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Vibration-based sound power measurements of arbitrarily curved panels

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ABSTRACT:
Vibration-based sound power (VBSP) measurement methods are appealing because of their potential versatility in application compared to sound pressure and intensity-based methods. It has been understood that VBSP methods have been reliant on the acoustic radiation resistance matrix specific to the surface shape. Expressions for these matrices have been developed and presented in the literature for flat plates, simple-curved plates (constant radius of curvature in one direction), and cylindrical- and spherical-shells. This paper shows that the VBSP method is relatively insensitive to the exact form of the radiation resistance matrix and that computationally efficient forms of the radiation resistance matrix can be used to accurately approximate the sound power radiated from arbitrarily curved panels. Experimental sound power measurements using the VBSP method with the simple-curved plate radiation resistance matrix and the ISO 3741 standard method are compared for two arbitrarily curved panels and are shown to have good agreement. The VBSP method based on the simple-curved plate form of the radiation resistance matrix is also shown to have excellent agreement with numerical results from a boundary element model, which inherently uses the appropriate form of the radiation resistance matrix.
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I. INTRODUCTION

Sound power is a standard metric commonly used to evaluate the amount of noise radiated from a source. The International Organization for Standardization (ISO) has published ten standards and two technical specifications describing methods for measuring sound power. The accuracy of the results from these ISO methods are rated as precision (Grade 1), engineering (Grade 2), and survey (Grade 3).[1] The ten ISO standards are pressure or intensity-measurement based and the two technical specifications are vibration measurement based. The vibration-based technical specifications only provide engineering (Grade 2) or survey (Grade 3) results. Vibration-based measurements are of interest if significant background noise is present, other methods are difficult to apply, and/or sound power measurements for a specific or partial source are required in the presence of other noise sources. Because of the potential versatility, the development of a vibration-based sound power (VBSP) measurement method that can provide precision (Grade 1) results is desirable. The motivation of this work is to further develop and validate a practical method for measuring sound power, based on vibration-based measurements, that could provide accurate *in situ* measurements of sound power.

VBSP methods are based on the theory of elementary radiators that was developed in the early 1990s.[2–8] In the past, volume velocity methods and lumped parameter models have also been used for computing sound power.[9–11] More recently, development of a VBSP method that has enabled sound power from flat plates, simple-curved plates (constant radius of curvature in one direction), and cylindrical-shells to be accurately measured has been presented in the literature.[12–14] As many commercial products and useful devices that radiate noise have more complex geometries than these simple shapes, the VBSP method needs to be extended to account for arbitrarily curved surfaces. This extension of the VBSP method is the focus of this paper.

The basic equation for computing sound power using the VBSP method is given by

$$P(\omega) = \mathbf{v}_e^H(\omega)\mathbf{R}(\omega)\mathbf{v}_e(\omega), \quad (1)$$

where $\mathbf{v}_e$ is a vector of the surface normal velocity of each elementary radiator, $(\cdot)^H$ is the Hermitian transpose, $\mathbf{R}$ is the acoustic radiation resistance matrix, and ω is the frequency.[8] Previous work has established specific forms of the radiation resistance matrix for basic surface shapes, but a specific form for arbitrarily curved surfaces has not yet been developed. However, work presented here indicates that a specific form of the radiation resistance matrix that closely matches the geometry for every useful surface shape may not be required to obtain accurate results.

In this paper, the sensitivity of sound power results to specific forms of the radiation resistance matrix is explored

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to support using an existing, computationally efficient form of the radiation resistance matrix to provide sound power results for arbitrarily curved panels. This exploration specifically compares the radiation resistance matrix expressions for flat plates, simple-curved plates, and cylindrical-shells. The results show that errors in the calculated sound power can be small when using any form of the radiation resistance matrix for many cases. Experimental sound power measurement results from two arbitrarily curved panels, obtained using the VBSP and ISO 3741[15] methods, show excellent agreement despite using the simple-curved plate form of the radiation resistance matrix in the VBSP calculations.

To further support the use of simple and computationally efficient forms of the radiation resistance matrix for arbitrarily curved panels, numerical models of the experimental arbitrarily curved-panels were created. The elemental velocities computed by the numerical models were used in the VBSP method to compute the sound power with the simple-curved plate radiation resistance matrix. These results were compared to the sound power computed directly from the numerical boundary element model (BEM), which inherently computes the appropriate radiation resistance matrix for the structure. Results from the two numerical data-based approaches show excellent agreement.

II. SOUND POWER SENSITIVITY TO BASIS FUNCTION

The VBSP method uses the radiation resistance matrix and surface normal velocity measurements to compute sound power. Previous research has established the form of the radiation resistance matrix for flat plates, cylindrical- and spherical-shells, and simple-curved plates.[8,12–14] After presenting the radiation resistance matrix expressions for these simple geometries, this section investigates the differences in sound power obtained using these expressions.

