Conformal Irregular-Grid Metasurfaces Comprising a Dense Array of Free-Standing Dipoles

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***Abstract*—For large or infinite metasurfaces, a design technique for a dense array of subwavelength resonators on an irregular grid is presented. For a given incident wave, the desired induced dipole moment distribution determines the local electric field that excites individual meta-atoms. The interaction field that accounts for mutual coupling is evaluated via a combination of discrete coupling from nearby resonators and continuous sheet current coupling from far-separated resonators. Meta-atoms placed on an irregular grid can be treated, greatly enhancing the flexibility in surface profile in practical conformal metasurfaces.**

I. Introduction

As a surface distribution of resonant inclusions, electromagnetic metasurfaces [1] have demonstrated a wide range of wave transformation capabilities in a thin form factor with low loss. Many metasurfaces are 2-D periodic structures, taking advantage of efficient and accurate numerical analysis of a unit cell under periodic boundary conditions. The condition of identical cells can be relaxed to allow position-dependent local transmission and reflection properties.

The unit-cell approach forces the grid for meta-atom placement to be regular and, as a result, limits the surface to a plane. For planar profiles, meta-atoms at non-periodic or random locations may be beneficial for field localization purposes [2] or for modeling self-assembly methods [3]. A regular grid may be deformed to conform to a cylindrical surface, but it is fundamentally incompatible with a doubly-curved surface. The discrete dipole approximation (DDA) has been applied to design meta-atoms and their aperiodic spatial distributions in metamaterials and metasurfaces of finite dimensions [4], [5].

In this paper, a design method is presented for metasurfaces comprising a dense array of free-standing resonators on an irregular grid of infinite or very large dimensions. Mutual coupling effects between meta-atoms are evaluated as a sum of discrete dipole and continuous current sheet contributions. Meta-atoms are individualized using the intrinsic polarizability relation. A large parabolic reflector as a conformal dipole array is designed and numerically analyzed for validation.

II. Current Homogenization

For scattering by a metasurface modeled as an electrically polarizable surface of zero thickness, the transmission and reflection characteristics are characterized by the induced electric surface current density $\mathbf{J}_s$. Such a current distribution may be realized by a dense array of subwavelength dipole resonators. For an x-polarized sheet current, Fig. 1 illustrates the equivalence between continuous and discrete sources,

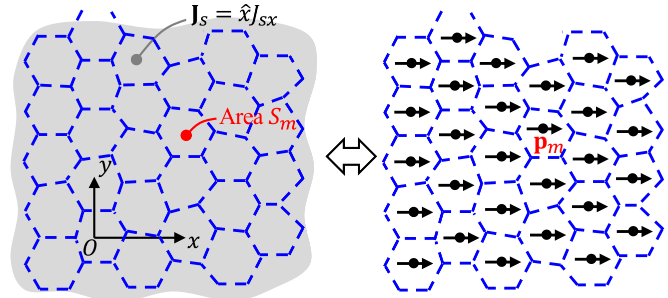


Fig. 1. Equivalence between a sheet electric current and a dipole array.

where the dipole moment $\mathbf{p}_m$ representing a cell with an area S_m is given by

$$\mathbf{p}_m = \frac{S_m}{j\omega}\mathbf{J}_s. \tag{1}$$

If a meta-atom has a dipole polarizability α, the dipole moment is given by

$$\mathbf{p} = \alpha\mathbf{E}^{\text{loc}}, \tag{2}$$

where $\mathbf{E}^{\text{loc}}$ is the local E-field that excites the dipole. For a given $\mathbf{J}_s$ derived from a desired scattering response, all $\mathbf{p}_m$'s are known. Determination of $\mathbf{E}^{\text{loc}}$ for a meta-atom will find α via (2), fixing the resonator design of choice for realizing the particular polarizability value.

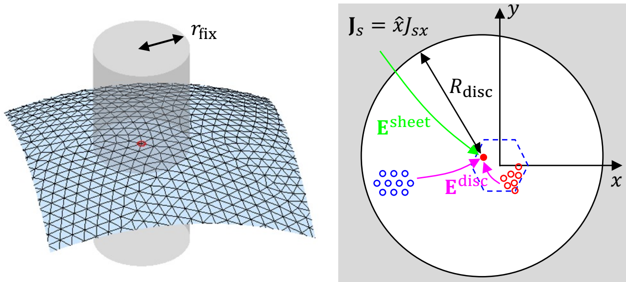
III. Individualization of Meta-Atoms

Let us consider a planar surface or a conformal surface with large radii of curvature of infinite extent or very large dimensions that can be approximated to be infinite. An unstructured mesher [6] provides a conformal grid as shown in Fig. 2(a), where meta-atoms are to be placed at nodes. For a meta-atom under consideration, the local conformal surface is approximated by the tangent plane at the node. Nearby nodes within a distance r_{fix} from the meta-atom are projected onto the tangent plane. Designing the meta-atom reduces to finding $\mathbf{E}^{\text{loc}}$ in the plane with neighboring meta-atom locations provided by the mesh generator.

