A Two-Sided Model of Paid Peering

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Abstract

Internet users have suffered collateral damage in tussles over paid peering between large ISPs and large content providers. Paid peering is a relationship where two networks exchange traffic with payment, which provides direct access to each other's customers without having to pay a third party to carry that traffic for them. The issue will arise again when the United States Federal Communications Commission (FCC) considers a new net neutrality order.

We first consider the effect of paid peering on broadband prices. We adopt a two-sided market model in which an ISP maximizes profit by setting broadband prices and a paid peering price. We analytically derive the profit-maximizing prices, and show that they satisfy a generalization of the well-known Lerner rule. Our result shows that paid peering fees reduce the premium plan price, increase the video streaming price and the total price for premium tier customers who subscribe to video streaming services; however, the ISP passes on to its customers only a portion of the revenue from paid peering. ISP profit increases but video streaming profit decreases as an ISP moves from settlement-free peering to paid peering price.

We next consider the effect of paid peering on consumer surplus. We find that consumer surplus is a uni-modal function of the paid peering fee. The paid peering fee that maximizes consumer surplus depends on elasticities of demand for broadband and for video streaming. However, consumer surplus is maximized when paid peering fees are significantly lower than those that maximize ISP profit. However, it does not follow that settlement-free peering is always the policy that maximizes consumer surplus. The peering price depends critically on the incremental ISP cost per video streaming subscriber; at different costs, it can be negative, zero, or positive.

Keywords: Broadband, Regulation, Net Neutrality, Two-sided Model, Interconnection, Paid Peering

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1. Introduction

It is no longer clear who should pay whom and how much for interconnection between Internet Service Providers (ISPs) and content providers. Large ISPs claim that large content providers are imposing a cost on ISPs by sending large amounts of traffic to their customers. ISPs claim that it is more fair that content providers pay for this cost than consumers, because then this cost will be paid only by those consumers with high usage. In contrast, large content providers (including CDNs) claim that when they interconnect with ISPs at interconnection points (IXPs) close to consumers, they are already covering the costs of carrying traffic through the core network, and that consumers are already covering the costs of carrying traffic through the ISP's access network. These disputes between large ISPs and large content providers have recurred often during the last 10 years. When not resolved, large ISPs have often refused to increase capacity at interconnection points with large content providers and transit providers, resulting in sustained congestion which has degraded users' quality of experience because of reduced throughput, increased packet loss, increased delay, and increased jitter.

The standard tiered interconnection model concerns the interconnection topology, interconnection services, and payment. Figure 1(a) illustrates the original tiered topology in which each small ISP interconnects with at least one transit provider. Transit providers interconnect with each other to provide full connectivity of the Internet. An end user obtains consumer broadband service from an ISP, in which the ISP offers to transport Internet traffic to and from all Internet endpoints. Similarly, a content provider obtains business broadband service from an ISP, in which the ISP offers to transport Internet traffic to and from all Internet endpoints. An ISP obtains *transit service* from a transit provider, in which the transit provider offers to transport Internet traffic between the ISP and all Internet endpoints that are not on the ISP's network. Transit providers offer each other *peering service*s, in which each agrees to accept and deliver traffic with destinations on its own network and on the networks of its customers. In this model, end users pay their ISPs for consumer broadband service, content providers pay their ISPs for business broadband service, and ISPs pay transit providers for transit service. Transit providers do not charge each other, called *settlement-free peering*, if and only if they perceive approximately equal value to the peering service they provide to each other.

