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Super-Planckian emission cannot really be 'thermal'

A heat-powered emitter can sometimes exceed the Planck thermal-emission limit. We clarify when such super-Planckian emission is possible, arguing that far-field super-Planckian emission requires a distribution of energy that is not consistent with a unique temperature, and therefore the process should not be called 'thermal emission'.

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The spontaneous release of thermal energy by emission of electromagnetic radiation ('thermal radiation' or 'thermal emission') is a fundamental property of matter. Together, Planck's law and Kirchhoff's law of thermal radiation describe the characteristic emission spectrum when an object has a well-defined internal temperature[1,2]. Over the past two decades, there has been a flurry of research activity to engineer the thermal emission from various materials and structures[3], for example to enhance the temporal[4] or spatial[5] coherence of thermal emission, or to enable dynamic control[6]. There have also been theoretical predictions[7] and experimental claims[8,9] of 'super-Planckian thermal emission', with spectral radiance apparently exceeding the blackbody limit over some wavelength range. The possibility of super-Planckian thermal emission has substantial implications for applications that rely on radiative energy transfer, including energy harvesting[10], cooling and thermoregulation[11], and light sources[12]. However, thermal emission exceeding the blackbody limit in the far field at first seems to be in contradiction with the laws of thermodynamics, and this can lead to confusion, some of which has been discussed and clarified in the literature[13,14]. In this Commentary, we aim to provide an intuitive perspective for several recent developments in this research area, further clarifying how and when super-Planckian emission can indeed be achieved without violating the laws of thermodynamics, and why it is helpful to consider the phenomenon as part of a broader class of heat-powered emission, as opposed to super-Planckian 'thermal emission'.

The two fundamental laws describing thermal emission are (1) Planck's law, which describes the far-field radiation spectrum from a blackbody[1], and (2) Kirchhoff's law of thermal radiation, which equates the emissivity and absorptivity of an object in thermal equilibrium, as a consequence of microscopic reversibility[2]. It follows from these laws that far-field thermal emission is bounded by the blackbody radiation spectrum. In Fig. 1a, we illustrate a simple, widely known thought experiment, a version of which was used by Kirchhoff in 1860 to develop what is now known as Kirchhoff's law of thermal radiation[2]. In this thought experiment, an object X is in thermal equilibrium with a blackbody via radiative heat transfer, with a lossless tunable narrowband filter positioned in between. Both object X and the blackbody are thermally isolated from the rest of the Universe. Consistent with the laws of thermodynamics, neither object can spontaneously heat up or cool down. Because the situation must hold for any filter:

$$I_X(\lambda, T_0)A = (1 - R_X(\lambda))I_{BB}(\lambda, T_0)A \quad (1)$$

where $I_X(\lambda,T_0)$ is the thermal-emission spectrum for object X, $R_X(\lambda)$ is its reflectance spectrum, A is the emitting area, and $I_{BB}(\lambda,T_0)$ is the blackbody radiation distribution given by Planck's law (note, of course, that this expression was not known to Kirchhoff in 1860):

$$I_{BB}(\lambda, T_0) = \frac{2hc^2}{\lambda^5}\frac{1}{e^{hc/(\lambda k_B T_0)} - 1}, \quad (2)$$

where h is Planck's constant, c is the speed of light and k_B is Boltzmann's constant.

Equation (1) states that the thermal radiance from any object is bounded by 0 and $I_{BB}(\lambda,T_0)$, since reflectance $R_X(\lambda)$ must be between 0 and 1. Figure 1b shows one example emission spectrum, which is always less than or equal to $I_{BB}(\lambda,T_0)$. Using the conventional definition of spectral emissivity as $\epsilon_X(\lambda) = I_X(\lambda, T_0)/I_{BB}(\lambda, T_0)$, equation (1) leads to Kirchhoff's law for an opaque, non-scattering object: $\epsilon_X(\lambda) = 1 - R_X(\lambda)$[2]. Note that this simple thought experiment is justified if (1) both objects are macroscopic and exchange energy only through far-field emission, and

