C} = (G, Q(u_o), \mathcal{P}, \epsilon)$;
Output: an $\epsilon$-Pareto instance set $\mathcal{Q}^*_\epsilon$.
1.  set $\mathcal{Q}^*_\epsilon$:= $\emptyset$; initializes Ł := $\{q_r\}$;
2.  BFExplore $(\mathcal{C}, q_r, Ł, \mathcal{Q}^*_\epsilon)$;
3.  return $\mathcal{Q}^*_\epsilon$;

Procedure BFExplore($\mathcal{C}, q_r, Ł, \mathcal{Q}^*_\epsilon$)
1.  if $q$.verified then return ;
2.  incVerify $(q, Ł, G)$; $q$.verified:= true;
3.  if !$q$.feasible then return ;
4.  Update $(q, \mathcal{Q}^*_\epsilon)$;
5.  set $\mathcal{Q}_F$ := Spawn $(q, \mathcal{C})$;
6.  for each $q' \in \mathcal{Q}_F$ do
7.      BFExplore $(\mathcal{C}, q', Ł, \mathcal{Q}^*_\epsilon)$;
```

Fig. 3. Algorithm RfQGen

We present our main result below.

Theorem 2: *Given a configuration $(G, Q(u_o), \mathcal{P}, \epsilon)$, there exists an algorithm that (1) correctly maintains an ϵ-Pareto instance set $\mathcal{Q}^i_\epsilon$ over all generated instances upon any time i the updater is invoked; (2) ensures a size-bounded $\mathcal{Q}_i$, where $|\mathcal{Q}^i_\epsilon| \leq \frac{\log(|V|)}{\log(1+\epsilon)}$, and (3) take $O(|\mathsf{adom_m}|^{|X|}(\frac{\log(|V|)}{\log(1+\epsilon)} + T_q))$ time to compute $\mathcal{Q}^*_\epsilon$, where $\mathsf{adom_m}$ refers to the largest active domain, and T_q is the cost of verifying a single instance.* □

We next present two efficient algorithms as a constructive proof of Theorem 2. Each implements QGen with different exploration strategies of the instance lattice Ł, and convergences to instances with high diversity, or a more balanced distribution of coverage, for different user preferences. Both have provable guarantees in Theorem 2.

A. Query Generation by Refinement

Our first algorithm, denoted as RfQGen, uses a "refine as always" strategy to compute $\mathcal{Q}^*_\epsilon$. Given a configuration $(G, Q(u_o), \mathcal{P}, \epsilon)$, it starts from the root q_r of the instance lattice Ł (which carries search predicates with the most "relaxed" conditions), and performs a depth-first exploration of Ł. The algorithm uses Lemma 2 (2) to achieve early pruning of infeasible instances, and reduce unnecessary updates.

Algorithm. Algorithm RfQGen is shown in Fig. 3. It initializes set $\mathcal{Q}^*_\epsilon$, and the lattice Ł with a single root q_r. It then invokes a recursive procedure BFExplore to perform depth-first exploration, which interacts spawn and update process and generates a front set $\mathcal{Q}_F$ to be explored at each level of Ł. RfQGen early terminates if no new instance can be spawned (as BFExplore backtracks), and returns set $\mathcal{Q}^*_\epsilon$.

Procedure BFExplore. The recursive procedure BFExplore starts by verifying an unvisited instance q from the current front set $\mathcal{Q}_F$. (1) It invokes a procedure incVerify (line 2; not shown) to incrementally update the match set $q(G)$ [17], along with the coordinates $(\delta(q), f(q))$ and boxing coordinates $(\delta_\epsilon(q), f_\epsilon(q))$. Following Lemma 2, incVerify only determines which matches should be excluded from the counterparts of the verified "parent" of q in Ł. (2) It then invokes a procedure Update (line 4) to maintain $\mathcal{Q}^*_f$ given a feasible instance q. (3) For a feasible instance q, it invokes a procedure Spawn (line 5) to generate the frontier set $\mathcal{Q}_F$ of refined instances

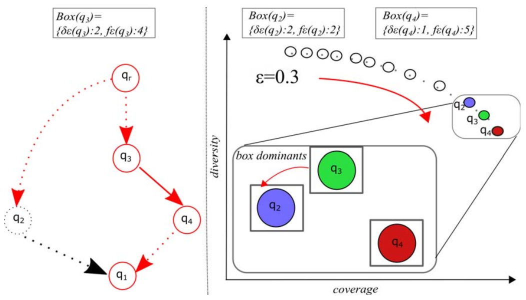

Box(q3)= {δε(q3):2, fε(q3):4}
Box(q2)= {δε(q2):2, fε(q2):2}
Box(q4)= {δε(q4):1, fε(q4):5}
qr
q3
q2
q4
q1
ε=0.3
diversity
coverage
box dominants

Fig. 4. Instance Lattice and “Refinement as always” exploration

(thus spawns a set of children of q in Ł), by modifying one variable at a time using the next closest active domain value. It then starts a next-level exploration for each instance in $\mathcal{Q}_F$. BFExplore backtracks whenever q is not feasible (line 3), as no refined counterparts are feasible (Lemma 2).

Procedure Update. Given a feasible instance q and current ϵ-Pareto instance set $\mathcal{Q}^*_\epsilon$, procedure Update maintains ϵ-dominance by verifying the dominance of Boxing coordinates $Box(q)$, with a case analysis below.

(Case 1) *Replacing boxes* (lines 1-5). This case verifies a box-level dominance relation. Using the boxing coordinates Box(q) = $(\delta_\epsilon(q), f_\epsilon(q))$ of q, it verifies if q introduces a box that also already dominates a set of boxes in the bi-objective space of diversity and coverage. If so, it removes all the representative instances of those boxes from $\mathcal{Q}^*_f$, and adds q.

(Case 2) *Replacing instances* (lines 6-7). If q falls into a box which is represented by another instance q', Update simply keeps the one that can dominate the other.

(Case 3) *Adding a non-dominated box* (lines 8-9). If no box can dominate Box(q), Update simply adds q to $\mathcal{Q}^*_f$ (which represents a new box). Here we use $Box(q') \succeq Box(q)$ to denote that $Box(q') \succ Box(q)$ or $Box(q') = Box(q)$.

Example 7: Fig 4 illustrates a case of the running of “update”. Starting from the root of the lattice q_r, RfQGen spawns and verifies instances following the refinement preorder. (1) In the first iteration, Update simply add q_2 to $\mathcal{Q}^*_\epsilon$. (2) Once q_3 is verified, Update removes q_2 under Case (1), as $Box(q_3) \succeq Box(q_2)$ (“Replacing boxes”). (3) In the next iteration, Update keeps q_4 in $\mathcal{Q}^*_\epsilon$, since q_4 and q_3 cannot dominate each other at the box level. (4) Update finally rejects q_1, since q_3 and q_4 both dominate q_1. RfQGen then returns $\mathcal{Q}^*_\epsilon$ as $\{q_3, q_4\}$. □

Procedure Spawn. To further reduce generation and verification costs, procedure Spawn uses the following strategy to actively refine the values. Each variable can take, and “simplifies” template $Q(u_o)$ when possible. The refined templates are restored when BFExplore backtracks to ensure correctness.