A. Radiation resistance matrix expressions

For simple flat plates, the radiation resistance matrix is given by

$$R_{\mathrm{flat},ij} = \frac{\rho_0 \omega^2 S_e^2}{4\pi c}\frac{\sin k d_{ij}}{k d_{ij}}, \qquad (2)$$

where ρ_0 is the density of the surrounding fluid, ω is the angular frequency, S_e is the area of a single discrete element, c is the speed of sound in the fluid, k is the acoustic wavenumber, and d_{ij} is the distance from the ith to the jth element.[8] This form of the radiation resistance matrix is relatively simple and computationally efficient, providing a useful basis function to apply the VBSP method for flat plates.[12]

The form of the radiation resistance matrix, or basis function, for cylindrical-shells has been recently derived and validated.[13] This expression is given by

$$R_{\mathrm{cyl},ij} = \frac{S_e^2 \omega \rho_0}{a\pi^2}\sum_{m=0}^{\infty}\cos\left[m(\theta_i - \theta_j)\right] \times \int_0^k \frac{1}{k_r}\mathrm{Im}\left\{\frac{H_m^{(2)}(k_r a)}{H_m^{(2)'}(k_r a)}\right\}\cos\left[k_z(z_i - z_j)\right]dk_z, \qquad (3)$$

where $S_e = a\Delta\theta\Delta z$ is the area of a single discrete element, a is the radius of curvature, θ_i, θ_j, z_i, z_j are the physical coordinates of the ith and jth elements, as shown in Fig. 1, k is the acoustic wavenumber, $k_r = \sqrt{k^2 - k_z^2}$, k_z is the axial wavenumber, and $H_m^{(2)}(x)$ is the m th-order Hankel function of the second kind. This form of the radiation resistance matrix is more computationally expensive than the flat plate expression.

Using eigenfunction expansion and the uniform theory of diffraction the cylindrical shell radiation resistance matrix can be modified to model simple-curved plates as given by

$$R_{ij} = -\frac{\rho_0 \omega S_e^2}{4\pi d_{ij}} Im\left\{V(\xi)e^{-jkd_{ij}}\right\}, \qquad (4)$$

where $V(\xi)$ is the hard Fock V coupling function with argument $\xi = t\left[k\cos^4\psi/(2a^2)\right]^{1/3}$, $t = \sqrt{(z_i - z_j)^2 + a^2\phi^2}$ is the distance traversed across the curved surface, $\psi = \tan^{-1}(z_i - z_j/a\phi)$ is the angle between the direction of propagation and the cylinder axis, $\phi = \theta_i - \theta_j$, and the non-subscripted $j = \sqrt{-1}$.[14] The hard Fock V coupling function, $V(\xi)$, has been sufficiently characterized to produce useful series representations with ten terms or less, with these terms given in an Appendix in Ref. 16. This form of the radiation resistance matrix is still relatively complex but is less computationally expensive than the form for cylindrical-shells and is useful for applying the VBSP method to simple-curved plates.

Computational efficiency is an important consideration in the development of VBSP methods. Given that the forms of the radiation resistance matrix are relatively complex for geometries with curvature, computation of the cylindrical-shell and simple-curved plate forms are less efficient than the flat plate form. If the specific form of the radiation

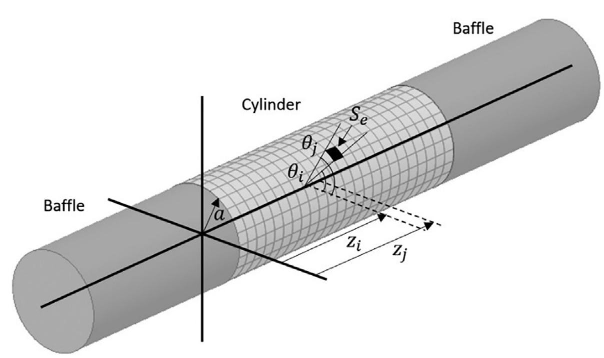

FIG. 1. A diagram of the infinitely baffled cylinder geometry with a discretization of the non-rigid portion of the cylinder shown. Variables of interest from Eq. (3) are illustrated with a single element of the discretized surface highlighted in black.

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resistance matrix does not significantly affect the accuracy of the sound power results, it may be preferable to use a computationally efficient form of the radiation resistance matrix.

B. Sensitivity of sound power computation to form of radiation resistance matrix

The sensitivity of the sound power calculations to the form of the radiation resistance matrix was investigated by comparing computational sound power results from the forms provided in Eqs. (2)–(4). The theoretical structure under test was a 2 mm thick, 31.5 cm × 31.5 cm, aluminum plate curved to various constant radii of curvature in one direction. Each plate was discretized with a 21×21 point grid. The velocity at each grid point was calculated with respect to frequency using the expression,

$$v_n(\theta, z) = \tilde{v}_{pq}\sin(p\pi\theta/\theta_{\max})\sin(q\pi z/h), \tag{5}$$

where $\tilde{v}_{pq}$ is the input amplitude, p and q are mode numbers, $\theta = x/a$ and $\theta_{\max} = w/a$ where x is the coordinate arc length along the width and a is the radius of curvature, w and h are the width and height of the plate when flat, and z is the coordinate location along the height of the plate. Equation (5) is a modified form of the surface velocity expression for simply supported flat plates. Figure 2 illustrates an example of the theoretical test structure and how the radius was applied, with other variables shown for clarity. Sound power was computed using the radiation resistance matrix for flat plates, cylindrical-shells, and simple-curved plates to apply the VBSP method based on the above theoretical inputs.