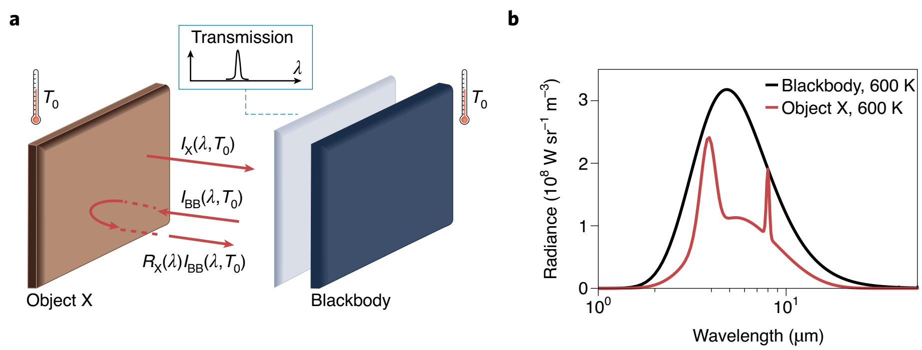

Fig. 1 | Thought experiment that explains Kirchhoff's law of thermal radiation. a, An object X with an arbitrary emission spectrum (red curve in **b**; the two peaks indicate the presence of two arbitrarily chosen resonances) is in thermal equilibrium with a blackbody via radiative heat transfer, where a lossless tunable narrowband filter is inserted between, allowing only light with certain wavelengths to pass through. **b**, The thermal radiance from objects much larger than the wavelength is bounded by the blackbody-radiation spectrum: $I_X(\lambda, T_0) \leq I_{BB}(\lambda, T_0)$; shown in red is just one example of a possible $I_X(\lambda, T_0 = 600\,\text{K})$.

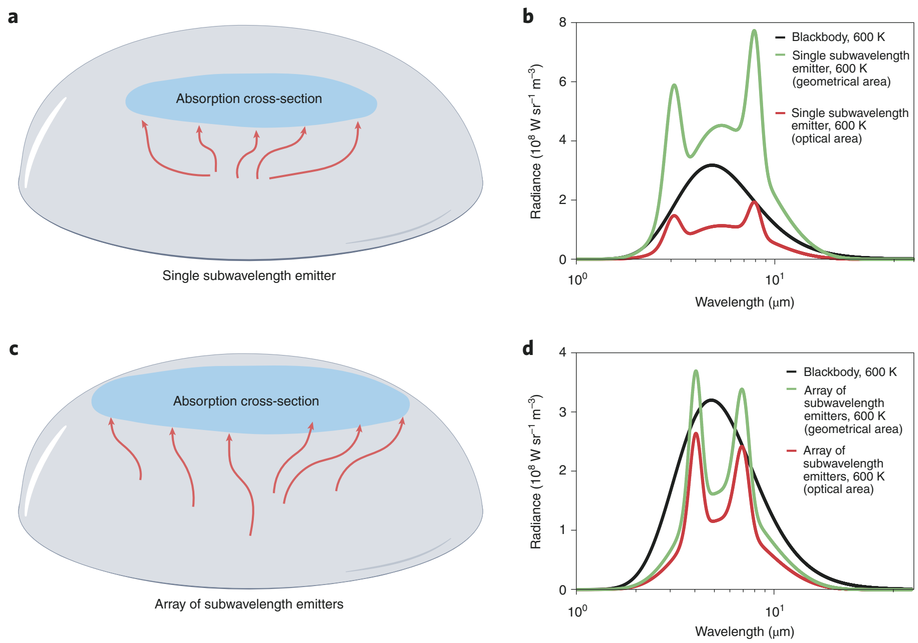

Fig. 2 | Subwavelength thermal emitters. a, A subwavelength thermal emitter can radiate toward a hemisphere with an optical absorption cross-section much larger than its geometrical cross-section near its resonance wavelength. **b**, Thermal emission from subwavelength objects at a particular temperature can appear super-Planckian if considering the geometrical cross-section as the emitting area, but cannot be super-Planckian if the optical-absorption cross-section is considered instead. **c**,**d**, The apparent enhancement of the absorption cross-section decreases if multiple subwavelength emitters are positioned together side by side, for example in a tightly packed array.

