Affordances of Fractions and Decimals for Arithmetic

Qiushan Liu

David Braithwaite

Florida State University

Author Note

Qiushan Liu, https://orcid.org/0000-0001-7219-624X

David W. Braithwaite, https://orcid.org/0000-0001-8111-607X

This research was supported by National Science Foundation under Grant No. 1844140 to the second author. We report how we determined our sample size, all data exclusions (if any), all manipulations, and all measures in the study. All data, analysis code, and research materials will be available for public access on OSF Network (https://osf.io/uzetx/?view_only=77e68c31d453422ba2e68ec639e0d1ce) upon acceptance of the manuscript. The design and analysis of experiment 2 is preregistered (https://osf.io/9sq5u?view_only=65b0a53912004002b7863ac11c5aa7fb). The result of the experiment will be presented at the Mathematical Cognition and Learning Society, 2021.

Correspondence concerning this article should be addressed to Qiushan Liu, Department of Psychology, Florida State University, 1107 W. Call Street, Tallahassee FL 32306, liu@psy.fsu.edu.

Abstract

Rational numbers are represented by multiple notations: fractions, decimals, and percentages. Whereas previous studies have investigated affordances of these notations for representing different types of information (DeWolf, Bassok, & Holyoak, 2015; Tian, Braithwaite, & Siegler, 2020), the present study investigated their affordances for solving different types of arithmetic problems. We hypothesized that decimals afford addition better than fractions do and that fractions afford multiplication better than decimals do. This hypothesis was tested in two experiments with university students (*N*s = 77 and 80). When solving fraction and decimal arithmetic problems, participants converted addition problems from fraction to decimal form more than vice versa, and converted multiplication problems from decimal to fraction form more than vice versa, thus revealing preferences favoring decimals for addition and fractions for multiplication. Accuracies paralleled these revealed preferences: Addition accuracy was higher with decimals than fractions, whereas multiplication accuracy was higher with fractions than decimals. Variations in notation preferences as a function of the types of operands involved (e.g., equal versus unequal denominator fractions) were more consistent with an explanation based on adaptive strategy choice (Siegler, 1996) than with one based on semantic interpretations associated with each notation.

Introduction

Symbolic notations are essential to the practice and pedagogy of math. On the one hand, they enable representation of mathematical information in a compact form. On the other hand, they facilitate problem solving by enabling the use of symbol manipulation algorithms, such as column addition and long division.

Rational numbers, unlike whole numbers, are commonly represented by multiple symbolic notations: fractions, decimals, and percentages. These notations have existed in their modern forms for over three hundred years (Cajori, 1928), and are pervasive across cultures, languages, and geographic regions. These observations suggest that the three notations are not redundant, but rather that each serves some functions uniquely well. What, then, are the functions best served by different rational number notations?

Previous research has focused on identifying types of information that each notation is best suited to represent. For example, fractions are preferred to decimals for representing ratios between small discrete sets, whereas decimals are preferred to fractions for representing ratios between continuous quantities (DeWolf et al., 2015; Tian et al., 2020). Further, when representing a given type of ratio, accuracy is generally highest when using the notation that is preferred for that type of ratio.

However, symbolic notations are used not only to represent information but also to solve problems. Are different notations best suited for solving different types of problems? To address this question, the present study investigated affordances of decimals and fractions for solving arithmetic problems involving addition and multiplication. We tested several predictions relating to the central hypothesis that decimals afford addition better than fractions do, whereas fractions afford multiplication better than decimals do.

Below, we review previous research on the suitability of different rational number notations for representing different types of information. Next, we elaborate on our hypothesis regarding affordances of different notations for solving different types of problems. Then, we describe the present study in more detail.

Suitability of Rational Number Notations for Representing Different Information

A rational number is the quotient of two integers, termed numerator and denominator (Rosen, 2012). Fractions, decimals, and percentages all allow the numerator to be any integer, but differ in whether and how they constrain the denominator. Fractions allow the denominator to be any nonzero integer, decimals require it to be a power of 10 (for example, the implied denominator of 0.321 is 1000), and percentages require it to be 100. These formal differences imply differences in the types of information that each notation is best suited to represent.

DeWolf et al. (2015) analyzed these differences from the perspective of semantic alignment theory (Bassok, Chase, & Martin, 1998). They reasoned that each notation aligns most naturally with situations possessing a conceptual structure similar to that of the notation. Specifically, because fractions allow the numerator and denominator to vary freely, fractions have a two-dimensional structure and therefore align well with ratios between cardinalities of sets. Because decimals allow only the numerator to vary freely, decimals are one-dimensional and therefore align well with the magnitudes of ratios between continuous masses.

Consistent with these hypotheses, DeWolf et al. (2015) found that university students preferred using fractions to represent ratios between cardinalities of sets, but preferred using decimals to represent ratios between continuous masses. Further, when judging the correspondence between a given ratio and a given rational number, participants performed best when the notation of the number (fraction or decimal) was the one preferred for the given type of

ratio. Subsequent studies obtained similar findings in the U.S., Korea, and Russia (Lee, DeWolf, Bassok, & Holyoak, 2016; Plummer, DeWolf, Bassok, Gordon, & Holyoak, 2017; Rapp, Bassok, Dewolf, & Holyoak, 2015; Tyumeneva et al., 2017). Gray et al. (2017) found that performance with percentages is similar to that with decimals, and concluded that the conceptual structure of percentages, like that of decimals, is one-dimensional.

Tian et al. (2020) analyzed preferences among rational number notations from a different theoretical perspective, the Strategy Choice Model (SCM; Siegler, 1996). Tian et al. (2020) reasoned that different notations afford different procedures for performing a task, so choosing a notation implies choosing the procedures afforded by that notation. Thus, notation choice can be viewed as a form of strategy choice. According to the SCM, strategy choices depend in part on the time and effort needed to execute each candidate strategy. Tian et al. (2020) accordingly assumed that preferences among notations for representing a given ratio should reflect differences in the time and effort required to represent the ratio with each notation.

Based on these assumptions, Tian et al. (2020) predicted that the preference for using fractions to represent ratios between discrete sets should decrease as the sizes of the sets increase, because determining the numerators and denominators of such ratios via counting requires more time and effort when the sets are larger. They also predicted that percentages should be preferred to decimals for representing ratios between large sets, and between continuous masses, if precision beyond two significant digits is not required. The basis for this prediction was that decimals, but not percentages, require a choice among implicit denominators (e.g., 10, 100, 1000), which incurs effort. Both predictions were confirmed (Tian et al., 2020).

In the next section, we apply the two theoretical perspectives described above to analyze affordances of fractions and decimals for solving different types of arithmetic problems.