Template refinement. Given a verified instance q, Spawn dynamically tracks the subgraph induced by d-hop neighbors of $q(G)$ (d is the diameter of $Q(u_o)$), denoted as G^d_q.

(1) For each literal $u.A$ op x of $Q(u_o)$, it refines the values x can take to $\{T(v.A)\} \subseteq \mathsf{adom}(A)$, where v ranges over the nodes in G^d_q and $L(v) = L(u)$.

(2) For each edge variable x_e on edge $e = (u, u')$ in $Q(u_o)$, it “fixes” x_e to be 0 if there is no path from any match of u_o in G^d_q with an edge (v, v') such that $L_Q(e) = L((v, v'))$.

```
Procedure Update ($q$, $\mathcal{Q}^*_\epsilon$)
1.  set $\mathcal{Q}_B$ := $\emptyset$;
    /* verify “box-level” dominance */
2.  for each $q' \in \mathcal{Q}^*_\epsilon$ do
3.      if $Box(q') \prec Box(q)$ then $\mathcal{Q}_B$:= $\mathcal{Q}_B \cup \{q'\}$;
4.  if $\mathcal{Q}_B \neq \emptyset$ then
5.      $\mathcal{Q}^*_\epsilon$ := $(\mathcal{Q}^*_\epsilon \setminus \mathcal{Q}_B) \cup \{q\}$;
    /* verify “instance-level” dominance */
6.  else if there is an instance $q' \in \mathcal{Q}^*_\epsilon$
    and $Box(q') = Box(q)$ then
7.      if $q' \prec q$ then $\mathcal{Q}^*_\epsilon$ := $(\mathcal{Q}^*_\epsilon \setminus \{q'\} \cup \{q\}$;
    /* adding a new instance and a non-dominated box */
8.  else if there is no instance $q' \in \mathcal{Q}^*_\epsilon$
    such that $Box(q') \succeq Box(q)$ then
9.      $\mathcal{Q}^*_\epsilon$:= $\mathcal{Q}^*_\epsilon \cup \{q\}$ ;
10. return $\mathcal{Q}^*_\epsilon$;
```

Fig. 5. Algorithm Update

Moreover, if e is a bridge of $Q(u_o)$, *i.e.,* removing e leads to two connected components in $Q(u_o)$, Spawn removes e and the entire connected component that does not contain u_o.

Example 8: Given a configuration $\mathcal{C} = (G, Q(u_o), \mathcal{P}, \epsilon)$ where $\mathcal{P}$ is defined on gender groups of users in G, and $\epsilon = 0.3$. Algorithm RfQGen starts with the root q_r in the lattice Ł, as illustrated in Fig. 4. (1) Following a depth first strategy, BFExplore reaches q_3 and invokes Spawn to refine q_3 to q_4. In particular, Spawn selects variable X_{l1} at node u_1. While the active domain of “yearsOfExp” suggests three values $\{10, 11, 12, 20\}$, Spawn recognizes that it suffices to explore only $\{10, 11, 12\}$ with the next available value, given that no neighbors of the current match have “yearsOfExp” more than 20. It then generates q_4 and adds it to the front set for further exploration. (2) As the exploration reaches q_2, it finds an ϵ-Pareto set $\{q_3, q_4\}$ and returns the set (See Example 9). □

Correctness. Algorithm RfQGen correctly maintains an ϵ-Pareto instance set $\mathcal{Q}^i_\epsilon$ over the generated instances $\mathcal{I}^i(Q)$ upon Update is invoked at time i. To see this, assume $\mathcal{Q}^i_\epsilon$ is not an ϵ-Pareto instance set. Then either (a) there exists an instance $q \in \mathcal{I}^i(Q) \setminus \mathcal{Q}^i_\epsilon$ that is not ϵ-dominated by any instance in $\mathcal{Q}^i_\epsilon$, or (b) $q \in \mathcal{Q}^i_\epsilon$ but not in the Pareto set of $\mathcal{I}^i(Q)$. For case (a), Update only removes q if there is another verified instance $q' \in \mathcal{I}^i(Q)$ that either dominates q (line 7), or ϵ-dominates q (line 3). In either case, it contradicts the assumption. Similarly, case (b) indicates that there exists at least an instance $q' \in \mathcal{I}^i(Q)$ that $q' \succeq q$. Thus q' is verified at some time and either remains in $\mathcal{Q}^i_\epsilon$ or leaves a box $Box(q'')$, where $q'' \succeq q'$. In either case, q should be excluded by Update from $\mathcal{Q}^i_\epsilon$ (at line 7 or line 3), which contradicts that $q \in \mathcal{Q}^i_\epsilon$.

Size bound. We next show that at any time i, $|\mathcal{Q}^i_\epsilon| \leq \frac{\log(|V|)}{\log(1+\epsilon)}$. To see this, observe that (a) Update ensures that each box is represented by a single instance; (b) there are in total $\frac{\log(|V|)}{\log(1+\epsilon)} \frac{C}{\log(1+\epsilon)}$ boxes in the bi-objective 2D space, thus at most $\frac{\log(|V|)}{\log(1+\epsilon)}$ instances having the same coverage that ϵ-dominate the rest (as $\delta(q) \in [0, |V|]$), or $\frac{\log C}{\log(1+\epsilon)}$ instances that ϵ-dominate the rest having same diversity (as $f(q) \in [0, C]$). As $C \leq |V|$ for disjoint groups, the size bound follows.

Time Cost. Following Theorem 1, Spawn generates up to $2^{|X_E|}|\mathsf{adom}_m|^{|X_L|}$ instances. Thus it takes in $O(|\mathsf{adom}_m|^{|X|})$ runs of BFExplore. For each instance q, it takes Update

```
Algorithm BiQGen
Input: configuration $\mathcal{C} = (G, Q(u_o), \mathcal{P}, \epsilon)$;
Output: an $\epsilon$-Pareto instance set $\mathcal{Q}^*_\epsilon$.
1.  queue $S_f := \{q_r\}$; queue $S_b := \{q_b\}$; set $\mathcal{Q}^*_\epsilon := \emptyset$;
2.  set $\mathcal{Q}_f := \emptyset$; set $\mathcal{Q}_b := \emptyset$; set SBounds:= $\emptyset$;
3.  while $S_f \neq \emptyset$ or $S_b \neq \emptyset$ do
    /* forward exploration */
4.      if $S_f \neq \emptyset$ then instance $q$:= $\mathcal{Q}_f$.dequeue();
5.          if $q$.verified or !$q$.feasible then continue;
6.          if SPrune(q,SBounds) then continue;
7.          incVerify ($q$); $q$.verified:=true;
8.          if $q$.feasible then Update ($q$, $\mathcal{Q}^*_\epsilon$);
9.              $\mathcal{Q}^f_F$ := SpawnF ($q, \mathcal{C}$); $S_f$.enqueue($\mathcal{Q}^f_F$);
    /* backward exploration */
10.     if $S_b \neq \emptyset$ then instance $q'$:= $\mathcal{Q}_b$.dequeue();
11.         if $q$.verified or !$q$.feasible then continue;
12.         if SPrune(q, SBounds) then continue;
13.         incVerify ($q'$); $q$.verified:=true;
14.         if $q'$.feasible then Update ($q'$, $\mathcal{Q}^*_\epsilon$);
15.             $\mathcal{Q}^b_F$ := SpawnB ($q, \mathcal{C}$); $S_b$.enqueue($\mathcal{Q}^b_F$);
    /*update "Sandwidch" bounds with feasible pair $(q, q')$*/
16. if $q' \succ_\mathcal{I} q$ and $(Box(q').\delta = Box(q').\delta$
    or $Box(q').f = Box(q').f)$ then
17.     update SBounds with $(q, q')$;
18. return $\mathcal{Q}^*_f$;
```

Fig. 6. Algorithm BiQGen

$O(\frac{\log(|V|)}{\log(1+\epsilon)} + T_q)$ time to verify q (in time T_q) and update $\mathcal{Q}^i_\epsilon$. Thus the total time cost is in $O(|\mathsf{adom_m}|^{|X|}(\frac{\log(|V|)}{\log(1+\epsilon)} + T_q))$ time. We found that $\mathsf{adom_m}$ and $|X|$ are usually small in real-world graphs. Moreover, the early pruning of infeasible instances reduces on average 40% of the generated ones, compared with the naive algorithm EnumQGen (Section V).