(2) both objects share one temperature that is characteristic of the equilibrium state of the isolated system. If these conditions are not met, Planck's law and Kirchhoff's law may not always apply.

For example, it is well understood, and has been clearly experimentally demonstrated, that exceeding the Planck thermal-emission limit is possible across subwavelength and wavelength-scale gaps (that is, in the near field), where radiative heat transfer can be greatly enhanced by evanescent waves[15,16]. These evanescent waves can also be extracted into the far field[17], but then the emitted power is bounded by Planck's law, considering the emission area of the entire structure comprising the emitter and the extractor.

It is also well understood that subwavelength thermal emitters can appear to be super-Planckian in the far field, because the absorption cross-section (and hence the emission cross-section) of subwavelength or wavelength-scale objects close to their resonances can be significantly enhanced[18]; if the emission cross-section becomes much larger than the geometrical cross-section, the emitter can radiate as if it were a much larger object (Fig. 2a). The thermal-emission power of a subwavelength emitter is proportional to its absorption cross-section, not its geometrical cross-section, and thus thermal emission from a subwavelength thermal emitter can appear super-Planckian if its geometrical size is taken as the emission area (green line, Fig. 2b). This effect has been demonstrated in theoretical calculations[19–21] and experiments[22,23] with subwavelength emitters such as nano- and microparticles and engineered resonant antennas. A recent theoretical work has also demonstrated that the emission cross-section can be made larger than the geometrical cross-section even if the geometrical size is much larger than the wavelength[24].

We argue that in most cases (though not all — see below), we should describe the emission area as the optical absorption cross-section rather than the geometrical cross-section. Then, even subwavelength thermal emitters are bounded by the usual Planck limit (red line, Fig. 2b). A clear argument for why the absorption cross-section should usually be taken as the emitting area is that the apparent enhancement of thermal radiance for subwavelength objects is not 'stackable': the enhancement will be reduced if many subwavelength emitters with enhanced absorption cross-sections are placed side by side (Fig. 2c,d). (Note that this argument also applies to the macroscopic emitters that have an emission area greater than their geometrical area, such as those described elsewhere[24].)

In the limiting case of a macroscopic surface consisting of many resonant subwavelength emitters, the enhancement will not exist anymore, as the absorption cross-section of the surface converges to the geometrical cross-section. For example, if designing an emitter for a thermophotovoltaic system, the existence of subwavelength elements that emit more strongly than would be expected from their geometrical area is unlikely to be helpful, as the effect cannot be stacked: overall thermal emission from the emitting region of the thermophotovoltaic cannot exceed the Planck limit. Similar arguments have been made before[25–27]. We do note, however, that consideration of the geometrical area can still be meaningful for some situations. For example, a single isolated emitter with a large absorption cross-section but a small geometrical volume and a correspondingly low heat capacity can enable high-speed modulation of thermal emission[26,28].

Unlike the previous example, super-Planckian emission is unambiguously possible in the non-equilibrium case, when the emitter and the receiver of the radiation are not at the same temperature. Although the invocation of equilibrium concepts (including Kirchhoff's law) is convenient, it is important to remember that almost every 'thermal-emission' experiment and application actually occurs under non-equilibrium conditions; examples include the hot Sun radiating toward the much colder Earth, or a hot thermophotovoltaic emitter facing a colder photovoltaic cell, or a grating-based selective emitter directed toward a detector in a laboratory. In these situations, temperature differences between the emitter and the receiver can give rise to non-equilibrium within the emitter; in other words, not all components or modes of the emitter are at the same well-defined temperature. Non-equilibrium can manifest in many forms, for example as a temperature gradient across the volume of the emitter[29], with different effective temperatures for the electrons and the lattice[30], or, more generally, in energetic distributions that cannot be described with a statistical notion of temperature. In turn, it is possible that this non-equilibrium can drive emission that can exceed the Planck limit for the temperature of the heat bath in contact with the emitter (Fig. 3a): that is, super-Planckian heat-powered emission. Said another way,

