Mechanical Computing

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Abstract

Mechanical mechanisms have been employed for information processing for millennia, with famous examples ranging from the Antikythera mechanism of the Ancient Greeks to the analytical machines of Babbage. More recently, electronic forms of computation and information processing have overtaken these mechanical forms, due to superior miniaturization and integration. Yet recently, a number of unconventional computing approaches have been introduced that blend ideas of information processing, materials science, and robotics. This has raised the possibility of novel mechanical systems that augment traditional electronic computing by interacting with and adapting to their environment in unprecedented ways. In this Perspective, we discuss the use of mechanical mechanisms, and associated nonlinearities, as a means of processing information with a view toward a new paradigm in which adaptable materials and structures can act as a distributed information processing network, even enabling "information processing" to be viewed as a material property alongside traditional material properties such as strength and stiffness. We focus on approaches to abstract digital logic in mechanical systems, discuss how these systems differ from traditional electronic computing, and highlight the challenges and opportunities that they present.

I. INTRODUCTION: "MECHANICS AS INFORMATION"

History provides a number of fascinating examples of computation via clever mechanical mechanisms, including the Antikythera mechanism of the Ancient Greeks [1], the analytical machines of Charles Babbage [2], and the differential analyzer of Vannevar Bush [3]. For the most part, these older mechanical forms of computation have long since been replaced by more efficient electronic forms. Recently there has been an explosion of unconventional computing approaches, blending ideas of information processing, chemistry, biology, materials science, and robotics into novel information processing platforms. Examples include neuromorphic computing [4], DNA computing [5], robotic materials [6], morphological computation [7–9], optical computing [10, 11], microwave-based quantum gates [12, 13], and pneumatic/microfluidic logic circuits [14–18]. There has also been a growing recognition that some natural systems (such as the Venus flytrap [19–21]) can also be viewed as unconventional computation platforms. These systems depart profoundly from the von Neumann architecture of classical computing and digital electronic hardware (see "Conventional computer" mapped from the Turing machine, a model for universal computation, down to the physical silicon substrate in Fig. 1. Further explanation is provided in the Sidebar). Also, these unconventional computing systems are capable of interacting with and adapting to their environment in unprecedented ways (see Fig. 1b).

As a case study, we focus on emerging research on the use of mechanical mechanisms as a means of processing information, a concept that has become plausible thanks to major advances in additive manufacturing, materials science, and structural engineering. Unlike the gears and linkages of ancient mechanical computers, these novel mechanical computing systems harness a variety of subtle mechanisms to sense, interact and process information from their environment. In this way, "information processing" itself can be viewed as a material property alongside traditional material properties such as strength and stiffness. However, with the information processing intrinsically part of the composition and geometry, new design rules and computing paradigms beyond traditional von Neumann architectures will be required (Fig. 1a).

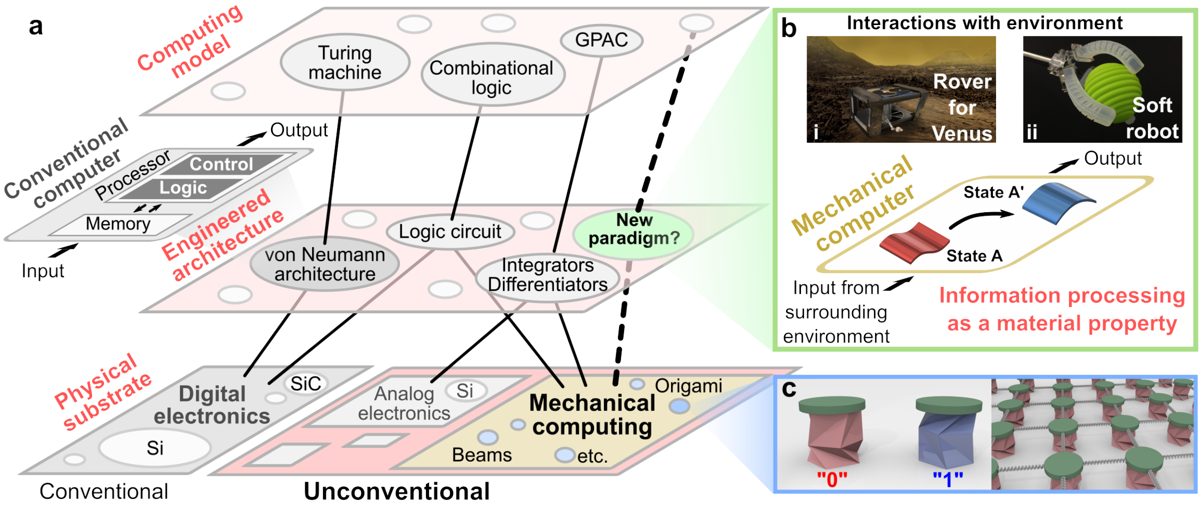

FIG. 1. **Three level hierarchy of a computational system:** (**a**) Building a computer through the three levels: (*Top layer*) "Computing model" (e.g., the Turing machine, combinatorial logic, and general purpose analog computer (GPAC.), (*Middle layer*) "Engineered architecture" which represents an abstract platform where a computing model is implemented (von Neumann architecture (see the left inset illustration) [22], logic circuit, etc.), and (*Bottom layer*) "Physical substrate" which realizes a design in a physical system. (**b**) A mechanical computing system, highlighting information processing as a material property, which can interact with environments and perform "computation", e.g., (i) a rover inspired by mechanical computers for extreme environments [23] (Image Credit: NASA/JPL-Caltech), and (ii) soft robotic grippers with embedded sensors which can sense pressure, temperature, etc. Reprinted with permission from Reference [24]. © 2018 Wiley. (**c**) A mechanical computing system can be realized by leveraging various mechanical building blocks (e.g., origami-inspired unit which can represent binary information ("0" or "1") depending on different deformation modes, and its 2D network); reprinted with permission from Reference [25].

In this Perspective, we employ a three-layer framework for computation to outline the process of information abstraction in computing systems to highlight innovations for mechanical computing in each layer. Using combinatorial logic as an instructive *computing model* (Fig. 1a), we first consider the abstraction of mechanical binary digits (bits) in the *physical substrate* layer (see Fig. 1c for origami-based example), highlighting both static and dynamic representations in Sec. II. Next, we consider how the above mechanisms may be combined or networked to achieve more complex computation (Sec. III), and to potentially implement specific *engineered architectures*. Then we consider how these systems interact (I/O) with the surrounding environment and/or other subsystems (Sec. IV), and the unique advantages this presents over conventional computing approaches. We conclude by summarizing the challenges and opportunities on the horizon and opportunities for broader community engagement going forward (Sec. V).