Affordances of Fractions and Decimals for Arithmetic

Arithmetic with rational numbers is exceptionally challenging, both with fractions (Gabriel et al., 2013; Hansen et al., 2015; Hecht & Vagi, 2012; Mack, 1995; Newton, Willard, & Teufel, 2014; Siegler & Pyke, 2013) and with decimals (Hiebert & Wearne, 1985; M. A. Hurst & Cordes, 2018; Kouba et al., 1988; Lortie-Forgues & Siegler, 2017; Ren & Gunderson, 2021; Rittle-Johnson, Star, & Durkin, 2009). Difficulties in this area often persist into adulthood (M. Hurst & Cordes, 2016, 2018; Newton, 2008; Siegler & Lortie-Forgues, 2015). For example, 138 university students in Braithwaite and Sprague (2021) correctly answered only 82% of twelve fraction and decimal addition and multiplication problems.

The central hypothesis of the present study was that fractions and decimals each best afford different arithmetic operations. Specifically, we hypothesized that decimals afford addition better than fractions do, whereas fractions afford multiplication better than decimals do. This hypothesis was motivated by both theoretical perspectives described in the previous section—semantic alignment and strategy choice.

Taking a semantic alignment perspective, rational numbers have multiple semantic interpretations: as measures, part-whole relations, ratios, quotients, and operators (Kieren, 1980). The measure interpretation is related to addition, in that "the joining of two measures to find a 'sum' measure exhibits the vector additions aspect of rational numbers" (Kieren, 1980, p. 136). Further, "using the metre as a [measurement] unit provides a natural entre to decimal notation" (Kieren, 1980, p. 136). Thus, decimals and addition are naturally aligned via their associations with measurement. In contrast, the quotient interpretation is closely related to multiplication because quotients involve division, the inverse of multiplication. Fractions are also associated with quotients, because the numerator and denominator of a fraction directly correspond to the

dividend and divisor of the corresponding quotient (i.e., $a/b = a \div b$; Wu, 2008). Thus, fractions and multiplication are naturally aligned via their associations with quotients. (See Dewolf, Son, Bassok, & Holyoak, 2017 for empirical evidence of an association between fractions and multiplication.)

Taking a strategy choice perspective, different notations afford different procedures, and fast or easy procedures should be preferred to slow or effortful ones. To apply these assumptions to the analysis of fraction and decimal arithmetic procedures, we assume that procedures requiring one whole number arithmetic operation tend to be faster and easier than procedures requiring multiple such operations. The basis for this assumption is that shifting between different whole number operations, such as between addition and multiplication, requires additional time and effort relative to repeating the same operation (Jersild, 1927).

First consider the standard procedures for adding fractions and decimals. To add fractions with equal denominators one may simply add the numerators (e.g., 3/4+6/4 = 9/4), but to add fractions with unequal denominators (e.g., 1/10+3/4), one must (1) convert the addends into equivalent fractions with a common denominator (e.g., 1/10 = 2/20, 3/4 = 15/20), before (2) adding the numerators (e.g., 2/20+15/20 = 17/20). Step 1 requires whole number multiplication, and step 2 requires whole number addition. Adding decimals, in contrast, requires whole number addition—to add the individual digits of the addends—but not multiplication (e.g., 0.1+0.25 = 0.35). Thus, other factors being equal, addition should be faster and/or easier in decimal than fraction format.

Now consider the standard procedures for multiplying fractions and decimals. To multiply fractions one may simply multiply their numerators and denominators (e.g., 3/10×1/4 = 3/40) , a procedure so intuitive that students often use it before being taught it (Byrnes & Wasik,

1991). To multiply decimals (e.g., 0.3×0.75) one must (1) multiply as with whole numbers (e.g., 3×75 = 225), then (2) place the decimal point so the answer has as many decimal digits as the total number of decimal digits in the operands (e.g., 0.3 and 0.75 have one and two decimal digits respectively, so the answer should have 1+2 = 3 decimal digits: 0.225). Step 1 requires whole number multiplication; if neither operand is a whole number, step 2 requires addition—to add the numbers of decimal digits in the operands. Thus, because fraction multiplication requires only multiplication of whole numbers whereas decimal multiplication requires both multiplication and addition of whole numbers, multiplication should be faster and/or easier in fraction than decimal format.

The Present Study

Both theoretical perspectives discussed in the previous section implied that when choosing to use either fractions or decimals to solve addition and multiplication problems, individuals should prefer decimals for addition problems and fractions for multiplication problems. To test this possibility, we leveraged the fact that adults sometimes convert fractions to decimals or vice versa when solving fraction and decimal arithmetic problems (Braithwaite & Sprague, 2021). We reasoned that converting from one notation to another reveals a preference for the latter notation for the problem at hand.

However, spontaneous conversion between fractions and decimals during arithmetic calculation is rare. For example, university students in Braithwaite and Sprague (2021) did so on 4-5% of trials. We therefore created contexts designed to encourage such conversions.

First, we presented problems in pairs involving the same arithmetic operation. In the blocked condition, each pair included two fraction problems or two decimal problems, whereas in the interleaved condition, each pair included one fraction problem and one decimal problem.

We predicted that in the interleaved condition, conversion between notations would occur more often than in the blocked condition (Prediction 1.1), and that addition problems would be converted from fractions to decimals more often than vice versa, whereas the opposite would occur for multiplication problems (Prediction 1.2).

Next, we presented mixed arithmetic problems involving one fraction operand and one decimal operand. Solving these problems using a standard procedure requires either converting the fraction operand into a decimal or vice versa. We predicted that conversions into decimals would be more common on addition than multiplication problems and conversions into fractions would be more common on multiplication than addition problems (Prediction 1.3).

The above predictions involve preferences, but our analysis of arithmetic procedures also has implications for accuracy, because procedures that are relatively cognitively taxing—that is, the fraction addition and decimal multiplication procedures—might also be relatively error-prone. To test this possibility with apples-to-apples comparisons, we created matched pairs of fraction and decimal problems involving the same arithmetic operation and very similar operands[1], such as (3/10×1/4, 0.3×0.75). We predicted that on these comparable fraction and decimal problems, addition accuracy would be higher with decimals than fractions, and multiplication accuracy would be higher with fractions than decimals (Prediction 2.1).

A final prediction involved effects of our blocking/interleaving manipulation on accuracy. Interleaving different types of problems during practice often improves learning outcomes, but at the cost of reduced accuracy during practice (Rohrer & Taylor, 2007; Taylor & Rohrer, 2010). However, we predicted that interleaved presentation in the present study would yield higher accuracy than blocked presentation (Prediction 2.2). The rationale for this prediction

[1] We used similar operands rather than exactly equal operands to prevent participants from solving one of the problems and then copying their answer to the other problem rather than separately solving both problems.

was that interleaving would encourage conversion of problems into whichever notation best affords the given arithmetic operation (Prediction 1.1), which would be associated with higher accuracy for that operation (Prediction 2.1).

