# Bending Seams – How to Create Couture Curves

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## Abstract

Adapting ideas from [1], we design pattern pieces for a scarf that exhibits negative curvature. We also discuss the methods and materials used for constructing versions in both wool and polyester.

## Introduction

**Figure 1:** *Negatively curved scarf.*

Making well-fitting clothing is a challenging geometry problem. Patterns must be made so flat two-dimensional pieces well-approximate some surface in three-dimensional space when sewn together. Most manufactured patterns seem to be chosen for their simplicity. For close-fitting clothing, it is often the physical properties of the fabric (such as stretchiness) that gives the fit. In this paper, we begin to explore design possibilities with more complicated seams. Our motivation for this is two-fold; complicated seams can be interesting from a design perspective, and they can lead to a better fit.

The ideas we use were inspired by a previous project [1], where the authors describe a method for building curved surfaces out of flat rigid materials such as paper. A brief sketch of the main idea is as follows: a polyhedral surface approximates a smooth surface. The Gaussian curvature of a polyhedron is concentrated in the vertices, and is equal to the sum of the *angle defect* over all vertices. At each vertex, the angle defect $k = 2\pi - \sum \alpha_i$ is $2\pi$ minus the total angle sum of the polygons meeting at that vertex. In the previous work, we created better approximations of smooth surfaces by redistributing the angle defect from the vertices to the edges of the polyhedral surface. We extend this idea to arbitrary curves in the plane and use them to create scarves with hyperbolic geometry.

Applying the ideas from [1] to actual garments presented both mathematical and sewing challenges. To begin our explorations, we choose to pattern a simple scarf design with one goal in mind: to use the bending technique on meandering seams to make a negatively curved surface out of cloth. This goal allowed us to focus on some of the sewing challenges that arise from non-traditional seams. How complex can these seams be, while still allowing sewing with a machine? How does seam allowance influence the overall shape of the garment? How do different fabrics behave? Figure 1 shows one of our initial creations.

## Bending

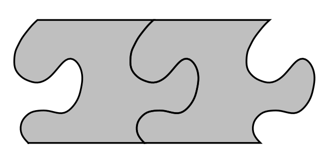


**Figure 2:** *Flat scarf*

We made scarves from eight identical copies of a pattern piece connected end to end. For scarves with negative curvature, we create pattern pieces using a process we call "bending". First, begin by drawing a curve. If we take a translate of that curve, we could create puzzle-like pattern pieces that could be sewn together to make a scarf, as shown in Figure 2. The meandering edge in the puzzle pattern shown is discrete, made from 101 points and the 100 line segments of equal length that connect them. Despite the smooth appearance of the sides, each puzzle piece shown in Figure 2 is a polygon.

Consider the local picture at a vertex $v$ contained in the seam (Figure 3a). Since the left and right edges of the pattern pieces are identical, a close up view of $v$ would look like Figure 3b. The total angle sum $\alpha + \beta$ at $v$ would be $2\pi$, hence the angle defect $k = 2\pi - (\alpha + \beta)$ at $v$ would be zero. As Gaussian curvature is concentrated at the vertices in polygonal surfaces, the resulting scarf would be a flat surface. If instead we joined polygonal pattern pieces where the angle sum at $v$ was less than $2\pi$, as seen in Figure 3c, the angle defect would be positive, and locally the garment would be positively curved. As our goal was to create a negatively curved scarf, we modified the initial curve to create pattern pieces so that the total angle at each vertex along a seam was slightly greater then $2\pi$, giving negative angle defect. See Figure 3d.