The above analysis completes the proof of Theorem 2.

B. *Bi-directional Query Generation*

Algorithm RfQGen achieves early convergences to an ϵ-Pareto instance set $\mathcal{Q}^*_\epsilon$, where a majority of instances may have high answer diversity (Lemma 3 (a)).

We next present a second algorithm, denoted as BiQGen. It adopts a bi-directional strategy that explores Ł from both ends. The forward exploration inspects instances with non-increasing diversity, and the backward exploration keeps "relaxing" instances towards early convergence to instances with high coverage. Following Lemma 3 (b), the computation has more chance to generate $\mathcal{Q}^*_\epsilon$ with a more "balanced" distribution on instances with high diversity and those with desired coverage, as also verified by our experiments (Section V).

Algorithm. The algorithm BiQGen (shown in Fig. 6) uses the same procedure Update as in RfQGen to maintain $\mathcal{Q}^*_\epsilon$ at any time, but specifies two separate spawners: SpawnF, same as Spawn in RfQGen, and SpawnB, a reversed "relaxation" counterpart that yields new instances by relaxing the search predicates. The procedure SpawnF and SpawnB are invoked to generate a front set $\mathcal{Q}^f_F$ (line 9) and $\mathcal{Q}^b_F$ (line 15) in a "forward" refinement-based exploration from q_r, and a "backward" relaxation-based exploration from q_b, respectively. It uses two queues S_f and S_b to control the iterative forward and backward exploration (line 2). It iteratively performs forward (lines 3-9) and backward exploration (lines 10-15), and terminates if no new instances can be generated (line 3).

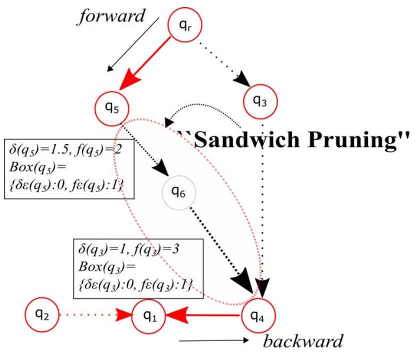

forward
q_r
q_5
q_3
"Sandwich Pruning"
$\delta(q_5)$=1.5, $f(q_5)$=2
$Box(q_5)$=
$\{\delta\varepsilon(q_5)$:0, $f\varepsilon(q_5)$:1$\}$
q_6
$\delta(q_3)$=1, $f(q_3)$=3
$Box(q_3)$=
$\{\delta\varepsilon(q_3)$:0, $f\varepsilon(q_3)$:1$\}$
q_2
q_1
q_4
backward

Fig. 7. Bi-directional Query Generation

"Sandwich" pruning. The bidirectional strategy enables an effective pruning strategy that exploits the monotonicity properties of diversity and coverage in Lemma 2 (2).

Lemma 3: *For any feasible instances $q \in \mathcal{Q}^f_F$ and $q' \in \mathcal{Q}^b_F$, if $q' \succeq_\mathcal{I} q$, and (a) $Box(q).\delta$ = $Box(q').\delta$ or (b) $Box(q).f$ = $Box(q').f$, then for any instance $q'' \in \mathcal{I}(Q)$ where $q \prec_\mathcal{I} q'' \prec_\mathcal{I} q'$, $q'' \notin Q^*_\epsilon$.* □

Proof sketch: Consider an instance $q'' \in \mathcal{I}(Q)$ where $q \prec_\mathcal{I} q'' \prec_\mathcal{I} q'$. Following Lemma 2, $\delta(q') \leq \delta(q'') \leq \delta(q)$, and $f(q') \geq f(q'') \geq f(q)$. For Case (a), if $Box(q).\delta$ = $Box(q').\delta$, then $Box(q'').\delta$ = $Box(q').\delta$ and $f(q') \geq f(q'')$. Thus $q' \succ q''$ or $q' \succ_\epsilon q''$. Similarly, for Case (b), if $Box(q).f$ = $Box(q').f$, then $Box(q'').f$ = $Box(q).f$. Thus $q \succ q''$ or $q \succ_\epsilon q''$. In either case, Update rejects q'' (line 5 or line 7). □

During the bi-directional exploration, BiQGen keeps track of the occurrences of "sandwich" pairs (q, q') that satisfy the condition in Lemma 3 in a set SBounds. Upon a new pair (q, q') is identified (line 16), it updates SBounds by (a) replacing any pair (q_1, q_2) with (q, q_2) (resp. (q_1, q')) if $q \prec_\mathcal{I} q_1$ (resp. $q_2 \prec_\mathcal{I} q'$), or (q, q') if $q \prec_\mathcal{I} q_1$ and $q_2 \prec_\mathcal{I} q'$; or (b) adding (q, q') (line 17). This in turn allows more instances to be pruned (by a procedure SPrune; lines 6 and 12), for both forward and backward exploration.

These pruning strategies are fast and effective. Checking the refinement preorder $\succ_\mathcal{I}$ takes $O(|X|)$ time per instance. We found on average 60% of the generated instances from EnumQGen are pruned by BiQGen (see Section V).

Example 9: Given $\mathcal{I}(Q)$ that contains $\{q_r, q_1, q_2, q_3, q_4\}$, BiQGen starts a forward search from q_r as in RfQGen, and a backward search from q_1. (1) In the first round of bidirectional search, SpawnF refines q_r to q_5, and SpawnB relaxes q_1 to q_4. Upon the verification of q_4 and q_5, BiQGen finds that $Box(q_5).f$ = $Box(q_4).f$ = 4. It then creates a pair (q_5, q_4) and adds it to SBounds. (2) In the next round, the backward search reaches q_2. Meanwhile, as $q_5 \prec_\mathcal{I} q_6 \prec_\mathcal{I} q_4$, q_6 is skipped in the forward search without further exploration. □

Analysis. The correctness of BiQGen follows from the correctness analysis of Update, SpawnF and SpawnB (following the analysis of Spawn) and the invariant that forward and backward exploration verifies and safely prunes $\mathcal{I}(Q)$ (Lemma 2 and Lemma 3). The size bound and time cost follows from a similar analysis as in BFExplore.