The differences between the various forms of the radiation resistance matrix are best observed by plotting the differences in calculated radiated sound power (in dB) between each of the matrices. This was done over the frequency range from 100 Hz to 2 kHz.

Figure 3 shows the results from comparing the sound power calculated using the simple-curved plate and flat-plate formulations of the radiation resistance matrix. The difference in sound power was computed by subtracting the flat plate result from the simple-curved plate result. As any difference shown is positive, the results indicate that the flat plate results represent a lower bound on sound power. The results show that there is little difference in using either formulation for larger radii of curvature and higher frequencies. There are larger differences in the sound-power results when the curvature is tight (below 20 cm) and the frequency is below 700 Hz but otherwise the differences are at most 2 dB and very often essentially zero. Figure 4 shows similar results from comparing the cylindrical-shell and flat-plate formulations of the radiation resistance matrix. Figure 5 shows the comparison between the simple-curved plate and cylindrical-shell formulations of the radiation resistance matrix. The basic trend of the largest difference in sound power results occurring only when the curvature is tight and the frequency is below 700 Hz holds for all three comparisons presented.

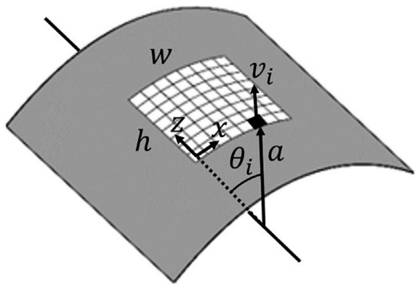

FIG. 2. A diagram of a simply supported curved plate with partial discretization of the surface shown and element i highlighted in black. This is an example of the theoretical structure applied, with various radii of curvature and surface velocity from Eq. (5), to produce the results in Figs. 3–5.

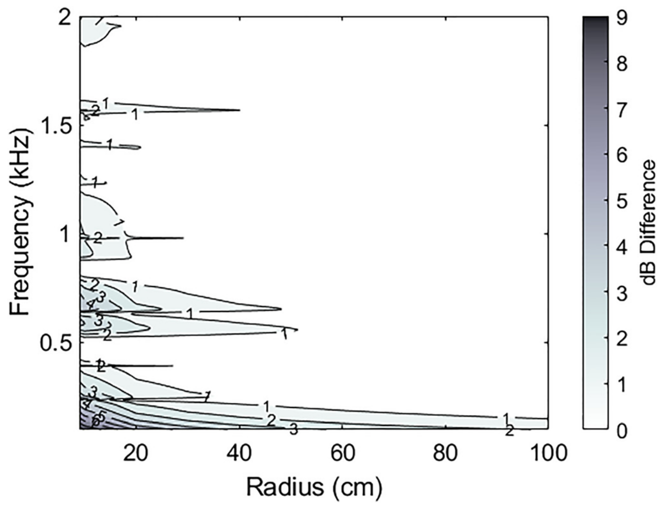

FIG. 3. (Color online) Difference in sound power between the simple-curved plate and flat plate formulations of the radiation resistance matrix using the VBSP method.

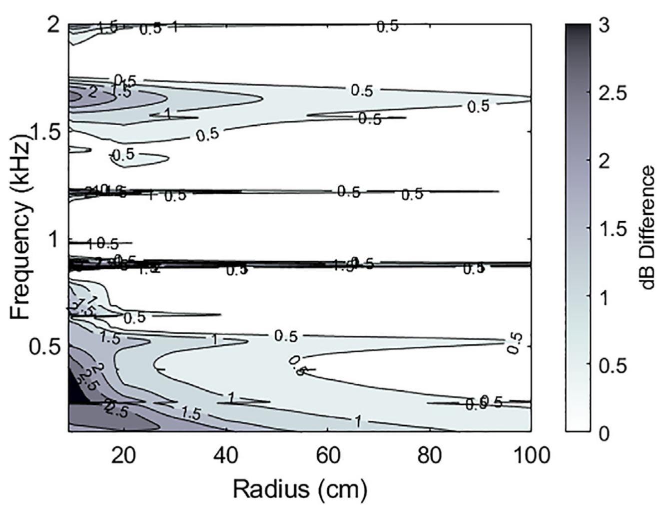

FIG. 4. (Color online) Difference in sound power between the cylindrical-shell and flat plate formulations of the radiation resistance matrix using the VBSP method.