**(a)** *Original Curve* **(b)** $\alpha + \beta = 0$ **(c)** $\alpha + \beta < 2\pi$. **(d)** $\alpha + \beta > 2\pi$

**Figure 3:** *Local bending by altering angle defect*

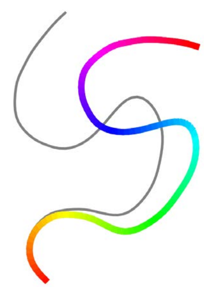


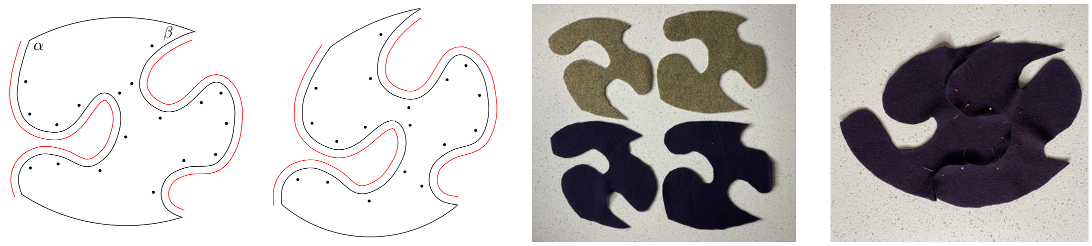
**Figure 4:** *Bending*

To create the right edge of our pattern piece, we started with the curve $C$ (seen in grey in Figure 4) which is made up of 100 line segments of unit length. Let $\mathbf{v_i} = \overrightarrow{P_iP_{i+1}} = \langle \cos\theta_i, \sin\theta_i \rangle$ be the vector made from consecutive points on the curve. Using these $\theta_i$, we create a new list of directions, $\hat{\theta}_i = \theta_i + i\Delta\theta$, where $\Delta\theta$ is some small constant angle. We now make our new "bent" curve by iteratively letting $\hat{P}_{i+1} = \hat{P}_i + \langle \cos\hat{\theta}_i, \sin\hat{\theta}_i \rangle$. The rainbow curve in Figure 4 shows the result when $\Delta\theta = \frac{\pi}{600}$. Note that if the angle at point $P_i$ on $C$ is $\alpha$, then then angle at $\hat{P}_i$ is $\alpha + \Delta\theta$. To create the left edge of our pattern piece, we do the same thing, except we use $-\Delta\theta$. These two new curves will make up the left and right edges of our pattern piece for a negatively curved scarf. An important point is that by altering just the angle, we do not change the overall length of the curve, which essential for sewing two pieces together. After sewing two pieces together, at each vertex $v$, the total angle defect will be $-2\Delta\theta$.

## Design

There were choices to be made in designing the pattern pieces which influenced the overall shape of the final scarf. First we chose the total bending angle $\theta$. We wanted to ensure that our final garment was noticeably negatively curved, but bending too much can overly distort the curve. For instance, the curve could self-intersect. Also, we had flexibility in choosing how to position the curves relative one another. The general principle is that the seam should be well distributed in the final scarf to avoid large patches of flat surface. We translated and rotated the curves with an aim of creating "skinny" pattern pieces. Technically, we looked to minimize the radius of the largest circular disk that could be embedded in the pattern piece, which we call the *pattern diameter*. Finally, once we chose the placement of the left and right edges, we connected the top and bottom endpoints with circle arcs. Given two points, there is a one-parameter family of circles passing through these points. For each boundary of our pattern we chose the circle arc for which the angle sum of $\alpha$ and $\beta$ shown in figure 5a was 180°, as this would give us a "straight" boundary to our scarf.

In Figure 5, we see the two distinct patterns we used to create the scarves shown in this paper. In both patterns, the left and right seams are the same. The difference in the shape of the pieces comes from the placement of the two curves relative to one another. Compared with Pattern A the right edge of Pattern B has




**(a)** *Pattern Piece A* **(b)** *Pattern Piece B* **(c)** *Cut outs of Pattern A & B* **(d)** *Pinned Pattern B*

**Figure 5:** *Pattern Pieces A & B*

been rotated by $\sim 28.5°$. The top boundary of our scarf B will clearly be shorter than the bottom boundary. This enabled us to create a design with the same curves as Pattern A, but with a smaller pattern diameter. The Pattern A pieces were not designed to be compatible with the Pattern B pieces. While the seam length is the same (when appropriately scaled), the angles at the corners made by the circle arcs from Pattern A and B are different. Sewing A to B would lead to an undesirable cornered effect on the boundary of the scarves.