C. *Online Query Generation*

Another need for query generation is to produce workloads with arbitrary size k with diversity and coverage requirements over interested groups for query benchmarking [37], [4], [5], [6]. We consider the following problem. Given a configuration $\mathcal{C}$=$(G, Q(u_o), \mathcal{P}, k)$, maintain an ϵ-Pareto instance set $\mathcal{Q}^t_{(\epsilon,k)}$ over a large set of instances (due to *e.g.,* active domains and G), such that at any time t, (a) $|\mathcal{Q}^t_{(\epsilon,k)}|$ = k, and (b) ϵ is as small as possible. This is nontrivial: as more instances arrive, one needs to compromise with a larger ϵ (thus worse approximation) for a smaller k (Theorem 2).

We extend QGen to an online algorithm, denoted as OnlineQGen, to maintain an ϵ-Pareto instance set with a fixed size k and a small ϵ at any time. To cope with large instance space, it treats $\mathcal{I}(Q)$ as a stream of instances from a query generator. Unlike RfQGen and BiQGen, it does not assume ordered processing (*e.g.,* "refinement"). Instead, (1) it uses a sliding window W_Q with a bounded size w to cache a certain number of instances that can help reduce ϵ, while keeping k fixed; and (2) it *incrementalizes* the maintenance of $Q^t_{(\epsilon,k)}$ upon the arrival of a new instance, and only perform necessary maintenance when Update causes size growth (in particular, Case (3) in Update).

Online Algorithm. The algorithm, as shown in Fig. 8, takes as input a stream of instances from an arbitrary generator that instantiates $Q(u_o)$, and a small initial constant $\epsilon_m > 0$. It starts by populating $Q^t_{(\epsilon,k)}$ with Update upon newly arrived instances (in arbitrary order), until $|Q^t_{(\epsilon,k)}|$ = k (lines 7-10). If an instance q is rejected by Update, OnlineQGen includes it to W_Q (line 9) for future consideration (up to at most w timestamps before it "expires"; lines 5-6). This is to "temporally" keep the instances that may be accepted by Update again, thus reducing $|Q^t_{(\epsilon,k)}|$. (and ϵ remains unchanged).

When $|Q^t_{(\epsilon,k)}|$ = k and a new instance q can be added, OnlineQGen incrementalizes Update by individually checking (a) if adding q increases $|Q^t_{(\epsilon,k)}|$ (Case (3)) or not (Case (1) and (2); see Update in Section IV). if the latter applies, it simply adds q (lines 12-13); otherwise, it first finds the nearest neighbor of q in $Q^t_{(\epsilon,k)}$ to be replaced by q. To this end, it enlarges ϵ as the Euclidean distance of the coordinates of q and q', to include q and q' in a larger box (line 16). It also verifies if a cached instance can be added without increasing $|Q^t_{(\epsilon,k)}|$ (lines 18-20). It returns a size-k ϵ-Pareto set $Q^t_{(\epsilon,k)}$ upon request (line 21) or no new instance is generated (line 22).

Analysis. OnlineQGen correctly maintains an ϵ-Pareto instance set with a fixed size k for the "seen" fraction of $\mathcal{I}(Q)$ at time t. We first observe the following property.

Lemma 4: *If $q \prec_\epsilon q'$, then $q \prec_{\epsilon'} q'$ for any $\epsilon' > \epsilon$.* □

The correctness follows from the following invariant: at any time t, (1) either Update correctly rejects an instance that is already ϵ-dominated by an instance in $|Q^t_{(\epsilon,k)}|$, including those "expired" in W_Q; or (2) ϵ is adjusted to a larger counterpart to reduce the size of $|Q^t_{(\epsilon,k)}|$ to k (Theorem 2), and preserves any previous ϵ-dominance relation (Lemma 4).

Delay time. OnlineQGen efficiently maintains the ϵ-Pareto instance set with a delay time in $O(T_q + w + k)$ time, where T_q is the cost of verifying a single instance. This verifies the practical application of OnlineQGen in query generation with desired diversity and coverage for large workloads.

```
Algorithm OnlineQGen
Input: a configuration $(G, Q(u_o), \mathcal{P}, k)$, $\epsilon_m$;
       a stream of instances $\mathcal{I}(Q)$, a cache size $w$;
Output: a size-$k$ $\epsilon$-Pareto instance set $\mathcal{Q}^t_{(\epsilon,k)}$ at any time $t$.
1.   set $\mathcal{Q}_{(\epsilon,k)}$ :=∅; set $W_Q$:= ∅; integer $t$:= 0; $\epsilon$:=$\epsilon_m$;
2.   while $\mathcal{I}(Q)$.hasNext() do
3.       instance $q$:=$\mathcal{I}(Q).getNext()$;
4.       verify $q$; $q$.ts:=$i$; $i$:=$i$+1;
     /* remove "expired" query instances */
5.       for each $q' \in W_Q$ do
6.           if $q'$.ts< $i - w + 1$ then $W_Q$:=$W_Q \setminus \{q'\}$;
7.       if $|\mathcal{Q}_{(\epsilon,k)}| < k$ then
8.           Update ($q$, $\mathcal{Q}_{(\epsilon,k)}$);
     /* cache an $\epsilon$-dominated instance for future update */
9.           if $q \notin \mathcal{Q}_{(\epsilon,k)}$ then $W_Q$ :=$W_Q \cup \{q\}$;
10.          else continue ;
11.      if $|\mathcal{Q}_{(\epsilon,k)}|$ = k then
12.          if Update accepts $q$ with Case (1) or (2) then
13.              $\mathcal{Q}_{(\epsilon,k)}$:=$\mathcal{Q}_{(\epsilon,k)} \cup \{q\}$; Continue;
14.          if Update accepts $q$ with Case (3) then
     /* replace an instance with q */
15.              $q'$ := NearestNeighbor($q$, $\mathcal{Q}_{(\epsilon,k)}$);
16.              $\epsilon$ := dist(($q.\delta, q.f$), ($q'.\delta, q'.f$));
17.              $\mathcal{Q}_{(\epsilon,k)}$ := $\mathcal{Q}_{(\epsilon,k)} \setminus \{q'\}$;
     /* check if a cached instance can be added without impact */
18.              if there is a $q_b \in W_Q$ such that Update accepts $q_b$
                 in Case (1) or Case (2) then
19.                  $\mathcal{Q}_{(\epsilon,k)}$:=$\mathcal{Q}_{(\epsilon,k)} \cup \{q_b\}$;
20.              $\mathcal{Q}_{(\epsilon,k)}$:=$\mathcal{Q}_{(\epsilon,k)} \cup \{q\}$;
21.      return $\mathcal{Q}_{(\epsilon,k)}$ upon request;
22.  return $\mathcal{Q}_{(\epsilon,k)}$;
```

Fig. 8. Algorithm OnlineQGen

V. Experiments

Using real-world graphs, we experimentally verify the effectiveness and efficiency of our algorithms. We investigate the following: **(RQ1)** The effectiveness of our algorithms in maximizing the diversity and ensuring the group coverage; **(RQ2)** The efficiency of our query generation algorithms; and **(RQ3)** The performance of our online generation algorithm.

Experiment Setting. We used the following setting.