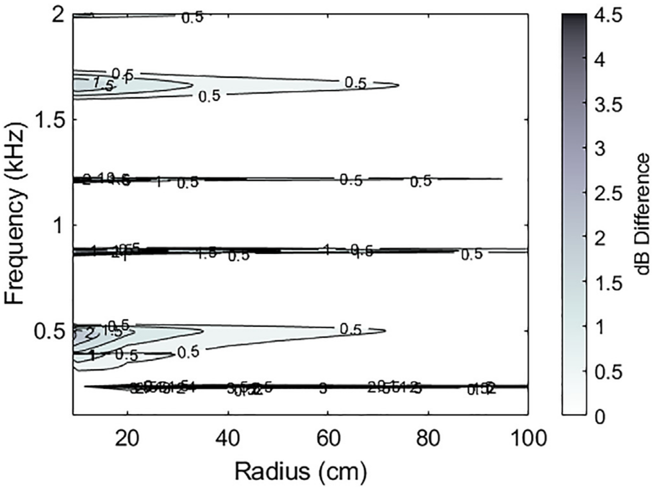

FIG. 5. (Color online) Difference in sound power between the curved-plate and cylindrical-shell formulations of the radiation resistance matrix using the VBSP method.

III. EXPERIMENTAL VERIFICATION OF SOUND POWER CALCULATIONS

To explore computing the sound power for arbitrarily curved plates using the simple-curved plate radiation resistance matrix, experimental results from the VBSP and ISO 3741 methods were compared. Two arbitrarily curved panels with significantly different shapes were used in the comparison. The experimental results are reported in one-third octave bands as per ISO 3741.

A. Experimental setup and measurement of two arbitrarily curved panels

The two arbitrarily curved panels used to obtain the experimental results presented in this work have similar boundary conditions but significantly different shapes of curvature. The arbitrarily curved panels are identified as the S-curved panel and M-curved panel, based on their basic curvature shape. These panels were shaped with a non-constant radius of curvature and are thus considered arbitrarily curved, but like a simple-curved plate, they have curvature in only one direction.

Each arbitrarily curved panel is composed of a thin aluminum panel formed in a curvature that is clamped along each straight edge in a heavy steel frame and baffled with solid aluminum caps along each curved edge (see Fig. 6). A silicone sealant bead was applied on the inside of the panel along the curved edges to prevent any sound radiation from the back of the panel escaping into the measured field (see Fig. 7). The basic dimensions of each arbitrarily curved panel are summarized in Table I.

The arbitrarily curved panels were mounted flush against one wall of a reverberation chamber with approximate dimensions 5 m × 6 m × 7 m. An infinite baffle was approximated by mounting the panels on the wall of the reverberation chamber. The edges of the frame holding the arbitrarily curved panels were sealed to the wall using Gaff tape to prevent acoustic radiation from the back of the panels escaping into the measured field. A piezoelectric transducer was attached to the inside surface of each panel within 4 cm of one corner. The piezoelectric transducer was activated with a pseudo random signal from 0 to 12.8 kHz.

A Polytec PSV-500-three-dimensional (3D), scanning laser Doppler vibrometer (SLDV), was used to measure the velocity response of the arbitrarily curved panels. One advantage of using an SLDV is that a large number of measurement points can be easily used to measure a surface. To enable the 3D features of the SLDV, precise registration points with known spatial locations with respect to a global coordinate system were placed on each panel using a 3D coordinate measurement arm. Each arbitrarily curved panel was scanned in three or four sections and then each section was stitched together to form a complete surface velocity response. The spatial resolution of scan points for each panel was a maximum of 6.7 mm, which gives a minimum of six scan points per structural wavelength for frequencies up to 10 kHz. The VBSP method requires the magnitude of the normal component of the surface velocity to compute sound power. The SLDV was also used to measure the

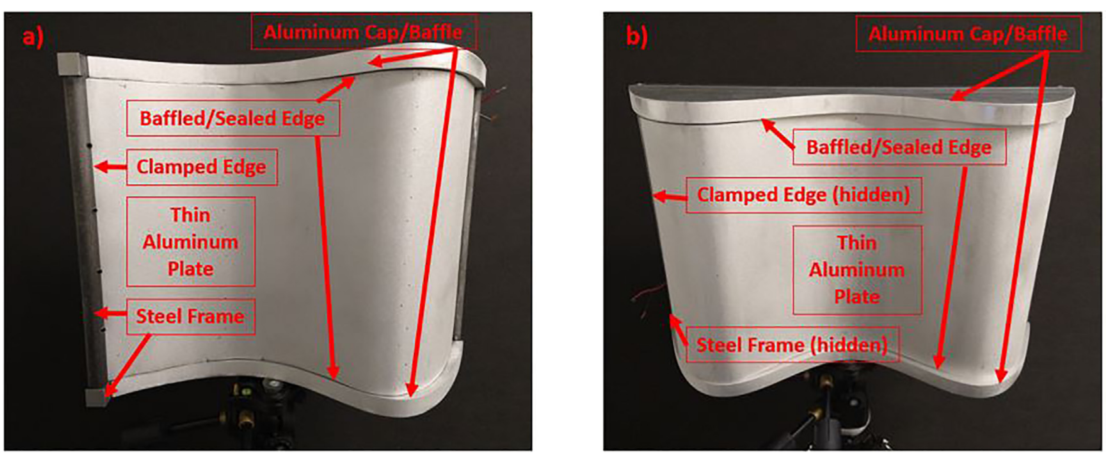

FIG. 6. (Color online) Images of the outside surfaces of the (a) S-curved panel and (b) M-curved panel to illustrate the general build of the arbitrarily curved plates used in this work. Coordinate frame included in (a) for reference, see Fig. 8.

