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On Bose–Einstein Condensation and Unruh–Hawking Radiation from a Quantum Optical Perspective

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Abstract
The use of quantum optical/laser physics techniques yields interesting insights into Bose–Einstein condensation and Unruh–Hawking radiation.

Keywords Quantum theory of the laser · Atom laser · BEC · HBAR radiation

1 Introduction

It is a pleasure to dedicate this article to professors David Lee and John Reppy. Both David and John derive from the excellent Yale low-temperature physics group of C. T. Lane. David focused on the superfluid properties of ^{4}He and ^{3}He, and won the Nobel prize for demonstrating the superfluidity of ^{3}He below 0.01 K due to pairing of ^{3}He atoms. John is famous, among other things, for his studies of rotating liquid helium. Professors Lee and Reppy are masters of thermodynamics—both classical and quantum. For example, the discovery of a superfluid phase of ^{3}He was found by Lee et al. by investigating the pressure-temperature diagram of ^{3}He, as it went from liquid to solid phases. Reppy and his group observed the first Bose–Einstein condensation (BEC) by studying the thermodynamic properties of ^{4}He atoms in the porous vycor glass which are "coated" by superfluid helium [1–3].

We are indebted to professors Lee and Reppy for their impact on our work. For example, from insightful discussions with David, we became interested in the entropy of black holes (BHs). John's BEC studies stimulated us in choosing a

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canonical ensemble model for our investigations of time dependence and fluctuations of ground-state occupation number of ideal and interacting Bose gases.

In this article, we shall sketch how the quantum master equations of laser physics and quantum optics are useful in such diverse fields as BEC on the one hand and BH radiation on the other—two topics we have been stimulated to study by Reppy and Lee. In the next few pages, we apply the laser/quantum optical master equation to John Reppy's BEC (atom laser) and to the problem David Lee posed: why is BH entropy proportional to the surface area rather than the volume of the BH?

2 Quantum (Photon) Theory of the Laser

Laser operation is, for most purposes, well described by using a classical (Maxwell) picture for the laser light and a quantum (Schrodinger) treatment of the atoms. The fully quantized (photon) picture of the laser is a more difficult problem, to wit the 1964 Glauber [4] quote:

> The only reliable method we have of constructing density operators, in general, is to devise theoretical models of the system under study and to integrate [the] corresponding Schrodinger equation, or equivalently to solve the equation of motion for the density operator. These assignments are formidable ones for the case of the laser oscillator and have not been carried out to date in quantum mechanical terms.

Taking up the Glauber challenge, we found [5, 6] the evolution equation for the field density matrix for a single-mode laser not too far above threshold

$$\begin{aligned}\dot{\rho}_{n,n'}(t) = &-[C_{n,n'}(n+1) + C_{n',n}(n'+1)]\rho_{n,n'} \\ &+ [C_{n-1,n'-1}\sqrt{nn'} + C_{n'-1,n-1}\sqrt{n'n}]\rho_{n-1,n'-1} \\ &- (\gamma/2)(n+n')\rho_{n,n'} + \gamma\sqrt{(n+1)(n'+1)}\rho_{n+1,n'+1},\end{aligned} \tag{1}$$

where $C_{n,n'} = \alpha/2 - (\beta/8)(n + 3n' + 4)$, and the parameters α, β, and γ are given by the equations

$$\alpha = \nu\bar{N}\frac{\wp^2}{\epsilon_0}\frac{1}{\gamma}, \quad \beta = \frac{3}{2}\bar{N}\left(\frac{\nu}{\epsilon_0}\right)^2\frac{\wp^4}{\gamma^3}, \quad \text{and} \quad \gamma = \frac{\nu}{Q}, \tag{2}$$

which are written in terms of the excitation density $\bar{N}$ (average number of excited atoms in the cavity), and atomic decay rate γ. The model system consists of a single-mode cavity of frequency ν and a finite quality factor Q into which excited two-level (a and b) atoms are injected sequentially (see Fig. 1). The laser $a - b$ transition has the dipole matrix element $\wp$ and is assumed to be in resonance with the cavity mode. Levels a and b can also decay with emission of nonlaser radiation to other states at a rate γ.