Datasets and Groups. We use three real-life data graphs (summarized in Table II), each reflects an application of query generation. (1) DBP [29] is a movie knowledge graph induced from DBpedia. Each node has a label (*e.g.,* movie, director, actors; in total 115 types) and attributes such as title, genre, and years. Each relation has a label (*e.g.,* directed, collaboration; in total 398 types). We induce up to 5 movie groups based on their genres (*e.g.,* "Action","Romance") or countries for diversified and fair movie recommendations. (2) For talent search, we use LKI [42]. with nodes denoting users and organizations, and edges denoting co-review and works. Each node has attributes such as "Major". We induce 2 gender groups $\mathcal{P}$ (male, female) with synthetic genders generated by gender inference tools [14]. (3) For diversified and fair academic recommendations, we use Cite [38] where nodes are papers and authors, and edges denoting citations and authorship. Each node has attributes such as "numberOfCitations" and "topic".

Dataset	$\|V\|$	$\|E\|$	avg. # attr	$\|\mathcal{P}\|$	$\|Q(u_o)\|$	C	$\|X\|$
DBP	1M	3.18M	10	2-5	3-5	100-800	3-5
LKI	3M	26M	7	2	3-5	200	3-5
Cite	4.9M	46M	6	2-4	3-4	200	3-4

TABLE II
OVERVIEW OF REAL-LIFE GRAPHS

We induce up to 4 groups $\mathcal{P}$ of papers with different topics (*e.g.,* "Machine Learning","Networking").

Queries and Templates. We developed a generator to produce query templates with practical search conditions, controlled by the number of variables $|X|$ (specifically, the number of edge variables and range variables), query size $|Q(u_o)|$ (in terms of the number of edges) and topologies.

For each dataset, we generated a set of $Q(u_o)$ and $\mathcal{P}$ and ensure the existence of feasible query instances. The largest set of instances $\mathcal{I}(Q)$ for DBP, LKI, and Cite are 1000, 1400 and 800, respectively. We quantify the diversity of two nodes with the normalized edit distances of their matching attributes.

Algorithms. We implemented the following algorithms in Java: (1) EnumQGen, which enumerates and verifies the instances in $\mathcal{I}(Q)$, and performs a simple nested loop to compute the ϵ-Pareto optimal instance set. (2) RfQGen, with "refine as always" strategy; (3) BiQGen, the algorithm that adopts bi-directional search with "Sandwich" pruning; and (4) the online query generation algorithm OnlineQGen, which maintains an ϵ-Pareto instance set with a fixed size k and small ϵ. (5) To verify the quality of query generation, we also implemented Kungs, an algorithm that enumerates and verifies the instances in $\mathcal{I}(Q)$, and invokes Kung's algorithm [13] to compute the Pareto optimal non-dominated set. (6) CBM [10], the constraint based bi-objective optimization algorithm. As default, we set $|\mathcal{P}| = 2$, $C = 200$, $|Q(u_o)| = 3$ with $|X| = 3$ and $\epsilon = 0.01$ for our experiments. We also summarized the parameter settings we adopted for our experiments in Table II. Our source codes and datasets are available online[1].

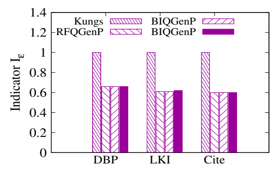
Experimental results. We next present our findings.

Exp-1: Effectiveness (RQ1). As the diversity and coverage of queries vary over different graphs, we quantify their effectiveness with two established relative measures: R-indicator, and ϵ-indicator[43], for a fair comparison.

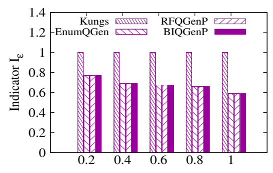
ϵ-Indicator (I_ϵ). Given a set of tuples $\mathcal{Q}$, the ϵ-indicator [43] finds the minimum ϵ, denoted by ϵ_m, for which $\mathcal{Q}$ is an ϵ_m-Pareto set. Given ϵ-Pareto instance set $\mathcal{Q}^*_\epsilon$ that conform to a given constant ϵ, we define a *normalized ϵ-Indicator* (denoted as I_ϵ), which is computed as $I_\epsilon(\mathcal{Q}^*_\epsilon)$ = 1 - $\frac{\epsilon_m}{\epsilon}$, where ϵ_m refers to the minimum constant such that for any instance $q \in \mathcal{I}(Q)$, there still exists an instance q' and $q' \succeq_{\epsilon_m} q'$, *i.e.,* $\mathcal{Q}^*_\epsilon$ remains to be an ϵ_m-Pareto instance set. The larger $I_\epsilon(\mathcal{Q}^*_\epsilon)$ is, the better. For the complete Pareto optimal set $\mathcal{Q}^*$, $I(\mathcal{Q}^*) = 1$.

R-indicator (I_R). For a set of tuples $\mathcal{Q}$, an R-indicator takes into consideration users' preferences, and maps $\mathcal{Q}$ to a score by aggregating the weighted attribute values [43]. We define a simple R-indicator with a preference factor $\lambda_R \in (0, 1)$, denoted as I_R, which is defined as $I_R(\mathcal{Q}^*_\epsilon) = \frac{(1-\lambda_R)\delta^* + \lambda_R \cdot f^*}{2}$,

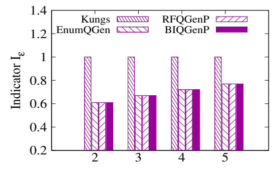
[1]https://github.com/PanCakeMan/QueryGen



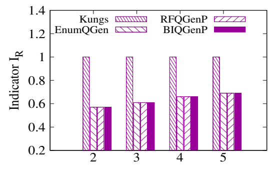
(a) Effectiveness: Real-life graphs



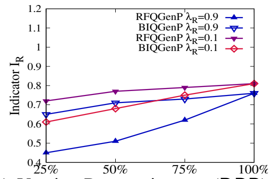
(b) Varying ϵ(LKI)



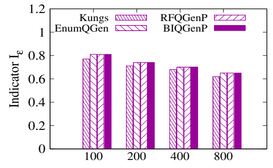
(c) Varying $|X_L|$ (DBP)



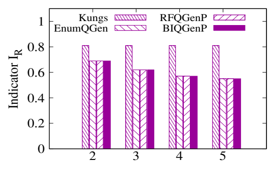
(d) Varying $|X_E|$ (LKI)



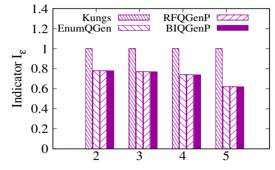
(e) Varying Pregressiveness (DBP)



(f) Varying C (DBP)



(g) Varying $|P|$ (DBP)



(h) Varying $|P|$ (DBP)

Fig. 9. Effectiveness of Subgraph Query Generation

where δ^* (resp. f^*) refers to the maximum diversity (resp. coverage) of an instance in $\mathcal{Q}^*_\epsilon$ (normalized to be in [0,1]). Here we use λ_R to "reward" the quality of $\mathcal{Q}^*_\epsilon$ in terms of coverage: a higher λ_R indicates the user's preferences that favor queries with a better coverage property; accordingly, a higher I_R under fixed λ_R suggests a query set $\mathcal{Q}^*_\epsilon$ that contains queries with more desired group coverage.