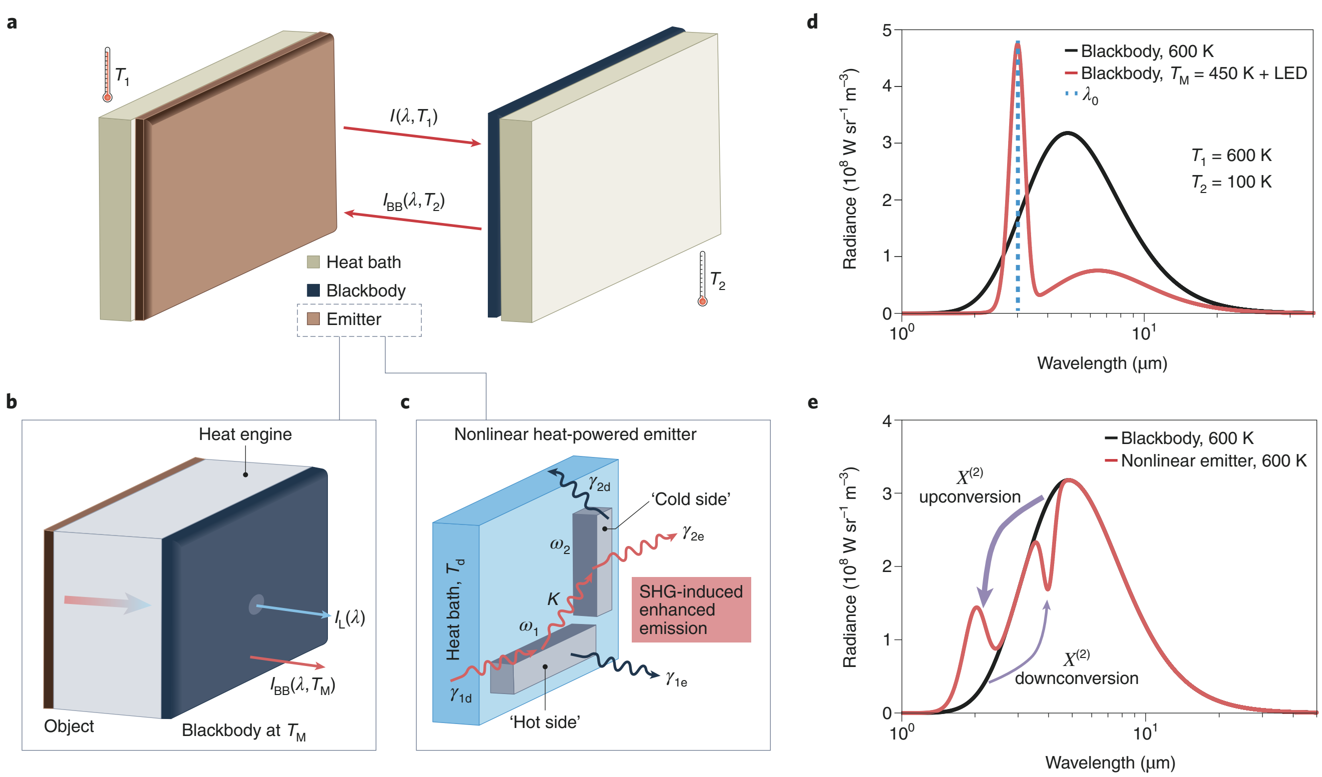

Fig. 3 | Non-equilibrium heat-powered emission. a, Non-equilibrium heat-powered emission between a hot emitter and a cold blackbody, which are in contact with heat baths at different temperatures. **b**, A heat engine runs between the hot object at T_1 and a blackbody at an intermediate temperature T_M, and the work generated from the heat engine is used to power a vanishingly small LED on the surface. In this configuration, the total light emission is the combination of blackbody radiation at T_M and LED light. **c**, Schematic of an emitter that can realize super-Planckian emission via nonlinear effects when radiating towards a cold environment. **d**, Super-Planckian heat-powered emission is realized near the LED centre wavelength λ_0 for the configuration shown in **b**: $I_{BB}(\lambda_0, T_M) + I_L(\lambda_0) > I_{BB}(\lambda_0, T_1)$. **e**, Modification of the blackbody radiation spectrum via nonlinear effects as shown in **c**: non-equilibrium conditions enable selective upconversion through second-harmonic generation, thus realizing super-Planckian heat-powered emission near 2 μm. Panels **c** and **e** adapted with permission from ref. [27], The Optical Society.

