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An adaptive cumulative sum method for monitoring integer-valued time-series data

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ABSTRACT
In this paper, we propose an adaptive CUSUM monitoring method for detecting step and linear trend changes in count-data time-series. The data is represented using a seasonal INGARCH time series model and an exponential smoother is used to estimate level or trend changes in the data in the cumulative-sum (CUSUM) detector. In a simulation study, the proposed approach is compared to existing CUSUM approaches that are tuned for a specific shift size and the ability of the methods to detect step shifts and linear trends is investigated. The application of the proposed method in public health surveillance is demonstrated using a real infectious disease count data set.

KEYWORDS
Cumulative sum; INGARCH time series models; public health surveillance

Introduction

In many industrial quality and public health surveillance applications the count data collected and monitored over time typically exhibit temporal correlations and seasonality which require a time-series model to properly represent its dynamic characteristics. For example, infectious disease counts may be influenced by those in the previous month or the number of defects in a manufacturing process driven by a specific process dynamics may be correlated over time when the sampling interval is small. The traditional Shewhart type statistical process control (SPC) charts that assume the observations are independently distributed result in too frequent false alarms than the design value when applied on observations from a positively autocorrelated process, requiring a time series modeling approach for the observations (Montgomery 2009). Much attention has been given to investigate integer-valued time series models that take into account of the non-negativity and the discreteness nature of their generator processes in monitoring of count data.

The cumulative sum (CUSUM) and the exponentially weighted moving average (EWMA) (Hawkins and Olwell 2012; Lucas and Saccucci 1990) are commonly used techniques for detecting small changes from a specific baseline (in-control) condition in both continuous and count data settings. A typical drawback of these approaches is that they are designed to detect a particular mean shift size, and could perform much worse for detecting shifts smaller or larger than this specific design value. For normally distributed continuous data, adaptive versions of the CUSUM and EWMA charts have been studied and demonstrated to provide an overall good detection over a range of mean shift sizes (Capizzi and Masarotto 2003; Jiang, Shu, and Apley 2008; Sparks 2000; Shu and Jiang 2006; Su, Shu, and Tsui 2011). Some recent research has focused on developing adaptive schemes for monitoring count data (Aly, Saleh, and Mahmoud 2021).

While the majority of the existing adaptive monitoring approaches are based on the assumption that the data are independently distributed, as we also illustrate in our case study, autocorrelation and seasonality are highly prevalent in count data applications and there is a need to develop adaptive monitoring methods for these settings. In this paper, we propose an adaptive CUSUM method for count data time series, in which an integer-valued generalized autoregressive conditional heteroskedastic (INGARCH) time series model is utilized to account for any autocorrelation structure and seasonality in the data. An exponentially weighted moving average (EWMA) smoother is used to update the reference

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value of CUSUM and a likelihood ratio test statistic that incorporates the INGARCH conditional mean function is used to detect step or linear trend shifts from the mean. It is shown how, simple EWMA and double EWMA estimators can be used in the proposed adaptive method for detecting step and linear trend shifts, respectively. We study by simulation the effect of seasonality and autocorrelation on the ability to detect step shifts and linear trends. A case study involving Salmonella outbreak data (Höhle and Paul 2008) was used to illustrate the application of the proposed method in public health surveillance.

The remainder of the article is organized as follows. Section "Review of relevant literature" reviews the relevant literature on count data time series and change detection. Section "Proposed methodology" presents the proposed adaptive CUSUM method. Section "Simulation study" presents a Monte Carlo simulation to study the efficacy of the the method. The illustration of the proposed method on real data is presented in the Section "Case study: German Salmonella infection data". Section "Conclusions" summarizes the contributions of the proposed research and discusses potential its future extensions.

Review of relevant literature

Prospective monitoring of time series of counts or attributes for detecting sustained changes in the mean function has been studied in industrial quality and public health surveillance fields by many authors (see e.g., the reviews by (Woodall 1997) and (Woodall et al. 2006)). In industrial quality control, defect rates, number of defects observed from a production line per the measurement unit are often described by an independent and identical distributed Poisson process. Periodic autoregressive models are a natural way to capture autocorrelation and seasonality in time series and are commonly used in infectious disease epidemiology (Corberán-Vallet and Lawson 2014) and econometrics (Bollerslev and Ghysels 1996). Seasonality and autocorrelation in count data is encountered in health care surveillance problems due to seasonal effects of infectious diseases or to time-varying population sizes, often modeled by using trigonometric functions of time (Höhle and Paul 2008), non-homogeneous Poisson models (Richards, Woodall, and Purdy 2015), or applying algebraic transformations of the data (Rossi, Lampugnani, and Marchi 1999).

Lucas Lucas (1985) studied the performance of the cumulative sum (CUSUM) introduced by Page (1954), for Poisson distributed counts and provided a detailed analysis of average run length. White and Keats (1996) proposed a Markov chain method to approximate the in-control average run length (ARL) of Poisson CUSUM. For modeling of correlated count data sequences, integer-valued generalized autoregressive conditional heteroscedasticity (INGARCH) models and integer-valued autoregressive moving average (INARMA) models are two main classes of time series models. INARMA models utilize a binomial thinning operation to adapt the standard, continuous-variable, ARMA models to discrete random variables (McKenzie 1988). INGARCH models (Ferland, Latour, and Oraichi 2006), by contrast, assume a linear structure for the conditional mean, that allows easy determination of the statistical properties of high order model structures. For monitoring of correlated count data sequences using CUSUM schemes, Weiß and Testik (2009) studied integer autoregressive (INAR) models to model AR(1)-like serial dependence. INAR models are effective in modeling first-order autocorrelation with Poisson marginals and from the properties of the thinning operator it is easy to compute the exact average run length (ARL) of the monitoring scheme. For applications of monitoring overdispersed counts with autoregressive serial dependence structures, Weiß and Testik (2012) proposed to use integer autoregressive conditionally heteroscedastic (INARCH) models in CUSUM monitoring. To handle seasonality and long term memory in count data, Vanli et al. (2019) extended the INARCH-based monitoring to monitoring with seasonal generalized autoregressive conditional heteroscedasticity (INGARCH) models. More recently, Ottenstreurer (2021) has studied Shiryaev-Roberts (SR) charts for monitoring INARCH(1) proesses with various marginal distributions, including Poisson, Negative Binomial and Binomial. By contrast to the CUSUM procedure, which, as explained below, uses the maximum of all log likelihoods upto the current time point as the alarm statistic, a SR procedure uses the sum of all log likelihoods.

In using a CUSUM approach, the practitioner typically assumes a value for the mean after a change, according to the smallest mean shift considered important enough to be detected quickly. In addition the basic CUSUM assumes sustained or persistent shifts, and not of a time-varying or intermittent shifts form (Capizzi and Masarotto 2012). The mean after a change is unknown, of course, and a possible solution is to use multiple monitoring statistics simultaneously, each optimized for a different size mean shift (Sparks 2000; Han et al. 2007). Another solution is to use a generalized likelihood ratio (GLR) formulation which uses maximum likelihood estimation to estimate the magnitude of the shift in addition to detecting the change (Tsui et al. 2012; Reynolds and Lou 2010). However, due to this additional optimization GLR charts have significantly higher computational burden than CUSUM or EWMA charts.

Another solution to deal with the assumption of a specific mean after the change is to use adaptive control charts. Adaptive CUSUM and adaptive EWMA charts have recently been proposed, for normally distributed data, to detect shifts with unknown magnitudes (Capizzi and Masarotto 2003; Jiang, Shu, and Apley 2008; Sparks 2000; Shu and Jiang 2006; Su, Shu, and Tsui 2011)). The unknown one-sided mean shift magnitude is first estimated based on some smoothing approach of the available observations, such as the absolute value of an exponentially weighted moving average (EWMA) statistic (Shu and Jiang 2006; Shu, Jiang, and Tsui 2008). Then to make the detection statistic sensitive to possibly time-varying "patterned" shifts, a CUSUM is defined as a weighted function of the shift estimate by using a certain type of weighting function (Shu, Jiang, and Tsui 2008; Yashchin 1989) representing the alarm limit of the adaptive chart. In order to apply likelihood ratio testing principles, a linear weighting function is typically recommended (Jiang, Shu, and Apley 2008). Sparks (2000) presented a regression approach to determine the alarm limit for a limited number of values. Shu and Jiang (2006) developed a two-dimensional Markov chain model for the adaptive CUSUM statistic and show that alarm limit can be approximated by a closed form expression.

Proposed methodology

In this section we present the proposed adaptive cumulative sum method to detect step and linear trend changes in the mean of count-data time-series models. We show how the count-data is represented with a seasonal INGARCH model, an EWMA is used to estimate step and linear trend shifts in the conditional mean function and the adaptive CUSUM statistic is forumlated as a function of the linearly weighted shift estimate.

Seasonal INGARCH(1,1) time series model

Conditional on the past data, the count y_t of an event (e.g., disease cases) in time periods $t = 1, 2, \dots$ is assumed to follow an integer-valued generalized autoregressive conditional heteroskedastic, or INGARCH(1,1), process (Ferland, Latour, and Oraichi 2006):

$$y_t|\mu_t \sim \text{Poisson}(\mu_t) \tag{1}$$

$$\mu_t = \delta + \alpha y_{t-1} + \gamma \mu_{t-1} \tag{2}$$

with conditional mean (or incidence rate) μ_t, intercept $\delta > 0$ and autoregressive parameters $\alpha \geq 0$ and $\gamma \geq 0$ for y_{t-1} and μ_{t-1}, respectively. The appropriate positivity or non-negativity requirements of the parameters are imposed to ensure nonnegativity of μ_t. INGARCH(1,1) process is considered to be an integer-valued analogue of the GARCH(1,1) process (Bollerslev 1986), because the model is conditional on the Poisson mean (which equals the conditional variance) and allow one to account for the heteroscedasticity in the variance.