However, changes in Internet topology have led to changes in interconnection topology, interconnection services, and payment. With the progression from dial-up ISPs to broadband ISPs, large numbers of small ISPs merged to create a small number of large ISPs. These large ISPs have also built their own backbone networks to connect their service territories. Large ISPs started peering with each other to avoid having to pass this traffic through a transit provider. Content providers often interconnected with transit providers instead of small ISPs. The resulting interconnection topology is illustrated in Figure 1(b). As before, end users pay their ISPs for consumer broadband service. Content providers now pay transit providers for transit service. ISPs do not charge each other for peering, if and only if they perceive approximately equal value to the peering service they provide to each other. ISPs transmit traffic with destinations on other ISP's networks through peering when possible,

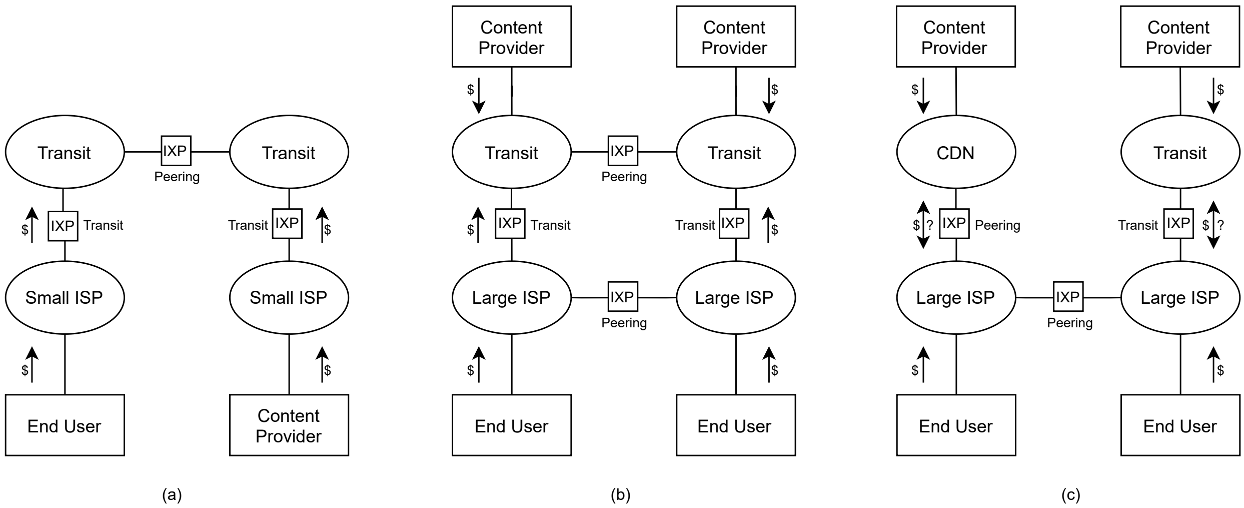


Figure 1: Evolution in Transit Market

and transmit all other traffic through transit providers. As a result, large ISPs continue to pay for transit service, but this constitutes a lower percentage of their traffic than previously.

The development and growth of content delivery networks (CDNs) led to further changes in interconnection topology, interconnection services, and payment. Third-party CDNs, wishing to deploy their servers close to consumers to improve network performance, started peering directly with large ISPs. Eventually, some large content providers built their own CDNs, and some similarly started peering directly with large ISPs. The resulting interconnection topology is illustrated in Figure 1(c). The majority of Internet traffic now consists of video flowing from CDNs operated by video streaming providers (e.g., Netflix) or third parties (e.g., Akamai) through direct interconnection to ISPs. As before, end users pay their ISPs for consumer broadband service. ISPs continue to transmit traffic through peering when possible, but continue to rely on transit service when needed. However, now it is unclear, when large content providers peer with large ISPs, whether large content providers should pay large ISPs, large ISPs should pay large content providers, or the peering between them should be settlement-free.

As a result, in some countries, including the United States, there has been an increasing number of disputes over interconnection between large ISPs, on one side, and large content providers and transit providers, on the other side. In 2013-2014, a dispute between Comcast and Netflix over terms of interconnection went unresolved for a substantial period of time, resulting in interconnection capacity that was unable to accommodate the increasing Netflix video traffic. In 2014, Netflix and a few transit providers brought the issue to the attention of the United States Federal Communications Commission (FCC), which was writing updated net neutrality regulations. The FCC discussed the dispute in the 2015 Open Internet Order (Federal Communications Commission, 2015).