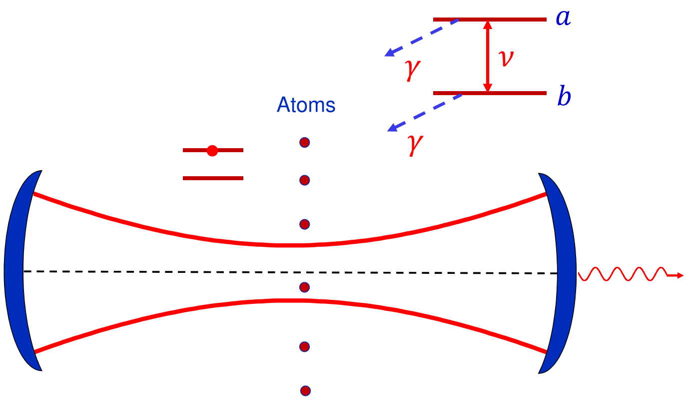

Fig. 1 Laser model. Excited atoms are sequentially injected into a single-mode cavity and emit photons into the cavity (laser) mode. In addition, atoms undergo spontaneous decay with emission of nonlaser radiation (Color figure online)

An important point is that only terms of the density matrix $\rho_{n,n'}$ of equal degree of "off-diagonality" (i.e., of equal $k = n - n'$) in Eq. (1) are coupled. Especially interesting are the diagonal equations implied by Eq. (1)

$$\dot{\rho}_{n,n} = \underbrace{-[\alpha - \beta(n+1)](n+1)\rho_{n,n} + [\alpha - \beta n]n\rho_{n-1,n-1}}_{\text{pumping}} - \underbrace{\gamma\left[n\rho_{n,n} - (n+1)\rho_{n+1,n+1}\right]}_{\text{damping}}. \tag{3}$$

The terms have been grouped to make the physical interpretation obvious. Equation (3) may be interpreted physically as a flow of probability between the nth level of the radiation oscillator and the $(n-1)$th and the $(n+1)$th levels, due to stimulated emission and finite cavity Q.

The steady-state solution of Eq. (3) yields the following probability distribution for finding n protons in the laser cavity

$$\rho_{n,n} = \mathcal{N}^{-1} \prod_{k=0}^{n} \frac{\alpha - \beta k}{\gamma}, \tag{4}$$

where $\mathcal{N}$ is a normalization factor. This distribution has a peak at $n_p = (\alpha - \gamma)/\beta$. For a sufficiently peaked distribution the average number of photons in the cavity obtained from Eq. (4) is $\langle n \rangle = n_p$. It is found that the photon statistics for the He-Ne laser in its normal operation region, which is only around 10 % above threshold, is much broader that the Poisson distribution characterizing coherent light.

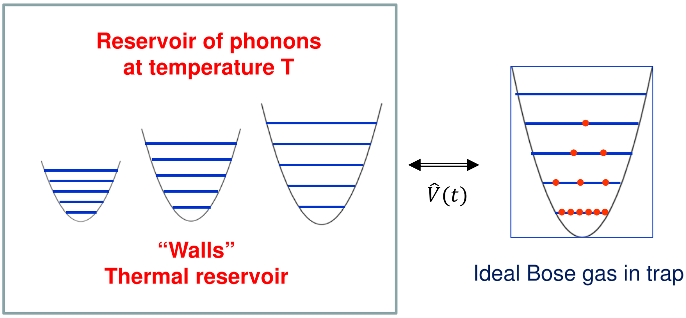

Fig. 2 Ideal Bose gas of N atoms in a harmonic trap interacting with a reservoir of harmonic oscillators (phonons in the walls). Interaction between the atoms and the walls is described by the Hamiltonian $\hat{V}(t)$ (Color figure online)

The quantum theory of the laser has been applied in many different quantum optics problems over the years, to wit the recent paper of the Franco Nori's group [7] entitled: "Scully–Lamb quantum laser model for parity-time-symmetric whispering-gallery microcavities: Gain saturation effects and nonreciprocity".

In the next sections we sketch the quantum theory of the laser in connection with the BEC and with the BH radiation.