Table 1 summarizes the predictions that were tested in the present study. Predictions 1.1, 1.2, and 1.3 involve notation preferences, as revealed by cross-notation conversions, and Predictions 2.1 and 2.2 involve accuracy.

Table 1. Predictions tested in the present study.

Number	Prediction
1.1	Conversion from one notation to the other will be more common when fraction and decimal problems are interleaved than when they are blocked
1.2	When fraction and decimal problems are interleaved, addition problems will be converted from fractions to decimals more than vice versa, whereas multiplication problems will be converted from decimals to fractions more than vice versa
1.3	On arithmetic problems with one decimal operand and one fraction operand, fractions will be converted to decimals more often on addition than multiplication problems, whereas decimals will be converted to fractions more often on multiplication than addition problems
2.1	On comparable fraction and decimal arithmetic problems, addition accuracy will be higher with decimals than fractions, whereas multiplication accuracy will be higher with fractions than decimals
2.2	Arithmetic accuracy will be higher when problems involving the same operation but different notations are interleaved than when problems are blocked by notation

Experiment 1

Participants in Experiment 1 first completed the Traditional Arithmetic Task, which involved addition and multiplication problems with either fraction or decimal operands but not both. Problems were presented in either the blocked condition or the interleaved condition, as described above. After completing the Traditional Arithmetic Task, participants completed the Mixed Arithmetic Task, which involved addition and multiplication problems with one fraction and one decimal operand. Written work on both tasks was analyzed to identify cases of

converting fractions into decimals or vice versa, affording tests of Predictions 1.1, 1.2, and 1.3. Predictions 2.1 and 2.2 were tested by analyzing accuracies on the Traditional Arithmetic Task.

Method

The experiment received approval from Florida State University Internal Review Board (IRB), study title is "Investigating Adults' Understanding of Rational Numbers and Rational Arithmetic" and the IRB approval ID is 00000429.

Participants

80 undergraduate students were recruited from a public university in the southeast US. Students participated in exchange for course credit. Three participants were excluded from analysis due to not completing either arithmetic task, leaving 77 participants, 39 in the blocked condition and 38 in the interleaved condition. Six of these (4 in the blocked condition, 2 in the interleaved condition) were excluded from analysis of the Mixed Arithmetic Task due to not completing that task. Participants were predominantly freshmen or sophomores (n = 51) and female (n = 55).

Materials

Traditional Arithmetic Task. Stimuli were 24 arithmetic problems, six for each combination of two arithmetic operations, addition and multiplication, and two notations, fraction and decimal (Table 2). The six fraction problems for each operation included two problems for each of three types of operand pairs: two fractions with equal denominators (ED), two fractions with unequal denominators (UD), and one whole number and one fraction (W-F). Similarly, the six decimal problems for each operation included two problems for each of three types of operand pairs: two decimals with equal numbers of decimal digits (EDD), two decimals with unequal numbers of decimal digits (UDD), and one whole number and one decimal (W-D).

The complexity of the problems was similar to those in a previous study of the same participant population (Braithwaite & Sprague, 2021). Accuracy in that study averaged around 80%, suggesting that such problems are appropriately difficult for this population.

Table 2. Stimuli for the Traditional Arithmetic Task.

		Notation	
Operation	Operands	Fraction	Decimal
Addition	ED/EDD	3/4+6/4	0.75+0.50
		3/5+1/5	0.6+0.8
	UD/UDD	4/5+1/4	0.8+0.75
		1/10+3/4	0.1+0.25
	W-F/W-D	3+1/5	3+0.6
		2+3/4	2+0.25
Multiplication	ED/EDD	4/5×1/5	0.8×0.6
		3/4×5/4	0.75×0.25
	UD/UDD	2/5×3/4	0.4×0.25
		3/10×1/4	0.3×0.75
	W-F/W-D	9×1/5	9×0.4
		2×3/4	2×0.25

Note. Problems appearing in the same row were matched problems. ED = fractions with equal denominators; EDD = decimals with equal numbers of decimal digits; UD = fractions with unequal denominators; UDD = decimals with unequal numbers of decimal digits; W-F = a whole number and a fraction; W-D = a whole number and a decimal.

Each fraction problem was matched with a decimal problem. Matched problems appear in the same row of Table 2. In the interleaved condition, matched problems were presented

together. To increase the likelihood that participants in that condition would convert one problem into the notation of the matched problem, matched problems involved the same arithmetic operation, the first operands in each matched pair were equal, and the second operands were related by a small multiplicative factor. For example, in the matched pair (3/4+6/4, 0.75+0.50), 3/4 equals 0.75 and 6/4 is three times 0.50. Further, ED problems were matched with EDD problems, UD problems with UDD problems, and W-F problems with W-D problems.

Mixed Arithmetic Task. Stimuli were 12 arithmetic problems with one fraction operand and one decimal operand, six addition problems (1/2+0.4; 4/5+0.3; 1/5+0.6; 0.8+6/8; 0.95+18/20; 0.44+84/100) and six multiplication problems (3/5×0.5; 9/10×0.7; 3/5×0.7; 0.5×16/20; 0.6×4/10; 0.4×2/8). Within the six problems for each arithmetic operation, the first operand was equally often a fraction or a decimal.

Other Tasks. Participants also completed a Conversion Task, which required converting fractions into decimals or vice versa, and two executive function tasks: Alpha Span (a measure of working memory; Craik, Bialystok, Gillingham, & Stuss, 2018), and Plus-Minus (a measure of shifting; Jersild, 1927). Because these tasks did not relate to our main predictions, the tasks and results from them are presented in the Supplementary Materials.

Procedure

Sessions were conducted in person and tasks were presented in paper-and-pencil format. For all arithmetic problem solving tasks, participants were instructed to solve the problem without using a calculator. Participants completed the tasks in a fixed order: Alpha Span, Plus-Minus, Traditional Arithmetic, Conversion, Mixed Arithmetic. Participants were randomly assigned to the blocked or interleaved condition, which differed only with respect to the Traditional Arithmetic Task.

In the Traditional Arithmetic Task, two problems were presented side-by-side on each page. Participants read both problems aloud, then solved them in whichever order they preferred. After solving both problems, they were asked which problem they solved first and why.

In the blocked condition, each problem was presented side-by-side with the other problem involving the same notation, arithmetic operation, and type of operands. For example, 3/4+6/4 and 3/5+1/5 were presented together. All fraction problems were presented before all decimal problems or vice versa, and within each notation, all addition problems were presented before all multiplication problems or vice versa. Orders of presentation of notations and operations were counterbalanced across participants.