The FCC first summarized the arguments of large content providers and transit providers. It noted that "[content] providers argue that they are covering the costs of carrying [their] traffic through the network, bringing it to the gateway of the Internet access service". Large content providers and transit providers argued that they should be entitled to settlement-free peering if the interconnection point is sufficiently close to consumers. The lack of willingness of large ISPs to offer settlement-free peering with large content providers, and to augment the capacity of existing interconnection points with transit providers with which they had settlement-free peering agreements, had led to the impasse. The FCC noted that "[s]ome [content] and transit providers assert that large [ISPs] are creating artificial congestion by refusing to upgrade interconnection capacity ... for settlement-free peers or CDNs, thus forcing [content] providers and CDNs to agree to paid peering arrangements."

The FCC then summarized the arguments of large ISPs. It noted that "large broadband Internet access service providers assert that [content] providers such as Netflix are imposing a cost on broadband Internet access service providers who must constantly upgrade infrastructure to keep up with the demand". The large ISPs explained that the network upgrades include adding capacity in the middle mile and access networks. The FCC noted that the large ISPs asserted that if they absorb these costs, then the ISPs would recoup these costs by increasing the prices for all subscribers, and that the large ISPs argued that "this is unfair to subscribers who do not use the services, like Netflix, that are driving the need for additional capacity".

Both large ISPs and large content providers agree that settlement-free peering is appropriate when both sides perceive equal value to the relationship. However, whereas large content providers assert that carrying their traffic to an interconnection point close to consumers is of value, large ISPs assert that "if the other party is only sending traffic, it is not contributing something of value to the broadband Internet access service provider".

In 2015, the FCC was concerned about the duration of unresolved interconnection disputes and about the impact of these disputes upon consumers. However, it concluded that in 2015 it was "premature to draw policy conclusions concerning new paid Internet traffic exchange arrangements between broadband Internet access service providers and [content] providers, CDNs, or backbone services." Thus, in 2015 the FCC adopted a case-by-case approach in which it would monitor interconnection arrangements, hear disputes, and ensure that ISPs are not engaging in unjust or unreasonable practices. However, in 2018, the FCC reversed itself and ended its oversight of interconnection arrangement, when it repealed most of the 2015 net neutrality regulations (Federal Communications Commission, 2018). It is almost certain that the FCC will revisit the issue in the next few years.

The goal of this paper is to evaluate the effect of paid peering fees on broadband prices and consumer surplus. Our principal approach is to model the interaction between an ISP and its subscribers, and between an ISP and large content provider, as a two-sided market model. We then consider the impact of an ISP-determined paid peering fee on both consumers and content providers. Finally, we consider what level of peering fee would maximize consumer surplus. To the best of our knowledge, this is the first work to use a two-sided market model to analyze the effect of paid peering fees on broadband prices and consumer surplus.

The rest of this paper is organized as follows. Section 2 summarizes the relevant research literature. Although two-sided market models have been widely used to examine issues relating to net neutrality or to other aspects of various telecommunication markets, there are few that examine the effects of interconnection agreements.

Section 3 proposes a model of user subscription to broadband service tiers and to video streaming. We consider a monopoly ISP that offers basic and premium tiers differentiated by bandwidth and price. We aggregate all video streaming providers that directly interconnect with the ISP. Consumers differ in the utilities they place on broadband service tiers and on video streaming, and each customer chooses the service which maximizes his/her surplus. We derive the demand of each broadband service tier and of video streaming services. We also derive the associated consumer surplus, ISP profit, and aggregated video service provider profit.