3 The BEC—The Atom Laser

Early BEC experiments were carried out by Reppy et al. in 1983 [1]. In these experiments, Helium II was placed in porous Vycor glass which keeps the atoms well separated. These experiments are characterized by a dilute gas of N atoms at temperature T. Thus, when BEC was observed in ultracold dilute alkali-metal gases, it was natural to use the simpler Reppy model to study the extent to which the BEC, "atom laser," was really "like" a laser. For example, one naturally asks "what is the atomic number distribution function for the ground-state atoms? Is there any similarity between the photon statistics for an ordinary laser and the "atom statistics" for an atom "laser"?

To answer this question, one naturally seeks to understand the connection between BEC of an ideal Bose gas, and the quantum theory of the laser, etc. In the latter context, we recall that the saturation nonlinearity in the radiation matter interaction (parametrized by β in Eq. 3) is essential for laser coherence. Is the corresponding nonlinearity in BEC due solely to atom–atom scattering, or is there a coherence generating nonlinearity even in an ideal Bose gas?

With this in mind, we extend our previous laser-phase transition analogy [8] to the problem of N ideal bosons in a 3D harmonic potential coupled to a thermal reservoir (see Fig. 2). The N particle constraint introduces the essential nonlinearity. To see this we derive a nonequilibrium master equation for the ground state of an ideal Bose gas in a 3D harmonic trap coupled to a thermal reservoir. Writing only the diagonal elements, we find for $T \ll T_c$ [9]

$$\frac{1}{\kappa}\dot{\rho}_{n_0,n_0} = -[N+1-(n_0+1)](n_0+1)\rho_{n_0,n_0} + [(N+1)-n_0]n_0\rho_{n_0-1,n_0-1} - \left(\frac{T}{T_c}\right)^3 N[n_0\rho_{n_0,n_0} - (n_0+1)\rho_{n_0+1,n_0+1}], \tag{5}$$

where $|n_0\rangle$ is the eigenstate of n_0 bosons in the ground state, κ is a rate constant, N is the total number of bosons, T is the temperature of the heat bath, and T_c is the BEC transition temperature.

At this point, it is useful to summarize the photon and atom laser equation of motion by writing the general quantum master equation for both

$$\dot{p}_n = -[A - B(n+1)](n+1)p_n + (A - Bn)np_{n-1} - C[np_n - (n+1)p_{n+1}], \tag{6}$$

where p_n is the probability to find n atoms in the condensate, or n photons in the cavity. Parameters A, B, and C for the case of the laser and BEC at $T \ll T_c$ are summarized in the Table.

	Laser	Bose gas at $T \ll T_c$
Linear gain A	α	$\kappa(N+1)$
Saturation B	β	κ
Loss C	γ	$\kappa N\left(\frac{T}{T_c}\right)^3$
Prob. Dist.	$p_n = \rho_{n,n}$ n photons in cavity	$p_{n_0} = \rho_{n_0,n_0}$ n_0 atoms in ground state

The steady-state photon/atom statistical distribution is given by

$$p_n = \mathcal{N}^{-1} \prod_{k=0}^{n} \frac{A - Bk}{C}.$$

Noting that for the laser the energy in the cavity goes as $\hbar\nu(A-C)/B$ and therefore the maximum energy occurs when there is no cavity loss, i.e., when $C = 0$. Thus, we may call $A/B = M$ the maximum "photon number," and write the photon distribution as

$$\rho_{n,n} = \mathcal{N}^{-1} \frac{\left[M\frac{\alpha}{\gamma}\right]^{M-n}}{(M-n)!}, \tag{7}$$

while the atomic distribution we may write as

$$\rho_{n_0,n_0} = \mathcal{N}^{-1} \frac{\left[N\left(\frac{T}{T_c}\right)^3\right]^{N-n_0}}{(N-n_0)!}. \tag{8}$$

In writing Eqs. (7) and (8) in this form, we are emphasizing the fact that the average number of photons in the cavity goes as

$$\frac{A-C}{B} \to M\left(1-\frac{\alpha}{\gamma}\right),$$

while the average number of atoms in the condensate

$$\frac{A-C}{B} \to N\left[1-\left(\frac{T}{T_c}\right)^3\right].$$

The clear message of our previous work is that BEC is very analogous to the laser [9]. In particular, we have obtained the probability distribution for finding n_0 atoms in the ground state to be identical with the probability of finding n photons in the laser cavity.