from a thermodynamic perspective, emitters can be engineered to act as heat engines that convert a non-equilibrium configuration into emission of light with spectra not bounded by Planck's law. (Note that the framing of Kirchhoff's law has recently been extended to non-equilibrium situations when there are well defined local temperatures, and this 'local Kirchhoff law' can be used to analyse a variety of non-equilibrium phenomena, including photoluminescence and electroluminescence[26].)

In Fig. 3, we describe two thought experiments that demonstrate how super-Planckian emission can emerge. In both cases, the emitter is in contact with a thermal bath at temperature T_1 on the left side, and the receiver is a blackbody in contact with a thermal bath at temperature T_2 (Fig. 3a). In Fig. 3b, we imagine that the emitter can be modelled as a combination of a heat engine operating at the Carnot limit and a small perfectly efficient light-emitting diode (LED) powered by the electronic potential energy (or work) generated by the heat engine that emits toward the receiving blackbody. The heat engine operates between temperature T_1 on the left (due to its contact with the thermal bath) and temperature T_M on the right side with Carnot efficiency $1\text{-}T_M/T_1$. The temperature difference giving rise to a thermal gradient within the emitter is $T_1\text{-}T_M$. We assume that the right side of the heat engine is coated such that it is a blackbody, but with a vanishingly small surface area that acts as an LED. The work generated by the heat engine powers the LED to produce non-thermal narrowband emission towards the cold blackbody. In this thought experiment, thermal energy from the hot object at T_1 is converted to a combination of blackbody radiation at T_M and non-thermal LED light. Even though the total power of this combined emission is smaller than thermal emission for a simple blackbody at T_1, super-Planckian emission can be observed around the centre wavelength of the LED, λ_0, that is, $I_{BB}(\lambda_0, T_M) + I_L(\lambda_0) > I_{BB}(\lambda_0, T_1)$ (Fig. 3d).

A second related thought experiment involves the use of nonlinear effects to realize super-Planckian emission (Fig. 3c,e). Traditionally, nonlinear effects are not relevant in the context of thermal emission owing to the relatively weak light intensity for modest temperatures. However, given recent development of highly nonlinear materials and high-Q resonators for enhancing nonlinear light–matter interactions, several theoretical papers have now shown that nonlinear 'thermal emission' can become significant[27,31,32]. In particular, super-Planckian emission has been discussed in the context of nonlinear frequency mixing[27], where electromagnetic excitations at ω_1 (here, corresponding to free-space wavelength λ_1=4 μm) are converted to emission at ω_2 (corresponding to λ_2=2 μm) via second-harmonic generation (Fig. 3c). Such a nonlinear process can redistribute the radiation spectrum, thus leading to super-Planckian emission near the upconverted wavelength (Fig. 3e). The super-Planckian emitter in ref. [27] considered modes at ω_1 and ω_2 that interact with each other via a $\chi^{(2)}$ interaction quantified by coupling coefficient κ. These modes also exchange energy with the heat bath (to the left) via conduction, with coupling coefficients γ_{1d} and γ_{2d}, and with the radiation field (to the right) with coupling coefficients γ_{1e} and γ_{2e} (The d subscript denotes energy transfer due to conduction, and e denotes emission). If the receiver or environment are colder than the heat bath in contact with the emitter, one can design the coupling coefficients to achieve a disparity in the activation for modes at ω_1 and ω_2, and hence change the ratio of the emitted power at these two frequencies compared with the Planck distribution.