## Construction

**Figure 6:** *Close up of seam.*

Once the patterns were designed, we strived to develop techniques to sew them together as simply as possible. The shift from paper to fabric required us to integrate common techniques in garment construction such as seam allowance, pinning, and stitch types to create the cleanest surface.

The preferred method to construct seams is using a straight stitch in conjunction with some degree of seam allowance. Seam allowance is the area in between the stitch and selvedge of the material that helps prevent fraying of fabric and serves to hide the actual stitches. In tandem, these effects both result in a better finish and increased longevity of the piece. In Figure 5, we see the seam allowance in red. To create this we added $\pi/2$ (left seam) or $-\pi/2$ (right seam) to each line segment after bending, and then we translated the curve. The problem with seam allowance around curved areas is that the excess fabric bunches on the underneath of the garment. A common remedy for this problem is to slit the seam allowance to make it more flexible. However, as our pattern pieces became drastically curvier with steeper angle defects, it became clear that we would be unable to continue to build these seams in this fashion.

Since the concept of seam allowance failed to translate from garment construction, we had to pivot and figure out another way to join our pattern pieces. We decided to utilize the zig-zag stitch which joins the selvedges of fabric together. Notice, in Figure 6, the stitches are now exposed, but it solves the issue of the fabric fraying. If the stitchwork is neat, the zig-zag stitch provides a decorative finish.

When using cardstock to build the surfaces, the stiffness of the paper ensures that the pattern pieces maintain their shape which makes construction easier. Fabric on the other hand is much more flexible so it requires meticulous attention to the state (taught or relaxed) in order to construct the desired surface. Thus, while sewing, we had to ensure we were stretching and relaxing each pattern piece properly. The unforgiving nature of the zig-zag stitch helped us identify this problem through puckering, seen in Figure 7, appearing from too much bunched fabric, holes or missed stitches from not enough relaxed fabric. To provide consistency and improve continuity in the stitchwork, we borrowed the idea of pinning from fashion design. Shown in Figure 5d, the pins serve to temporarily hold the garments together until they are properly sewn together. After pinning the pattern pieces together, we checked our accuracy by ensuring that the vertices of the surfaces were flush, which increased confidence while sewing.

**Figure 7:** *Puckering.*

## Results and Next Steps

Our final product were scarves made from two types of material; a more stretchy polyester and a stiffer wool. These elastic properties changed the techniques we needed to use to effectively sew the scarves. We sewed a total of four scarves, using both pattern pieces found in Figure 5 with both materials. The end results were a success. The garment was clearly negatively curved, the seams do add to the aesthetic appeal, and there are multiple ways to wear them. The pattern pieces are completely asymmetric and the chiral properties differ depending on if they are worn in an "upright" (with the right side of the fabric facing outwards) or "flipped" orientation (with the wrong side of the fabric facing outwards), as seen in the figure below.

**(a)** *Polyester Pattern A Flipped* **(b)** *Polyester Pattern A Upright* **(c)** *Polyester Pattern B Flipped* **(d)** *Polyester Pattern B Upright*

**(e)** *Wool Pattern A Flipped* **(f)** *Wool Pattern A Upright* **(g)** *Wool Pattern B Flipped* **(h)** *Wool Pattern B Upright*

**Figure 8:** *Our four finished scarves worn both upright and flipped.*

The results were promising enough to encourage us to pursue applying these ideas to more complicated garments which are modeled on surfaces of non-constant curvature. The general principle we hope to apply is that by choosing longer meandering seams for our pattern pieces, we can well-approximate our target surface while creating clothing that is novel and geometrically interesting.

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[1] Kelly Delp and William P. Thurston. Playing with surfaces: Spheres, monkey pants, and zippergons. *Proceedings of Bridges*, 2011.