The constant intercept in (2) can be modified to incorporate secular (linear) and seasonal time trends (Kleinman 2005) by using linear and trigonometric functions of time to obtain a seasonal INGARCH(1,1) model:

$$\mu_t = \delta + \Psi_t + \alpha y_{t-1} + \gamma \mu_{t-1} \tag{3}$$

where Ψ_t combines the effects of secular and seasonal trends:

$$\Psi_t = \rho t + \sum_{j=1}^{K} \{\eta_j \cos(2\pi jt/T) + \psi_j \sin(2\pi jt/T)\} \tag{4}$$

$$= \rho t + \sum_{j=1}^{K} R_j \sin(2\pi jt/T + \Omega_j) \tag{5}$$

where ρ is the coefficient modeling the linear trend over time, $\eta = (\eta_1, \dots, \eta_K)$ and $\psi = (\psi_1, \dots, \psi_K)$ are the coefficients of the trigonometric functions of time modeling the seasonal trend, K is the number of trigonometric functions needed, and T is the length of the season. In Equation (5), $R_j = \sqrt{\eta_j^2 + \psi_j^2}$ is the amplitude and $\Omega_j = \arctan(\psi_j/\eta_j)$ is the phase shift of each sinusoidal wave.

This approach is similar to (Höhle and Paul 2008) who used trigonometric functions to model seasonality in count data for prospective outbreak detection, however, their model assumed count data is distributed independently over time and hence does not account for temporal autocorrelation. Modeling seasonal variation in infectious disease counts through the inclusion of set of trigonometric functions has been used in various studies, including (Held and Paul 2012). Bentarzi and Bentarzi Bentarzi and Bentarzi (2017) provided closed-form expressions for the marginal mean and variance functions for seasonal INGARCH(1,1) processes under the condition of periodical stationarity:

$$E[y_t] = \beta_t = \frac{\delta + \Psi_t}{1 - (\alpha + \gamma)} \tag{6}$$

$$V[y_t] = \frac{\beta_t(1 - (\alpha + \gamma)^2 + \alpha^2)}{1 - (\alpha + \gamma)^2}. \tag{7}$$

Existing methods to detect specific magnitude shifts in count-data time-series

Before introducing the proposed CUSUM method we first review the existing INGARCH(1,1) based CUSUM scheme for detecting fixed magnitude changes in count-data time series, previously studied by (Vanli et al. 2019; Weiß and Testik 2012). The approach is based on a likelihood-ratio test that assumes the count-data follows the INGARCH(1,1) model with conditional mean $\mu_{0,t} = \mu_t(\delta_0, \eta, \psi, \alpha, \gamma, \rho)$ upto a time point $\tau - 1$, τ being the changepoint, and with conditional mean $\mu_{1,t} = \mu_t(\delta_1, \eta, \psi, \alpha, \gamma, \rho)$ after time point τ. That is, it is assumed that only the intercept δ is subject to change and all other parameters remain constant.

The intercept of the process when it is subject to a step shift with an unknown magnitude κ is represented as

$$\begin{aligned} \delta &= \delta_0, \ \text{for } t \le \tau \\ &= \delta_0 + \kappa, \ \text{for } t = \tau, \tau + 1, \ldots \end{aligned} \tag{8}$$

Similarly, if the intercept is subject to linear trend shifts with slope ω then

$$\begin{aligned} \delta &= \delta_0, \ \text{for } t \le \tau \\ &= \delta_0 + \omega \times (t - \tau + 1), \ \text{for } t = \tau, \tau + 1, \ldots \end{aligned} \tag{9}$$

Weiß and Testik (2012) and Vanli et al. (2019) considered detecting step shifts in the intercept of INGARCH(1,1) processes, by testing the following hypotheses

$$\begin{aligned} &H_0 : \delta = \delta_0 \ \text{for all } t \\ &H_1 : \delta = \delta_0 \ \text{for } t = 1, 2, \ldots, \tau - 1 \\ &\qquad \delta = \delta_1^*(t) = \delta_0 + \kappa^* \ \text{for } t = \tau, \tau + 1, \ldots \end{aligned}$$

where κ^* is a specified step shift magnitude. For detecting linear trends, this method can be modified to test the following hypotheses:

$$\begin{aligned} &H_0 : \delta = \delta_0 \ \text{for all } t \\ &H_1 : \delta = \delta_0 \ \text{for } t = 1, 2, \ldots, \tau - 1 \\ &\delta = \delta_1^*(t) = \delta_0 + \omega^*(t - \tau + 1) \ \text{for } t = \tau, \tau + 1, \ldots \end{aligned}$$

Note that for detecting linear trends an estimate of the change-point needs to be obtained, as will be reviewed below. The log likelihood ratio for the joint probability distribution $f(y_1, \ldots, y'_t|.)$ of the data observed up to a time instant t' under the hypotheses H_1 and H_0 is:

$$\begin{aligned} L(\mu_0, \mu_1^*, t') &= \log \frac{f(y_1, \ldots, y_{t'}|\mu^*_{1,t'})}{f(y_1, \ldots, y_{t'}|\mu_{0,t'})} \\ &= \sum_{t=1}^{t'} \log \frac{f(y_t|\mu^*_{1,t})}{f(y_t|\mu_{0,t})} \end{aligned} \tag{10}$$

where $\mu_{0,t}$ and $\mu^*_{1,t}$ are the conditional mean functions under the null H_0 hypothesis (i.e., with intercept δ_0) and the alternative H_1 hypothesis (i.e., with the specified intercept $\delta_1^*(t)$, based either on step or linear trend), respectively. The second equality is obtained because the data are independent given the conditional mean at time period t per Equations (1) and (2), and writing the log likelihood as the sum of the log likelihood ratios of respective time instants. The alarm statistic is the maximum of all log likelihoods upto time t, that is, $S_t = \max_{1 \le t' \le t} L(\mu_0, \mu_1^*, t')$. Accordingly, the alarm statistic based on Equation (10) can be written as a cumulative sum (CUSUM) (Lorden 1971).

$$S_t = \max\left(0, S_{t-1} + \log \frac{f(y_t|\mu^*_{1,t})}{f(y_t|\mu_{0,t})}\right). \tag{11}$$

For a Poisson probability mass function $f(y|\mu) = e^{-\mu}y^{\mu}/\mu!$, the CUSUM statistic is simplified as:

$$S_t = \max\left(0, S_{t-1} + y_t \log \frac{\mu^*_{1,t}}{\mu_{0,t}} - (\mu^*_{1,t} - \mu_{0,t})\right) \tag{12}$$

where the conditional means $\mu^*_{1,t}$ and $\mu_{0,t}$ are obtained by evaluating Equation (3), with $\delta_1^*(t)$ and δ_0, respectively. For detecting step shifts with magnitude κ^*, the out-of-control mean $\mu^*_{1,t}$ is evaluated with $\delta_1^* = \delta_0 + \kappa^*$. For detecting trend shifts with slope ω^* the out-of-control mean is obtained using $\delta_1^*(t) = \delta_0 + \omega^*(t - \hat{\tau} + 1)$. These are the existing cusum approaches for step and trend shifts, which will be termed, respectively, as SCUSUM and TCUSUM, and will be studied to compare with the proposed adaptive CUSUM method.

As the estimator of change-point using a CUSUM, Page (1954) proposed to use the starting point of the last Wald sequential test (starting point of the rejection test):

$$\hat{\tau} = \max_{1 \le t \le t_a} \{t | S_t = 0\}. \tag{13}$$

where t_a is the alarm time, $t_a = \min_{1 \le t' \le t}\{t' | S_{t'} > h\}$. This is the approach we will use in implementing the TCUSUM.

Proposed adaptive CUSUM method to detect shifts in count-data time-series

A disadvantage of the existing CUSUM approach is that the shift magnitude, either κ^* or ω^*, need to be specified, while the actual shift may have a different magnitude from the specified value. Another drawback is that, the process change may follow a profile different than a specified pattern, for example instead of a step change a linear trend change may occur. The proposed method is an adaptive cumulative sum (CUSUM) monitoring statistic to detect step or linear trend changes in seasonal INGARCH(1,1) processes by using an exponential smoother to estimate the shift magnitude. The proposed Adaptive CUSUM method, unlike the existing approach, does not require a shift size be specified and can be formulated for detecting both step and linear trend changes.

A one-step ahead forecast of the count data y_t of the in-control process given all observations $y_1, y_2, \ldots, y_{t-1}$ upto the current time $t-1$ is the conditional mean of the INGARCH process model (Bollerslev and Ghysels 1996) at time t, written as:

$$\mu_{0,t} = \delta_0 + \Psi_t + \alpha y_{t-1} + \gamma \mu_{0,t-1} \quad \text{for } t = 1, 2, \ldots \tag{14}$$

where Ψ_t is defined with parameters ρ, ψ_j, η_j and T using Equation (4) and the equation is initialized at $\mu_{0,0} = \delta_0/(1-\alpha-\gamma)$.

Suppose, as a result of a shift in the intercept, the conditional mean changes from $\mu_{0,t}$ to $\mu_t = \mu_{0,t} + \theta$ where θ is the shift magnitude. Let $\hat{\theta}_t$ be the exponentially weighted moving average (EWMA) estimate of the shift. The one step ahead forecast of the process y_t accounting for the possible shift is

$$\hat{\mu}_t = \mu_{0,t} + \hat{\theta}_t. \tag{15}$$

Since all parameters of the in-control mean $\mu_{0,t}$ are known we do not use a "hat" notation for this term. However, if the in-control model is also estimated from data then $\mu_{0,t}$ is replaced with its estimate $\hat{\mu}_{0,t}$. In the case study that is presented in "Case study: German Salmonella infection data" we consider a problem where the in-control mean is estimated.

We study both simple EWMA and double EWMA to estimate the shifts in mean. For detecting step shifts, a simple EWMA is used to estimate the shift:

$$\begin{aligned}\hat{\theta}_t &= (1-\lambda)\hat{\theta}_{t-1} + \lambda(y_t - \mu_{0,t-1}) \\ &= \hat{\theta}_{t-1} + \lambda e_t\end{aligned} \tag{16}$$

where $0 \le \lambda \le 1$ is the smoothing constant, $e_t = y_t - \mu_{0,t-1} - \hat{\theta}_{t-1} \equiv y_t - \hat{\mu}_{t-1}$ is the prediction error where in the second equality $\hat{\mu}_t$ comprises both the time-series model forecast andthe EWMA estimate, according to Equation (15). The EWMA is initialized as $\hat{\theta}_0 = 0$. The EWMA equation can be written more generally, using a monotone score function

$$\hat{\theta}_t = \hat{\theta}_{t-1} + \phi(e_t) \tag{17}$$

which reduces to (16) when $\phi(e_t) = \lambda e_t$. To ensure that the procedure (17) tracks large shifts quickly, Capizzi and Mazorotto (2003) propose using Huber's score function defined as

$$\phi(e) = \begin{cases} e + (1-\lambda)\xi & \text{if } e < -\xi \\ \lambda e & \text{if } |e| \le \xi \\ e - (1-\lambda)\xi & \text{if } e > \xi \end{cases} \tag{18}$$

where $\xi \ge 0$ is a thresholding constant specified by the user. The Markovian-type statistic (Lorden 1971) with the Huber function includes the EWMA statistic $\phi(e_t) = \lambda e_t$ as a special case when $\xi \to \infty$. When $\xi = 0$ or $\lambda = 1$, Huber's function reduces to $\phi(e) = e$, and the statistic (Lorden 1971) with the Huber function is essentially a Shewhart statistic.