In the interleaved condition, each problem was presented side-by-side with the matched problem of the other notation. For example, 3/4+6/4 and 0.75+0.50 were presented together. All addition problems were presented before all multiplication problems or vice versa, and the problem appearing on the left side of the page was either always a fraction problem or always a decimal problem. Which operation was presented first, and whether the problems on the left were fraction or decimal problems, were counterbalanced across participants. In this experiment and the next one, presenting problems in pairs precluded separately measuring how long participants took to solve each problem.

In the Mixed Arithmetic Task, participants were told they could solve each problem using decimals or fractions or in any other way they preferred and could write the answer in decimal or fraction form. Addition and multiplication problems were interleaved. The arithmetic operation of the first problem, and the notation of the first operand in the first problem, were counterbalanced across participants.

Coding and Analysis

To test the predictions regarding conversion between notations (Table 1, Predictions 1.1, 1.2, and 1.3), each trial of each arithmetic task was coded according to whether the participant converted fractions into decimals, or vice versa, either while solving the problem or in their final answer. To test the predictions regarding accuracy (Table 1, Predictions 2.1 and 2.2), responses were coded as accurate if they were numerically correct, regardless of the notation in which the responses were given. Analyses were conducted in R (R Core Team, 2020). Here and in the next experiment, all significant effects are reported.

Transparency and Openness

We report all data exclusions (if any), all manipulations, and all measures in this experiment and in Experiment 2. All data, analysis code, and research materials will be available for public access on OSF Network upon acceptance of the manuscript (https://osf.io/uzetx/?view_only=77e68c31d453422ba2e68ec639e0d1ce) .

Results

Traditional Arithmetic Task

Descriptive statistics for the Traditional Arithmetic Task are presented in Table 3.

Table 3. Mean (SD) percent trials in which conversion occurred, and accuracy, on the Traditional Arithmetic Task for each combination of operation, notation, and condition (Experiment 1).

		Conversion		Accuracy	
Operation	Notation	Blocked condition	Interleaved condition	Blocked condition	Interleaved condition
Addition	Fraction	2 (7)	12 (23)	92 (18)	84 (29)
Addition	Decimal	3 (16)	1 (6)	94 (14)	92 (17)
Multiplication	Fraction	7 (21)	14 (26)	65 (40)	75 (33)
Multiplication	Decimal	13 (29)	21 (33)	58 (33)	64 (32)

Cross-Notation Conversion. As predicted (Prediction 1.1), cross-notation conversion was more common in the interleaved condition than in the blocked condition (12% vs. 6%). This effect was significant in a mixed ANOVA with notation (fraction or decimal) and operation (addition or multiplication) as within-subjects factors and condition (blocked or interleaved) as a between-subjects factor, $F(1, 75) = 4.05$, $p = .048$, $\eta_g^2 = .02$. Conversion occurred more often on multiplication than addition trials (14% vs. 4%), $F(1, 75) = 29.00$, $p < .001$, $\eta_g^2 = .04$. Further, a notation×operation interaction was found, such that conversion occurred more often on fraction than decimal addition problems (7% vs. 2%), but occurred more often on decimal than fraction multiplication problems (17% vs. 10%), $F(1, 75) = 7.50$, $p = .008$, $\eta_g^2 = .02$.

Because Prediction 1.2 related specifically to the interleaved condition, conversion in that condition was submitted to a separate ANOVA with operation and notation as within-subjects factors. Consistent with Prediction 1.2, conversion in the interleaved condition was more common on fraction than decimal addition problems (12% vs. 1%), but was more common on decimal than fraction multiplication problems (21% vs. 14%), $F(1, 37) = 7.35$, $p = .010$, η_g^2

= .04. Conversion in the interleaved condition was also more common on multiplication than addition problems (18% vs. 7%), $F(1, 37) = 19.92$, $p < .001$, $\eta_g^2 = .05$.

Order of Solving Problems. As another test of our hypothesis regarding notation preferences, we submitted the proportion of problem pairs in the interleaved condition on which participants chose to solve the fraction problem before the decimal problem to ANOVA with arithmetic operation as a within-subjects factor[2]. The fraction problem was solved first more often on pairs involving multiplication than addition (64% vs. 45%), $F(1,35) = 13.23$, $p < .001$, $\eta_g^2 = .06$, consistent with the preferences revealed in our analyses of conversion.

Accuracy. As predicted (Prediction 2.1), in the Traditional Arithmetic Task, addition accuracy was higher with decimals than fractions (93% vs. 88%), whereas multiplication accuracy was higher with fractions than decimals (70% vs. 61%). This notation×operation interaction was significant in an ANOVA on accuracy with condition as a between-subjects factor and notation and operation as within-subjects factors, $F(1, 75) = 8.39$, $p = .005$, $\eta_g^2 = .02$. This analysis also found an effect of operation, $F(1, 75) = 62.65$, $p < .001$, $\eta_g^2 = .16$, indicating that accuracy was higher on addition than multiplication problems (90% vs. 65%).

Contrary to Prediction 2.2, accuracy did not differ between the blocked and interleaved conditions (77% vs. 79%), $p = .762$. The only other effect found was a condition×operation interaction, $F(1, 75) = 4.44$, $p = .038$, $\eta_g^2 = .01$, indicating that for addition problems, accuracy in the blocked condition was higher than in the interleaved condition (93% vs. 88%), whereas the opposite pattern appeared with the multiplication problems (70% vs. 62%). This unexpected interaction was not replicated in Experiment 2, so we refrain from interpreting it.

[2] Two participants were removed from the analysis due to missing data on which problem they chose first.

Mixed Arithmetic Task

Descriptive statistics for the Mixed Arithmetic Task are presented in Table 4. As predicted (Prediction 1.3), fraction-to-decimal conversion was more common on addition than multiplication problems (52% vs. 38%), whereas decimal-to-fraction conversion was more common on multiplication than addition problems (66% vs. 52%). One-way ANOVAs with conversion of fractions to decimals and conversion of decimals to fractions as the dependent variables and arithmetic operation as a within-subjects factor revealed effects of operation for fraction-to-decimal conversion, $F(1, 70) = 27.50$, $p < .001$, $\eta_g^2 = .03$ and for decimal-to-fraction conversion, $F(1, 70) = 24.04$, $p < .001$, $\eta_g^2 = .03$.

Table 4. Mean (SD) percent fraction-to-decimal conversion, percent decimal-to-fraction conversion, and accuracy for each operation on the Mixed Arithmetic Task (Experiment 1).

Operation	Fraction-to-decimal conversion	Decimal-to-fraction conversion	Accuracy
Addition	52 (40)	52 (40)	79 (29)
Multiplication	38 (41)	66 (39)	63 (38)

Note. The sum of fraction-to-decimal conversion and decimal-to-fraction conversion exceeds 100% because both types of conversion sometimes occurred within the same trial.