The meta-atom under design is shown as a solid red dot in Fig. 2(b). The local E-field at this meta-atom is decomposed as a sum of external and interaction fields, $\mathbf{E}^{\text{loc}} = \mathbf{E}^{\text{ext}} + \mathbf{E}^{\text{int}}$, where

$$\mathbf{E}^{\text{int}} = \mathbf{E}^{\text{sheet}} + \mathbf{E}^{\text{disc}}. \tag{3}$$

Centered at the meta-atom under design, a circular disc of radius R_{disc} is formed, as shown in Fig. 2(b). Coupling from



(a) (b)

Fig. 2. Local E-field determination for a meta-atom. (a) A cylinder of radius r_{fix} for incorporating accurate positions of nearby meta-atoms in discrete coupling evaluation in the tangent plane. (b) Partition of the infinite planar array into discrete and continuous source regions.

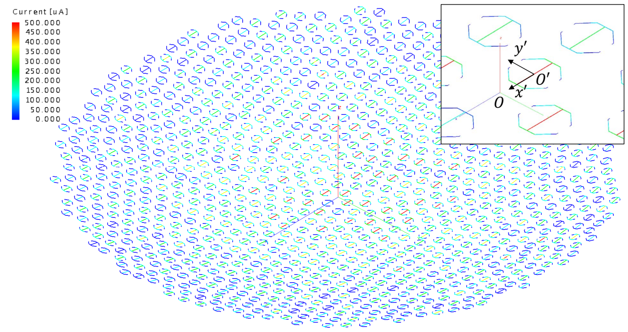


Fig. 3. The induced current distribution on dipole meta-atoms for a metasurface parabolic reflector of a 10λ diameter and a 10λ focal length. The inset shows a magnified view near the origin (paraboloid vertex).

outside the disc is evaluated as that due to a continuous current, giving [7]

$$\mathbf{E}^{\text{sheet}} = -\mathbf{J}_s \frac{\eta}{4}\left(1 - \frac{1}{jkR_{\text{disc}}}\right) e^{-jkR_{\text{disc}}}, \tag{4}$$

where k and $\eta \approx 377\ \Omega$ are free-space wavenumber and intrinsic impedance, respectively. The DDA is applied to determine coupling from individual meta-atoms inside the disc, treating each meta-atom as a point dipole. Since physical meta-atoms behave differently from ideal point dipoles, (2) tends to slightly overestimate the required resonator size. A reference periodic metasurface using identical meta-atoms can be analyzed and a calibration factor is introduced by comparing the DDA-based estimation via (2) and the prescribed physical meta-atom size.

The proposed technique allows accurate and efficient evaluation of the local E-field, and therefore meta-atom design, accounting for irregular meta-atom positions. An unstructured mesh provides a conformal grid, where meta-atom spacing is more-or-less uniform. The design time scales linearly with the total number of meta-atoms.

IV. Numerical Example

For a doubly-curved metasurface for full reflection, a parabolic reflector of a diameter $D = 10\lambda$ (λ = free-space wavelength) and a focal length $F = 10\lambda$ is designed for a prime-focus reflector antenna application. At a frequency $f = 300$ MHz, a paraboloidal surface mesh containing 874 nodes is generated, where the nominal edge length is 0.32 m.

The meta-atom is a thin-wire dipole of wire radius $a_{\text{wire}} = 1$ mm with a wire load at both ends along an octagon perimeter contained in a circle of radius $a = 0.1$ m. The length of the wire end loads is determined for a reference periodic metasurface on a hexagonal grid to have resonance at 300 MHz, fully reflecting a normally incident plane wave. For a feed wave generated by an x-directed electric current source with a $\cos^q\theta$-type pattern adjusted for a −10 dB edge illumination, the axes of the dipole-meta atoms are oriented to align with the incident E-field. Using $r_{\text{fix}} = 1.12$ m, the meta-atom size a is individualized for each resonator following the procedure in Section III. It ranges from 0.09871 m to 0.10170 m, with an average of 0.100515 m. The retrieved dipole dimensions are subsequently reduced by 0.44% via the calibration process.

The metasurface reflector illuminated by the pattern-defined feed wave is modeled and numerically analyzed using Altair FEKO. The induced current magnitude distribution together with geometry details is shown in Fig. 3. The predicted maximum directivity is 29.09 dBi, practically reproducing 29.05 dBi when a solid conductor reflector is used.

V. Conclusion

A design method for dense metasurfaces on an irregular or unstructured grid for meta-atom placement has been presented. Local fields that excite individual dipoles are efficiently evaluated as a sum of nearby discrete dipole and far-separated sheet current contributions. The proposed technique provides a pathway for conformal metasurfaces to achieve performance comparable to that of planar periodic counterparts.

ACKNOWLEDGEMENT

This work was supported by National Science Foundation grant ECCS-1930032.

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