For estimating constant shifts in the mean of a process, a simple EWMA typically provides adequate performance. However, for processes that drift according to a linear trend, the simple EWMA estimate often "lags" behind the actual shift and a double EWMA, which uses two exponential smoothers, provides a superior performance (Del Castillo 1999). In a double EWMA, a smoother F_t is used to estimate the level and another smoother G_t is used to estimate the slope of the data. The double EWMA estimate $\hat{\theta}_t$ of the shift in the mean of the INGARCH process is then obtained using the following equations:

$$\hat{\theta}_t = F_t + G_t \tag{19}$$

$$\begin{aligned}F_t &= (1-\lambda)(F_{t-1} + G_{t-1}) + \lambda(y_t - \mu_{0,t-1}) \\ &= (F_{t-1} + G_{t-1}) + \lambda(y_t - \mu_{0,t-1} - F_{t-1} - G_{t-1})\end{aligned} \tag{20}$$

$$\equiv \hat{\theta}_{t-1} + \phi(e_t) \tag{21}$$

$$e_t = y_t - \mu_{0,t-1} - \hat{\theta}_{t-1} \tag{22}$$

$$\begin{aligned}G_t &= (1-\eta)G_{t-1} + \eta(F_t - F_{t-1}) \\ &= G_{t-1} + \eta(F_t - \hat{\theta}_{t-1}) \\ &\equiv G_{t-1} + \eta\phi(e_t)\end{aligned} \tag{23}$$

where $\phi(e_t)$ is the score function defined as in Equation (18) with $0 \le \lambda \le 1$ as the smoothing constant for the level and $0 \le \eta \le 1$ as the smoothing constant for the slope.

The proposed one-sided adaptive CUSUM for detecting increases based on the EWMA estimator $\hat{\theta}_t$ of the shift is defined as

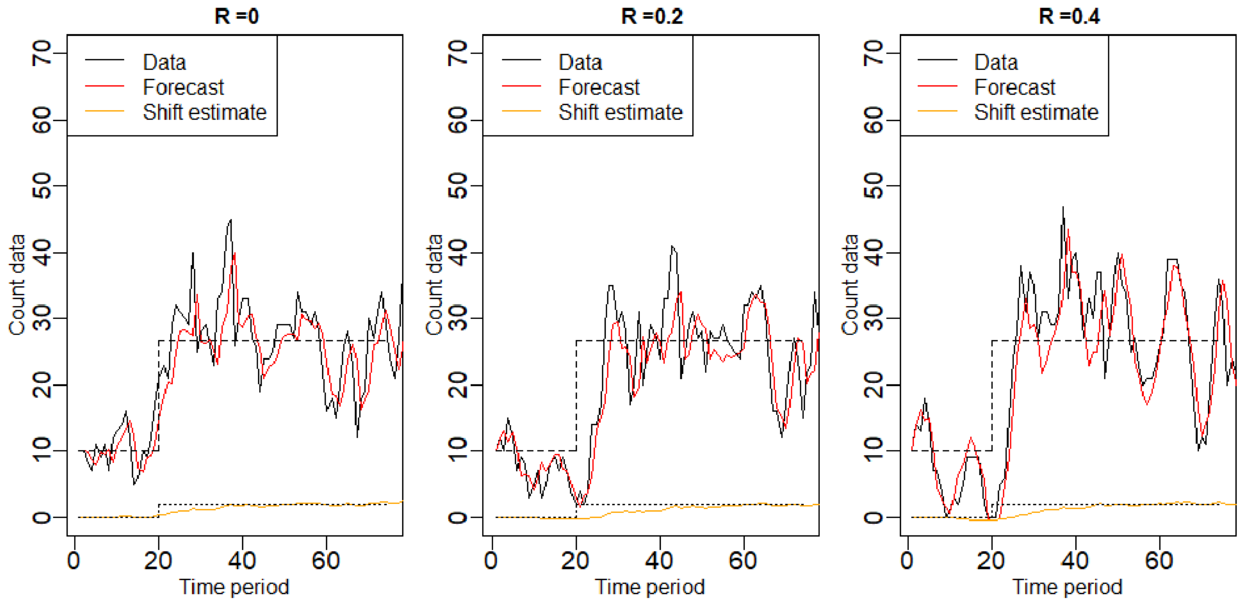

Figure 1. Single realization of an INGARCH(1,1) process with seasonalities $R = 0, 0.2$ and 0.4 under step shift. The shift estimate and the forecast are obtained with a simple EWMA utilizing $\lambda = 0.2$ and $\xi = \infty$.

$$S_t = \max\left(0, S_{t-1} + \varphi(\hat{\theta}_t)\left[y_t \log\frac{\hat{\mu}_t}{\mu_{0,t}} - (\hat{\mu}_t - \mu_{0,t})\right]\right) \tag{24}$$

where $\mu_{0,t}$ is the one-step-ahead forecast of the in-control process mean found by Equation (14) and $\varphi(\hat{\theta}_t)$ is a weight function of the estimated shift magnitude that defines the control limit of the CUSUM statistic. The shift estimate $\hat{\theta}_t$ and the forecasted conditional mean $\hat{\mu}_t$ are found using either a simple EWMA (Lorden 1971) or a double EWMA (Lucas and Saccucci 1990). The adaptive CUSUM is initialized at $S_0 = 0$. The single EWMA is initialized at $\hat{\theta}_0 = 0$ and the double EWMA smoothers are initialized at $F_0 = \delta_0/(1 - \alpha - \gamma)$ and $G_0 = 0$.

In this study, two new INGARCH-based adaptive CUSUMs are proposed. The first method, referred to as the single exponential smoother CUSUM, or SESCUSUM, is based on a simple EWMA and used to detect step shifts in the mean. The second method, referred to as the double exponential smoother CUSUM, or DESCUSUM, is based on a double EWMA and used to detect linear trend shifts in the mean. Without loss of generality, we focus in this paper on one-sided CUSUMs to detect increases in the mean, however, the proposed approach is applicable for constructing two-sided CUSUMs as well.

In this study, we consider the linear weight function $\varphi(\hat{\theta}_t) = \hat{\theta}_t$, which was shown to provide superior performance in adaptive CUSUM charts in continuous data in previous studies (Jiang, Shu, and Apley 2008). Weighted CUSUM is a generalization of the basic cumulative sum control scheme and has been studied to make the method more sensitive to time-varying and dynamic patterned shifts (Shu, Jiang, and Tsui 2008; Yashchin 1989).

The performance of the monitoring scheme will be measured by how quickly an alarm is signaled by the monitoring scheme when the process moves out-of statistical control. The speed of signaling an alarm is measured by the average run length (ARL), the expected number of samples required by the method to signal. It is desirable to have a small out-of-control ARL, denoted ARL_1, the ARL when there is a significant change in the process, so that the change is detected quickly or with minimum detection delay. By contrast, it is desirable to have a large in-control ARL, denoted ARL_0, the ARL when the process is in a state of statistical control, so that the rate of false alarms is low. The monitoring scheme signals an alarm if the CUSUM exceeds an alarm threshold h, that is, when $S_k > h$. The alarm threshold h is determined by Monte Carlo simulation of the process under the null hypothesis so that the in-control ARL of the chart is close to a pre-specified value.

Simulation study

In this section we study by simulation the performance of the proposed adaptive CUSUM method in detecting step and trend shifts in the mean of count-data time-series. The in-control process is represented with a seasonal INGARCH(1,1) with $K = 1$ harmonic

Table 1. ARL of SESCUSUM and SCUSUM under step shifts.