Discussion

Revealed preferences between fractions and decimals for solving problems involving different arithmetic operations were consistent with the hypothesis that fractions afford multiplication better than decimals do, whereas decimals afford addition better than fractions do. Further, accuracies on different problem types paralleled these revealed preferences. In Experiment 2, we sought to replicate these key findings with a preregistered design.

Cross-notation conversion on the Traditional Arithmetic Task occurred more often in the interleaved than in the blocked condition, and conversion was most often in the direction of the notation that was associated with higher accuracy for the problem at hand. However, accuracy on the Traditional Arithmetic Task did not differ between conditions. This null finding might reflect the fact that even in the interleaved condition, conversion between notations on the Traditional Arithmetic Task was still rather rare (12% of trials). To address this possibility, in Experiment 2, we attempted to strengthen the blocking-interleaving manipulation by instructing participants explicitly to compare problems within each pair on the Traditional Arithmetic Task.

Experiment 2

Comparison highlights both differences and similarities between the items being compared (Gentner, 1983; Loewenstein, Thompson, & Gentner, 2003; Markman & Gentner, 1993; Star et al., 2015). On this basis, we reasoned that comparing problems involving the same operation but different notations, as in the interleaved condition of the Traditional Arithmetic Task, could improve accuracy. Highlighting differences between such problems could enable participants to notice that the standard procedure for one of the problems is easier than the standard procedure for the other, and highlighting similarities could enable participants to notice that the easier procedure could be used to solve both of the problems. For example, comparing 4/5+1/4 with 0.8+0.75 might lead one to notice that the standard procedure for adding decimals is easier than that for adding fractions, and that the decimal addition procedure could be used to solve 4/5+1/4 after converting it into decimal form (0.8+0.25). Consistent with this possibility, comparing different solutions to algebra problems improves students' accuracy and flexibility in problem solving (Rittle-Johnson & Star, 2007).

In Experiment 2, we tested modified versions of the interleaved and blocked versions of the Traditional Arithmetic Task in which participants were instructed to compare the problems in each pair before solving either of them. We expected that with this modification, Experiment 2 would replicate the main findings of Experiment 1 and would yield larger effects of condition on cross-notation conversion on the Traditional Arithmetic Task than in Experiment 1. Accordingly, we also expected to find the predicted effect of condition on accuracy that did not appear in Experiment 1. Experiment 2 was preregistered at https://osf.io/9sq5u?view_only=65b0a53912004002b7863ac11c5aa7fb.

Method

Participants

Our preregistered target sample size was 80. This target was powered to detect an effect of condition on accuracy of $d = 0.50$ or larger. However, as described below, no such effect was found. With respect to the predicted effects of operation and notation, post-hoc power analysis with G*Power 3.1 (Faul, Erdfelder, Lang, & Buchner, 2007) indicated that a sample of 80 had at least 80% power to detect effects of $\eta_p^2 = 0.043$ or larger. All significant effects reported below fell into this range.[3]

Due to a management error, over-recruitment occurred, leading to a sample of 98 participants. To be consistent with our preregistration, we created a limited sample consisting of the first 80 participants completed. All planned analyses yielded the same significant effects using data from either the limited sample or the full sample. Analyses of the limited sample are reported below, and analyses of the full sample are reported in the Supplementary Materials.

[3] We report generalized eta-squared below, but this effect size is not supported by G*Power. Thus, for the power analysis described here, we calculated partial eta-squared for all effects that are reported in the Results.

The limited sample included 40 participants in the blocked condition and 40 in the interleaved condition. Participants were predominantly freshmen or sophomores (n = 58). Following our preregistered exclusion criteria, participants who did not complete a task or skipped any questions in a task were excluded from analyses of that task, including one participant for the Traditional Arithmetic task (in the blocked condition) and 25 participants for the Mixed Arithmetic Task. A few additional participants were excluded from particular analyses, as detailed in the Supplementary Materials.

Materials

Stimuli were the same as in Experiment 1, with three exceptions. First, in the Traditional Arithmetic Task, in the blocked condition, 0.6×0.8 was presented instead of 0.8×0.6 due to a programming error. Second, the Plus-Minus Task was omitted. Third, a survey of math attitudes was included. Tasks were presented in the following order: Traditional Arithmetic Task, Conversion Task, Mixed Arithmetic Task, Math Attitude Survey, and Alpha Span. Methods and results for tasks other than the two arithmetic tasks are described in the Supplementary Materials.

Procedure

The procedure was the same as in Experiment 1, with the following exceptions. First, due to the ongoing COVID-19 pandemic, interviews were conducted via Zoom rather than in person. Materials were presented on Qualtrics, and participants were asked to use their own paper to solve problems before typing in their answers. After each trial, participants showed their work to the camera and the experimenter took a screenshot to use for coding cross-notation conversions.

Second, in the Traditional Arithmetic Task, after presenting each pair of problems and before participants solved either of the problems, participants were asked which problem in the pair was easier and why. They were then instructed to solve the easier problem, after which they

were shown both problems again and asked, “What similarity do you see between these two problems, if any?” and then “Is there any way the first problem you solved would be helpful for solving the other problem?”. They then solved the second problem. When participants were solving the problems, only the problem they were solving at the moment was visible.

Third, in the Traditional Arithmetic Task, for both the blocked and the interleaved condition, the side where each problem was presented was counterbalanced across participants.

Coding and Analysis

Coding and analyses were the same as in Experiment 1. All reported analyses were preregistered unless marked as exploratory, and all preregistered planned analyses are reported. Preregistered exploratory analyses are reported in the Supplementary Material. We also reported some additional data exclusion in the Supplementary Material.

Results

Traditional Arithmetic Task

Descriptive statistics for the Traditional Arithmetic Task are presented in Table 5.

Table 5. Mean (SD) percent trials in which conversion occurred, and accuracy, on the Traditional Arithmetic Task for each combination of operation, notation, and condition (Experiment 2).

		Conversion (n = 77)		Accuracy (n = 79)	
Operation	Notation	Blocked condition	Interleaved condition	Blocked condition	Interleaved condition
Addition	Fraction	5 (19)	35 (34)	87 (26)	89 (24)
Addition	Decimal	5 (21)	6 (13)	95 (17)	96 (9)
Multiplication	Fraction	7 (24)	15 (28)	79 (35)	74 (38)
Multiplication	Decimal	17 (34)	38 (43)	69 (32)	71 (34)

Cross-Notation Conversion. As predicted (Prediction 1.1), conversion on the Traditional Arithmetic Task was more common in the interleaved condition than in the blocked condition (23% vs. 9%). This difference of 14% was larger than the corresponding difference in Experiment 1 (12% vs. 6%, a difference of 6%), suggesting that instructing participants to compare problems may have increased cross-notation conversion in the interleaved condition. This effect of condition was significant in an ANOVA on conversion with operation, notation, and condition as factors, $F(1, 75) = 13.71$, $p < .001$, $\eta_g^2 = .06$.