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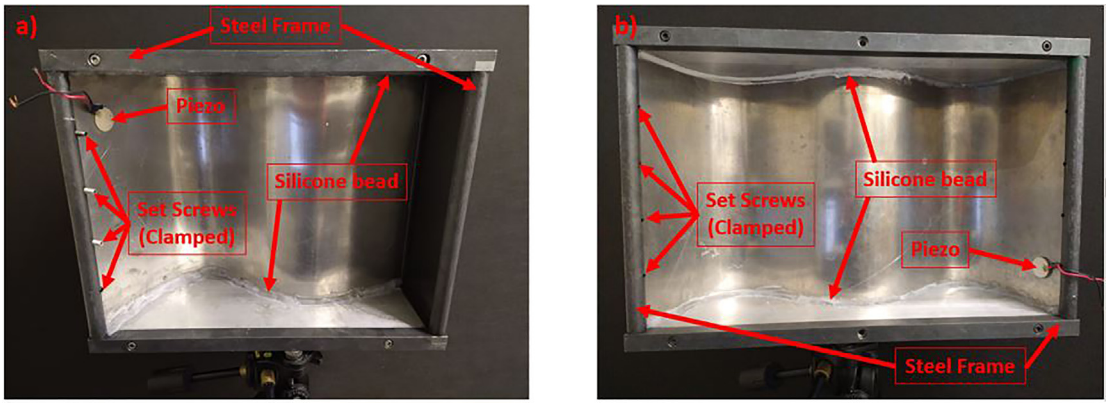

FIG. 7. (Color online) Images of the inside surfaces of the (a) S-curved panel and (b) M-curved panel to illustrate the general build of the arbitrarily curved panels used in this work.

surface geometry, from which the normal direction at each scan point was determined and then used to compute the normal component of velocity.

Experimental sound-power results were computed using the VBSP (SLDV velocity and geometry data) and ISO 3741 (microphone sound-pressure data) methods. These results are presented and compared in the following section. Both sets of results are presented and compared in one-third octave bands as per ISO 3741. The VBSP method was employed using the simple-curved plate radiation resistance matrix given by Eq. (3).

B. Sound power results of two arbitrarily curved panels

1. Applying the simple-curved plate radiation resistance matrix

The form of the simple-curved plate radiation resistance matrix assumes a constant radius of curvature, a, and also includes an angle ϕ that is determined using this constant radius of curvature [see Eq. (4)]. When applying the simple-curved plate form of the radiation resistance matrix to arbitrarily curved panels an appropriate constant radius value must be determined. This was done by applying a constant-radius curve fit to the basic curvature of each panel, constraining the width to match the width of the actual panels (see Fig. 8 for an example using the S-curved panel). The resulting radius of curvature for each panel was used in computing the radiation resistance matrix. The rest of the computations proceeded using the experimental data as measured and with no further modifications.

It was found that sound power computed using the simple-curved plate form of the radiation resistance matrix is not highly sensitive to the radius of curvature. Figure 9 shows results from investigating the impact of the radius on sound power using the same theoretical setup presented in Sec. II B. A radius of 50 cm was selected as the base comparison radius. Sound power was then calculated using the simple-curved plate form of the radiation resistance matrix for a range of radius values from 5 to 100 cm. The results are shown as sound power difference in dB between each radius case and the comparison radius case of 50 cm. These results indicate that sound power calculated using the simple-curved plate form of the radiation resistance matrix is not highly sensitive to the constant radius value except at low frequency (below 200 Hz) and when the radius is tight (below 20 cm).

TABLE I. Dimensions of the S and M arbitrarily curved panels.

Panel	Height (cm)	Width (cm)	Thickness (mm)	Range of radius of curvature (cm)
S-Curved	30	40	1.59	6–63
M-Curved	30	48	1.59	6–51

2. S-curved panels

Figure 10 presents the sound power results comparison between the VBSP and ISO 3741 methods for the S-curved panel. The results show excellent agreement between the two methods for the center band frequency range from

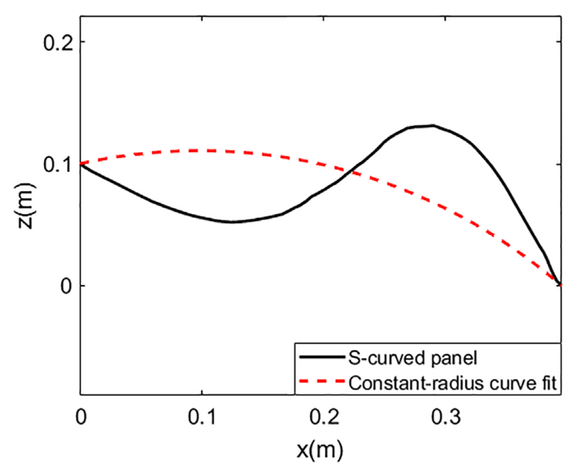

FIG. 8. (Color online) Plot of a constant-radius curve fit applied to the curvature of the S-curved panel, to compute a resultant radius to use in applying the simple-curved plate form of the radiation resistance matrix. The curvatures are shown with x and z coordinates, with y normal to the page, see Fig. 6.