Results of [9] are valid at $T \ll T_c$. They have been extended for all temperatures in [10]. In [10], the model interaction Hamiltonian describing statistical dynamics of the condensate was taken to be

$$V = \sum_{\mathbf{k}} g_{\mathbf{k}}\, \hat{a}_0^{\dagger}\, \hat{b}_{\mathbf{k}}^{\dagger}\, \hat{a}_{\mathbf{k}} + \text{adj.}, \tag{9}$$

where $\hat{a}_{\mathbf{k}}$ is the atom annihilation operator which satisfies the particle number conservation constraint, while $\hat{b}_{\mathbf{k}}$ is operator for quasiparticle (phonons in the thermal bath) which number is not conserved, $\hat{a}_0$ is the condensate annihilation operator and $g_{\mathbf{k}}$ is the corresponding coupling strength for the collision of a ground-state atom and an atom having momentum $\mathbf{k}$ scattering into the BEC. Hamiltonian (9) describes processes which add (remove) atoms from the condensate with the annihilation (creation) of an excited atom and the emission (absorption) of a quasiparticle. Quasiparticles $\hat{b}_{\mathbf{k}}$ are treated as a thermal reservoir which is traced over to obtain the density matrix equation for the ground state. Such approach yields the following equation of motion for the probability to find n_0 particles in the condensate $\rho_{n_0,n_0} = P_{n_0}$ [10]

$$\frac{1}{\kappa}\dot{P}_{n_0} = -K_{n_0}(n_0+1)P_{n_0} + K_{n_0-1}n_0P_{n_0-1} - H_{n_0}n_0P_{n_0} + H_{n_0+1}(n_0+1)P_{n_0+1}. \tag{10}$$

In this equation, the K_{n_0} and K_{n_0-1} terms describe cooling of the gas which increases the condensate number, while the heating terms H_{n_0} and H_{n_0+1} decrease it. The constant κ is an uninteresting overall rate factor. The cooling and heating coefficients are given by

$$K_{n_0} = \sum_{\mathbf{k}\neq\mathbf{0}} \langle n_{\mathbf{k}}\rangle_{n_0}\left(1+\left\langle \hat{b}_{\mathbf{k}}^{\dagger}\hat{b}_{\mathbf{k}}\right\rangle\right), \tag{11}$$

$$H_{n_0} = \sum_{\mathbf{k}\neq\mathbf{0}} \left\langle \hat{b}_{\mathbf{k}}^{\dagger}\hat{b}_{\mathbf{k}}\right\rangle\left(1+\langle n_{\mathbf{k}}\rangle_{n_0}\right), \tag{12}$$

where $\langle n_{\mathbf{k}}\rangle_{n_0} \equiv \left\langle \hat{a}_{\mathbf{k}}^{\dagger}\hat{a}_{\mathbf{k}}\right\rangle_{n_0}$ is the thermal average taken under the condition that there are n_0 atoms in the condensate. In quasithermal approximation, we write the conditional thermal average as

$$\langle n_{\mathbf{k}} \rangle_{n_0} = (N - n_0) \frac{\bar{n}_{\mathbf{k}}}{\sum_{\mathbf{k} \neq \mathbf{0}} \bar{n}_{\mathbf{k}}}, \tag{13}$$

where the usual atomic thermal average is given by

$$\bar{n}_{\mathbf{k}} = \langle \hat{b}_{\mathbf{k}}^{\dagger} \hat{b}_{\mathbf{k}} \rangle = \frac{1}{e^{\beta E_k} - 1} \equiv \eta_k, \tag{14}$$

$\beta = 1/k_B T$ and E_k is the energy of an atom with momentum $\mathbf{k}$. Such approximation guarantees satisfaction of the particle number constraint in average, that is $\sum_{\mathbf{k} \neq \mathbf{0}} \langle n_{\mathbf{k}} \rangle_{n_0} = N - n_0$.