II. MECHANICAL BIT ABSTRACTIONS

To leverage materials for information processing, the physical material must be structured to instantiate an abstract computational process. Developing these material-to-computation abstractions are core issues to defining the meaning and opportunity space of physical computation [26, 27]. As the complexity of the targeted abstract computation increases, so does the complexity of the design required to instantiate it. In light of this, binary operations are the dominant computational abstractions utilized in modern computing systems due to their relative simplicity, robustness, and scalability. In electronic systems, transistors function as a binary digit (bit) (Fig. 1a), systematically switching between the "on" and "off" state to represent, process, and store information. It is noteworthy that novel unconventional computing systems operate on alternative architectures that do not necessarily require digital representation [28]. In fact, a variety of exciting new research areas such as morphological computing [7–9], wave-based mechanical metamaterials [29–31], and neuromorphic systems [4] explicitly make use of analog computing principles.

Following the goal of illustrating pervasive challenges, we limit the scope to mechanical computing approaches that embody digital abstractions of information. One of the empowering aspects of mechanical computing is the diverse opportunities to define digital abstractions of information from the physical system. In this section, we discuss two different strategies for representing digital states in mechanical systems: *non-volatile systems*, which undergo quasi-static deformation between equilibrium states, thereby storing discrete state information without external energy; and *volatile systems*, which are abstractions from dynamic systems and require external energy to maintain the information state.

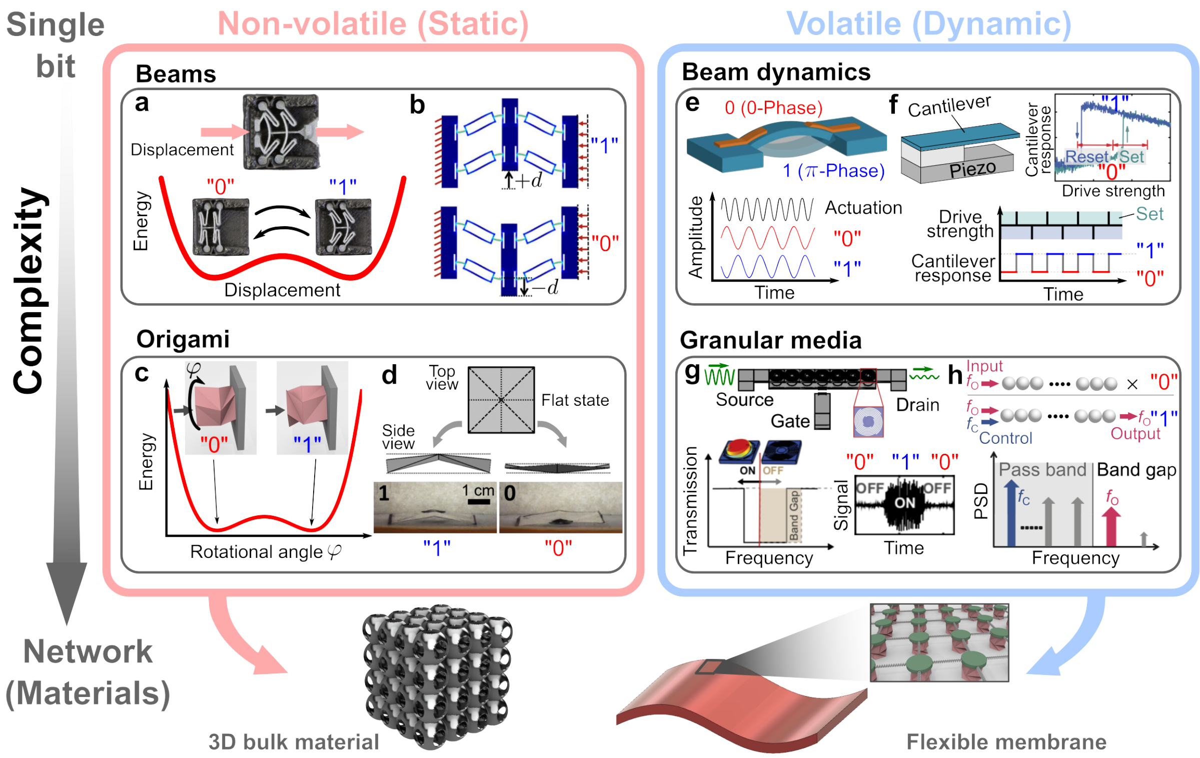

FIG. 2. **Non-volatile and volatile mechanical realizations/implementations of abstract bits**. One of the approaches to retain information without external power source is to utilize bistable behavior based on geometrical nonlinearities, such as (**a**) a unit cell composed of clamped beams, which can transform between undeformed ("0") and deformed ("1") configurations (reprinted with permission from Reference [32]), and (**b**) a bistable flexure mechanism. Reprinted with permission from Reference [33]. Origami can also be used to design non-volatile mechanical memory, e.g. (**c**) triangulated cylindrical origami-based structure (reprinted with permission from Reference [25]) and (**d**) waterbomb origami. Reprinted with permission from Reference [34]. Volatile logic can be encoded in beam dynamics, as demonstrated in (**e**) electromechanical beams (reprinted with permission from Reference [35], © 2008 Springer Nature) and (**f**) microcantilevers with stiffening behavior (reprinted with permission from Reference [36], © 2010 AIP Publishing). Other examples of volatile mechanical devices include (**g**) a 1D array of spiral spring cells with a magnetic mass (reprinted with permission from Reference [37]) and (**h**) granular chains (reprinted with permission from Reference [38], © 2014 Springer Nature).

A. Non-volatile systems

Mechanical realizations of non-volatile, digital computing have predominantly assumed a binary form through harnessing bistable configurations. Such bistability can be readily obtained by introducing geometrical nonlinearity into a mechanical structure. Under certain loading and constraints, even simple beams can be designed to support two stable configurations. As an example of loading constraints that support this behavior, if planar tilted beams are confined perpendicular to their loading direction (Fig. 2a-b), they may snap between two stable configurations, which can be assigned a '0' or '1' state, respectively. By leveraging mechanical snap-through between these two states, one can manipulate the binary information. When the deformation is limited to the elastic regime, this transition to bistability is governed by scale-independent geometric parameters and boundary conditions rather than the material properties. Hence, beam-based bistabilities have been exploited in a number of materials (silica, soft materials, etc.) and form factors to realize mechanical bits [32, 33, 47, 48]. Similarly, bistability can be realized in origami-based structures [25, 49–64], enabling the structure to possess two distinct ‘0’ and ‘1’ states as above. For example, a mechanical bit has been defined in triangulated cylindrical origami (TCO) structures by transitioning between two stable states through cross-sectional rotation. (see Fig. 2c, [25]). Another origami example is the waterbomb fold pattern (see Fig. 2d, [34]), which leverages bistability to “pop” between up (1) and down (0) equilibrium states of the center vertex of the fold pattern. The multistable energy landscape of the origami structures, and their ability to form modular assemblies, serves as a helpful intuition-building construct for identifying and developing mechanical computing devices.

Binary representations are of central importance in electronic computation, and have facilitated immense information densities through the miniaturization and computational scaling of a single bit. While some mechanical bit implementations may be compatible with a miniaturization approach, increasing the number of stable configurations [65–67] (i.e., changing the base of the computation) is likely a more tractable path to increasing information density. For example, mechanical mechanisms that are tristable (e.g., rotating squares [67]) or quadstable (e.g., origami [53]) could be utilized as non-volatile computing devices with superior information density to binary equivalents (see Supplementary Information for additional discussion).