This ANOVA also found that participants converted between notations more often on multiplication than addition problems (19% vs. 13%), $F(1, 75) = 7.60$, $p = .007$, $\eta_g^2 = .01$. This effect was qualified by an operation×notation interaction, $F(1, 75) = 34.55$, $p < .001$, $\eta_g^2 = .07$, such that conversion occurred more often on fraction than decimal addition problems (21% vs. 5%) whereas conversion occurred more often on decimal than fraction multiplication problems (28% vs. 11%). Finally, an operation×notation×condition interaction was found, $F(1, 75) = 14.73$, $p < .001$, $\eta_g^2 = .03$, indicating that the operation×notation interaction, though directionally similar in both conditions, was larger in the interleaved condition.

Although the operation×notation interaction described above was consistent with Prediction 1.2, our preregistered prediction pertained specifically to the interleaved condition. As predicted, cross-notation conversion in the interleaved condition was more common on fraction than decimal addition problems (35% vs. 6%), but was more common on decimal than fraction multiplication problems (38% vs. 15%). ANOVA on conversion in the interleaved condition found a notation×operation interaction, $F(1, 39) = 34.97$, $p < .001$, $\eta_g^2 = .15$.

Order of Solving Problems. In an exploratory analysis, we submitted the proportion of matched problem pairs in the interleaved condition on which participants said that the fraction

problem was easier than the decimal problem, and therefore solved the fraction problem first, to ANOVA with arithmetic operation as a within-subjects factor. An effect of operation was found, $F(1,39) = 77.81$, $p < .001$, $\eta_g^2 = .45$. On multiplication trials, most participants (80%) said the fraction problem was easier, whereas on addition trials, fewer than half (35%) said so.

Accuracy. As predicted (Prediction 2.1), in the Traditional Arithmetic Task, addition accuracy was higher with decimals than fractions (96% vs. 88%), whereas multiplication accuracy was higher with fractions than decimals (77% vs. 70%). This notation×operation interaction was significant in an ANOVA with notation and operation as within-participants factors and condition as a between-participants factor, $F(1, 77) = 11.47$, $p = .001$, $\eta_g^2 = .02$. This analysis also found that accuracy was higher on addition than multiplication problems (92% vs. 73%), $F(1, 77) = 30.31$, $p < .001$, $\eta_g^2 = .10$. Contrary to Prediction 2.2, accuracy did not differ between the blocked and interleaved conditions (83% vs. 82%), $p = .943$.

Mixed Arithmetic Task

Descriptive statistics for the Mixed Arithmetic Task are presented in Table 6. As predicted (Prediction 1.3), fraction-to-decimal conversion was more common on addition than multiplication problems (56% vs. 35%), whereas decimal-to-fraction conversion was more common on multiplication than addition problems (71% vs. 48%). Each of these effects was significant in an ANOVA with arithmetic operation as a within-subjects factor, $F(1, 53) = 35.35$, $p < .001$, $\eta_g^2 = .08$ for fraction-to-decimal conversion and $F(1, 53) = 40.64$, $p < .001$, $\eta_g^2 = .11$ for decimal-to-fraction conversion.

Table 6. Mean (SD) percent fraction-to-decimal conversion, percent decimal-to-fraction conversion, and accuracy for each operation on the Mixed Arithmetic Task (Experiment 2).

Operation	Fraction-to-decimal conversion	Decimal-to-fraction conversion	Accuracy
Addition	56 (34)	48 (35)	80 (30)
Multiplication	35 (37)	71 (34)	73 (35)

Note. The sum of fraction-to-decimal conversion and decimal-to-fraction conversion exceeds 100% because participants sometimes performed both types of conversion within a single trial.

Discussion

Experiment 2 replicated the key findings of Experiment 1 regarding both cross-notation conversion and accuracy. Preregistered replication of these findings lends confidence in their reliability. Discussion of these central findings is deferred to the General Discussion.

Instructing participants explicitly to compare problems on the Traditional Arithmetic Task appears to have increased cross-notation conversion in the interleaved condition (23% in Experiment 2 vs. 12% in Experiment 1). Although assessing effects of comparison was not a goal of the present study, the findings echo those of previous research suggesting that comparison is an effective method of encouraging students to make connections between related problems (Gentner, 1983; Markman & Gentner, 1993; Rittle-Johnson & Star, 2007).

Nevertheless, as in Experiment 1, accuracy did not differ by condition. Interleaving was hypothesized to improve accuracy by encouraging participants to convert a problem into the notation associated with higher accuracy for the given arithmetic operation. The absence of such an effect may reflect conversion occurring rarely even in the interleaved condition, conversion sometimes occurring in the direction of lower accuracy (e.g., participants sometimes converted multiplication problems from fraction to decimal form even though multiplication accuracy was

lower in decimal than fraction form), effects of notation on accuracy for a given arithmetic operation being relatively small; or a combination of these factors.

Exploratory Analyses

Effects of Operands on Cross-Notation Conversion

In both experiments, effects of notation and arithmetic operation on cross-notation conversion in the interleaved condition appeared to vary depending on the types of operand pairs (ED/EDD, UD/UDD, or W-F/W-D) appearing in matched problem pairs. Specifically, the tendency to convert fractions to decimals more than vice versa for addition appeared attenuated on ED/EDD pairs, and the tendency to convert decimals to fractions more than vice versa for multiplication appeared attenuated on W-F/W-D pairs (Table 7). Although we did not predict effects of operand type, we reasoned that such effects might be informative regarding the reasons underlying notation preferences. Accordingly, we conducted exploratory analyses using pooled data from the two experiments. (Separate analyses for each experiment yielded similar results, and are reported in the Supplementary Materials.)

Table 7. Mean (SD) percent trials in which conversion occurred for different operand pair types on the Traditional Arithmetic Task in the interleaved condition.

		Types of Operands in Matched Problem Pairs		
Operation	Notation	ED/EDD	UD/UDD	W-F/W-D
		Experiment 1		
Addition	Fraction	7 (21)	13 (32)	16 (37)
Addition	Decimal	1 (8)	0 (0)	3 (16)
Multiplication	Fraction	9 (28)	9 (26)	22 (36)
Multiplication	Decimal	29 (43)	20 (39)	16 (33)
		Experiment 2		
Addition	Fraction	23 (39)	33 (47)	49 (45)
Addition	Decimal	10 (28)	10 (30)	0 (0)
Multiplication	Fraction	9 (27)	16 (33)	25 (44)
Multiplication	Decimal	44 (47)	35 (47)	30 (46)

First, we submitted cross-notation conversion on addition trials in the interleaved condition to ANOVA with experiment as a between-subjects factor and notation and operand pair type as within-subjects factors. Conversion rates varied by operand pair type (ED/EDD: 10%, UD/UDD: 12%, W-F/W-D: 17%), $F(2, 152) = 3.89$, $p = .023$, $\eta_g^2 = .01$. As expected, conversion was more common on fraction than decimal addition problems (23% vs. 4%), $F(1, 76) = 30.59$, $p < .001$, $\eta_g^2 = .10$. Critically, this effect of notation was smaller on ED/EDD pairs (15% vs. 6%) than on UD/UDD pairs (20% vs. 3%) or W-F/W-D pairs (33% vs. 1%), as indicated by a notation×operand pair type interaction, $F(2, 152) = 6.58$, $p = .002$, $\eta_g^2 = .02$.

