# Remaining Useful Life Estimation for Ball Bearings Using Feature Engineering and Extreme Learning Machine

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**Abstract**: Rotating machines, such as pumps and compressors, are critical components in refinery and chemical plants used to transport fluids between processing units. Bearings are often the critical parts of rotating machinery, and their failure could result in economic loss and/or safety issues. Therefore, estimation of the remaining useful life (RUL) of a bearing plays an important role in reducing production losses and avoiding machine damage. Because bearing failure mechanisms tend to be complex and stochastic, data-driven RUL estimation approaches have found more applications. This work proposes a novel RUL estimation method based on systematic feature engineering and extreme learning machine (ELM). The PRONOSTIA dataset is used to demonstrate the effectiveness of the proposed method. *Keywords*: remaining useful life, ball bearing, machine learning, feature engineering, extreme learning machine.

## 1. INTRODUCTION

Rotating machines, such as pumps and compressors, are critical components in refinery and chemical plants used to transport fluids between processing units. Rotating machines generally cost more to maintain than static equipment in the plants because they generally operate at high speed and under harsh conditions (e.g., high pressure and corrosive fluids), and they consist of many moving parts, leading to frequent failures. Failure of bearings, one of the critical parts of rotating machinery, could result in economic loss and/or safety issues. Thus, estimation of the remaining useful life (RUL) of a bearing plays an important role in reducing production losses and avoiding machine damage. This estimation can be done through the prognostics and health management (PHM) of bearings. To improve the reliability of process operation, accurate estimation of RUL of a bearing is crucial. This enables process engineers to inspect performance and conduct maintenance in a timely manner.

The most commonly used measurements in RUL estimation include vibration, oil debris and acoustic emission. In this work, we focus on RUL estimation using vibration measurements. RUL estimation techniques can be roughly classified into two categories: model-based and data-driven (Heng et al., 2009; Lee et al., 2014). The former describes physics of a system and failure mechanisms through mathematical models. If the physics of the system is well understood and the model explains failure mechanisms well, this approach can provide accurate RUL estimation. In reality, however, failure mechanisms tend to be complex and stochastic, making the underlying physical principles difficult

to understand or model. In comparison, data-driven approaches apply statistical/machine learning or deep learning to historical data and estimate RUL from the data. Therefore, data-driven approaches have found more applications in estimating RUL of bearings in complex systems. Since data-driven methods require historical data, their usage can be limited to the systems that have sufficient historical data for model training. Also, it is not often to obtain the whole failure data because the degraded bearings usually get replaced before reaching complete failure. Finally, bearing failure process can be highly stochastic, *e.g.*, due to material defects and/or operation condition changes.

Some representative data-driven RUL estimation techniques proposed in the literature include nonlinear regression (Gebraeel et al., 2005), hidden Markov model (HMM) (Dong & He, 2007), particle filter (PF) (Qian & Yan, 2015), support vector machines (SVM) (Benkedjouh et al., 2013; Yan et al., 2020), and artificial neural networks (ANN) and deep learning (DL) based methods (Ali et al., 2015; Chen et al., 2020; Hinchi & Tkiouat, 2018; Pan et al., 2020; Zhu et al., 2018), and many others. Interested readers are referred to the review papers (H.-Z. Huang et al., 2015; Liu et al., 2018; Si et al., 2011) for more thorough coverage of different methods in the field. Among all the above-mentioned techniques, ANN/DL based approaches have drawn the most research interests, which is also the focus of this work. We have found that the performances of these approaches are not always satisfactory. Therefore, this study proposes a new framework for RUL estimation of bearings using systematic feature engineering (FE) and extreme learning machine (ELM), with the goal of achieving better performance than the existing approaches.

ELM is a feedforward neural network with a single hidden layer. It has been shown that the learning speed of ELM is extremely fast, and it has better generalization performance than the gradient-based learning such as backpropagation in most cases (G.-B. Huang et al., 2006). Although ELM has only one hidden layer, in theory, ELM can approximate any continuous functions. In this work, we use ELM to model the complex nonlinear relationship between features extracted from a bearing's vibration signals and the RUL of the bearing. Due to limited space, the detailed mathematical description of the ELM algorithm is omitted. Interested readers are referred to (G.-B. Huang et al., 2006, 2011).

The remainder of this work is organized as follows. Section 2 describes the novel framework for RUL estimation. Section 3 presents the RUL estimation results using a publicly available real bearing failure dataset to demonstrate the effectiveness of the proposed method. Section 4 draws conclusions of this work.