B. Volatile systems

In the non-volatile examples of the previous section, digital abstraction is tied to quasi-static transitions between equilibrium configurations of a multistable structure. However, digital abstraction of information and manipulation of the bit state can also be achieved via the dynamic response of a mechanical system, e.g., phase, frequency, amplitude and other metrics. One of the well-studied examples is the clamped beam under harmonic excitation [35, 68–72] which behaves as a mechanical resonator. Figure 2e shows structural oscillations of a clamped-clamped beam integrated with a piezoelectric actuator. The bit information is expressed by the two stable phases, 0 and π [35]. Another example based on beam vibration is a microcantilever with stiffening behavior that arises due to geometric nonlinearities at large amplitudes [36]. This nonlinear behavior results in distinct dynamic responses depending on whether there is a forward or backward sweep in the input drive voltage (i.e., a hysteretic response as shown in the upper right inset in Fig. 2f). Therefore, if the system is operated at a certain drive strength in this hysteretic response regime and the input drive voltage is modulated, the dynamic response can be one of the two distinct stable states, i.e., high-amplitude or low-amplitude, depending on whether a forward or backward sweep in the input voltage is used.

The burgeoning field of mechanical metamaterials presents a large toolset of methods and building blocks to control the flow of mechanical energy, guide mechanical waves, and tune the frequency band structure [73–80]. Precise control of these dynamic phenomena, both through advances in conceptual design and experimental validation, constitute a rich testbed for novel mechanical computing abstractions. For example, Bilal et al. studied a pop-up structure which exhibits tunable transmission depending on its structural configuration [37] (i.e., a pop-up state which allows the propagation of input signals, or a flat state where elastic waves are prohibited, see Fig. 2g). By constructing an array of the unit cells, they designed a mechanical transistor and demonstrated various logic gate operations based on transmission dynamics. Similarly, granular acoustic switches have been proposed [38], which digitize the state information by harnessing the system's nonlinearity to tune the frequency response (Fig. 2h). The use of multi-frequency information, together with phase and amplitude control discussed above, could be exploited to abstract and manipulate multiple mechanical bits in parallel. In addition to the use of elastic waves, acoustic logic operations based on non-reciprocal propagation of sound pressure have also been proposed [81, 82]. The above examples highlight the diverse digital abstractions possible in dynamic mechanical systems, and offer an alternative view of mechanical information processing.

Bit retention in volatile systems requires sustained energy input, typically through a continuous harmonic excitation or other driving force. The volatility provides flexible bit manipulation, such as driving multi-bit logic operations as discussed above, and flexible bit abstraction, as the bit state can be (re-)assigned for different driving frequencies, amplitudes, etc. In contrast, the bistable mechanisms of non-volatile systems retain bit information without additional energy input, but require additional mechanisms to reconfigure the system (e.g., control of loading or constraint conditions in a beam-based system). New metrics are needed to map the trade-off between computational versatility and mechanical energy consumption in mechanical computing devices. Hybrid systems present an opportunity to harness the strengths of both, by combining the programming flexibility and operational sensitivity of volatile systems with the stable memory storage of non-volatile systems. While simple hybrid approaches could leverage non-volatile subsystems as memory and volatile subsystems as processors (analogous to the classic von Neumann architecture of Fig. 1a), it remains an open question how these subsystems could be combined in more creative ways to attain novel functionality. The discovery of new mechanical logic networking principles and architectures that implement hybrid bit information is an open challenge.

III. MECHANICAL COMPUTING ARCHITECTURES

In order to perform more complex computing operations, the mechanical computing units discussed above require assembly into larger, integrated networks. While replicating electronic computers is not the underlying goal of research in alternative computing approaches such as mechanical computing, the principles of digital logic design from electronic computing systems provide a robust foundation of theory and circuit simplification schemes to guide the development of mechanical logic analogs. AND, OR, and NOT gates can be combined to achieve universal logic; NAND and NOR are each able to achieve universal logic merely by combinations of themselves (functionally complete). The design of universal gates in mechanical logic systems is an important benchmark for demonstrating computational utility and for revealing the physical constraints of networking these building blocks in one, two, and three dimensions.

The simplest examples of mechanical computing systems are 1D chains of mechanical bits, such as linkage systems [72, 83–86] or granular chains [37, 38]. For example, if two units composed of spiral springs with lumped masses (see Fig. 2g for the single element) are connected in series, this 1D chain structure can exhibit an AND gate behavior, i.e., no output signal is obtained unless input signals ("1") are applied to both units (see the upper inset in Fig. 3a) [37]. On the other hand, if the two units are connected in parallel, the system can serve as an OR gate (the lower inset in Fig. 3a). In addition, NOR/XOR/NAND/NOT gate behaviors can be achieved by combining multiple units. The above examples are volatile, but 1D non-volatile logic systems have also been constructed, including functionally-complete logic gates (a NAND gate example in [33]). In these 1D systems, the output of one unit is connected to the input of the next unit. Therefore, input information is typically processed unidirectionally from one end of the chain to the other.

The limitation of linear information paths in 1D systems motivates the development of 2D and 3D systems, where signal branching and interactions beyond nearest neighbors are possible. Several 2D systems have been demonstrated [25, 34, 46, 87, 88]. The blue box in Fig. 3 illustrates examples of 2D planar systems comprising constrained beams [32] (Fig. 3b) or waterbomb origami [34] (Fig. 3c). For example, modules composed of constrained beams (see Fig. 2a) can be arranged as a grid-like planar system (Fig. 3b), which allows the implementation of multiple logic operations. Parallel connections of two modules could coordinate to pass/block a signal or emulate an AND gate by propagating the snap-through behavior [32]. Similarly, waterbomb origami can be connected side by side to form a system of multiple bits that perform simple logic operations, depending on the configurations of the unit cells [34]. Unlike 1D systems, the mechanical computing units can interact with multiple nearest neighbors along both dimensions, allowing information to propagate across the 2D plane, instead of only along one dimension. This feature can be exploited to control multiple bits in parallel, and could enable new functionality or mechanical computing architectures. Extending to 2D and 3D not only increases the degrees of freedom (DOFs) of mechanical systems but also allows new logic state assignments arising from the coupling of DOFs. For example, mechanical substrates that are effectively 2D in nature, such as lattice or origami structures, can take on complex and multistable 3D conformations due to the coupling of twisting and bending motions, as well as in-plane deformations. The mapping between the sequence and structure of cell deformation and global, stable configurations can also emulate logic, as recently demonstrated in an elastomeric sheet with embedded bistable domes [89]. Therefore, 2D and 3D systems can offer not only a simple extension or tiling of 1D logic elements but also a platform to assign new kinematic mechanisms and 3D deformations with a logic state.