More specifically, the configuration of γ_{1d}, κ, $\gamma_{2e} > \gamma_{2d}$, γ_{1e} favours energy flow from the heat bath to the mode at ω_1 via conduction, then from ω_1 to ω_2 via nonlinear coupling, and finally from the mode at ω_2 to free space via radiation (see more discussions in Supplementary Section 3). Effectively, the scheme acts like a heat engine, with the 'hot side' being the mode at ω_1 and the 'cold side' being the mode at ω_2 (Fig. 3c). Note, however, that as with the LED example in Fig. 3b,d, this super-Planckian effect can only be observed when the emitter is out of equilibrium with the receiver or environment[27]. When the emitter is in equilibrium with the environment, every energy-transfer process is balanced by a reverse process with the same rate: that is, microscopic reversibility applies, and the emission spectrum must be bounded by Planck's law, as described in Fig. 1.

The two thought experiments in Fig. 3 help to clarify how non-equilibrium can result in super-Planckian emission, which is not possible in equilibrium. For a given temperature difference between the heat bath in contact with the emitter and its environment, one can calculate thermodynamic bounds on how much an emission spectrum can exceed Planck's law. For example, in the engine/LED thought experiment, there are clear restrictions from energy considerations: the heat-engine efficiency and the temperature of the intermediate blackbody together bound the total power of the LED (Supplementary Section 2). However, the combined emission power over a narrow wavelength range can substantially exceed the power of blackbody radiation. For example, we show in Supplementary Fig. 3 that for the same temperatures as in Fig. 3d, the combined emission power from a 0.25-μm-wide spectral band near 10 μm can be more than five times as large as that of a blackbody. Further constraints on super-Planckian emission can be derived by calculating the entropy of the radiated light with respect to the requirements of the second law of thermodynamics. In particular, we show in Supplementary Section 1 that there is a fundamental requirement on the temperature difference for a given amount of frequency conversion. For example, to realize the nonlinear radiation spectrum shown in Fig. 3e, the receiving blackbody temperature T_2 must be smaller than 580 K. These second-law arguments are also helpful for defining the limiting behaviour of the frequency-conversion example as the emitter approaches thermal equilibrium with its environment (Supplementary Section 1). We note that second-law arguments apply even when microscopic reversibility is absent, for example if a system includes non-reciprocal elements[33,34].

A thermodynamic perspective also explains why, historically, it has usually not been necessary to consider the possibility of super-Planckian emission even though one would be hard-pressed to find an experiment that was actually performed with an emitter in equilibrium with its environment. When heat is the only energy source directly driving radiative emission (in contrast with photoluminescence, electroluminescence and so on), usually thermal equilibration within an object occurs much faster than different avenues for radiative emission. The behaviour of an emitter at a single, uniform temperature, even if the temperature is different from that of its environment, is still well described by Planck's law, because there are no microscopic energy gradients to drive thermodynamic heat engines, and therefore no non-equilibrium emission. However, as control over radiative emission properties continues to develop, it is becoming possible to design increasingly sophisticated architectures that lead to exotic super-Planckian behaviour, for example the nonlinear scheme mentioned above[27].

We summarize conclusions based on thermodynamics considerations that we hope will provide a helpful perspective as this exciting research area advances. First, when invoking the concept of super-Planckian emission, we encourage authors to consider explicitly the difference between geometrical cross-sections and emission cross-sections (see Fig. 2) as it relates to some desired application. Will the emitter operate in a regime such that the geometrical cross-section is a useful metric for understanding the intended functionality? In equilibrium, it is not possible for macroscopic surfaces to display super-Planckian emission when the overall structure is much larger than the emission cross-sections of the individual elements comprising it. That said, there are ongoing investigations about the enhancement of absorption cross-sections of 'individual' emitters[35] and efforts to establish an upper limit[36].

Second, the terms 'thermal emission' and 'thermal radiation' have been used colloquially, including by the authors of this Commentary in the past[3,29,30], to mean any radiation that is due to heat energy. The discussion here makes clear that the 'thermal' designation should only be used to describe emission from objects at a well-defined, uniform temperature. By definition, the thermal-emission spectrum must be bounded by Planck's law if the emission area is defined as the absorption cross-section. In contrast, super-Planckian emission can occur when the distribution of internal energy inside an emitter is not consistent with a unique temperature, and hence is non-thermal. In these circumstances, we believe 'heat-powered emission' may be a helpful, distinguishing term. ❐

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Competing interests

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