		SESCUSUM (ξ,λ)									SCUSUM (κ^*)					
		1.5			4			∞								
R	κ	0.4	0.8	0.95	0.4	0.8	0.95	0.4	0.8	0.95	0.25	0.6	1.8	2.5	6	12
0	0	398.95	402.69	399.85	399.59	401.15	399.83	400.89	400.46	399.42	400.41	400.14	401.54	401.01	402.75	416.41
	0.25	149.73	147.62	148.10	151.80	146.55	148.05	141.83	145.64	147.39	148.56	151.36	192.67	209.47	242.58	239.25
	0.5	86.33	84.42	84.59	86.89	83.77	84.37	78.47	82.18	84.13	86.26	81.14	103.72	118.86	163.26	158.48
	1	42.95	42.20	42.38	42.74	41.70	42.28	39.11	40.96	41.89	51.23	42.01	42.91	48.84	77.97	79.89
	1.5	27.33	26.99	27.15	27.32	26.71	27.09	25.70	26.47	26.95	39.16	30.01	24.52	26.26	42.46	47.48
	2	19.74	19.48	19.59	19.70	19.27	19.53	19.14	19.18	19.45	32.74	23.88	16.98	16.96	25.96	30.67
	2.5	15.32	15.14	15.19	15.27	14.98	15.19	15.29	15.00	15.14	28.59	20.25	13.04	12.29	16.72	21.29
	3.5	10.36	10.23	10.26	10.36	10.14	10.25	10.87	10.29	10.24	23.30	15.69	9.00	7.97	8.47	11.39
	6	5.55	5.54	5.55	5.52	5.51	5.55	6.30	5.68	5.57	16.62	10.31	5.23	4.38	3.36	3.68
	9	3.50	3.53	3.56	3.41	3.50	3.55	4.19	3.68	3.58	12.58	7.39	3.59	2.99	2.11	1.87
	12	2.55	2.61	2.63	2.46	2.57	2.63	3.13	2.74	2.65	10.09	5.74	2.79	2.35	1.62	1.35
0.2	0	391.15	399.31	399.03	400.14	399.85	399.20	400.59	400.33	400.11	403.06	400.66	399.35	399.76	393.26	397.11
	0.25	163.68	158.24	156.44	164.51	158.63	157.39	155.93	159.46	156.46	136.34	145.21	183.59	200.00	229.01	232.54
	0.5	96.09	93.06	91.36	96.26	92.00	91.82	87.61	91.71	91.86	81.88	78.21	99.40	113.72	147.93	153.71
	1	48.08	46.63	46.07	47.21	46.24	46.30	43.77	45.68	46.11	50.99	41.45	41.67	47.14	72.78	76.74
	1.5	30.24	29.56	29.15	29.63	29.16	29.27	28.34	28.91	29.09	39.82	30.29	23.82	25.40	40.27	45.74
	2	21.56	21.06	20.86	21.24	20.84	20.95	21.00	20.86	20.88	33.84	24.58	16.86	16.67	24.51	29.71
	2.5	16.55	16.19	16.04	16.36	16.08	16.14	16.69	16.19	16.11	29.99	21.05	13.18	12.33	16.02	20.62
	3.5	11.08	10.89	10.81	11.00	10.82	10.85	11.75	11.03	10.89	25.04	16.89	9.41	8.32	8.49	11.09
	6	6.13	6.06	6.03	6.01	6.03	6.06	6.95	6.28	6.09	18.84	11.60	5.98	5.01	3.68	3.76
	9	3.95	3.97	3.96	3.80	3.93	3.97	4.77	4.16	4.01	14.82	8.79	4.24	3.46	2.31	1.95
	12	2.89	2.94	2.94	2.75	2.90	2.94	3.59	3.10	2.97	12.21	7.11	3.30	2.68	1.74	1.38
0.4	0	400.57	399.66	399.86	399.58	399.26	400.05	400.72	400.91	400.10	392.98	399.16	400.96	394.30	388.89	396.78
	0.25	163.68	155.45	154.60	162.83	154.13	152.32	144.66	149.51	154.50	102.08	110.33	154.40	169.28	223.22	253.16
	0.5	94.16	88.92	87.88	92.32	87.60	87.58	79.58	84.41	87.68	61.49	58.53	77.80	89.00	135.68	165.90
	1	46.20	43.83	43.53	44.88	42.97	43.08	39.25	41.62	43.51	40.96	33.20	33.01	37.51	63.80	84.45
	1.5	28.60	27.59	27.44	28.04	27.21	27.29	25.68	26.47	27.41	34.39	25.54	20.12	21.19	35.80	49.07
	2	20.50	19.65	19.57	20.15	19.41	19.43	19.05	18.99	19.59	30.88	21.93	14.66	14.42	21.72	31.45
	2.5	15.64	15.17	15.08	15.54	14.99	14.99	15.18	14.78	15.14	28.36	19.48	11.88	11.16	14.65	21.65
	3.5	10.57	10.26	10.22	10.44	10.15	10.16	10.75	10.16	10.29	24.79	16.36	9.07	8.06	8.19	11.54
	6	6.07	6.00	6.00	5.94	5.94	5.97	6.64	6.08	6.05	19.75	11.90	6.35	5.32	3.89	4.08
	9	4.06	4.05	4.06	3.90	3.99	4.04	4.71	4.17	4.11	16.29	9.37	4.68	3.79	2.48	2.15
	12	3.00	3.01	3.02	2.85	2.96	3.01	3.58	3.13	3.06	13.74	7.91	3.68	2.93	1.86	1.50

component for monthly observations with an annual season ($T=12$):

$$y_t|\mu_{0,t} \sim \text{Poisson}(\mu_{0,t}) \tag{25}$$

$$\mu_{0,t} = \delta_0 + \alpha y_{t-1} + \gamma\mu_{0,t-1} + \psi\cos(2\pi t/12) + \eta\sin(2\pi t/12). \tag{26}$$

Processes with equal harmonic coefficients (i.e., with $\eta=\psi$) and various seasonality values, represented by $R=\sqrt{\psi^2+\eta^2}$, are considered.

The adaptive CUSUM methods are compared to the existing CUSUM procedures. All methods assume the correct INGARCH(1,1) representation of the process as the baseline model $\mu_{0,t}$. The alarm thresholds for the methods are determined to achieve $ARL_0 = 400$ from Monte Carlo simulations of the baseline process replicated 10,000 times for any configuration. Similarly, ARL_1 is calculated from 10,000 Monte Carlo simulations for each out of control configuration. The relative mean index (RMI), studied by Han and Tsung (2006), is used to summarize the performance of a chart over a range of shifts. RMI for a chart is calculated using

$$RMI = \frac{1}{L}\sum_{i=1}^{L}\left(\frac{ARL_1^{(i)} - ARL_1^{*(i)}}{ARL_1^{*(i)}}\right) \tag{27}$$

where L is the total number of shift sizes considered, $ARL_1^{(i)}$ is the out of control ARL of the chart at the i-th shift size and $ARL_1^{*(i)}$ is the minimum of the ARLs attained by all charts at the i-th shift size ($i=1,2...,L$). A monitoring scheme with a smaller RMI is considered to have a better overall performance than a competing method.

Detecting step shifts

The process with $\delta_0=1.2, \alpha=0.6, \gamma=0.28$ and $T=12$, but varying $\psi=\eta$ values are considered to investigate different seasonalities. Figure 1 shows a single realization of the process where a step shift with magnitude $\kappa=2$ is introduced in the intercept at time $\tau=20$ based on Equation (8). The cases of $R=0$ (non seasonal) and $R=0.2$ and 0.4 are shown. The forecasted mean $\hat{\mu}_t$ (red line) and estimated shift $\hat{\theta}_t$ (orange line) are found with the seasonal INGARCH(1,1) model. A simple EWMA with

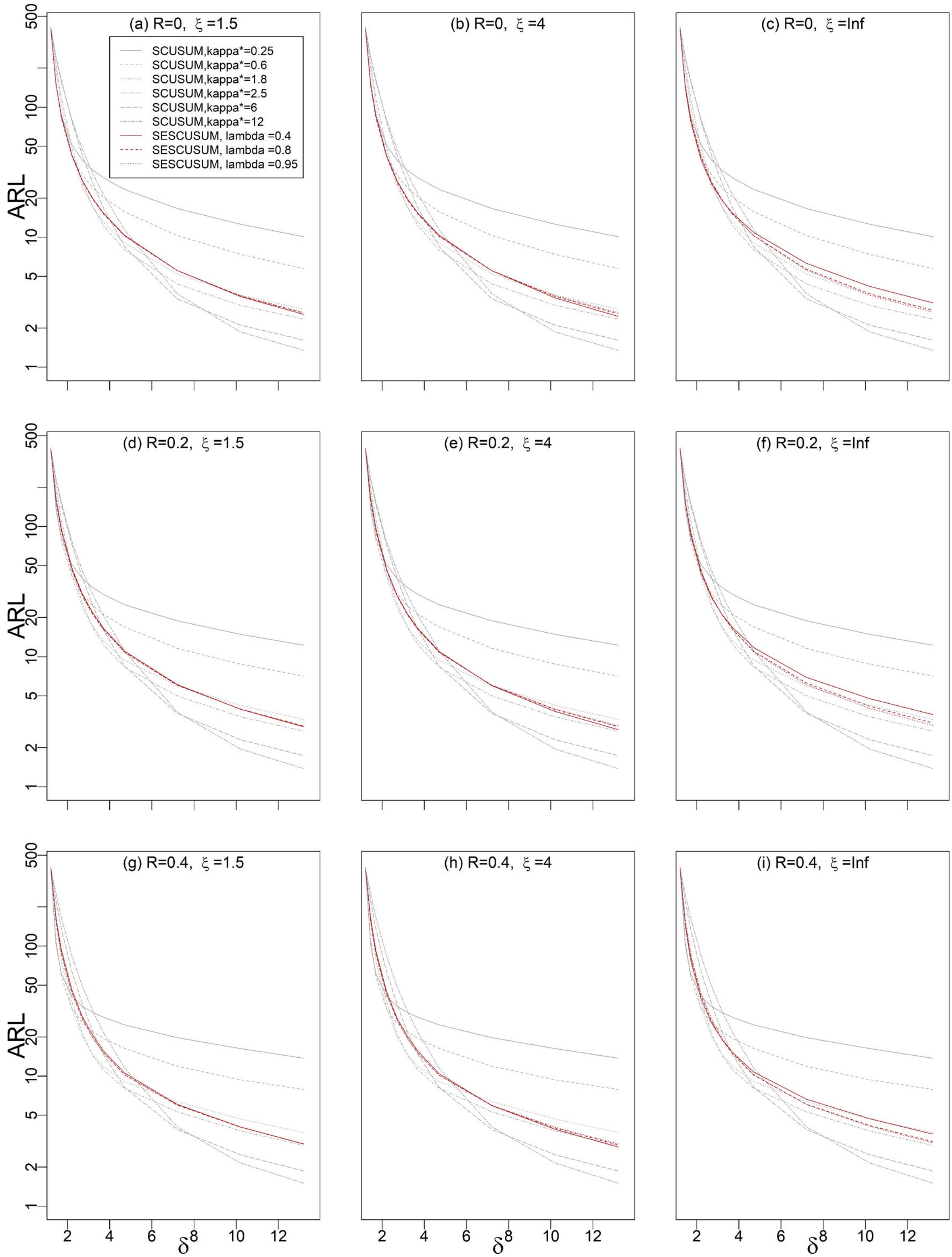

Figure 2. ARLs of SESCUSUM and SCUSUM under step shifts and increasing seasonality ($R = 0, 0.2, 0.4$). SESCUSUM uses thresholds $\gamma = 1.5, 4, \infty$ and smoothers $\lambda = 0.2, 0.4, 0.8, 0.95$. SCUSUM uses $\kappa^* = 0.6$ and 1.8.

smoothing parameter $\lambda = 0.2$ and threshold $\xi = \infty$ is used to estimate the shift. The dashed lines show the true mean and the shift before and after the change. Based on Equation (6), the initial marginal mean is 10 and a shift of $\kappa = 2$ units in the conditional mean causes it to increase by $\kappa/(1 - \alpha - \gamma) = 2/(1 - 0.6 - 0.28) = 16.67$ units to 26.67. The EWMA estimate $\hat{\theta}_t$ of the shift converges to the true value, however, the convergence is slower with seasonal processes than the non-seasonal process.

The proposed adaptive SESCUSUM statistic (Page 1954) and the existing SCUSUM statistic (Hawkins

Table 2. RMI of SESCUSUM and SCUSUM under step shifts.