3D mechanical computing systems have not been studied extensively. However, a number of previously reported 1D and 2D architectures could naturally be extended to 3D [45], and could be exploited to control the mechanical flow of information in unprecedented ways. Recent advances in 3D printing could allow fabrication of more complex 3D mechanical systems that have been recently conceptualized. For example, by utilizing a combinatorial approach, a metacube structure composed of cubic unit cells has been proposed [90]. This structure exhibits a programmed pattern on its side surface under axial compression (Fig. 3d). Not only linear motions, but also coupling between axial and rotational deformations, have been demonstrated [91] (Fig. 3e), allowing vertical deformation to induce transverse/lateral motions in 3D space. In addition to these static responses, there are also opportunities to process information using the dynamic properties of a mechanical system, such as topological phases or phase transitions which were originally studied in condensed matter physics. These emerging, so-called "topological mechanical metamaterials" can be designed to provide robust control of wave dynamics in 2D planar networks and 3D volumetric systems [37, 92–98], (e.g., 3D systems with elastic polarization [99] (Fig. 3f)). Due to localization of waves (e.g., topological edge mode), such systems could enable various operations relevant to information processing, e.g., mechanical diodes, which can be tailored to route mechanical signals in a specific direction, to switch/reroute signals, or to isolate a complex routing pathway.

The development of mechanical computing architectures involves several challenges, which will require both clear understanding of the fundamental abstraction layer discussed above (Sec. II) and new design rules for circuit and component-level integration. For example, the kinematics of the bit abstraction place constraints on the gate assembly, as input and outputs may be mechanically incompatible for certain gate combinations. Due to these constraints, circuit designs from electronic digital logic may not translate to "bottom-up" gate assembly in a mechanical logic system. One approach to this challenge inspired by the electronics community is to develop design tools for these constraints. For example, instead of a single AND gate design, perhaps the design of an AND gate structure is optimized based on the gate types connected to it. Similarly, a "top-down" design approach may be more tractable for certain mechanical logic implementations, where higher level functionality (e.g., a full or half adder) could be designed directly rather than assembling the individual logic gates that are known to collectively produce the equivalent functionality. Topology optimization, pseudo-rigid body models, and graph-based techniques for mechanism design [100–103] are promising approaches to these more complex logic structures, with the potential benefit of reducing gate inter-connections, incompatibilities, and overall energy requirements of the mechanical computing devices.

Mechanical logic networks are also constrained by the number of accessible interactions between gates, limiting the number of inputs that an output signal can drive (also known as the problem of "fan-out" in electronic circuits). Damping and other losses may also limit the distance of force propagation, which could constrain the overall size of the mechanical computing network. These limitations also afford approaches where the order or sequence of mechanical loading may enable multiple mechanical logic networks to co-exist within the same structure, effectively increasing the computational utility for the same size of network. For example, Faber et al. [89] demonstrated that an elastomeric sheet populated with bistable domes exhibits distinct 3D conformations based on the order of dome inversion, not just the specific combination of inverted domes. Sequence-dependent effects of this nature could lead to complex and branched logic networks, which may redefine the current understanding of these mechanical networking constraints. Mechanical computing systems also have the advantage of a direct interface with the environment, which can include a large set of physics and timescales of interaction. Leveraging this additional design dimension of computing physics has the potential to relax the fan-out constraint (using long range interaction - magnetics) and recoup energy losses (harvesting environmental sources - thermal cycles), all while simultaneously integrating these cues into the computing task of the device. In the following section, we explore how new computing paradigms, enabled by integration of stimuli-responsive materials and additional physics into the logic flow, present a possible strategy for seamless embodiment of computation and function within mechanical systems.

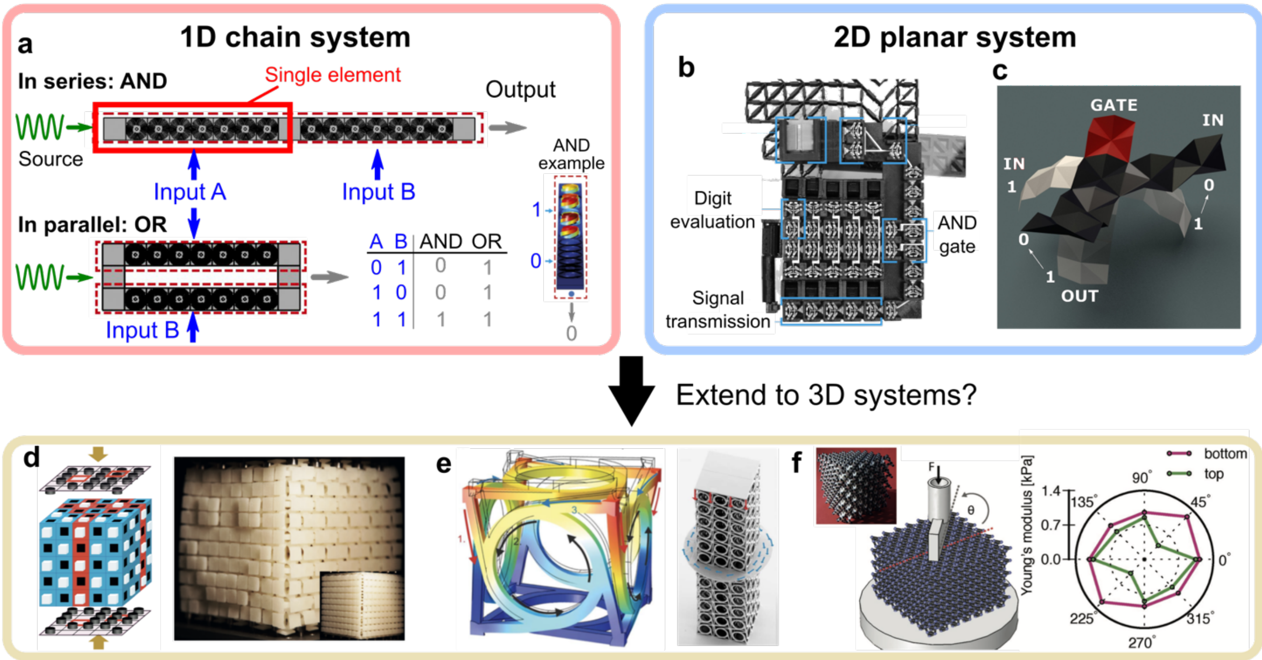

FIG. 3. **Networking mechanical computing units for digital logic**. By using single-bit mechanical memory units as a building block, we can construct 1D chains (denoted by a red box) and 2D planar structures (blue box) to create classic digital logic gates and networks of these. (**a**) 1D array of spiral spring cells with a magnetic mass (reprinted with permission from Reference [37]). 2D planar configurations have been designed using (**b**) modules composed of constrained beam elements (reprinted with permission from Reference [32]), and (**c**) tessellation of waterbomb origami unit cells (reprinted with permission from Reference [34]). Though 3D networks for mechanical information processing have not yet been widely explored, the deformation mechanisms and unconventional properties of 3D mechanical metamaterials suggest strategies for their implementation, e.g., (**d**) a combinatorial design for programmed shape change (reprinted with permission from Reference [90], © 2016 Springer Nature), (**e**) 3D chiral metamaterials with compression-twist coupling behavior (reproduced with permission from Reference [91], © 2017 American Association for the Advancement of Science), and (**f**) topological materials with elastic polarization (reproduced with permission from Reference [99], © 2017 Wiley).