## 2. PROPOSED RUL ESTIMATION METHOD

#### 2.1 Overview

The proposed method consists of the following four steps: health indicator (HI) construction, health status determination, feature engineering, and RUL estimation. The first step is to construct a reliable HI that is not only closely linked with bearing health, but also comparable between different bearings. The second step is to determine bearing health status based on the constructed HI, i.e., to determine whether the bearing is operating normally or there is degradation happening. Once a degradation is detected, the third step is to employ systematic FE to generate features that are not only physically meaningful but also predictive in machine learning. Finally, the engineered features are used to train ELM models, which are used to predict RUL of test bearings. In this study, the PHM 2012 Challenge dataset obtained on the PRONOSTIA platform is used to demonstrate effectiveness of the proposed method. The goal of the PRONOSTIA platform is to provide useful experimental data that describe the degradation behavior of ball bearings through their whole operation life (Nectoux et al., 2012). The same dataset is used to illustrate the steps involved in the proposed method.

# 2.2 Health indicator construction

During the health indicator (HI) construction step, we first extract the root mean square (RMS) values from the mechanical vibration signal, which is 0.707 times the peak value by assuming a sine wave. Previous studies have found that RMS values are robust indicators that reflect the degradation behavior of ball bearings. According to the international standard ISO 2372, when the RMS value of the medium mechanical vibration signal reaches 2.0 to 2.2 g, the equipment is in a dangerous state (Shiroishi et al., 1997). However, it is challenging to determine the RMS failure threshold because the bearings, even under the same operation condition, can have drastically different degradation trajectories, which significantly affect the RMS values. This is illustrated in Figure 1 where different bearings have different

baseline or nominal RMS values during the normal operation periods.

Therefore, the standardized RMS (SRMS) (Pan et al., 2020) is utilized to reduce the variation of RMS values between different bearings based on the following equation

$$x_{srms}(i) = \frac{x_{rms}(i)}{\frac{1}{m} \sum_{j=m_1}^{m_1+m} x_{rms}(j)}$$
 (1)

where  $\frac{1}{m}\sum_{j=m_1}^{m_1+m}x_{rms}(j)$  calculate a nominal value of the RMS when the bearing is working normally (e.g., after bearing replacement or maintenance). Figure 2 shows SRMS of the same bearings as in Figure 1. It can be seen that SRMS values fluctuate around 1 during normal operations for all bearings. The peak values of SRMS are also more balanced than RMS as shown in Figure 1. As a result, SRMS is used as HI in this work.

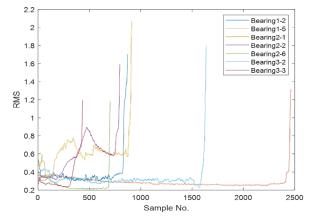


Figure 1. Different bearings have different baseline or nominal RMS values during the normal operation periods

As can be seen in Figure 2, SRMS as HI can be noisy, which would negatively affect the RUL estimation. To reduce the fluctuation of HI (i.e., SRMS), a moving average (MA) filter is used to smooth HI values. In this work, window width of 30 is used. Figure 3 compares the unsmoothed and smoothed HI for Bearing 1-1.

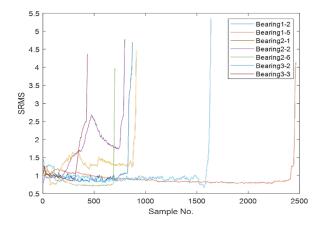


Figure 2. After standardization, SRMS have similar baselines and peak values for different bearings

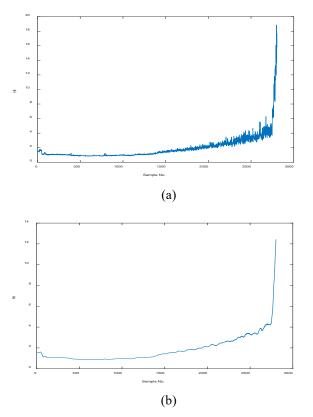


Figure 3. (a) original SRMS; (b) MA smoothed SRMS

Since the full trajectory would not be available when estimating RUL for a failing (but not yet completely failed) bearing, its HI trajectory will be estimated by either polynomial or exponential fitting using the available data, whichever gives the smaller fitting error.