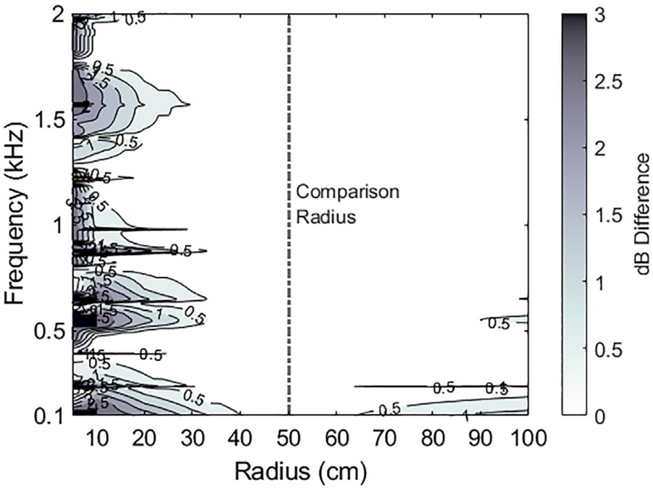

FIG. 9. (Color online) Difference in sound power between different radius value cases and the base comparison radius case of 50 cm (dash and dot vertical line). Sound power was computed using the simple-curved plate form of the radiation resistance matrix and the same theoretical setup presented in Sec. II B.

400 Hz to 10 kHz. In this frequency range, the mean difference was 0.4 dB with a standard deviation of 1.3 dB, and the maximum absolute difference was 3.2 dB at center band frequency 8 kHz. For the full frequency spectrum from 100 Hz to 10 kHz, the total sound-power level based on the VBSP method was 77.9 dB re 10^{-12} W and, based on ISO 3741, the sound power was measured as 79.9 dB re 10^{-12} W for a total absolute difference of 2.0 dB. At frequencies below 400 Hz, the noise floor of the reverberation chamber introduces significant error in the ISO 3741 results. This is due to the piezoelectric transducer not exciting the S-curved plate significantly at these lower frequencies, resulting in the plate not radiating above the background noise in the reverberation chamber. In this low frequency regime, it is likely that the VBSP method provides more accurate results than the ISO 3741 method because the background noise has less effect on the VBSP method.

3. M-curved panel

Figure 11 shows the comparison between the VBSP and ISO 3741 sound power results for the M-curved panel. Again, the results show excellent agreement for the one-third octave center band frequency range from 400 Hz to 10 kHz. In this frequency range, the mean difference was 0.3 dB with a standard deviation of 1.2 dB, and with a maximum absolute difference of 3.0 dB at center band frequency 4 kHz. For the full frequency spectrum from 100 Hz to 10 kHz, the total sound power level based on the VBSP method was 72.2 dB re 10^{-12} W and based on ISO 3741 was 71.8 dB re 10^{-12} W, for a total absolute difference of 0.4 dB. The sound power results below 400 Hz again indicate that there is significant error introduced into the ISO 3741 results due to the presence of background noise in the chamber.

Table II provides a summary of the difference in the sound-power results between the VBSP and ISO 3741 methods for both arbitrarily curved panels. The results in this table are presented with one-third octave band frequencies and the sound power difference in dB, with a total difference for each arbitrarily curved panel presented on the bottom row.

IV. COMPUTATIONAL VERIFICATION OF SOUND POWER CALCULATIONS

To provide additional evidence that supports the use of the simple-curved plate radiation resistance matrix to calculate sound power from arbitrarily curved panels, a BEM computational model was created for each arbitrarily curved panel presented above. Each model was setup for coupled analysis with structural finite elements and a fluid boundary element mesh. Sound power results from the BEM models were determined by applying data recovery surfaces around the radiating surface of each model. The surface velocity of each element of the radiating surface was exported from the models to compute sound power using the VBSP method. The VBSP results are based on the simple-curved plate radiation resistance matrix and BEM computed velocities and compared with numerical results for the sound power from

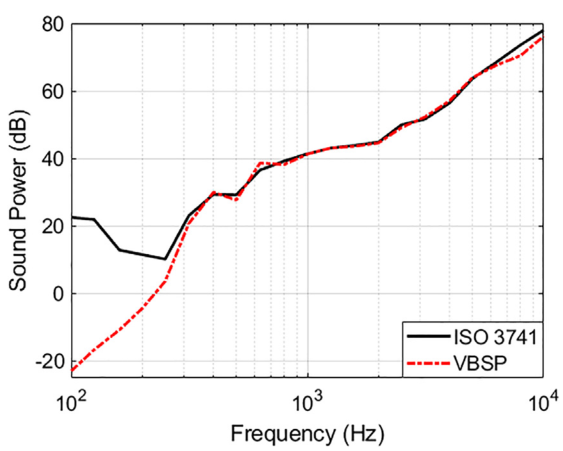

FIG. 10. (Color online) Results of the sound-power measurements of the S-curved panel using the VBSP and ISO 3741 methods.

