Three-Way Frequency Beamsplitter

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Abstract: We perform classical unitary conversion between three frequencies mediated by Bragg-scattering four-wave mixing with three pump fields. In the quantum regime, such a scheme can be scaled to realize *N*-frequency-bin *W*-states and boson sampling.

Frequency-domain processing presents advantages in both classical and quantum photonic networks, since the ability to multiplex many frequencies allows for resource-efficient transfer of information across numerous channels. For quantum information processing applications, the frequency degree of freedom has been used to define frequency-bin qubits with the potential to scale favorably with respect to losses as compared with their spatial/polarization qubit counterparts [1]. Manipulation of these frequency-bin qubits necessitates the realization of controlled unitary interactions between the frequency modes. Recent progress has seen the development of frequency-domain processing for qubits represented by two frequency modes (e.g., [2-4]), as well as a frequency-domain tritter [5]. Experiments have used two approaches: i) nonlinear parametric processes and ii) electro-optic modulation. With parametric processes, Bragg scattering four-wave mixing (BS-FWM) has been effective in producing high-fidelity quantum operations and near-unity efficiencies. In this nonlinear process, two strongly-pumped frequency modes induce noiseless frequency translation between two weak-amplitude frequency modes (signal/idler), equivalent to rotating a qubit on the Bloch sphere [6]. The process phase-matches interactions between signal/idler frequencies mirroring the pump frequencies symmetrically about the zero-dispersion frequency, as in Fig. 1a. The conversion process can be made frequency-selective due to higher-order dispersion engineering, which leads to a finite conversion efficiency bandwidth shown in Fig. 1b. Although two-pump BS-FWM has been well-studied (e.g. [4,6,7]), the case of $N > 2$ pumps has received limited experimental treatment [8].

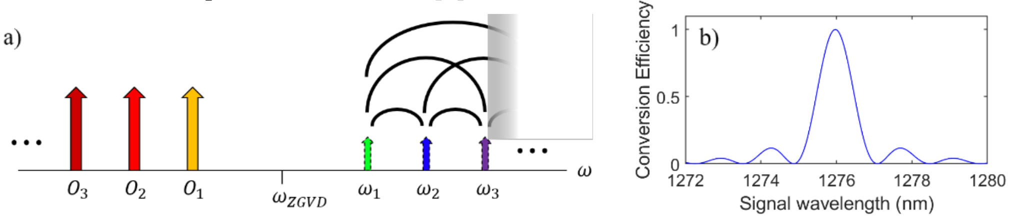

Fig. 1. a) Classical pumps (left) lead to pairwise coupling among the weak fields (right). b) Simulated phasematching for two-pump BS-FWM.

In this work, we present analysis of *N*-pump parametric mixing and show, to the best of our knowledge, the first experimental demonstration of three-pump BS-FWM using an attenuated coherent state as input signal and low-loss (< 2 dB [1]) conversion between the signal field and the two idlers. These results represent a significant step towards achieving *W*-states not only for three frequencies but for arbitrarily higher *N*, limited eventually by heralding probability and fiber dispersion.

Our theoretical analysis for *N*-pump BS-FWM assumes the phase-matching principle of symmetric frequency placement extends to every pair of pumps, each of which induces interactions between a symmetrically situated pair of signal/idler fields. The unitary evolution for *N*-pump four-wave mixing after propagating a distance L may be written as $a_i(L) = e^{-i\Delta_i L} \sum_j (e^{iHL})_{ij}\, a_j(0)$, where $a_i(z)$ is the i^{th} field at propagation distance z, Δ_i is the phase mismatch between weak fields 1 and i (and the corresponding pumps), and H is a Hermitian matrix with elements

$$H = \begin{pmatrix} \ddots & & 2\gamma A_i^* A_j \\ & \Delta_i & \\ 2\gamma A_i^* A_j & & \ddots \end{pmatrix}, \qquad (1)$$

with A_i representing the slowly-varying complex amplitude of the i^{th} pump field at $z = 0$ and γ the effective nonlinearity (units of $W^{-1}m^{-1}$). The off-diagonal elements of H can be interpreted as the couplings between the modes, controlled by the pump amplitudes and phases. As H is Hermitian, this evolution is manifestly unitary, and therefore *N*-pump BS-FWM effects rotations belonging to the *N*-dimensional unitary group U(*N*). However, while the most general rotation is parametrized by N^2 real numbers, the above matrix H is determined by only $O(N)$ parameters.