## 2.3 Health status determination

Once HI is smoothed by a MA filter, it is ready for health status determination – whether the bearing is operating normally, or it has started degradation. During the health status determination step, we aim to find the degradation onset point to start predicting bearing's RUL. The degradation onset of the bearing (a.k.a., time to start prediction or  $t_{SP}$ ) can be determined by investigating the gradient value of HI in a given window. In this work  $t_{SP}$  is detected when the HI gradient exceeds a predetermined degradation threshold of 0.005. The HI gradient is defined as

$$k_i = \frac{y_i(n) - y_i(1)}{n - 1} \tag{2}$$

where  $y_i$  is a set of HI values in the *i*th window,  $k_i$  is the gradient value in the *i*th window, n is the window size, which is 30 in this study. One example of  $t_{SP}$  determination is shown in Figure 4.

## 2.3 Feature engineering

Usually, HI becomes smooth after MA filtering as shown in Figure 3. However, there are cases where HI trajectory still fluctuate significantly as shown in Figure 5. In this work, monotonicity is used to quantify monotonic trend in HI trajectory of training bearings, which is defined as follows.

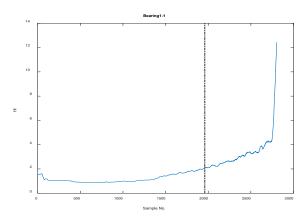


Figure 4.  $t_{SP}$  is determined based on HI gradient

$$\mu = \left| \frac{n_{d_{HI} > 0} - n_{d_{HI} < 0}}{N - 1} \right| \tag{3}$$

where  $d_{HI}$  denotes the series of differenced HI values with length N-1, given a series of HI values of length N.  $n_{d_{HI}>0}$  and  $n_{d_{HI}<0}$  denote the counts of positive and negative values in the  $d_{HI}$  series, respectively. It can be seen that  $\mu \in [0,1]$ , and the closer the  $\mu$  value is to 1, the better is the monotonicity of the HI trajectory. For training bearings,  $\mu$  is calculated using all HI values after  $t_{SP}$ . For a testing bearing,  $\mu$  is calculated every time when a new HI is made available, *i.e.*, during each prediction step. If  $\mu$  is below a certain threshold, which is chosen as 0.7 in this work, the following exponential smoothing is implemented to improve the monotonicity of the HI trajectory.

$$y = a \cdot exp(bx) + c \cdot exp(dx) \tag{4}$$

where x and y denote the HI values before and after smoothing. a, b, c, and d are the fitting parameters. The smoothing process is recursive -i.e., the smoothed HI value at t is fitted using all HI values between  $t_{SP}$  and t. However, the

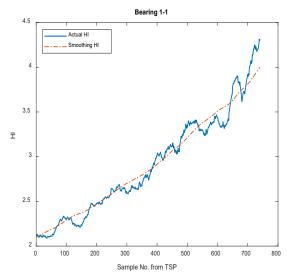


Figure 5 The fluctuations in HI are reduced after exponential smoothing, with  $\mu$  increasing from 0.2 (before smoothing) to 1 (after smoothing)

smoothed HI values between  $t_{SP}$  and t-1 are not updated. Figure 5 shows the HI trajectory of bearing 1-1 before and after smoothing, where  $\mu$  is improved from 0.2 to 1.

Once  $t_{SP}$  is determined, HI values are normalized based on its value at  $t_{SP}$ . In this work, the bearing complete failure is defined when HI (i.e., smoothed SRMS) reaches 4.0.

$$\overline{\mathrm{HI}}(t) = \frac{\mathrm{HI}(t) - \mathrm{HI}(t_{SP})}{4.0 - \mathrm{HI}(t_{SP})} \tag{5}$$

In this way, starting from  $t_{SP}$  to complete failure,  $\overline{\text{HI}}(t)$  varies from 0 to 1. One example of a complete  $\overline{\text{HI}}(t)$  trajectory of a training bearing is shown in Figure 6. Note that for test bearing, HI can be scaled in the same way once  $t_{SP}$  is determined (i.e., fault is detected).

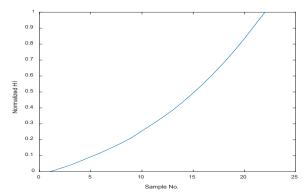


Figure 6.  $\overline{HI}(t)$  varies from 0 (at  $t_{SP}$ ) to 1 (at time of complete failure) for a training bearing

In addition, RUL varies from bearing to bearing. To develop a universal model across different bearings, this dependence needs to be minimized. To do so, we propose relative RUL at time *t*:

$$R_{RUL}(t) = \frac{t_{life} - t}{t_{life} - t_{SP}} \times 100\% = \frac{RUL(t)}{RUL(t_{SP})} \times 100\%$$
 (6)

where  $t_{life}$  is the bearing's entire lifetime, RUL(t) is the RUL at time t,  $RUL(t_{SP})$  is the RUL at  $t_{SP}$ .  $R_{RUL}$  varies from 100% (when  $t=t_{SP}$ ) to 0% (when  $t=t_{life}$ ). Note that since the model will be trained using  $R_{RUL}$ , the prediction of a test bearing will also be  $R_{RUL}$ , which can be back calculated for RUL based on Eqn. (6).