Overall Effectiveness (ϵ-indicator). We compare the effectiveness of Kungs, EnumQGen, RfQGen and BiQGen over the three real life datasets (Fig. 9(a)). We set $|Q(u_o)| = 3$ with 3 variables (1 edge variable, and 2 range variables), $|\mathcal{P}| = 2$, $\epsilon = 0.01$, and C=200. We use an "Equal opportunity" scenario and set $c = 100$ for both groups. (1) Kungs always can achieve scores as 1 over all the graphs as it computes the exact Pareto-optimal sets. (2) Over all the datasets, EnumGen, RfQGen and BiQGen achieve I_ϵ at least 0.6, which indicates that they constantly achieve an "actual" approximation of Pareto optimal set with an ϵ_m constantly smaller than $0.4 \cdot \epsilon$ for a predefined ϵ. (3) RfQGen and BiQGen approximate the complete Pareto set almost equally good as EnumGen, which enumerates all query instances. On the other hand, RfQGen and BiQGen on average inspect 40% and 60% less instances compared with EnumGen and Kungs. We also found that RfQGen and BiQGen approximate Pareto optimal sets with a representative subset of 10% of their sizes.

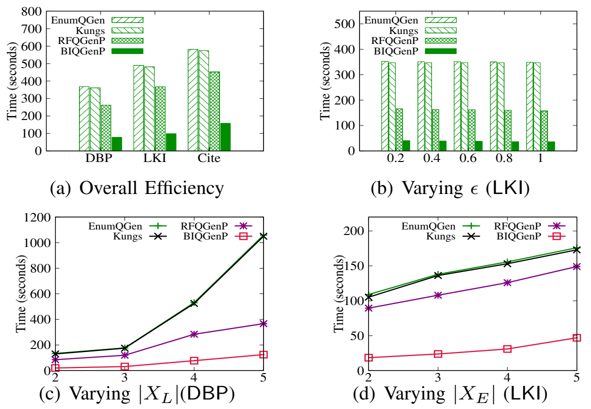
Varying ϵ (ϵ-indicator). Fixing $|Q(u_o)|$ = 4, $|X|$ = 3 (with 1 range variable and 2 edge variables), and C = 200, we varied ϵ from 0.2 to 1 and evaluate its impact over LKI. Fig. 9(b) verifies the following. (1) EnumGen, RfQGen and BiQGen approximate Pareto optimal set with larger ϵ_m (all bounded by ϵ). This is due to the trade-off between the enforced tolerance ϵ and the output size. The larger ϵ is, the fewer boxes and representative ϵ-dominance instances are verified by RfQGen and BiQGen. Thus, it is more difficult to use less amount of representative instances to approximate the Pareto set. (2) In all cases, RfQGen and BiQGen can approximate Pareto optimal set with a small ϵ_m up to 0.4 of predefined ϵ in all cases ($I_R \leq 0.6$), and achieve the same performance as EnumGen. These suggest our methods can generate a good approximation of the Pareto set over various settings of ϵ.

Varying $|X_L|$ (ϵ-indicator). We use DBP to evaluate the impact of range variables given that the nodes have more attributes on average. Fixing $|Q(u_o)|$ = 4, $|\mathcal{P}| = 2$, $C = 200$, $\epsilon = 0.01$, we varied the number of range variables from 2 to 5, and evaluated the impact to the effectiveness of Kungs and our algorithms. As shown in Fig. 9(c), EnumQGen, RfQGen and BiQGen approximate the Pareto set better for larger $|X_L|$. Interestingly, the larger $|X_L|$ is, the more ϵ-dominating query instances are verified to approach Pareto optimal set; on the other hand, the increased query complexity leads to less number of matches, which reduces the number of feasible instances and the sizes of Pareto instance sets, thus making it "easier" to approximate Pareto sets with fewer instances.

Varying $|X_E|$ (ϵ-indicator). We use LKI to evaluate the impact of edge variables, given its dense social structures. Fixing $|Q(u_o)|$ = 5, $|\mathcal{P}|$ = 2, C = 200, ϵ = 0.01, we varied the number of range variables from 2 to 5, and evaluated its impact to the effectiveness of Kungs, EnumQGen, RfQGen and BiQGen. We observe a consistent trend (Fig. 9(d)) for the algorithms over larger $|X_E|$ as for their counterparts in Fig. 9(c). Similarly, more edge variables lead to more dominating instances and better approximations of Pareto sets.

The above results verify that our methods suggest better approximation for higher template complexity (in terms of the number of range and edge variables), due to the reduced number of feasible instances, and a larger instance space that can be efficiently explored by RfQGen and BiQGen.

"Any time" quality with user preference (R-indicator). We evaluate the convergence property of RfQGen and BiQGen in response to different user preferences (controlled by λ_R) in Fig. 9(e). Fixing $|Q(u_o)|$ = 4, $|\mathcal{P}|$ = 2, C = 200, $|X|$ = 3, and ϵ = 0.01, we report I_R when different fractions of $\mathcal{I}(Q)$ are explored over DBP, with λ_R = 0.9 (favoring high coverage) and λ_R = 0.1 (favoring answer diversity). We observe the following (1) RfQGen converges faster to a set of instances with high answer diversity than BiQGen when $\lambda_R = 0.1$, as the refinement strategy probes feasible instances (2) On the other hand, BiQGen promotes the discovery of instances with desired group coverage when $\lambda_R = 0.9$, due to the bi-directional search bringing more feasible queries with higher coverage from the backward exploration. (3)



(a) Overall Efficiency (b) Varying ϵ (LKI)
(c) Varying $|X_L|$(DBP) (d) Varying $|X_E|$ (LKI)

Fig. 10. Efficiency of Subgraph Query Generation

Consistently, BiQGen and RfQGen converge to query sets with higher diversity and coverage, respectively.

Varying C (R-indicator). Fixing $|Q(u_o)|$ = 4, $|\mathcal{P}|$ = 3, $|X|$ = 3 and λ_R = 0.5 which represents an equal preference over diversity and coverage, We evaluate the impact of coverage requirement(Fig. 9(f)). We follow equal opportunity and evenly distribute C to each group, and report I_R over DBP. As more nodes are required to be covered, less instances become feasible. This reduces the chance for EnumGen, RfQGen and BiQGen to identify ϵ-dominating instances.

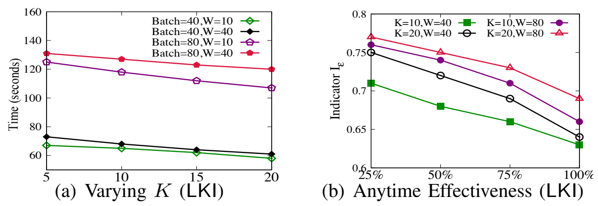
Varying $|\mathcal{P}|$. In this test, we set $|Q(u_o)|$ = 4, $|X|$ = 3, λ_R = 0.5, C = 240 and vary $|\mathcal{P}|$ from 2 to 5 and evenly distribute C to each group. We evaluate the impact of the number of groups (Fig. 9(h) and 9(g)) over DBP. I_ϵ and I_R decrease as the number of groups increases. This is because as more groups are required to be covered, less instances become feasible. As a result, EnumGen, RfQGen and BiQGen identify less ϵ-dominating instances to the approximate Pareto set.