	SCUSUM (κ^*)							SESCUSUM (ξ, λ)									
								1.5			4			Inf			
R	0.25	0.6	1.8	2.5	6	12	Avg	0.4	0.8	0.95	0.4	0.8	0.95	0.4	0.8	0.95	Avg
0	2.14	1.07	0.35	0.30	0.48	0.58	0.82	0.35	0.34	0.35	0.34	0.33	0.35	0.42	0.35	0.35	0.40
0.2	2.40	1.24	0.40	0.32	0.42	0.53	0.88	0.46	0.44	0.43	0.43	0.43	0.44	0.55	0.46	0.44	0.59
0.4	2.46	1.23	0.43	0.35	0.54	0.89	0.98	0.56	0.52	0.51	0.53	0.50	0.50	0.55	0.50	0.52	0.65

and Olwell 2012) are implemented to detect step shifts $\delta = \delta_0 + \kappa$ with κ varying between 0 and 12. The SESCUSUM statistic is implemented with smoother constants $\lambda = 0.4, 0.8, 0.95$ and thresholds $\xi = 1.5, 4, \infty$. The SCUSUM statistic is implemented with $\kappa^* = 0.6$ and 1.8. The detection performance of the methods is studied by computing the ARL for the processes shown in Figure 1, however, a step shift with magnitude κ is introduced in the intercept at time $\tau = 1$, that is, the zero-state ARL is computed.

Table 1 and Figure 2 show the ARLs of the methods. The ARLs under all shift sizes are larger with larger seasonalities indicating that change detection is more difficult under larger seasonalities for both methods. The SCUSUM with $\kappa^* = 0.6$ generally gives smaller ARLs for shifts smaller than about 2.5 and that with $\kappa^* = 1.8$ gives smaller ARLs for shifts larger than about 2.5 and less than about 9. The SESCUSUM, by contrast, which is not tuned for a specific shift, provides a more uniformly good ARL performance throughout the shift sizes considered regardless of λ and ξ, and for shift sizes larger than 9 the SESCUSUMs outperform the SCUSUMs.

Table 2 gives the RMI values of the methods computed based on the ARLs reported in Table 1. The proposed SESCUSUM under thresholds, $\xi = 4$ and ∞ and smoothers $\lambda = 0.8$ and 0.95 and the SCUSUM with $\kappa^* = 2.5$ provides the best performance under all seasonalities. Further, the performance of SCUSUM varies significantly with the choice of κ^*, while the performance of SESCUSUM is less variable across the choices of ξ and λ. For non-seasonal data, the SESCUSUM with λ larger than 0.4 (any ξ) outperforms the SCUSUM with $\kappa^* = 1.8$ for the entire range of shifts (smaller RMI). By contrast, when the data is seasonal, the SESCUSUM with a larger threshold $\xi = \infty$ and a larger smoother $\lambda = 0.8$ or 0.95, which puts more weight to the current data (less smoothing) is needed for better performance.

The columns labeled "Avg" in Table 2 give the average RMI under each seasonality from all model parameters (i.e., all κ^* values for the SCUSUM, all ξ and λ values for the SESCUSUM). Based on the smaller average RMI values attained, it can be seen that the SESCUSUM provides better overall performance than the SCUSUM under all seasonalities. In summary, the existing SCUSUM performance depends very strongly on the choice of κ^* (reflected by a higher average RMI value) and the choice of this parameter may not be obvious for a practitioner to set in applications. By contrast, the proposed adaptive SESCUSUM has a more uniform RMI performance regardless of the choice of its parameters ξ and λ (reflected by a lower average RMI value) and hence is easier to use by practitioners.

Detecting linear trend shifts

We considered a non-seasonal process with $\delta_0 = 1.2, \alpha = 0.6, \gamma = 0.28$ and $\psi = \eta = 0$ in which a linear trend shift in the intercept with slope ω is introduced based on Equation (9). Figure 3 shows a single realization of the case with $\omega = 0.5$ and $\tau = 20$. A simple and a double exponential smoother is used to compute the estimated shift $\hat{\theta}_t$ according to Equations (17) and (19), respectively (orange line). The forecasted mean $\hat{\mu}_t$ (red line) is obtained with Equation (15) and the dashed lines show the true mean and the shift magnitude. The simple exponential smoother with $\lambda = 0.2$ lags behind the change, however double exponential smoother with $\lambda = 0.2$ and $\eta = 0.2$ more adequately tracks the shift. Note that a double exponential smoother with $\lambda = 0.2$ and $\eta \to 0$ is equivalent to a simple exponential smoother with $\lambda = 0.2$.

The ARL performance of the proposed adaptive DESCUSUM statistic (Page 1954), based on a double EWMA, and the existing TCUSUM statistic (Hawkins and Olwell 2012), that assumes a fixed slope for a linear trend change, are compared for their efficacy in detecting linear trend shifts with slope ω varying between 0 and 12.8. A linear trend shift in the intercept is introduced at time $\tau = 1$ based on Equation (9) (i.e., zero-state ARL is computed). The double EWMA is implemented with parameters $\lambda = 0.2, 0.4, 0.8$, $\eta = 0.01, 0.05, 0.2$ and $\xi = 1.5, 4, \infty$. Note that a DESCUSUM with $\eta \to 0$ is equivalent to a SESCUSUM. A smaller value for the slope smoother than the level smoother (i.e., $\eta < \lambda$) was found to achieve more stable forecasts. The TCUSUM statistic is implemented with trend slopes $\omega^* = 1.7$ and 6.7.

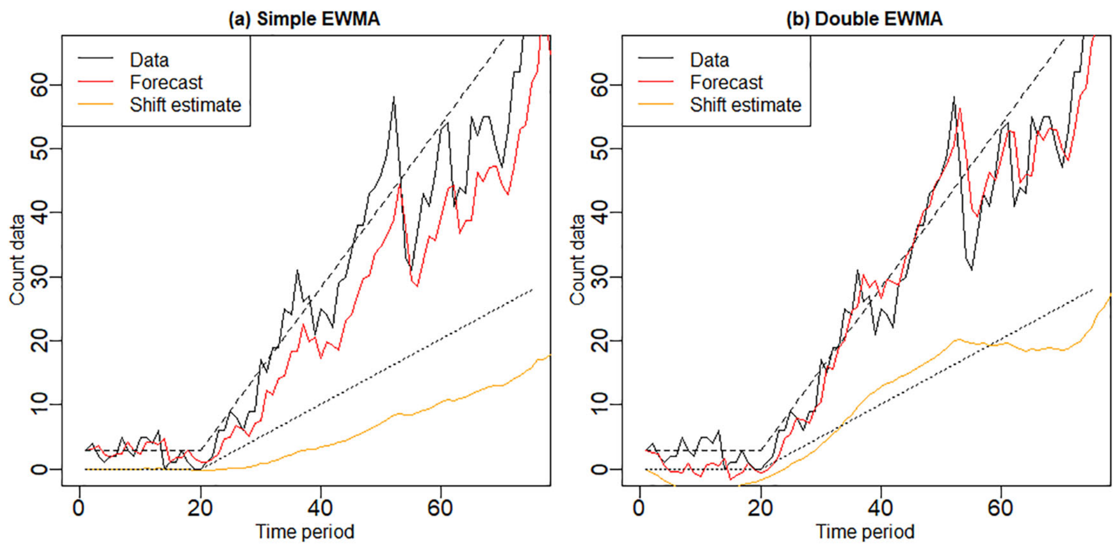

Figure 3. Single realization of an INGARCH(1,1) process under trend shift. Shift estimates and forecasts are obtained with a simple EWMA with $\lambda = 0.2$ and $\xi = \infty$ and a double EMWA with $\lambda = 0.2, \eta = 0.05,$ and $\xi = \infty$.

Table 3. ARLs of DESCUSUM and TCUSUM under linear trend shifts.