# 2.4 RUL estimation

In this work, extreme learning machine (ELM) is used to correlate  $R_{RUL}(t)$  with  $\overline{\mathrm{HI}}(t)$ . In other words, the training bearings'  $\overline{\mathrm{HI}}(t)$  will be used as the inputs to ELM, while their known  $R_{RUL}(t)$  will be used as the outputs for training ELM models. Once ELM models are trained, they can be used to predict  $R_{RUL}(t)$  for given  $\overline{\mathrm{HI}}(t)$  from testing bearings.

To improve the robustness of the proposed method, this work employs ensemble modeling. Specifically, to estimate  $R_{RUL}(t)$  for each testing bearing, the other ten bearings are used as the training bearings. Each training model will generate an ELM model and be used to predict the  $R_{RUL}(t)$  of the test bearing. The final  $R_{RUL}(t)$  estimation of the test bearing is calculated by taking average of the ten  $R_{RUL}(t)$  estimations:

$$R_{RUL}(t) = \frac{1}{10} \sum_{j=1}^{10} R_{RUL}(t,j)$$
 (7)

where  $R_{RUL}(t, j)$  is the test bearing's  $R_{RUL}$  at time t estimated based on the ELM model trained by bearing j.

## 3. RESULTS AND DISCUSSIONS

#### 3.1 Dataset

In this study, we use the PHM 2012 Challenge dataset for evaluating the performance of the proposed method and comparing to other methods. The dataset is publicly available NASA's Repository: from **Prognostics** Data https://ti.arc.nasa.gov/c/18/, which consists of experimental data on eleven ball bearings' accelerated life tests provided by FEMTO-ST Institute, Besançon, France. The vibration sensors are two accelerometers placed on the vertical and horizontal axes. The dataset consists of three operating conditions and the sampling frequency is 25.6kHz for 0.1 second and sample interval is 10 seconds. More information on the experimental setup and the dataset can be found in (Nectoux et al., 2012).

#### 3.2 Evaluation metrics

The following three performance metrics are used to evaluate and compare the performance of different RUL estimation methods: mean absolute error (MAE), root mean square error (RSME) and mean absolute percentage error (MAPE). Their mathematical definitions are provided below:

$$MAE = \frac{\sum_{i=1}^{n} \left| R_{RUL_i} - \hat{R}_{RUL_i} \right|}{n} \tag{8}$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n} \left(R_{RUL_i} - \hat{R}_{RUL_i}\right)^2}{n}}$$
 (9)

$$MAPE = \frac{\sum_{i=1}^{n} \left| \frac{R_{RUL_i} - R_{RUL_i}}{R_{RUL_i}} \right|}{n} \times 100\%$$
 (10)

where  $R_{RUL_i}$  and  $\hat{R}_{RUL_i}$  are the  $i^{th}$  actual and predicted  $R_{RUL}$ , respectively, and n is the total number of  $R_{RUL}$  measurements available from  $t_{SP}$  (when RUL prediction starts) to total failure (when  $\overline{\text{HI}}$  reaches 1), which is the same as the total number of samples available during that period.

#### 3.3 Performance comparison

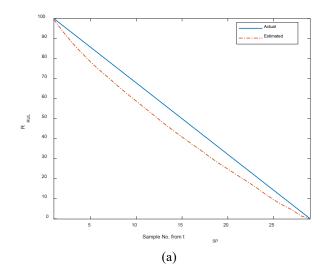
In this work, we compare the  $R_{RUL}$ 's estimated based on the proposed method to those based on a deep neural network (DNN) and a DNN with multi-scale feature extraction (DNN-MSFE).

For DNN, a conventional neural network (CNN) is implemented, where two fully connected (FC) layers with 128 neurons are used for feature extraction of each sequence, and two more FC layers with 64 and 1 neurons are further adopted for final  $R_{RUL}$  regression (Li et al., 2019).

For DNN-MSFE, the time series vibration signals are transformed into time-frequency domain using STFT. Three convolutional layers are adopted for feature extraction. The generated feature maps in different layers are then

concatenated to obtain the features of multiple scales. One more convolutional layer with one filter is further implemented for information compression. After the flatten layer, a fully-connected layer with 128 neurons is used, and one neuron is finally attached at the top of the network for the RUL estimation (Li et al., 2019).