IV. ENVIRONMENTAL INTERACTIONS AND I/O

In Sections I-III, we have discussed an operational framework in which abstract computational models can be physically realized in networked mechanical systems. We discussed how mechanical mechanisms enabled by geometric nonlinearity could produce mechanical systems with switchable, discrete information states. However, to this point we have not discussed what, beyond mechanical loading, might induce the mechanical systems to change state. In this section, we consider how these unconventional computing systems might interface with their environment and with other subsystems. What are the "inputs and "outputs" relevant to mechanical or material computing systems with coupled physics? How can mechanical computing augment digital electronic systems to improve performance of engineered systems? What new computing architectures are needed to fully integrate multiple, diverse environmental inputs? To navigate these questions, we evaluate environmental interactions in the physical substrate and architecture levels, highlighting future opportunities for mechanical computing in the process. Figure 4a provides examples of relevant interactions (either with the external environment or with other subsystems). Note that this *interaction* can be triggered via stimuli-responsive materials/structures within a layer. In mechanical systems such active materials serve as an analog to conventional sensors/actuators. In this framework, a specific computation (e.g., logic gate operations) can be performed by connecting Physical substrate and Engineered architecture layers.

In conventional digital computers, silicon serves as a substrate for electronic components but is not itself designed to change or respond to the environment. Instead, environmental inputs are obtained via modular sensors, distinct from the computing device, that transduce physical quantities such as temperature or light intensity into an electronic signal that the computer can subsequently operate on. In contrast, mechanical computing systems can be constructed from a large palette of adaptive materials, which can directly *respond* (bend, twist, etc.) to environmental inputs corresponding to the active materials used in the system. Examples include electronic signals (e.g., using dielectric elastomer actuators [104, 105] or liquid metal [106]), mechanical stimuli [32, 107], chemical stimuli [21, 108], acoustic pressure [87], and humidity gradients [34]. In addition, mechanical deformation can be triggered in shape memory polymers and liquid crystal elastomers in response to temperature changes [109, 110] and/or light [111]; polymers can be designed to mechanically respond to pH [112] and magnetic fields [113–115]. Moreover, multiple input sources can be combined for operation (e.g., mechanical force and magnetic field to manipulate bit information [107]; see panel **i** in Fig. 4b). This could enable computation in new form factors and operating environments [116]. Multi-responsive systems can also be designed to account for stimuli order, allowing *time* to serve as another design parameter to logically couple or decouple stimuli [21].

In distinct contrast with I/O in traditional digital electronics, the changes that occur to the mechanical computing system due to environmental inputs are not limited solely to the physical substrate layer—they can also manipulate the engineered architecture layer. As a simple example, the application of external force can be used to morph a mechanical logic gate from an AND gate to an OR gate, and vice versa (panel **ii** in Fig. 4b) [48]. Evolving the computing architecture in response to environmental input represents a novel tool for reprogramming mechanical computing platforms, with the potential of intra- and inter-switching within and between architecture classes. Collectively, these examples highlight the novelty of mechanical computing concepts, not only in granting access to new operational environments, but more importantly, expanding the definition and methods of how information is abstracted and processed.

Understanding materials in terms of their information processing capabilities could impact every aspect of automation systems that interact with their environment. In particular, robotic systems can be expected to be equipped with classical centralized computing when physically feasible; yet, for a variety of scenarios, this may not be plausible, nor optimal. For example, it is typically not possible for micron-scale robots [117] to rely entirely on traditional electronic computing. Even with classical computing available, robots will rely on physical properties to perform material pre-processing to reduce the centralized computational load. As an example, Zhao et al. [118] use a soft robotic hand to assess fruit ripeness through a temporal-spatial integration of the mechanical deformation during contact, effectively augmenting the computing task of the robot through a form of mechanical filtering (see the top insets in panel **iii** of Fig. 4b). This filtering concept can be expanded to other features, such as texture, temperature, and shape, as demonstrated by Truby et al. in another soft robotic gripper example (see the middle insets in panel **iii** of Fig. 4b) [24]. Together, these examples highlight the opportunity to consolidate sensing and computing into the structure and physics of the device, performing 'materials-enabled computation' in the relevant physics and timescales of the target application. This congruence between a computing task, physics modeling/computation, and physical task execution motivates the augmentation of conventional computing with unconventional computing substrates, to improve both energy consumption and information collection (see further discussion in Sec. V).