Further, conversion on addition problems was more frequent in experiment 2 than experiment 1 (21% vs. 7%), $F(1,76) = 14.40$, $p < .001$, $\eta_g^2 = .05$. Experiment also interacted with notation, $F(1,76) = 6.26$, $p = 0.015$, $\eta_g^2 = 0.02$, and with notation×operand, $F(2,152) = 3.33$, $p = 0.038$, $\eta_g^2 = 0.01$. The latter interaction indicated that the notation×operand interaction that was found in both experiments was larger in experiment 2.

Next, we submitted cross-notation conversion on multiplication trials in the interleaved condition to ANOVA with the same factors as the previous analysis. Conversion was more common on decimal than fraction multiplication problems (29% vs 14%), $F(1,76) = 6.02$, $p = .016$, $\eta_g^2 = .03$. As in the analysis of addition trials, the effect of notation was qualified by an interaction with operand pair type, $F(2, 152) = 13.46$, Greenhouse-Geisser corrected $p < .001$, $\eta_g^2 = .02$. Conversion occurred about equally often on decimal and fraction multiplication problems (21% vs. 23%) on W-D/W-F problem pairs, but was more common on decimal than fraction problems for EDD/ED pairs (37% vs. 9%) and UDD/UD pairs (28% vs 13%).

The implications of these exploratory analyses are discussed in the General Discussion.

Effects of Cross-Notation Conversion on Accuracy

Additional analyses tested whether cross-notation conversion affected accuracy. Using pooled data from both experiments, we submitted accuracy on each trial of the Traditional Arithmetic Task to mixed logistic regression, with experiment (Experiment 1 = -1, Experiment 2 = 1), condition (blocked = -1, interleaved = 1), the interaction of experiment with condition, and whether the participant converted notations (did not convert = 0, converted = 1) as fixed effects and participant as a random effect. One regression was conducted for each problem type—fraction addition, fraction multiplication, decimal addition, and decimal multiplication. An effect of conversion was found for decimal multiplication problems, indicating that participants were

more accurate when they converted such problems into fraction form than when they did not (81% vs. 60%), estimate = 1.32, SE = 0.35, $z = 3.80$, $p < .001$. Thus, such cross-notation conversions appear to have been adaptive. No effect of conversion on accuracy was found in the other regressions, *p*s > .05.

Effect of Condition on Mixed Arithmetic Task Accuracy

To determine whether problem sequence in the Traditional Arithmetic Task affected subsequent performance, using pooled data from both experiments, we submitted the accuracy from the Mixed Arithmetic Task to a mixed ANOVA with experiment and condition as between-subjects factors and operation as a within-subjects factor. A condition×experiment interaction was found, $F(1,121) = 4.31$, $p = .040$, $\eta_g^2 = 0.03$, qualified by a condition×experiment×operation interaction, $F(1,121) = 4.23$, $p = .042$, $\eta_g^2 = 0.01$. Post-hoc *t*-tests comparing accuracies in the two conditions separately for each operation in each experiment found that in Experiment 2, accuracy on multiplication problems was higher in the blocked condition than the interleaved condition (87% vs. 59%), $t(285) = 5.92$, $p < .001$. This effect runs counter to the benefits of interleaving found in other studies, because it was not predicted and was not found in both studies, we refrain from interpreting it. The only other effect found was higher accuracy on addition than multiplication problems (80% vs. 67%), $F(1,121) = 14.40$, $p < .001$, $\eta_g^2 = 0.03$.

General Discussion

Affordances of Fractions and Decimals for Arithmetic

Fractions, decimals, and percentages all represent rational numbers, but do so in different ways that have different strengths and weaknesses. Recent research aimed at clarifying these differences has focused on the types of information that each notation is best suited to represent. For example, fractions are well suited for representing ratios between small discrete sets,

whereas decimals are well suited for representing ratios between continuous quantities (DeWolf, Bassok, & Holyoak, 2015; Gray, DeWolf, Bassok, et al., 2017; Tian, Braithwaite, & Siegler, 2020). The present study extended this line of work by investigating the affordances of fractions and decimals for solving different types of arithmetic problems.

In both experiments, participants converted fractions to decimals more frequently than vice versa on addition problems and converted decimals to fractions more frequently than vice versa on multiplication problems. Accuracies paralleled these conversion patterns, with higher accuracy on decimal than fraction addition problems and on fraction than decimal multiplication problems. Together, these findings support our central hypothesis that fractions afford multiplication better than decimals do, whereas decimals afford addition better than fractions do.

Our claim that fractions afford multiplication particularly well converges with findings of DeWolf and colleagues (2017), who asked college students to evaluate equations involving multiplication of a whole number by a fraction, such as $3 \times 4/3 = 4$. A priming effect was observed such that performance was facilitated when successive trials involved structurally inverse equations, such as $4 \times 3/4 = 3$ followed by $3 \times 4/3 = 4$. DeWolf et al. (2017) interpreted the results to indicate that fractions facilitate reasoning about multiplicative relations. Decimals were not included in DeWolf et al.'s (2017) stimuli, but our findings suggest that decimals are not as well suited as fractions for performing tasks involving multiplication.

As for addition, difficulties adding fractions—especially fractions with unequal denominators—are well-documented among both children (Siegler & Pyke, 2013; Siegler, Thompson, & Schneider, 2011) and adults (Newton, 2008; Siegler & Lortie-Forgues, 2015). To our knowledge, the present findings are the first evidence that decimals afford addition better than fractions do. However, the assumption that addition is easier with fractions than decimals

appears implicit in many math curricula. For example, the grade 4 and 5 teacher manuals of Eureka Math (Great Minds, 2015) devote 395 pages to fraction addition and subtraction but only 37 pages to decimal addition and subtraction. The corresponding numbers in Houghton Mifflin Harcourt's Go Math! (Dixon, Adams, Larson, & Leiva, 2012) are 140 pages for fraction addition/subtraction and 83 pages for decimal addition/subtraction. It is striking that despite greater instructional resources being devoted to fraction addition, accuracy remained higher on decimal addition in the present study.