The comparison is summarized in Table 1. It can be seen that the proposed method outperforms DNN and DNN-MSFE for the estimation of  $R_{RUL}$  for all bearings using all performance metrics except MAE for bearing 1-5 where the proposed method has slightly higher MAE than DNN-MSFE. For seven out of eleven bearings, the proposed method reduces MAE, RMSE and MAPE in RUL estimation by over 50%. Figure 7 shows the estimated  $R_{RUL}$  vs. actual  $R_{RUL}$  for bearings 1-4 and 2-4. It can be seen that  $R_{RUL}$  can be underestimated (Figure 7 (a)) or underestimated (Figure 7 (b)).



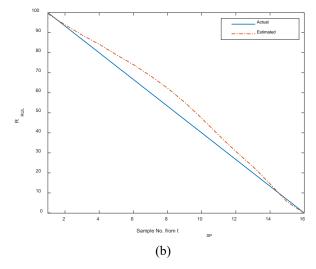


Figure 6 Comparison of estimated and actual RUL's for (a) bearing 1-4 and (b) bearing 2-4.

It is worth noting that  $t_{SP}$  estimation for DNN and DNN-MSFE is based on the kurtosis calculated from the vibration data (Li et al., 2019), while  $t_{SP}$  estimation for the proposed method is based on the MA (and exponential if monotonicity

 $\mu$  < 0.7) smoothed SRMS as described in Section 2.3. Therefore, differences in  $t_{SP}$  are expected. However, the differences do not introduce bias in this comparison as they do not favor any one particular approach.

Table 1 MAE, RMSE and MAPE of  $R_{RUL}$  estimation based on the proposed method and two other existing methods

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	DNN			DNN-MSFE			Proposed method		
	MAE	RMSE	MAPE	MAE	RMSE	MAPE	MAE	RMSE	MAPE
Bearing 1-1	30.4	44.5	174.2	21.4	26.3	62.3	2.38	2.74	9.09
Bearing 1-2	28.5	31.2	113.4	14.7	17.5	52.7	11.73	13.32	29.11
Bearing 1-3	16.3	21.5	59.2	7.8	9	15.8	6.62	8.17	12.79
Bearing 1-4	32.1	34.2	95.8	21.8	24.4	73.4	4.86	5.28	14.82
Bearing 1-5	28.7	33.5	104.3	18.5	22.2	61	18.62	21.4	56.55
Bearing 1-7	16.6	19.6	179.6	8.3	10.3	52.4	6.4	6.97	18.35
Bearing 2-4	22.3	25.2	215.3	6.1	8.9	33.3	5.74	6.87	15.26
Bearing 2-6	29.4	38.5	198.2	20.7	25.7	47.8	4.06	4.74	13.62
Bearing 2-7	38	43.2	212.4	27.4	30.6	114.5	8.94	11.35	23.63
Bearing 3-2	30.4	38.7	110.5	23.1	27.1	52.8	13.15	15.11	41.44
Bearing 3-3	38.5	43.5	156.3	30.4	35.7	69.9	5.82	6.64	21.27

#### 4. CONCLUSIONS

In this work we propose a novel RUL estimation method based on systematic feature engineering and extreme learning machine (ELM). The proposed method consists of four steps. The first step is to construct a reliable HI that is not only closely linked with bearing health, but also directly comparable between different bearings. A standardization approach and a MA filter are employed to reduce between-, and within-bearing variations, respectively. The second step is to determine bearing health status based on the constructed HI. In this step, the trend of HI is used to determine whether the bearing is operating normally or it has started degradation. If a degradation is detected, the degradation onset of the bearing (a.k.a., time to start prediction or  $t_{SP}$ ) is determined as the time of detection. Before RUL estimation, the third step is to employ systematic FE to generate features that are not only physically meaningful but also predictive for machine learning models to accurately estimate RUL. Finally, the engineered features of training bearings are used to train ELM models, which are then used to predict RUL of test bearings.

The PHM 2012 Challenge dataset is used for evaluating the performance of the proposed method and comparing it to other two DNN based methods. The comparison results show that the proposed method outperforms DNN and DNN-MSFE for the estimation of  $R_{RUL}$  for all bearings using all performance metrics, except MAE for bearing 1-5 where the proposed method has slightly higher MAE than DNN-MSFE. For seven out of eleven bearings, the proposed method reduces MAE, RMSE and MAPE in RUL estimation by over 50%.

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