Performance of CBM (not shown). Following the same setting in Fig. 9(a), we evaluate the performance of CBM over DBP. On average, Kungs outperforms CBM by 1.2 times in efficiency, as CBM iterates over a more expensive bi-level optimization procedure. Nevertheless, BiQGen outperforms CBM in I_R by 1.1 times on average. We thus report Kungs as a better alternative and omit the result of CBM.

These results verify the application of our methods for generating favorable queries for different user preferences.

Exp-2: Efficiency (RQ2). We next evaluate the efficiency of Kungs, EnumQGen, RfQGen andBiQGen.

Efficiency over real-life graphs. Using the same setting as in Fig. 9(a), we report the efficiency of Kungs, EnumQGen, RfQGen and BiQGen, over the real datasets in Fig. 10(a). (1) BiQGen achieves the best performance for all the datasets. On average, it outperforms EnumQGen and RfQGen by 4.4 and 2.5 times, respectively, due to the bi-directional search, and the pruning from both forward and backward exploration. (2) Query generation with diversity and coverage is feasible for large graphs. For example, it takes BiQGen (resp. RfQGen) 78s (resp. 367s) over LKI with $3M$ nodes and $26M$ edges.



(a) Varying K (LKI) (b) Anytime Effectiveness (LKI)

Fig. 11. Performance of OnlineQGen

Varying ϵ. Using the same setting as in Fig. 9(b), we report the efficiency of the algorithms in Fig. 10(b). (1) EnumGen and Kungs are not sensitive due to that their main bottleneck is the enumeration and verification of all instances. (2) BiQGen achieves the best performance among all the algorithms due to effective pruning. BiQGen (resp. RfQGen) outperforms EnumGen by 6 times (resp. 2.2 times) on average. While not very sensitive to ϵ, BiQGen and RfQGen take slightly less time over large ϵ due to that more instances are ϵ-dominated and captured by Update, thus are early pruned.

Varying $|X_L|$ and $|X_E|$. Following the setting as in Fig. 9(c), we report the efficiency of the algorithms in Fig. 10(c). BiQGen achieves the best performance among all the algorithms, and is the least sensitive compared with others. BiQGen (resp. RfQGen) outperforms EnumQGen (resp. QGenEq$_n$) by 7.5 times (resp. 5.6 times) on average over DBP.

Using the same setting as in Fig. 10(d) over LKI, Fig. 10(d) verifies that BiQGen achieves the best performance among all the algorithms. BiQGen (resp. RfQGen) outperforms EnumQGen by 3 times (resp. 2.1 times) on average due to the pruning over LKI. RfQGen and BiQGen are less sensitive to $|X_L|$. This is because an increase of the number of edge variables (and by enforcing them to '1') significantly reduces feasible instances that are effectively captured by spawn procedure during template refinement (Section IV).

Exp-3: Online Generation (RQ3). We next evaluate the performance of OnlineQGen over LKI. We simulate instance streams by randomly instantiating fixed query templates.

Delay time: Varying k and Batch sizes. Fig. 11(a) reports the delay time of OnlineQGen to process a batch of instances (with size 40 or 80) from the input stream. We varied k from 5 to 20 and set window size w as 10 and 40, respectively. While OnlineQGen takes around 1 second per instance to maintain $\mathcal{Q}_{(\epsilon,k)}$ of size k, on average it takes 63 seconds for the batch with size 40 and 121 seconds for the batch with size 80. It takes less time for larger k and smaller w. Indeed, the cached instances and incremental updates reduce the chance of k to be enlarged; on the other hand, the larger w is, the more unexpired instances in the cache need to be verified.

Anytime Effectiveness (ϵ-indicator). Keeping the setting in Fig. 11(a), we evaluate the anytime effectiveness of OnlineQGen by setting $k = 10, 20$ and $w = 40, 80$, respectively. Fig. 11(b) verifies the following. (1) I_ϵ decreases as the OnlineQGen evaluates more instances from the stream. This is consistent with our observation in Fig. 9(b) even when k is not fixed. Indeed, OnlineQGen compromises ϵ (a case in

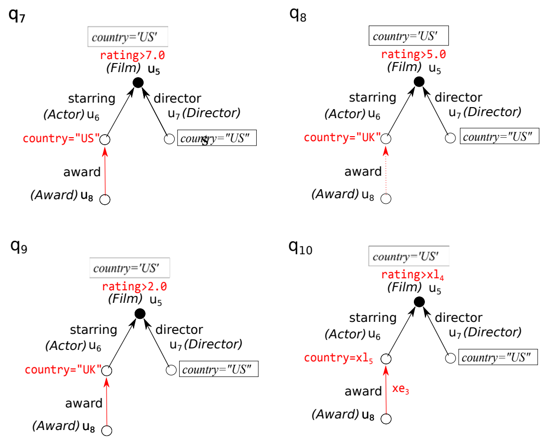


Fig. 12. Case study: Query Generation

Fig. 9(b)) in trade for smaller k as more instances arrive from the stream. On the other hand, it retains an $I_R \geq 0.63$ at any time. (2) OnlineQGen effectively exploits larger w over larger k to achieve higher I_ϵ. The incremental updating and caching strategy reduce the unnecessary growth of ϵ as well as k, keeping both smaller to maintain high-quality queries.

Exp-4: Case Study. We also conducted a case analysis to evaluate how our algorithms adapt to users' preferences. The query template q_{10} (with parameterized ratings, country and award information) and three instances are shown in Fig. 12 for movie search (DBP). Our method automatically generates feasible queries and only requires users to specify output node type "movie" and coverage constraints *e.g.,* "(100,100)". An initial query (not shown) that searches for high rating, award wining US movies (rating >7) with US actors returns 350 romance movies and 120 horror movies. Upon enforcing an equal coverage over genres, BiQGen prefers q_7 and q_8, achieving more desired coverage. For example, q_8 refines the results to 112 romance movies and 103 horror movies. On the other hand, RfQGen returns q_7 and q_9, where q_9 has more skewed but more diversified results compared with q_7.

VI. Conclusions

We have studied a bi-objective subgraph query generation problem with group coverage constraints. We have provided two feasible algorithms that approximate the Pareto-optimal set with ϵ-Pareto instance set, with effective pruning strategies, as well as an online algorithm that maintains the ϵ-Pareto instance set with a fixed size and high quality (small ϵ). As verified analytically and experimentally, our methods are feasible for large graphs, and can achieve desirable diversity and coverage properties over targeted groups. A future topic is to study parallel query generation over large graphs with diversity and group fairness. Another topic is to extend our work to multiple output nodes, attributes with large domains and other query classes such as RPQs.

Acknowledgement. This work is supported by NSF under CNS-1932574, OIA-1937143, ECCS-1933279, CNS-2028748, OAC-2104007 and DoE under DE-EE0009353.

References

[1] Full version. *https://github.com/PanCakeMan/SQGen/blob/main/full.pdf*.

[2] Z. Abbassi, V. Mirrokni, and M. Thakur. Diversity maximization under matroid constraints. In *KDD*, 2013.

[3] A. Asudeh and H. Jagadish. Fairly evaluating and scoring items in a data set. *Proceedings of the VLDB Endowment*, 13(12):3445–3448, 2020.

[4] G. Bagan, A. Bonifati, R. Ciucanu, G. H. Fletcher, A. Lemay, and N. Advokaat. Controlling diversity in benchmarking graph databases. *arXiv preprint arXiv:1511.08386*, 11, 2015.