Introducing notations

$$\mathcal{H} = \sum_{\mathbf{k} \neq \mathbf{0}} \eta_k \text{ and } \eta = \frac{\sum_{\mathbf{k} \neq \mathbf{0}} \eta_k^2}{\mathcal{H}} \tag{15}$$

one can rewrite the K_{n_0} and H_{n_0} coefficients as

$$K_{n_0} = (N - n_0)(1 + \eta), \tag{16}$$

$$H_{n_0} = \mathcal{H} + (N - n_0)\eta. \tag{17}$$

The steady-state solution of the master Eq. (10) yields the following recursion relation for the condensate distribution function

$$P_{n_0+1} = \frac{K_{n_0}}{H_{n_0+1}} P_{n_0} \tag{18}$$

which can be solved analytically in a closed form

$$P_{n_0} = \frac{1}{Z_N} \frac{\left(N - n_0 + \mathcal{H}/\eta - 1\right)!}{(\mathcal{H}/\eta - 1)!(N - n_0)!} \left(\frac{\eta}{1 + \eta} \right)^{N - n_0}. \tag{19}$$

Knowing P_{n_0} one can also find central moments analytically [10, 11]. Results for the condensate particle number and its fluctuations, obtained by the master equation approach of Ref. [10] (CNB2), are in excellent agreement with exact numerical simulations in the canonical ensemble for the ideal gas [12], as indicated in Fig. 3. Results of the recent experiment on observation of atom number fluctuations in BEC [13] are shown in Fig. 3c.

4 On Black Hole Radiation

Thermodynamics is a powerful tool in the hands of the low-temperature physicist. No better example of this than David Lee, whose beautiful experiments demonstrating the superfluidity of helium-3 won him, Richardson, and Osheroff the Nobel

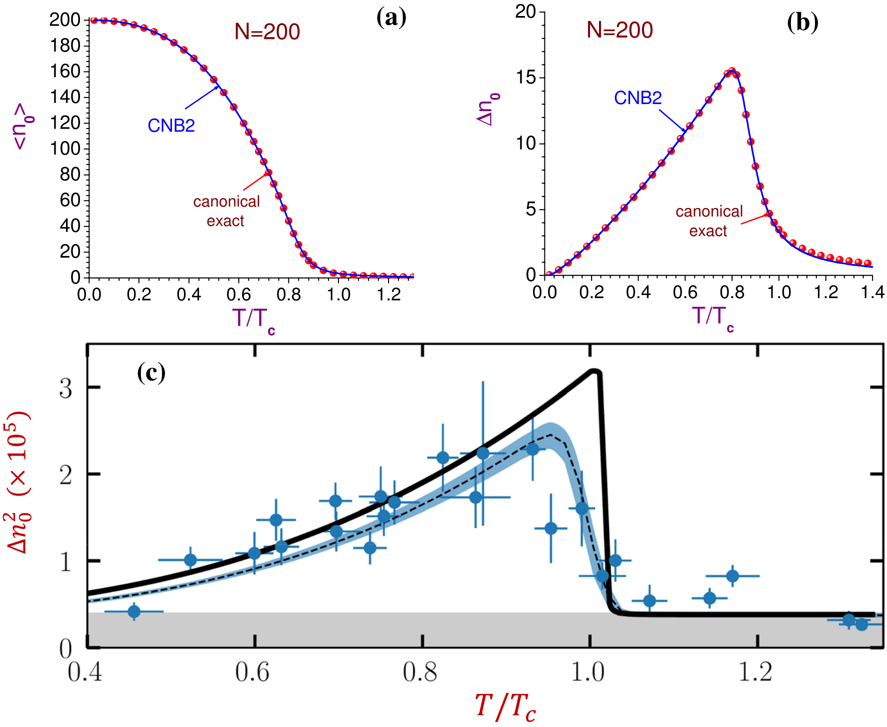

Fig. 3 Mean number of condensate particles (**a**), and variance of the BEC atom number (**b**) as a function of temperature for $N = 200$ noninteracting particles in a harmonic trap calculated in the CNB2 approach [10]. Large dots are the exact numerical results obtained in the canonical ensemble for the ideal Bose gas [12]. **c** Experimental variance of BEC atom number as a function of temperature (blue points). The dashed line is a fit, where the blue band represents the uncertainty of the fit. The gray area indicates the offset due to technical fluctuations. The solid black line is exact canonical ensemble calculations for a noninteracting gas [12]. The results are plotted as a function of the temperature rescaled with the temperature at peak fluctuations. Adopted from [13] (Color figure online)

Prize. They were studying thermodynamic phase transitions in liquid helium-3 when they discovered superfluidity.