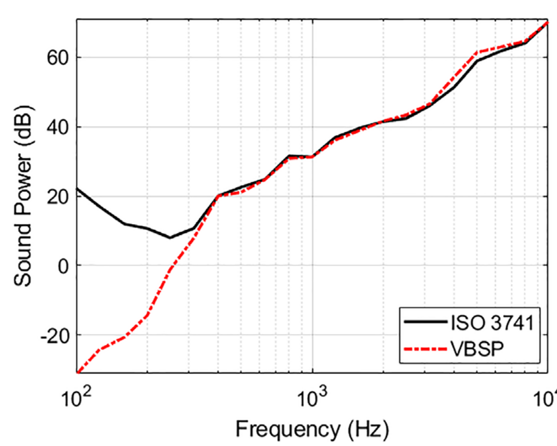

FIG. 11. (Color online) Results of the sound power measurements of the M-curved panel using the VBSP and ISO 3741 methods.

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TABLE II. Differences in sound power measurements between the VBSP and ISO 3741 methods for the S- and M- curved panels.

	Sound power difference (dB)		
Arbitrarily curved panel:	S	M	
Third octave band by center band frequency (Hz)	100	45.5	53.6
	125	38.5	41.3
	160	23.7	32.6
	200	15.9	25.0
	250	6.6	9.2
	315	2.3	2.8
	400	−0.6	0.0
	500	1.5	1.5
	630	−2.1	0.1
	800	1.1	0.7
	1000	0.1	0.0
	1250	0.0	0.8
	1600	0.3	0.6
	2000	0.2	−0.2
	2500	0.8	−1.0
	3150	−0.7	−0.5
	4000	−0.6	−3.0
	5000	−0.3	−2.5
	6300	0.9	−1.2
	8000	3.2	−0.5
	10 000	1.9	0.0
	Total	2.0	−0.4

the BEM models, which inherently uses the correct radiation resistance matrix. The same S- and M-surface shapes used in the experimental measurements were used in the numerical models. The numerical models were developed using VA One^{TM}, a commercial software package produced by the ESI Group.

The comparison was performed for each arbitrarily curved panel (S-curved panel then M-curved panel) with a setup that was similar to that used in the experimental testing. An infinite baffle was applied to simulate the rigid wall of the reverberation chamber. Rigid plates were modeled at the edges of the radiating surface such that the back side of the radiating surface was sealed against the infinite baffle and could not radiate. A 0.1% damping loss factor was assumed and applied to the radiating surface. The straight edges of the radiating surface were set to a clamped boundary condition and the curved edges were set to a simply supported boundary condition. A constant point force was applied in a similar location to that used in the experimental testing setup. The spacing between nodes applied for each model was a maximum of 13 mm, which is sufficient to resolve the response up to 4 kHz. For reference, the S-curved model was meshed with a total 10 082 triangle elements and 10 903 nodes. The M-curved model was meshed in the same way with triangle elements and a similarly refined mesh. The highest frequency was limited to 4 kHz, with a spacing of 2 Hz, as this was considered sufficient evidence and higher frequency models require increased nodal density that greatly increases the time to solve the models.

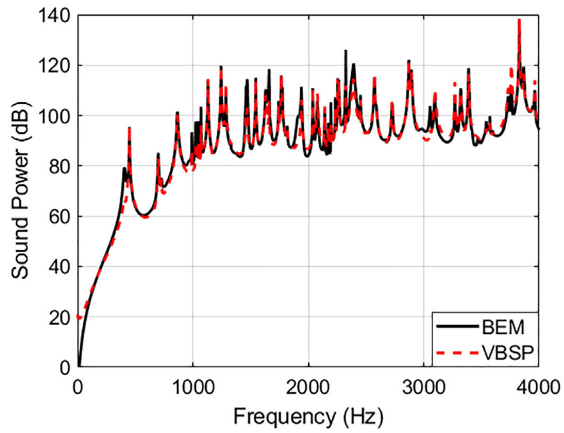

FIG. 12. (Color online) Numerically derived sound power of the S-curved panel using the VBSP and BEM methods.

The BEM models were used to compute the sound power for the panels and to compute the surface velocity at each node. The surface velocity and geometry data were then used to apply the VBSP method, which was based on the simple-curved plate radiation resistance matrix. The results are presented for the sound power (in dB) over the narrow band frequency range from 0 to 4 kHz.