		DESCUSUM (ξ, λ, η)									TCUSUM (ω^*)					
		0.2			0.4			0.8								
ω	ξ	0.01	0.05	0.2	0.01	0.05	0.2	0.01	0.05	0.2	0.8	1.7	3.3	6.7	10	15
0	1.5	399.45	400.39	399.28	399.73	400.16	401.47	401.74	399.52	400.00	400.30	399.53	400.49	399.28	400.92	401.71
0.2		20.63	20.64	21.12	20.29	20.34	20.75	20.12	20.23	20.93	18.36	16.78	16.24	16.77	17.33	17.99
0.6		10.93	10.93	11.13	10.80	10.82	10.99	10.76	10.81	11.10	12.32	10.45	9.11	8.54	8.49	8.64
1		7.99	8.00	8.13	7.92	7.93	8.04	7.90	7.93	8.12	10.53	8.69	7.32	6.50	6.25	6.23
1.8		5.52	5.52	5.58	5.47	5.47	5.53	5.45	5.47	5.59	8.93	7.17	5.83	4.97	4.60	4.37
2.6		4.38	4.38	4.43	4.34	4.34	4.39	4.33	4.34	4.43	8.10	6.40	5.13	4.27	3.89	3.62
3.4		3.68	3.68	3.72	3.65	3.65	3.69	3.65	3.65	3.72	7.56	5.91	4.69	3.85	3.47	3.19
4.2		3.22	3.22	3.24	3.20	3.20	3.22	3.19	3.20	3.24	7.19	5.54	4.34	3.55	3.19	2.91
5.8		2.63	2.64	2.65	2.61	2.62	2.63	2.61	2.62	2.65	6.66	5.08	3.96	3.13	2.86	2.52
7.4		2.27	2.27	2.28	2.26	2.26	2.27	2.26	2.26	2.28	6.23	4.80	3.68	2.95	2.59	2.25
9.8		1.91	1.91	1.92	1.91	1.91	1.91	1.91	1.91	1.91	5.95	4.35	3.25	2.72	2.23	2.04
12.8		1.66	1.66	1.66	1.66	1.66	1.66	1.66	1.66	1.66	5.66	4.04	3.02	2.35	2.03	1.99
0	4	401.10	400.87	401.73	401.37	399.59	400.62	400.14	400.33	401.89	400.30	399.53	400.49	399.28	400.92	401.71
0.2		21.60	22.23	25.64	20.41	20.63	21.60	20.08	20.05	20.42	18.36	16.78	16.24	16.77	17.33	17.99
0.6		11.21	11.52	13.11	10.77	10.85	11.27	10.72	10.70	10.87	12.32	10.45	9.11	8.54	8.49	8.64
1		8.17	8.37	9.41	7.85	7.93	8.20	7.85	7.83	7.96	10.53	8.69	7.32	6.50	6.25	6.23
1.8		5.59	5.72	6.35	5.42	5.46	5.62	5.43	5.41	5.48	8.93	7.17	5.83	4.97	4.60	4.37
2.6		4.44	4.52	4.98	4.31	4.34	4.45	4.31	4.31	4.35	8.10	6.40	5.13	4.27	3.89	3.62
3.4		3.72	3.80	4.17	3.61	3.64	3.74	3.63	3.62	3.66	7.56	5.91	4.69	3.85	3.47	3.19
4.2		3.25	3.31	3.63	3.17	3.19	3.26	3.18	3.17	3.20	7.19	5.54	4.34	3.55	3.19	2.91
5.8		2.66	2.70	2.93	2.59	2.60	2.66	2.60	2.60	2.61	6.66	5.08	3.96	3.13	2.86	2.52
7.4		2.29	2.32	2.51	2.21	2.23	2.29	2.23	2.22	2.26	6.23	4.80	3.68	2.95	2.59	2.25
9.8		1.92	1.93	2.10	1.85	1.86	1.92	1.86	1.86	1.91	5.95	4.35	3.25	2.72	2.23	2.04
12.8		1.66	1.66	1.80	1.58	1.58	1.66	1.58	1.58	1.66	5.66	4.04	3.02	2.35	2.03	1.99
0	∞	400.59	400.12	399.67	400.30	399.20	400.68	400.33	399.75	399.83	400.30	399.53	400.49	399.28	400.92	401.71
0.2		18.92	18.93	21.86	19.60	19.75	20.91	20.00	20.06	20.30	18.36	16.78	16.24	16.77	17.33	17.99
0.6		10.21	10.20	11.26	10.36	10.39	10.75	10.66	10.68	10.77	12.32	10.45	9.11	8.54	8.49	8.64
1		7.65	7.66	8.37	7.66	7.68	7.88	7.81	7.82	7.87	10.53	8.69	7.32	6.50	6.25	6.23
1.8		5.50	5.52	6.00	5.37	5.38	5.49	5.41	5.41	5.42	8.93	7.17	5.83	4.97	4.60	4.37
2.6		4.49	4.51	4.89	4.33	4.34	4.42	4.30	4.31	4.31	8.10	6.40	5.13	4.27	3.89	3.62
3.4		3.86	3.88	4.21	3.68	3.70	3.76	3.63	3.63	3.63	7.56	5.91	4.69	3.85	3.47	3.19
4.2		3.43	3.45	3.75	3.25	3.27	3.31	3.19	3.19	3.19	7.19	5.54	4.34	3.55	3.19	2.91
5.8		2.87	2.89	3.12	2.70	2.73	2.77	2.61	2.61	2.61	6.66	5.08	3.96	3.13	2.86	2.52
7.4		2.50	2.53	2.76	2.35	2.37	2.41	2.26	2.26	2.26	6.23	4.80	3.68	2.95	2.59	2.25
9.8		2.11	2.15	2.32	2.00	2.02	2.06	1.91	1.91	1.91	5.95	4.35	3.25	2.72	2.23	2.04
12.8		1.89	1.93	2.02	1.79	1.79	1.84	1.66	1.66	1.66	5.66	4.04	3.02	2.35	2.03	1.99

Table 3 and Figure 4 show the ARLs of the methods. Similar to the step shift results, the performance of the TCUSUM is highly variable and sensitive on the choice of the slope parameter ω^*. By contrast the performance of DESCUSUM is more stable regardless of the choices of its parameters η, λ and ξ. The

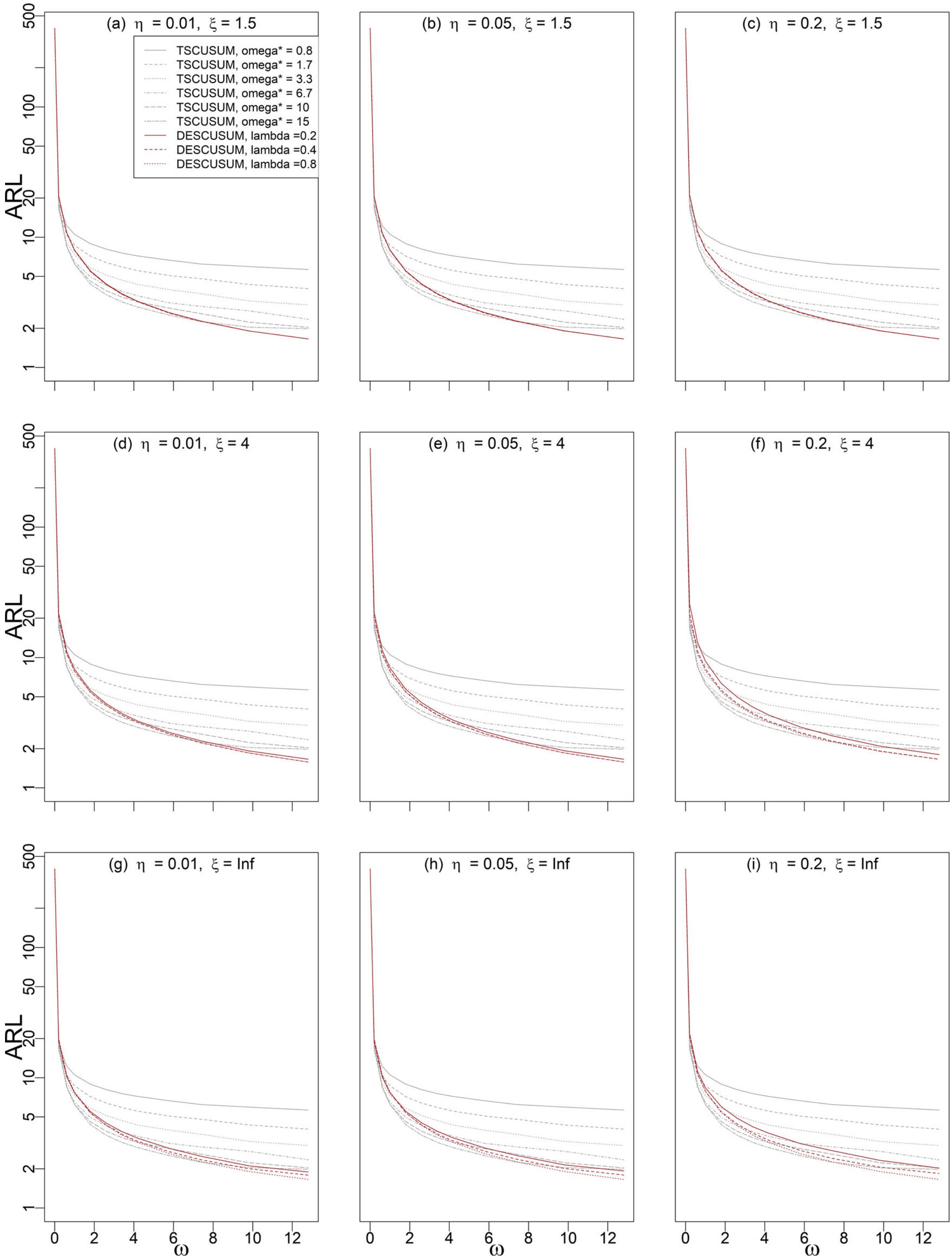

Figure 4. ARL of DESCUSUM and TCUSUM under linear trend shifts. DESCUSUM uses smoothing parameters $\eta = 0.01, 0.05$ and 0.2, thresholds $\xi = 1.5, 4, \infty$ and level smoothing parameters $\lambda = 0.2, 0.4, 0.8$ used. TCUSUM uses $\omega^* = 1.7$ and 6.7.

DESCUSUM with level smoothing constant $\lambda = 0.8$, slope smoothing constant $\eta = 0.01$ and 0.05 and the thresholds $\xi = 4$ and ∞ gives smaller ARLs. From Figure 4, an adaptive scheme outperforms the fixed slope CUSUMs when the actual slope ω is outside the range of specified ω^* values 1.7 and 6.7. Within the specified range, the fixed slope CUSUM with a larger ω^* tends to give smaller ARLs.

Table 4 gives the RMI values of the methods using the ARLs reported in Table 3 and Figure 4. The DESCUSUM with threshold $\xi = \infty$, level smoother $\lambda = 0.8$ and slope smoothers $\eta = 0.01$ and 0.05

Table 4. RMI of DESCUSUM and TCUSUM under linear trend shifts.

DESCUSUM (λ, η)										TCUSUM (ω^*)					
ξ	0.2			0.4			0.8			0.8	1.7	3.3	6.7	10	15
	0.01	0.05	0.2	0.01	0.05	0.2	0.01	0.05	0.2						
1.5	0.154	0.155	0.167	0.146	0.146	0.157	0.143	0.146	0.165	1.372	0.833	0.458	0.239	0.124	0.072
4	0.175	0.195	0.327	0.129	0.136	0.177	0.127	0.125	0.149	1.372	0.833	0.458	0.239	0.124	0.072
∞	0.185	0.195	0.307	0.155	0.160	0.194	0.138	0.139	0.143	1.372	0.833	0.458	0.239	0.124	0.072

provides better performance (reflected by smaller RMI values). By contrast, the TCUSUM method performs better with large slope values, such as $\omega^* = 15$. To assess the robustness of the methods on the choice of the parameter, the average of all RMI values for the two methods are calculated. The DESCUSUM has an average RMI of 0.209 and the TCUSUM has an average RMI of 0.389, showing that the DESCUSUM has a more robust performance over the choice of its parameters η, λ and ξ than TCUSUM over the choice of its parameter ω^*. Overall, a DESCUSUM method with a medium to large threshold value (4 or larger), a small slope smoother (between 0.01 and 0.05) and a relatively large level smoother (0.8 or larger) can be recommended for detecting linear trend shifts.

Choice of the parameters

In this section, we provide general guidance for selecting the parameters ξ and λ of SESCUSUM and ξ, η and λ of DESCUSUM. Capizzi and Masarotto (2003) presented an optimization approach and Jiang, Shu, and Apley (2008) presented a graphical approach to determine the best smoothing and threshold parameters for Adaptive EWMA monitoring of normal data. However, no studies for Poisson data or count-data time series were presented. In our study, we will follow a strategy similar to (Jiang, Shu, and Apley 2008), since as those authors also discussed, Capizzi and Masarotto (2003) optimization approach would be too complicated, especially given the additional time series structure and discreteness of data have to be considered for our problem.