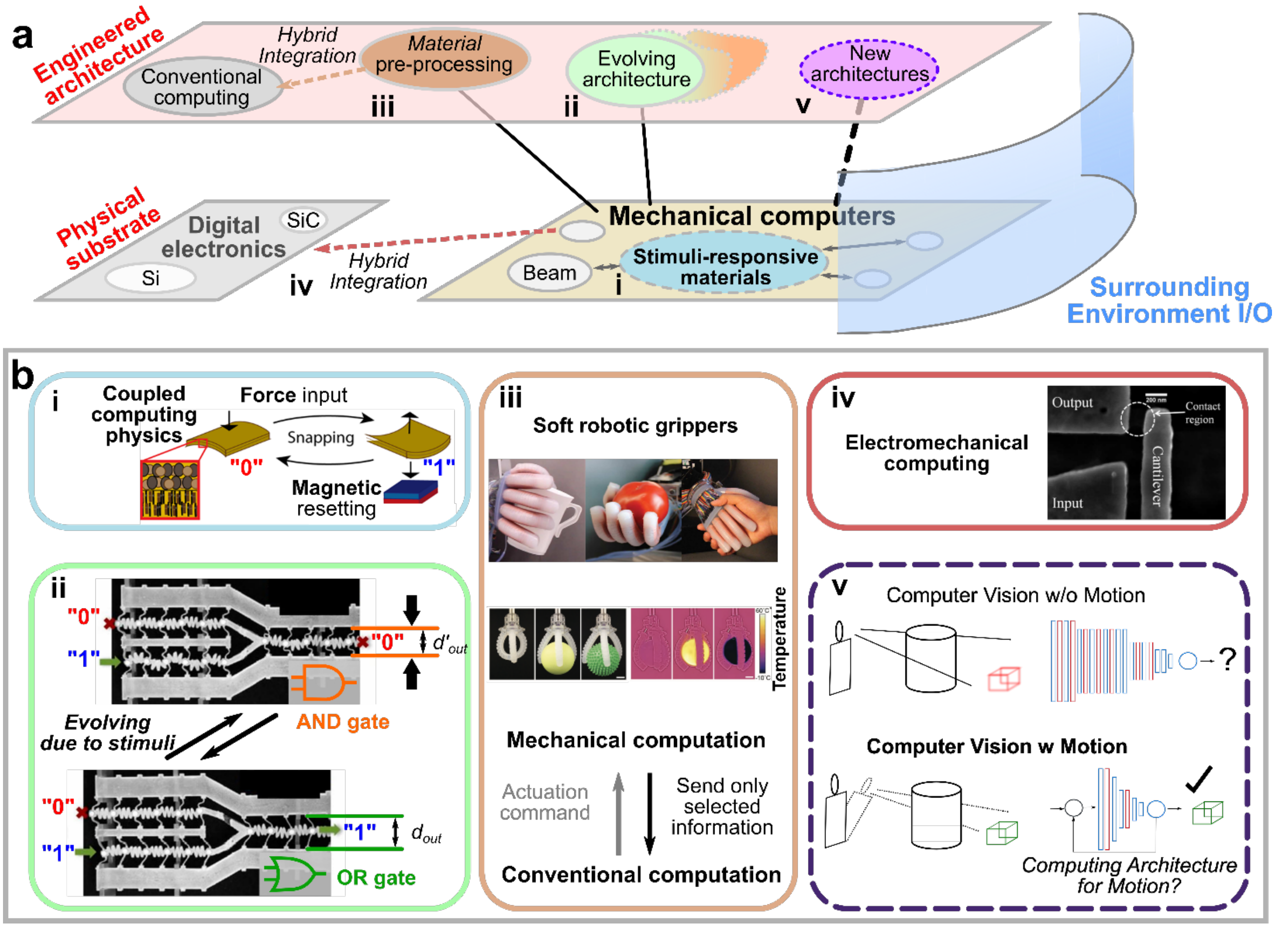

FIG. 4. **Environmental Interactions:** (**a**) Conceptual schematic of the advantages and opportunities of mechanical computing to directly couple with the physical environment, followed by representative examples in subpanel (**b**). **Coupled Computing Physics:** Opportunity to combine physics and sensory input in the abstraction layer of the physical substrate - (**i**) Multiple inputs (force and magnetic field) to manipulate the binary state (reprinted with permission from Reference [107], ©2019 American Chemical Society). **Evolving Architecture:** environmental stimuli can reprogram the computing architecture - (**ii**) example of a mechanical logic gate switching between AND and OR behavior in response to external mechanical loading (reprinted with permission from Reference [48]). **Material Pre-Processing:** leverage mechanics to synthesize environmental input for integration with conventional architecture - (**iii**) Examples of soft robotic grippers with embedded sensory functions, e.g., (*top*) detecting target object shapes (reprinted with permission from Reference [118], ©2016 American Association for the Advancement of Science), and (*middle*) processing different textures and temperatures (reprinted with permission from Reference [24], ©2018 Wiley). **Electromechanics:** (**iv**) image of electromechanical SiC switch highlighting coupled mechanics and electrostatics for high temperature computing applications (reprinted with permission from Reference [119], ©2010 American Association for the Advancement of Science). **New Architectures:** abstractions and mappings to higher computing layers are needed to precisely define the computational contribution of new substrates and physics - (**v**) Illustration of a computer vision task to classify the shape of a partially occluded cube with, and without, the aid of mechanical motion. Motion to avoid a visual occlusion reduces the conventional computing cost of a machine learning vision classifier for this task by enabling a camera to see all of an object. However, it is unclear presently what architecture and computing model should be employed for assessing the tradeoffs between conventional computation and mechanical motion.

V. CHALLENGES AND OPPORTUNITIES

Although many recent publications have shown the feasibility and potential for storing and processing binary information as a material property, there remain both challenges and associated opportunities for advancing the field of mechanical computing. In this Section, we explore some current and future research directions related to the realization of unconventional computing in mechanical systems, leveraging the three layer model of computation (Fig. 1) to guide the discussion.

A. Beyond binary abstraction

Major advances in additive manufacturing, materials science, and mechanical metamaterials have led to new ways of thinking about materials. As presented here, the research community has begun to think about ways in which *information processing* itself can be thought of as a material property. Abstracting information processing is a ubiquitous and underutilized opportunity in mechanical systems. The mechanical mechanisms described in Section II underscore this point and serve as an instructive guide to identify new ways to embed and abstract information. Extending the number of states, such as tristable mechanisms in which discrete states could take values of 0, 1, 2, (or -1,0,1) is one simple example of a promising next step. Exploiting the frequency response spectrum affords another. Far more complex multistable or volatile mechanisms are also possible, allowing representation of more than just binary information. These non-traditional discrete representations present opportunities for the mechanics and materials communities to interface with computational theorists to explore new abstractions and mappings between computing layers.

B. "Compilers"

In conventional computing, the choice of architecture and substrate is biased by the inherent (and clearly justifiable) demand for a universal computing platform, which has focused investment (and achieved remarkable success) into a handful of core technologies. However, is a universal computing machine optimal for every application? The mechanical computing examples highlighted above demonstrate that even simple logic calculations could enhance the operation of a device without serving as a general purpose computer. To tap into this computing potential, design tools are needed to move both up and down between computing layers in Fig. 4a, not only to fit new materials and physics into established computing models, but also to identify new computing abstractions that are most compatible with the physical substrate, whether localized, dispersed, or some hierarchical combination. This relates to conventional compilers, which translate a higher level program language into a lower level language more closely tied to the operation of the physical substrate (i.e., silicon-based digital electronics for traditional computing systems). This is a key step in telling a "universal computer" how it should specifically operate. In contrast, an analogous "compiler" for a mechanical computing system would need to play the role of algorithmically generating an appropriate computational substrate-layer (Fig. 1 and Fig. 4): i.e., it must generate a suitable design of a 3D mechanical system, reconciling its mechanical kinematics and energy constraints, and ensuring the system properly embodies sensing, computing, and actuating functions in its arrangement of potentially multiple active materials. An initial example of a mechanical logic compiler is included in Ion et al [32], which presents a design editor to minimize the size of the mechanical logic network to achieve a target logic operation. Expanding the capability of the compiler to integrate diverse environmental I/O, computing models, spatially dispersed nodes, hybrid integration with conventional electronics, and fabrication constraints presents a challenge, and potential bottleneck, for the advancement of mechanical computing concepts. Most unconventional computing systems, including mechanical logic, are programmed at a very low level, since substrate-specific design and abstraction rules have not had time to mature. In light of this, codifying the "compiler" design rules for these unconventional substrates is an open challenge for the materials, design, and computing communities.

C. Exploring new unconventional computing

Opportunities to innovate exist at all three layers of the computing framework (Fig. 1). In the Physical Substrate, novel abstractions are beginning to be identified through combinations of materials, physics, geometry, and timing to access new operation regimes. For example, by combining the physics of electrostatics with contact mechanics, sub-micron electromechanical switches made from silicon carbide (SiC) enable digital logic computations at extreme temperatures (>500 °C) [119], typically outside the operating temperature of conventional electronics (panel **iv** in Fig. 4b). The Engineered Architecture layer can also interact directly with the environment (panel **ii** in Fig. 4b), presenting an opportunity to embed self-reconfigurable computing architectures in mechanical systems. The range of computational tasks this will enable has yet to be investigated. For instance, could a periodic, temporal cue from the environment trigger the material computing system to convert from a digital to an analog interpretation or to produce some form of digital-analog hybrid? Lastly, innovations in the Computational Model layer will have the dual benefits of establishing new computing constructs for guiding the discovery of unconventional computing materials, and also stimulating new ways of characterizing and thinking about materials. For example, multistable beam networks are physically continuous, with temporally- and spatially-varying internal stress and strain states under deformation. However, it is the discrete configurations of the multistabilities, not the continuous state variables, that are leveraged to emulate logic operations in the examples of this Perspective. The focus on the discrete properties of the beam array motivates the application of discrete mathematics techniques, such as graph theory, not only to scan for computing potential, but to provide a new lens to characterize and benchmark the behavior of the underlying material structure.