[5] G. Bagan, A. Bonifati, R. Ciucanu, G. H. Fletcher, A. Lemay, and N. Advokaat. gmark: Schema-driven generation of graphs and queries. *IEEE Transactions on Knowledge and Data Engineering*, 29(4):856–869, 2016.

[6] A. Bonifati, G. Fletcher, J. Hidders, and A. Iosup. A survey of benchmarks for graph-processing systems. In *Graph Data Management*, pages 163–186. 2018.

[7] A. Bonifati, I. Holubová, A. Prat-Pérez, and S. Sakr. Graph generators: State of the art and open challenges. *ACM Computing Surveys (CSUR)*, 53(2):1–30, 2020.

[8] S. Bouhenni, S. Yahiaoui, N. Nouali-Taboudjemat, and H. Kheddouci. A survey on distributed graph pattern matching in massive graphs. *CSUR*, 54(2):1–35, 2021.

[9] S. Chaudhuri, H. Lee, and V. R. Narasayya. Variance aware optimization of parameterized queries. In *SIGMOD*, 2010.

[10] K. Chircop and D. Zammit-Mangion. On epsilon-constraint based methods for the generation of pareto frontiers. *J. Mech. Eng. Autom*, 2013.

[11] J. Chomicki, P. Ciaccia, and N. Meneghetti. Skyline queries, front and back. *ACM SIGMOD Record*, 42(3):6–18, 2013.

[12] P. Ciaccia and D. Martinenghi. Reconciling skyline and ranking queries. *Proceedings of the VLDB Endowment*, 10(11):1454–1465, 2017.

[13] L. Ding, S. Zeng, and L. Kang. A fast algorithm on finding the non-dominated set in multi-objective optimization. In *The 2003 Congress on Evolutionary Computation, 2003. CEC '03.*, 2003.

[14] Y. Dong, Y. Yang, J. Tang, Y. Yang, and N. V. Chawla. Inferring user demographics and social strategies in mobile social networks. In *KDD*, 2014.

[15] T. Draws, N. Tintarev, and U. Gadiraju. Assessing viewpoint diversity in search results using ranking fairness metrics. *ACM SIGKDD Explorations Newsletter*, 23(1):50–58, 2021.

[16] W. Fan, X. Wang, and Y. Wu. Diversified top-k graph pattern matching. *VLDB*, 2013.

[17] W. Fan, X. Wang, and Y. Wu. Incremental graph pattern matching. *ACM Transactions on Database Systems (TODS)*, 38(3):1–47, 2013.

[18] M. Feldman, S. A. Friedler, J. Moeller, C. Scheidegger, and S. Venkatasubramanian. Certifying and removing disparate impact. In *proceedings of the 21th ACM SIGKDD international conference on knowledge discovery and data mining*, 2015.

[19] Y. Ge, S. Zhao, H. Zhou, C. Pei, F. Sun, W. Ou, and Y. Zhang. Understanding echo chambers in e-commerce recommender systems. In *SIGIR*, pages 2261–2270, 2020.

[20] S. Gershtein, T. Milo, and B. Youngmann. Multi-objective influence maximization. *algorithms*, 2021.

[21] S. C. Geyik, S. Ambler, and K. Kenthapadi. Fairness-aware ranking in search & recommendation systems with application to linkedin talent search. In *KDD*, 2019.

[22] S. Gollapudi and A. Sharma. An axiomatic approach for result diversification. In *WWW*, 2009.

[23] C.-L. Hwang and A. S. M. Masud. *Multiple objective decision making—methods and applications: a state-of-the-art survey*. Springer Science & Business Media, 2012.

[24] N. Jayaram, A. Khan, C. Li, X. Yan, and R. Elmasri. Querying knowledge graphs by example entity tuples. *IEEE Transactions on Knowledge and Data Engineering*, 27(10):2797–2811, 2015.

[25] N. Koudas, S. Sarawagi, and D. Srivastava. Record linkage: similarity measures and algorithms. In *SIGMOD*, 2006.

[26] M. Laumanns, L. Thiele, E. Zitzler, and K. Deb. Archiving with guaranteed convergence and diversity in multi-objective optimization. In *Proceedings of the 4th Annual Conference on Genetic and Evolutionary Computation*, pages 439–447, 2002.

[27] W. Le, A. Kementsietsidis, S. Duan, and F. Li. Scalable multi-query optimization for sparql. In *ICDE*, pages 666–677, 2012.

[28] M. Lissandrini, D. Mottin, T. Palpanas, and Y. Velegrakis. Graph-query suggestions for knowledge graph exploration. In *The Web Conference*, 2020.

[29] J. Lu, J. Chen, and C. Zhang. Helsinki Multi-Model Data Repository. https://www2.helsinki.fi/en/researchgroups/unified-database-management-systems-udbms/, 2018.

[30] H. Ma, S. Guan, and Y. Wu. Diversified subgraph query generation with group fairness. In *WSDM*, 2022.

[31] T. Ma, S. Yu, J. Cao, Y. Tian, A. Al-Dhelaan, and M. Al-Rodhaan. A comparative study of subgraph matching isomorphic methods in social networks. *IEEE Access*, 6:66621–66631, 2018.

[32] S. Mitchell, E. Potash, S. Barocas, A. D'Amour, and K. Lum. Prediction-based decisions and fairness: A catalogue of choices, assumptions, and definitions. *arXiv preprint arXiv:1811.07867*, 2018.

[33] D. Mottin, F. Bonchi, and F. Gullo. Graph query reformulation with diversity. In *KDD*, 2015.

[34] Z. Moumoulidou, A. McGregor, and A. Meliou. Diverse data selection under fairness constraints. In *ICDT*, 2021.

[35] M. H. Namaki, Q. Song, Y. Wu, and S. Yang. Answering why-questions by exemplars in attributed graphs. In *SIGMOD*, 2019.

[36] C. H. Papadimitriou and M. Yannakakis. On the approximability of trade-offs and optimal access of web sources. In *FOCS*, pages 86–92, 2000.

[37] M. Poess and J. M. Stephens Jr. Generating thousand benchmark queries in seconds. In *VLDB*, pages 1045–1053, 2004.

[38] A. Sinha, Z. Shen, Y. Song, H. Ma, D. Eide, B.-J. P. Hsu, and K. Wang. An overview of microsoft academic service (mas) and applications. WWW, 2015.

[39] Q. Song, M. H. Namaki, and Y. Wu. Answering why-questions for subgraph queries in multi-attributed graphs. In *ICDE*, 2019.

[40] J. Stoyanovich, K. Yang, and H. Jagadish. Online set selection with fairness and diversity constraints. In *Proceedings of the EDBT Conference*, 2018.

[41] Y. Wang, Y. Li, J. Fan, C. Ye, and M. Chai. A survey of typical attributed graph queries. *World Wide Web*, 24(1):297–346, 2021.

[42] Y. Zhang, J. Tang, Z. Yang, J. Pei, and P. S. Yu. Cosnet: Connecting heterogeneous social networks with local and global consistency. In *KDD*, 2015.

[43] E. Zitzler, J. Knowles, and L. Thiele. *Quality Assessment of Pareto Set Approximations*. 2008.