It is natural, therefore, that many of our group sessions involved ^{3}He and thermodynamics in exotic systems. Indeed, during one such discussion, David remarked that it would be nice to have a simple "back of the envelope" demonstration that Bekenstein–Hawking BH entropy goes as the area (not the volume) of the BH. Little did we know that our simple calculations on the subject would "stir up the proverbial hornet's nest"!

Since we have had so much fun with this BH problem, we would like to dedicate this essay section to our studies and the reaction(s) of our friends in quantum optics and general relativity. A simple argument runs as follows. First recall that a two-level atom undergoing uniform acceleration a will emit Unruh radiation of temperature $T_U \cong \hbar a/k_B c$. This result can be obtained from a textbook quantum optics calculation and does not require general relativity. From simple Newtonian mechanics, it follows that particles held at the BH event horizon $r_g \simeq MG/c^2$ experience an acceleration $a_g \cong c^4/MG$. In general relativity, the acceleration is infinite, if held at the horizon, but is just the Newtonian value if looked at from infinity, because of the redshift factor between the horizon and infinity. Now suppose the atom falls into the BH. While GR says such a freely falling atom has no acceleration, as seen from far away, the particle looks as though it is accelerating into the BH. One might ask if it is emitting radiation because of that apparent acceleration. According to most relativists, since it is not accelerating, it should not be emitting radiation. However, this result depends on the state of the radiation field. If one chooses the background field state so that an observer or a detector far away sees no radiation either coming into the BH or going out of the BH (the Boulware state for BHs, or the Rindler vacuum for accelerating observes in flat spacetime), then that freely falling atom does emit radiation. In the case of a BH, this radiation at radiation frequencies higher than the BH Hawking temperature has a thermal spectrum (if one measures the number of photons at a certain frequency) with a temperature of the Hawking radiation. Note that this behavior is very different from that for the normal Hawking process. There an atom or a detector at a fixed radius will find itself excited as though immersed in a thermal bath. Here, the atom radiates outgoing photons with a thermal spectrum in the number representation, but the state is actually a pure state. The number representation amplitudes are coherent across the frequencies, not incoherent as a true thermal state would have them. However, if many atoms fall in, at times which are random and incoherent with each other, the outgoing state would look thermal, since the coherence of each single atom's radiation would be canceled by the incoherence between the atoms.

Some of us showed the above argument to some of our general relativity friends and received pushbacks like:

(1) This is not Hawking entropy—he had no atoms in his calculations;
(2) For atoms in free fall, the 4−acceleration is zero;
(3) This discussion is for one atom, you need to consider a cloud of atoms.

To answer these (and other) issues we wrote a paper together with our critics and friends, the paper's abstract reads [14]:

"We show that atoms falling into a BH emit acceleration radiation which, under appropriate initial conditions, looks to a distant observer much like (but is different from) Hawking BH radiation. In particular, we find the entropy of the acceleration radiation via a simple laser-like analysis. We call this entropy horizon brightened acceleration radiation (HBAR) entropy to distinguish it from the BH entropy of Bekenstein and Hawking. This analysis also provides insight into the Einstein principle of equivalence between acceleration and gravity."

In the paper, we consider a BH bombarded by a beam of two-level atoms with transition frequency ω which fall into the event horizon at a rate κ (see Fig. 4). A cavity mirror held at the event horizon shields infalling atoms from the Hawking radiation. The equivalence principle tells us that an atom falling in a gravitational field does not "feel" the effect of gravity, namely its 4−acceleration is equal