D. Metrics to assess mechanical computers

New computing and material performance metrics are needed to classify and benchmark the collective innovations across these computing layers (see, e.g., Ref. [120] for discussion on quantifying unconventional computing 'resources'). Conventional metrics are largely focused on processing speed, bit density, and I/O package miniaturization. Mechanical computing performs poorly against these benchmarks. While miniaturization has been pursued for mechanical computing using micro-/nano-electromechanical systems (MEMS/NEMS) [121–124] and could provide benefits (such as robustness against harsh environments or high temperatures [119]), the relevant fabrication approaches for MEMS/NEMS come with their own set of constraints that would limit the complexity of a mechanical logic network and the types of materials (and hence sensors) that could be integrated. Instead, alternative metrics are needed to better capture the unique strengths of mechanical and other unconventional computing concepts, and to assess the impact of hybridization with conventional electronics. For example, the intrinsic integration of the computation within the physical material or device offers distinct efficiencies and insertion opportunities that would be challenging for conventional approaches. Metrics reflecting this advantage could include the number of data type conversions between input and output computations, spatial proximity of the computation to the input signal, and relevance of the computing physics and timescales to the computing application. Does a dynamic mechanical load operating on the timescale of Hz require state assessment on the order of MHz or greater? Is it more efficient to continually query for the current configuration or to have the material/structure directly detect, assess, and process the mechanical event? Efficiency and integration benefits of this nature lack the precision and concreteness of the benchmarks currently employed for conventional computing, but are necessary for placing mechanical computing concepts in an appropriate context.

Developing methods to establish the computational equivalence of these alternative metrics in augmenting conventional computing systems is also an important next step. For instance, machine vision—and vision-based object classification—rely heavily on sophisticated algorithms to robustly handle occlusion, distortion, and other environmentally-driven image degradation. These algorithms come at high computational and, implicitly, energetic expense. However, vision systems that can move are able to meet the same object identification requirements through mechanical motion by looking around an occlusion rather than using classifiers intended for limited data. In addition, mechanical motion augments the view of the object relative to previously collected images, which can also improve the efficiency of classification [125]. Panel **v** in Fig. 4b shows an illustration of such a situation, where a camera must either identify an object—the cube—based on a partial image or must move to avoid the visual occlusion created by the cylinder. That machine learning uses mechanical motion to improve data collection and learning efficiency [126] highlights the need for new architectures and computational models to precisely define the interactions between new material substrate mechanical properties and computational requirements.

Integration, efficiency, and material compatibility metrics will also provide clear evaluation criteria for the merits of using stimuli-responsive materials to directly harness environmental interactions in the computational abstraction. Bottlenecks in information processing often occur at the points of data conversion between physical type (mapping sensor physics to computation physics) or computational representation (analog to digital). Mechanical computing may mitigate this bottleneck by merging the sensing and computing physics into a single domain. However, timescale incompatibilities are likely to arise as additional physical stimuli are integrated into the computation, due to the distinct timescales associated with each stimuli-responsive phenomenon. For example, a sudden change in temperature or voltage may equilibrate throughout the system more rapidly than a change in the chemical environment due to diffusion (which also introduces time dependence based on feature size). This could be harnessed to produce exciting new effects, such as spatially and temporally distributed reprogramming in response to local environmental cues, but this will also require careful design at the architecture level to retain the meaning and utility of the computation. Understanding the advantages of sensory consolidation at the physical substrate layer will be key to deciding whether to use conventional, unconventional, or hybrid computing approaches.

E. Conclusion

Treating information processing as a material property will introduce multidisciplinary challenges that will require both new theoretical approaches and practical design tools as discussed above; solutions are therefore likely to be found at the interfaces between materials science, information theory, computer science, additive manufacturing, and robotics. The converging path ahead for these research communities is an exciting one. Our intent is that the framework highlighted in this Perspective, along with the specific mechanical computing examples reviewed, will serve as a catalyst for discovery of new material computing paradigms and will invite the community to view information processing as a material/structure behavior.

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Contributions

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Competing interests

The authors declare no competing interests.

Figure legends

FIG. 1. **Three level hierarchy of a computational system:** (**a**) Building a computer through the three levels: (*Top layer*) "Computing model" (e.g., the Turing machine, combinatorial logic, and general purpose analog computer (GPAC.), (*Middle layer*) "Engineered architecture" which represents an abstract platform where a computing model is implemented (von Neumann architecture (see the left inset illustration) [22], logic circuit, etc.), and (*Bottom layer*) "Physical substrate" which realizes a design in a physical system. (**b**) A mechanical computing system, highlighting information processing as a material property, which can interact with environments and perform "computation", e.g., (i) a rover inspired by mechanical computers for extreme environments [23] (Image Credit: NASA/JPL-Caltech), and (ii) soft robotic grippers with embedded sensors which can sense pressure, temperature, etc. Reprinted with permission from Reference [24]. © 2018 Wiley. (**c**) A mechanical computing system can be realized by leveraging various mechanical building blocks (e.g., origami-inspired unit which can represent binary information ("0" or "1") depending on different deformation modes, and its 2D network); reprinted with permission from Reference [25].

FIG. 2. **Non-volatile and volatile mechanical realizations/implementations of abstract bits**. One of the approaches to retain information without external power source is to utilize bistable behavior based on geometrical nonlinearities, such as (**a**) a unit cell composed of clamped beams, which can transform between undeformed ("0") and deformed ("1") configurations (reprinted with permission from Reference [32]), and (**b**) a bistable flexure mechanism. Reprinted with permission from Reference [33]. Origami can also be used to design non-volatile mechanical memory, e.g. (**c**) triangulated cylindrical origami-based structure (reprinted with permission from Reference [25]) and (**d**) waterbomb origami. Reprinted with permission from Reference [34]. Volatile logic can be encoded in beam dynamics, as demonstrated in (**e**) electromechanical beams (reprinted with permission from Reference [35], © 2008 Springer Nature) and (**f**) microcantilevers with stiffening behavior (reprinted with permission from Reference [36], © 2010 AIP Publishing). Other examples of volatile mechanical devices include (**g**) a 1D array of spiral spring cells with a magnetic mass (reprinted with permission from Reference [37]) and (**h**) granular chains (reprinted with permission from Reference [38], © 2014 Springer Nature).