We perform three-pump BS-FWM, achieving strong conversion between three signal frequencies in the O-band. The pump fields are generated via amplified pulses (6 ns, 500 kHz repetition rate) in the C-band and sent in conjunction with a strongly attenuated CW signal (1279 nm) into the nonlinear fiber. The nonlinear fiber is cooled in liquid nitrogen to suppress noise due to Raman scattering. After propagating 100 m in the fiber, the pump fields are filtered from the signal/idler fields, which are then sent through free-space filters followed by superconducting nanowire single-photon detectors (detector efficiency >90%). We observe no parasitic processes in the 2-pump process of largest bandwidth, suggesting the three-pump process only couples three frequencies. Raw count rates are corrected for wavelength-dependent losses such that the counts add to a constant, which follows from the unitary evolution of the fields.

The resulting measured three-wave conversion is depicted in Fig. 2, along with a numerical fit to a matrix of the form of Eqn. (1). The analytical solutions can be written $a_i(z) = \sum_{j,k} c_{ijk} e^{i\lambda_j z} a_k(0)$, where the c_{ijk} and λ_j are constants determined by the nonlinearity, pump powers, and phase mismatch terms. The probability of detecting a given frequency is thus determined by the square of the sum over complex exponentials, giving rise to the rich sinusoidal dependence seen in our simulations and confirmed by our measurements (Fig. 2). This system represents an efficient three-port beamsplitter than can be used for a number of quantum applications.

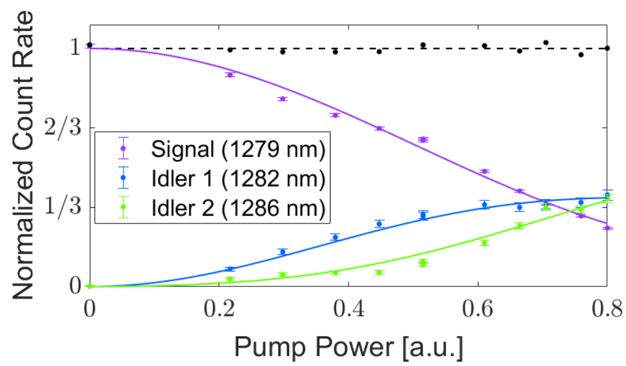

Fig. 2. Normalized count rates along with theoretical fit. The black curve is the sum of all three rates.

One of the key applications for this three-pump frequency mixer with a single-photon input is the generation of W-states in the frequency domain (up to a trivial phase): $|100\rangle \rightarrow_{z=L} 1/\sqrt{3}\left(e^{i\phi}|100\rangle + |010\rangle + |001\rangle\right)$, where each 'qubit' corresponds to a frequency mode. This occurs deterministically up to N=4 pumps, and beyond this N-dimensional frequency-bin W states may be heralded with probability $4/N(1-1/N)$. Although generation becomes probabilistic as $1/N$, the optimal conversion point occurs for $2\gamma PNL = \pi$ (where P is the power of one pump): that is, for *constant* total power overhead as N increases. This reflects the intuition that all N pumps contribute to the mixing. Another application of such an N-frequency mixer is the ability to perform N-dimensional unitary transformations that can be used for boson sampling [9], qudit gates, and quantum interference with 2 and more photons. Our setup is low-loss, and there is no fundamental restriction to scaling the number of interacting frequencies by multiplexing more pumps. Finally, this scheme can be integrated, potentially combining more than one pump configuration in succession with the goal of implementing arbitrary unitary transformations.

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