In order to gain an understanding of the relationship between the ARL_1 values for detecting different sizes of shifts and the design parameters of the monitoring schemes, we fit and visualize polynomial response surfaces to the observed values. Second order polyomial response surfaces were fitted to the ARL_1 results of SESCUSUM shown in Table 1 and those of DESCUSUM shown in Table 3. For step shifts, the shift sizes $\kappa = 0.5$ and $\kappa = 2.5$ were considered as small and large shifts; for trend shifts, the shift sizes $\omega = 0.6$ and $\omega = 5.8$ were considered as small and large shifts.

Figure 5 shows the contour plots of the surfaces for SESCUSUM and step shifts. The results suggest that the smoothing parameter λ and threshold ξ of SESCUSUM should be chosen based on the shift sizes targeted. For detecting shifts with small sizes (Figure 5a and c), a larger smoothing parameter λ, between 0.8 to 0.9, is required to minimize ARL, while for large shift sizes (Figure 5b and d), a somewhat smaller smoothing parameter λ, between 0.6 to 0.7, is needed. The choice of ξ depends on whether or not the data is seasonal. For small shifts and non-seasonal data (Figure 5a), a small thresold ξ, of about 1 to 2, is needed, however, for large shifts (Figure 5b and d) or with seasonal data (Figure 5c), a very large thresold, such as ∞, is called for. Note that for large shifts and non-seasonal data (Figure 5b), the threshold parameter do not appear to have any impact (see the stationary ridge) however, to be consistent with seasonal case we recommend setting ξ to a very large value.

Figure 6 shows the contour plots of the response surfaces for DESCUSUM and trend shifts. The threshold $\xi = \infty$ (Figure 6e and f) results in larger regions for λ and η in which ARL is minimized and therefore is preferred. For detecting shifts with small slopes ($\omega = 0.6$), Figure 6e suggests that the region of $\lambda < 0.6$ and $\eta < 0.1$ minimizes the ARL. For shifts with larger slopes ($\omega = 5.8$), Figure 6f suggests the region of $0.6 < \lambda < 0.8$ and $\eta < 0.2$ minimizes the ARL. Therefore, for detecting trend shifts with DESCUSUM, we recommend to set $\xi = \infty$; for shifts with small slopes, we recommend to set λ between 0.5 and 0.6 and η to about 0.1, and for shifts with large slopes, we recommend to set λ between 0.7 and 0.8 and η to about 0.2.

Case study: German Salmonella infection data

In this section we present the application of the proposed adaptive CUSUM method on the German Salmonella case data set that was previously studied in many public health surveillance studies, including (Höhle and Paul 2008) and the R package surveillance (Meyer, Held, and Höhle 2017). The calculations are implemented in R programming language (R Development Core Team 2021) and the R codes

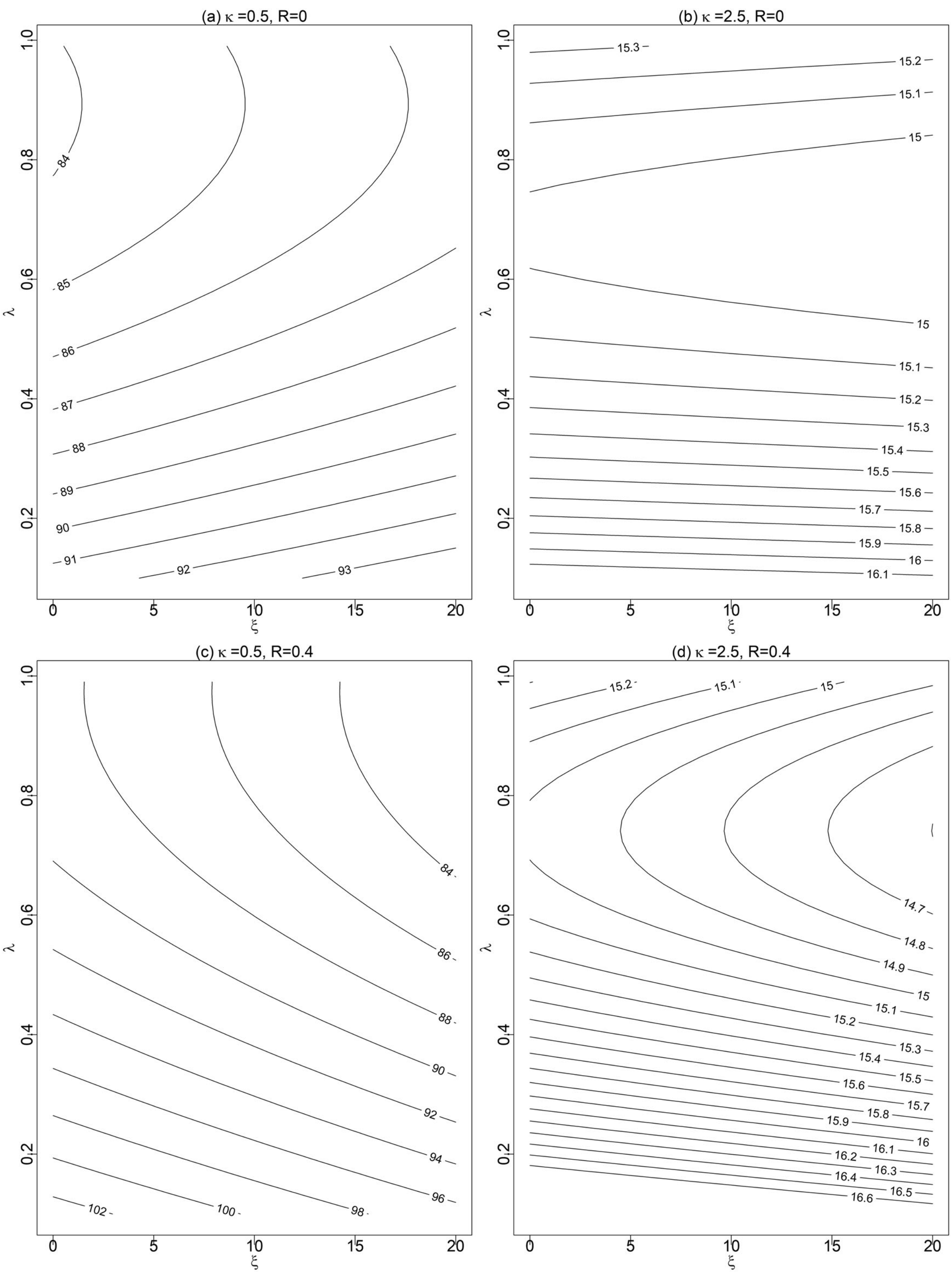

Figure 5. Contours of the response surfaces for ARLs of SESCUSUM.

developed for this study can be obtained at https://github.com/avanli/Adaptive-CUSUM-Salmonella.

The data set, plotted in Figure 7, contains the weekly counts of Salmonella Hadar disease cases observed in Germany, from 2001 to 2006, for a total of 295 weeks. The data exhibits an annual seasonal trend (with $T = 52$) superimposed on a decreasing linear secular trend. It is evident that the disease counts gradually decrease up to around week 280, which is followed by a continuous increase starting around this time. The exact time point at which this outbreak started is not clear. In order to detect the outbreak time, we consider the data observed in the time period between week 1 and week 240 (well in

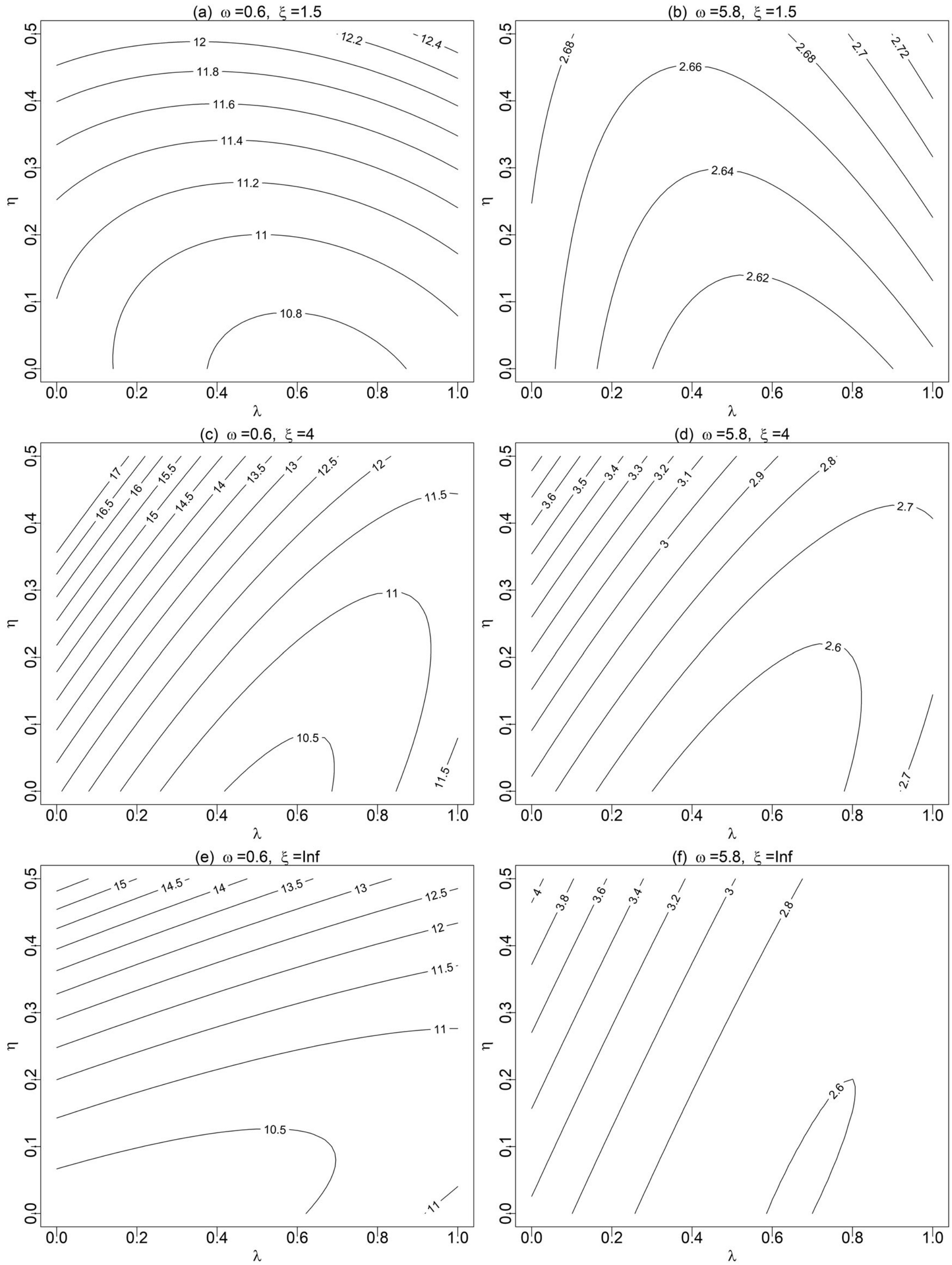

Figure 6. Contours of the response surfaces for ARLs of DESCUSUM.

advance of a potential outbreak time period) to estimate a baseline INGARCH model, using the method presented in (Vanli et al. 2019), and use this estimated model to monitor the cases observed in weeks 241 onwards. We compare the proposed adaptive SESCUSUM and DESCUSUM methods (utilizing simple and double exponential smoothers) with the existing SCUSUM method (considering a fixed step size) for detecting outbreaks using this data.