FIG. 3. **Networking mechanical computing units for digital logic**. By using single-bit mechanical memory units as a building block, we can construct 1D chains (denoted by a red box) and 2D planar structures (blue box) to create classic digital logic gates and networks of these. (**a**) 1D array of spiral spring cells with a magnetic mass (reprinted with permission from Reference [37]). 2D planar configurations have been designed using (**b**) modules composed of constrained beam elements (reprinted with permission from Reference [32]), and (**c**) tessellation of waterbomb origami unit cells (reprinted with permission from Reference [34]). Though 3D networks for mechanical information processing have not yet been widely explored, the deformation mechanisms and unconventional properties of 3D mechanical metamaterials suggest strategies for their implementation, e.g., (**d**) a combinatorial design for programmed shape change (reprinted with permission from Reference [90], © 2016 Springer Nature), (**e**) 3D chiral metamaterials with compression-twist coupling behavior (reproduced with permission from Reference [91], © 2017 American Association for the Advancement of Science), and (**f**) topological materials with elastic polarization (reproduced with permission from Reference [99], © 2017 Wiley).

FIG. 4. **Environmental Interactions:** (**a**) Conceptual schematic of the advantages and opportunities of mechanical computing to directly couple with the physical environment, followed by representative examples in subpanel (**b**). **Coupled Computing Physics:** Opportunity to combine physics and sensory input in the abstraction layer of the physical substrate - (**i**) Multiple inputs (force and magnetic field) to manipulate the binary state (reprinted with permission from Reference [107], ©2019 American Chemical Society). **Evolving Architecture:** environmental stimuli can reprogram the computing architecture - (**ii**) example of a mechanical logic gate switching between AND and OR behavior in response to external mechanical loading (reprinted with permission from Reference [48]). **Material Pre-Processing:** leverage mechanics to synthesize environmental input for integration with conventional architecture - (**iii**) Examples of soft robotic grippers with embedded sensory functions, e.g., (*top*) detecting target object shapes (reprinted with permission from Reference [118], ©2016 American Association for the Advancement of Science), and

(*middle*) processing different textures and temperatures (reprinted with permission from Reference [24], ©2018 Wiley). **Electromechanics:** (**iv**) image of electromechanical SiC switch highlighting coupled mechanics and electrostatics for high temperature computing applications (reprinted with permission from Reference [119], ©2010 American Association for the Advancement of Science). **New Architectures:** abstractions and mappings to higher computing layers are needed to precisely define the computational contribution of new substrates and physics - (**v**) Illustration of a computer vision task to classify the shape of a partially occluded cube with, and without, the aid of mechanical motion. Motion to avoid a visual occlusion reduces the conventional computing cost of a machine learning vision classifier for this task by enabling a camera to see all of an object. However, it is unclear presently what architecture and computing model should be employed for assessing the tradeoffs between conventional computation and mechanical motion.

FIG. S1. (**a**) The number (β) of n-ary digits required to express a positive integer (k) can be calculated as $\beta = \log_n k$. The grey horizontal line indicates 2^{32}, which corresponds to the maximum value of a 4-byte integer representation. (**b**) To express a large integer number efficiently, we assume that the efficiency of information storage is proportional to n and define the efficiency (f) as $f = an + \log_n k$ where a is a weight coefficient. We consider the case of $k = 10^5$, and plot the result for a = 1, 5, and 10. The black triangle markers represent the local minimum state, which means the most efficient storage, i.e., smaller n and smaller number of digits. (**c**) We calculate and plot various positive integer numbers as a function of the efficient n-ary storage. Based on this calculation, to express smaller numbers (e.g., 10^3), $n = 3, 4$ can be advantageous, compare to binary or decimal digits. Please note that this calculation is a rough estimation without considering fabrication challenges, operation speed, or robustness. Example multistable mechanical structures include (**d**) ternary memory based on rotating squares (reprinted with permission from Reference [67], © 2020 American Physical Society) and (**e**) quaternary memory based on origami (reproduced with permission from Reference [53], © 2015 American Physical Society). See also Ref. [66] for a discussion of why base $n = 3$ (i.e., ternary digit) can be optimal for storage efficiency.

BOX / SIDEBAR: "Information Processing"

When thinking about building a computational system, a 'computer', it is helpful to describe three levels: the model of computation, the architecture, and the physical substrate.

The model of computation.

The model of computation is an abstract, usually mathematical, model of how the computational process unfolds. There are many models of computation. Classically, there is a progression of models of increasing computational power: combinational logic, finite state machines, pushdown automata with an unbounded memory stack, and Turing Machines with an unbounded memory tape. Other classical models, such as lambda calculus, are equivalent in power to the Turing Machine model. These models are discrete state space (symbols) and discrete time. Other discrete space/discrete time models, such as Cellular Automata, have the same theoretical computational power as Turing Machines, but may map to an architecture more suited to different implementations and problems. Quantum computational models have greater efficiency, but not greater computational power, than Turing Machines (they can solve some problems faster, but they cannot solve non-Turing computable problems).

There are also continuous space/discrete time computational models, such as Coupled Map Lattices, and some Neural Network models, typically based on underlying difference equations. There are continuous space/continuous time models, such as some Spiking Neural Network models, reaction-diffusion models, Shannon's General Purpose Analog Computer (GPAC), Rubel's general purpose extended analogue computer, and continuous time quantum computational models, typically based on underlying differential equations.

The architecture.

An architecture is an abstract design for how a model of computation may be realized (implemented) in hardware. It focuses on a set of basic components, and how they are connected. For example, the combinational logic model maps naturally to an architecture comprising a universal set of logic gates connected into a circuit. The classical von Neumann architecture, describing how a CPU controls and performs computational operations, with random-access memory containing a stored program and data, is not itself a natural mapping of the Turing Machine, with its sequential memory access, but has a more natural mapping to an efficient hardware implementation. Other architectures, such as those underlying GPUs and FPGAs, are alternate designs for classical computing. An architecture need not be realized directly in hardware; it may be a form of 'virtual machine' implemented in software in another architecture. For example, cellular automata and neural networks are typically implemented in classical architectures.

The physical substrate.

The physical substrate (hardware) realizes an architecture and its model of computation: it forms the physical computer. The standard substrate for realizing the von Neumann architecture is digital electronics. (Technically, since the von Neumann architecture, both in principle and its realizations in practice, does not have unbounded memory, it has the computational power of a finite state machine, not a Turing Machine. This tension between theoretical computational power and finitary physical limitations tends to be glossed over in practice.)

There are many other substrates supporting a range of architectures, including non-linear materials, analogue electronics, magnetic materials, optics, chemicals, biochemicals, biological organisms, and mechanical devices. Indeed, the earliest engineered computers were mechanical clockwork systems, including the Antikythera device, Babbage's Difference Engine, and the Differential Analyzer. In recent decades all of the above approaches have been referred to as "unconventional computing" due to the enormous success of conventional silicon-based digital electronics. Yet thanks to numerous advances in manufacturing, materials, and design, unconventional computing has recently begun to receive a great deal of attention. In this Perspective we focus primarily on digital architectures that enable information processing via mechanical mechanisms and stimuli-responsive materials.

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