The results of the S-curved panel model are presented in Fig. 12 and show good agreement. The total sound power computed from the BEM results was 141.7 dB and the total sound power computed from the VBSP method based on the simple-curved plate radiation resistance matrix was 141.3 dB, for a total absolute difference of 0.4 dB. The results from each method do not agree perfectly with respect to a number of specific frequencies but the overall sound power levels do indicate that the approximation of using the simple-curved plate formulation of the radiation resistance matrix is accurate enough to produce useful sound power results in practice.

The sound power results of the M-curved panel model are presented in Fig. 13 and also show good agreement. The total sound power result computed by the BEM was 141.8 dB and the total sound power result computed by the

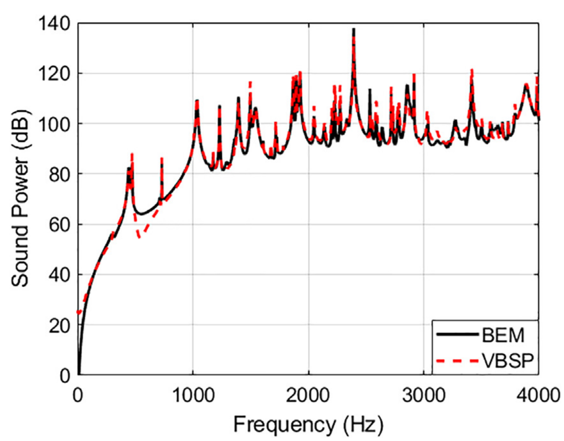

FIG. 13. (Color online) Numerically derived sound power of the M-curved panel using the VBSP and BEM methods.

VBSP method was 141.1 dB, for a total absolute difference of 0.7 dB. As noted above, these results do not show perfect agreement at every discrete frequency but the total results do indicate the likelihood that the use of the simple-curved plate radiation resistance matrix for the VBSP method for arbitrarily curved panels is accurate enough to produce useful sound power results in practice.

V. CONCLUSIONS

The VBSP method is a useful tool for measuring sound power from acoustically radiating surfaces. Critical to the VBSP method is the use of the acoustic radiation resistance matrix. Previous work has established the form of the radiation resistance matrix for flat plates, cylindrical- and spherical-shells, and simple-curved plates. There is no established form of the radiation resistance matrix for arbitrarily curved panels, such as the panels experimentally tested and presented in this paper.

The sound power sensitivity of the VBSP method to different forms of the radiation resistance matrix was explored to assess whether established, computationally efficient forms could be used to approximate sound power results for arbitrarily curved panels. For numerical model-based test cases where the radius of curvature is greater than 20 cm and the frequency is above 500 Hz, there is less than 1 dB difference in the sound power results computed using the flat plate, simple-curved plate, and cylindrical-shell radiation resistance matrices.

Experimental results showing excellent agreement between the VBSP and ISO 3741 methods were also presented. For the VBSP method, surface velocity measurements were obtained using a SLDV. Surface shape geometry data were also obtained using the SLDV and were used to calculate the normal velocity components. The experimental results showed good agreement over the 400 Hz to 10 kHz one-third octave band spectrum for both arbitrarily curved panels measured. The mean sound power difference in third-octave bands was 0.4 dB with a standard deviation of 1.3 dB for the S-curved panel, and 0.3 dB with a standard deviation of 1.2 dB for the M-curved panel. Below 400 Hz, the lack of good agreement was due to background noise in the reverberation chamber and that the arbitrarily curved panels radiated relatively little noise in this low frequency range. The VBSP method was less sensitive to background noise than the ISO 3741 method, and in this case, may likely be more accurate in the low frequency range.

To further investigate the use of the simple-curved plate radiation resistance matrix to compute the sound power for arbitrarily curved panels, numerical BEM and VBSP results were compared. The BEM is capable of providing computationally correct radiation resistance matrices for arbitrarily curved panels and computing the correct sound power. Surface velocity results from the BEM models were used to compute sound power using the VBSP method with the simple-curved plate radiation resistance matrix. The results showed that the total sound power difference between the BEM and VBSP methods was less than 1 dB. These results indicate that useful sound power results may be obtained from arbitrarily curved panels using a form of the radiation resistance matrix that is not specific to the arbitrarily curved surface.

The results presented in this paper have shown validation for extending the VBSP method to accurately measure sound power from arbitrarily curved panels and continue to establish the potential usefulness of the method. Furthermore, the results indicate that an approximate and or general form of the radiation resistance matrix could be used for many different cases and basic shapes of acoustically radiating surfaces. In considering more complex systems, such as ribbed plates, the sound power would still be given by Eq. (1) since the radiation resistance matrix is primarily dependent on the geometry of the radiating surface and the physical properties of the fluid surrounding the surface. This indicates that the VBSP method is applicable for more complex systems. With the form of the radiation resistance matrix being available for several ideal geometries, a user can choose the radiation resistance matrix form that most closely approximates the general geometry being considered. The capability of utilizing a single form of the radiation resistance matrix for many cases would greatly simplify the VBSP method in further applications and extension to more complex acoustically radiating surfaces of interest, perhaps ultimately leading to establishing a standardized VBSP method.

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