Notation preferences and accuracy likely depend not only on arithmetic operation, but also on other factors that affect the difficulty of solving a problem. We speculate that increasing problem difficulty would tend to amplify differences between notations in the time or effort required to solve the problems, resulting in even stronger notation preferences (and effects on accuracy) than those observed in the present study. For example, most individuals would likely strongly prefer to solve 0.425+0.375 rather than the equivalent fraction problem 17/40+9/24. However, the notation preferences observed in the present study might be reduced or reversed for problems that are more complex in the usually-preferred notation than in the other notation. For example, despite decimals usually being preferred to fractions for addition, most individuals would likely prefer to calculate 3/8+1/8 than the equivalent decimal problem 0.375+0.125. Future research may clarify boundary conditions of the notation affordances for arithmetic identified in the present study.

Theoretical Explanations of Notation Affordances

Our central hypothesis that fractions afford multiplication and decimals afford addition was motivated by two theoretical perspectives: semantic alignment (Bassok et al., 1998) and strategy choice (Siegler, 1996). From the former perspective, fractions could be aligned with

multiplication via division, and decimals could be aligned with addition via measurement. Notation preferences for each arithmetic operation could reflect these semantic alignments. From the latter perspective, notation preferences may be viewed in terms of strategy choice, in that choosing a notation implies choosing the procedures afforded by that notation. From this perspective, fractions are preferred for multiplication, and decimals for addition, because in each case the preferred notation affords a quicker and/or easier procedure than does the other notation (see Gray et al., 2006 for a similar perspective on strategy choice).

Both explanations were plausible *a priori*, but our results favor the explanation based on strategy choice, for at least two reasons. First, this interpretation implies that accuracies should parallel notation preferences, a prediction that was confirmed. In contrast, although semantic alignments have clear implications for notation preferences, it is unclear whether or how such alignments would impact accuracy on an arithmetic calculation task. Braithwaite and Sprague (2021) recently found that when solving fraction and decimal arithmetic problems, university students generally rely entirely on procedures and only rarely refer to semantic interpretations of fractions and decimals.

Our exploratory analyses of conversion rates on problems involving different types of operands also favor the strategy choice interpretation. If notation preferences for each arithmetic operation reflected semantic alignments between notations and operations, then these preferences should not differ based on the operands. To the contrary, however, preferences did vary depending on the operands, and this variation was consistent with strategy choice considerations. Adding fractions is easier when equal denominators are involved than in other cases (Braithwaite, Pyke, & Siegler, 2017; Siegler & Pyke, 2013) because equal denominators obviate the need to convert the addends to a common denominator. Correspondingly, the preference

favoring decimals over fractions for addition was reduced when the fraction addition problems involved ED operands. Similarly, multiplying decimals is easier when a whole number operand is involved than in other cases (Braithwaite, Sprague, & Siegler, n.d.; Hiebert & Wearne, 1985) because the presence of a whole number operand simplifies the placement of the decimal point in the answer (i.e., the answer has the same number of decimal digits as the one decimal operand). Correspondingly, the preference favoring fractions over decimals for multiplication was reduced when the decimal multiplication problems involved W-D operands.

The notation preferences revealed in the present study were often larger than the effects of notation on accuracy. For example, in the interleaved condition of the Traditional Arithmetic Task of Experiment 2, cross-notation conversion was much more common on fraction than decimal addition problems (35% vs. 6%), whereas accuracy was only slightly higher on decimal than fraction addition problems (96% vs. 89%). Tian et al. (2020) and DeWolf et al. (2015) similarly found notation preferences to be generally stronger than effects of notation on accuracy when matching fractions or decimals to non-symbolic ratios. Notation preferences may be larger than effects of notation on accuracy because notation preferences reflect not only the likelihood of obtaining a correct answer with each notation, but also the time and effort required to solve a problem using each notation, as in the SCM (Siegler, 1996).

However, the present findings may also have underestimated notation effects on accuracy because our accuracy data are potentially polluted by cross-notation conversion. For example, accuracies on fraction problems included trials in which individuals converted fractions into decimals, and thus may differ from the accuracies that would be achieved if every fraction problem was solved in fraction form. This possibility could be tested by assessing accuracy

separately from preferences and requiring use of the given notation when assessing accuracy, as in the choice/no-choice method (e.g., Lemaire & Lecacheur, 2002; Siegler & Lemaire, 1997).

Implications Regarding Representational Fluency

Representational fluency refers to "the ability to work within and translate among representations" (Bieda & Nathan, 2009). Previous research on representational fluency has emphasized its importance in algebra, for example when translating between equations, tables, and graphs representing functions (Herman, 2007; Krawec, 2014; Nathan & Kim, 2007). The present findings suggest that representational fluency may also be important in learning about rational numbers. Specifically, the fact that different rational number notations are associated with higher accuracy on different arithmetic problems suggests that students should perform better on such problems if they are skilled at translating between notations and selecting the most suitable notation for the problem at hand. Consistent with this possibility, Braithwaite et al. (2022) recently found that accuracy comparing fractions versus decimals—a task that likely involves translation between notations—predicted children's fraction and decimal arithmetic accuracy when controlling for accuracy on within-notation comparisons. Rational number instruction could be enhanced by emphasizing representational fluency with different rational number notations.

Implications Regarding Effects of Interleaving Versus Blocking

Interleaving similar fraction and decimal problems increased cross-notation conversions, especially when interleaving was accompanied by an instruction to compare matched problems. These findings suggest that interleaving helped participants to make connections between problems. The absence of a positive effect of interleaving on accuracy could reflect beneficial effects of interleaving on conversions being counteracted by negative effects of interleaving on

performance, as found in previous studies (Rohrer et al., 2020; Rohrer & Taylor, 2007; Taylor & Rohrer, 2010). Alternatively or additionally, the absence of an interleaving benefit could reflect positive effects of blocking, which have previously been found in categorization studies involving low within-category similarity (Carvalho & Goldstone, 2014). In the present study, blocking could have highlighted similarities between problems involving the same notation and operation, making the use of correct solution strategies mutually reinforcing within each pair of problems.

Interleaved sequence often improves learning outcomes despite negative effects on practice accuracy (Rohrer et al., 2020; Rohrer & Taylor, 2007; Taylor & Rohrer, 2010). Thus, in the context of an intervention (unlike the present study, which provided no instruction or feedback), interleaving fraction and decimal arithmetic problems during practice may improve learning despite the absence of an immediate performance benefit. This possibility seems worth investigating in the future.

Conclusion

Different rational number notations differentially afford not only representing different types of information, but also solving different types of problems. Fractions are preferred to decimals for multiplication, whereas decimals are preferred to fractions for addition. These preferences are adaptive, in the sense of favoring the notation that is associated with less complex solution procedures and higher accuracy for the problem at hand. Thus, the theoretical perspective of adaptive strategy choice (Siegler, 1996) provides valuable insights regarding preferences among symbolic representations in math.

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