Let $1, \ldots, N$ weeks denote the period of data used to estimate the baseline model, and we consider $N = 200$ and 240 as two possible data set sizes to investigate the impact of estimation on monitoring performance. With $N = 240$, the estimated seasonal INGARCH model is:

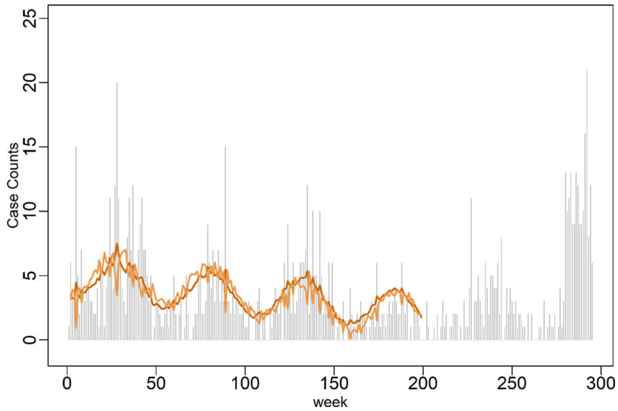

Figure 7. Salmonella case count data and the forecasts of INGARCH models fitted with $N = 200$ (orange line) and $N = 240$ (brown line) observations.

$$\hat{\mu}_{0,t} = 4.0392 - 0.0105t + 0.2129y_{t-1} - 0.0544\hat{\mu}_{0,t-1} - 1.2113\cos(2\pi t/52) - 0.4745\sin(2\pi t/52), \tag{28}$$

and with $N = 200$, the estimated model is:

$$\hat{\mu}_{0,t} = 5.3589 - 0.0184t + 0.2209y_{t-1} - 0.2288\hat{\mu}_{0,t-1} - 1.4060\cos(2\pi t/52) - 0.8116\sin(2\pi t/52). \tag{29}$$

Figure 7 shows the one-step ahead forecasts $\hat{\mu}_{0,t}$ for weeks $t = 2, 3, \ldots, 200$ with both estimated models. The smaller data set results in more variable forecasts, as expected.

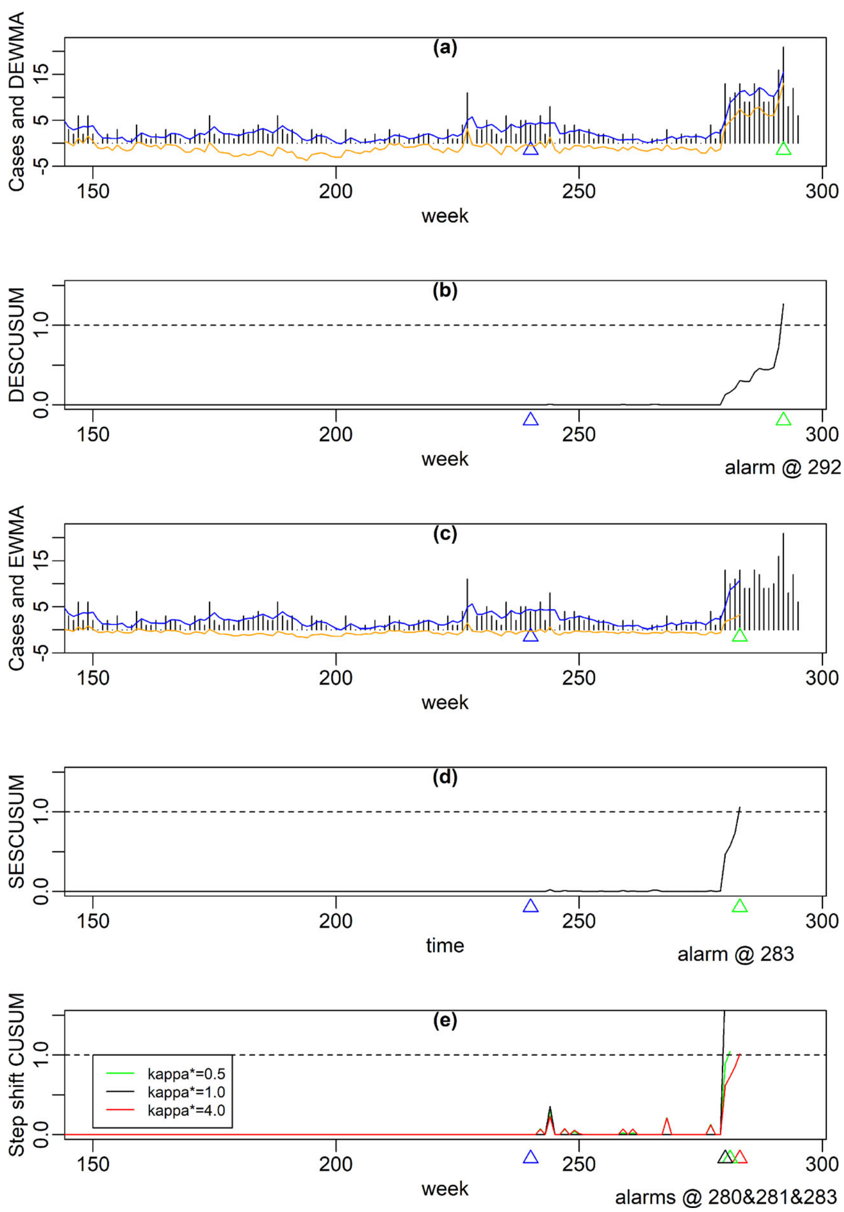

Figure 8. Outbreak detection results using DESCUSUM, SESCUSUM and SCUSUM using $\lambda = 0.8, \xi = \infty, \eta = 0.05$ and models estimated with $N = 240$. Blue triangle: moment monitoring began, green triangle: alarm time. (In the EWMA plots) Orange line: shift estimate, blue line: one step ahead forecast.

Table 5. Alarm times of the methods applied on the Salmonella data.

Method	Parameter	*N* 200	240
DESCUSUM (λ)	0.4	292	292
	0.8	292	292
	0.95	292	292
SESCUSUM (λ)	0.4	286	286
	0.8	286	283
	0.95	283	283
SCUSUM (κ^*)	0.5	286	283
	1	282	281
	4	280	280

All monitoring methods are tuned to achieve $ARL_0 = 400$. The DESCUSUM and SESCUSUM are tuned with the smoothing constants $\lambda = 0.4, 0.8, 0.95$ and the threshold $\xi = \infty$ and only $\eta = 0.05$ is used for DESCUSUM. The SCUSUM is tuned for an increase in the intercept from $\hat{\delta}_0$ to $\hat{\delta}_0 + \kappa^*$ of $\kappa^* =$ 0.5 cases, 1 case, 4 cases.

The results of monitoring using the model estimated with $N = 240$ data is shown in Figure 8 (between weeks 150 and 295). The figure shows the shift estimate $\hat{\theta}_t$ and the forecast $\hat{\mu}_t$ using a simple EWMA (Panel a) and a double EWMA (Panel c), the corresponding adaptive SESCUSUM statistic (Panel b) and DESCUSUM statistic (Panel d) and the SCUSUM statistic (Panel e). In order to show multiple monitoring statistics in the same graph, the scaled statistics, defined $\tilde{S}_t = S_t/h$, are plotted and the alarm limit for the scaled statistic is 1 (i.e., the method signals an alarm when $\tilde{S}_t > 1$).

Table 5 summarizes alarm times of the methods using INGARCH models estimated with different data set sizes *N*. The DESCUSUM consistently signals at week 292 regardless of the smoothing parameter or the data set size. The SESCUSUM alarm time varies between 283 and 286, and the SCUSUM alarm times vary between 280 and 286. While a smaller data set size *N* causes larger variability in alarm times of SCUSUM, the alarm times of SESCUSUM are less variable. Assuming that the alarm times of the SESCUSUM and SCUSUM are more reliable than the alarm time of DESCUSUM, since they are sooner, the likely outbreak form can be decided as that of a step shift rather than that of a linear trend and is happening around between weeks 283 and 286. The likely time of the outbreak is determined by relying more heavily on the alarm times of SESCUSUM since this method's alarm times are less sensitive to the choice of its parameters and the data set size.

Conclusions

This paper presented an adaptive cumulative sum (CUSUM) method for detecting step shift and linear trends changes in count-data time-series represented as seasonal integer-valued generalized autoregressive conditional heteroskedastic (INGARCH) time series models. While the applications of detecting change considered were in the context of public health and disease outbreaks, the method is equally applicable in other contexts including industrial quality. The simulation study showed that the proposed adaptive CUSUM approach has a better overall performance in detecting changes than existing fixed magnitude CUSUM methods under various seasonality, step and linear trend settings. In particular it is shown that guidelines on the selection of the smoothing parameter of the adaptive scheme is easier to develop than the specification of the shift magnitude used in the fixed magnitude CUSUM methods, with less variation in the resulting detection performance depending on the choice. In addition, a case study utilizing real data set from a public health monitoring problem illustrated the effectiveness of the proposed adaptive method with estimated models.

This research has not considered effect of estimation error on detection performance. As a future work of interest, optimal choice of smoothing parameters based on phase I sample size can be considered. Optimal choice of smoothing parameters has been studied for adaptive CUSUM with known model parameters (Capizzi and Masarotto 2003) however the effect of estimation error has not been considered. As we have illustrated in the case study the models estimated with different data set sizes and the resulting smoother performance heavily depends on the sample size. Other potential areas of extensions would include considering spatial dimension in addition to temporal dimension in surveillance (in particular for healthcare problems) or to include exponential or more complex trend forms in addition to linear trends in the detection methodology.

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