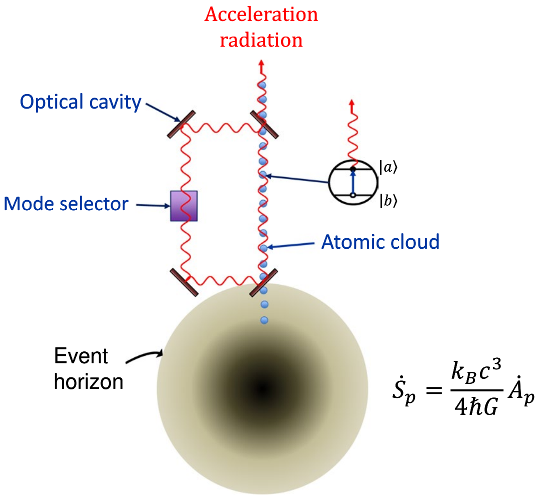

Fig. 4 A BH is bombarded by a pencil-like cloud of two-level atoms falling radially from infinity. A cavity is held at the event horizon which shields infalling atoms from the Hawking radiation and the mode selector picks one cavity mode (or a few modes) counterpropagating relative to the atoms that are in the relevant frequency range for significant excitation probabilities. The relative acceleration between the atoms and the field yields generation of acceleration radiation. The physics of the acceleration radiation process corresponds to the excitation of the atom together with the emission of the photon (Color figure online)

to zero. However, there is relative acceleration between the atoms and the field modes. This leads to the generation of acceleration radiation.

In the classic works [15–22], the atom (or other Unruh-DeWitt detector) was accelerated through flat spacetime. Our work differs in that the atom is in free fall and the cavity is accelerated (or supported in a gravitational field) and contains a Boulware-like ground state of the quantized field. Qualitatively, the principle of equivalence suggests that the results should be analogous to those in [15–23], but the notion that an atom in free fall should emit radiation is surprising to many people.

The above calculations were done for the state of the field being the universal Boulware vacuum. Of course for a real BH formed by collapse, Hawking taught us that it will contain a flux of particles out of the horizon, which would give entirely different results to the above (the freely falling detector would not emit thermal radiation, nor would it see its surroundings as thermal). However, some of us believe that one could model the Boulware state by creating a stationary cavity, whose walls are impermeable (e.g., as if they were perfect mirrors) to the field. Either by cooling the inside of the cavity, or by adiabatically growing it from a tiny cavity that stretched from just outside the horizon, to far away from the BH, one could create a state inside the cavity that is suitable approximation to the Boulware vacuum. The walls are assumed to be penetrable to the atoms, but not the field of interest. Dropping the atoms through the cavity would, under this argument, also result in an outgoing flux of radiation from the freely falling atoms, just as for the pure Boulware vacuum. Another issue is the edge effect—radiation created as the atoms go through the walls which are impenetrable by the field, which could produce excess radiation. This problem is thus still one of research interest.

Assuming that the intuition of the first group is correct, we obtain an evolution equation for the density matrix of the radiation in the cavity (see Fig. 4) following the approach used in the quantum theory of the laser and find rate of change of the radiation field density matrix to be

$$\begin{aligned}\frac{1}{R}\frac{d\rho_{n,n}}{dt} &= -\frac{\kappa g^2}{\omega^2}e^{-\xi}\left[(n+1)\rho_{n,n} - n\rho_{n-1,n-1}\right] \\ &\quad - \frac{\kappa g^2}{\omega^2}e^{\xi}\left[n\rho_{nn} - (n+1)\rho_{n+1,n+1}\right],\end{aligned} \tag{20}$$

where g is the atom-field coupling constant, $\xi = 2\pi\nu r_g/c$, $R = \xi/\sinh(\xi)$, and ν is the photon frequency far from the BH. Using $S = -k_B\,\mathrm{Tr}[\hat{\rho}\ln\hat{\rho}]$, we find that the von Neumann entropy generation rate of the HBAR to be given by

$$\dot{S}_p = \frac{k_B c^3}{4\hbar G}\dot{A}_p. \tag{21}$$

Here $\dot{A}_p$ is the rate of change of the BH area due to photon emission which we are interested in.

The quantum master equation from the HBAR radiation answered the objections, of the original critics and led to further work. For example, conformal quantum theory techniques were applied by Ordóñez–Camblong school who say in the abstract to their paper [24]:

“A two-level atom freely falling toward a Schwarzschild BH was recently shown to detect radiation in the Boulware vacuum in an insightful paper [14]. In this paper, we show that this acceleration radiation is driven by the near-horizon physics of the BH. We additionally highlight the conformal aspects of the radiation that is given by a Planck distribution with the Hawking temperature.”

What is clear is that, as so often happens in physics, taking a question seriously often results in finding answers which are not only surprising to the questioners, but also to anyone in the field who thought that they thoroughly understood the field itself.

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