To focus on the effect of peering fees, in Section 4 we propose a two-sided model in which a monopoly ISP maximizes its profit by choosing broadband prices as well as a peering price. An ISP earns revenue by increasing its peering price, but this will also trigger a decrease in the demand for the ISP's premium tier. An ISP also earns revenue by increasing its premium tier price, but this will also trigger a decrease in demand for video streaming and thus in the revenue from paid peering. We derive numerical model parameters based on public data about broadband and video streaming prices and subscription. We prove that an ISP maximizes its profit by choosing prices that satisfy a generalization of the well-known Lerner rule, which specifies how these prices are related to a matrix of elasticities and cross-elasticities of demand.

In Section 5, we consider the effect of paid peering on broadband prices as well as ISP profit. ISPs assert that paid peering revenue is offset by lower broadband prices, and that ISP profits remain unchanged. Content providers assert that peering prices do not result in lower broadband prices, but simply increase ISP profits. Using our model, we find that the basic tier price is almost unaffected by peering fees, but that the premium tier price is lower when an ISP chooses the paid peering price to maximize profit than when settlement-free peering is used. Also, we find that positive peering prices result in increased ISP profit and in decreased video streaming profit.

In Section 6, we consider the impact of paid peering on consumer surplus. ISPs assert that paid peering fees increase aggregate consumer surplus because they eliminate an inherent subsidy of consumers with high video streaming use by consumers without such use. However, content providers assert that paid peering fees decrease consumer surplus because they are passed onto consumers through higher video streaming prices without a corresponding reduction in broadband prices. To address this question, we consider the peering price to be an independent variable set by a regulator with the goal of maximizing consumer surplus. We show that consumer surplus is a uni-modal function of the peering price, and that the peering price that maximizes consumer surplus is substantially less than the peering price that maximizes ISP profit and less than the incremental ISP cost per video streaming subscriber. In Section 7, we show that the peering price depends critically on this cost, and that at different costs it can be negative, zero, or positive. Then we formulate an optimization problem in which a regulator maximizes consumer surplus by choosing not only the peering price but also the broadband prices and the aggregate video streaming price. We show that the resulting peering price is the ISP cost per video streaming subscriber plus the desired rate of return.

2. Research Literature

A few papers examine the effects of interconnection agreements in the Internet backbone by using two-sided market models.

Kim (2020) is concerned with whether an ISP that is vertically integrated with a content provider may use peering fees to gain advantages over unaffiliated content providers. It proposes a two-sided market model with one monopoly ISP, one affiliated content provider, and one unaffiliated content provider. The ISP is assumed to provide direct interconnection with its affiliated content provider for free, but can choose a peering price to charge the unaffiliated content provider. The two-sided model also incorporates indirect interconnection between the unaffiliated content provider and the ISP through a transit provider. The paper finds that, when the cost of direct interconnection is low, the ISP sets the peering price at the maximum amount that the unaffiliated content provider is willing to pay, so that it earns the maximum possible revenue from direct interconnection. However, when the cost of direct interconnection is high, the ISP sets the peering price above the maximum amount that the unaffiliated content provider is willing to pay, so that the affiliated content provider has an advantage over the unaffiliated content provider. This outcome suggests that a vertically integrated ISP might exert leverage through direct interconnection in order to favor its affiliated content provider. They find consumer welfare may or may not be maximized by direct interconnection; however, this conclusion is strongly dependent on the two-sided model. The research problem addressed in Kim (2020) differs from that which we consider here. First, Kim (2020) is focused on the effect of a peering price on competition between content providers, while we focus on the effect on both content providers and consumers. Second, Kim (2020) adopts a game theoretic approach, while we consider both profit maximization and consumer surplus maximization.

Laffont et al. (2003) is concerned with how interconnection fees between a pair of ISPs affect the allocation of network costs between consumers and content providers. It considers a two-sided model in which there is perfect competition between two ISPs, each of which can serve any customer or content provider. The model assumes that interconnection fees are symmetric between the two ISPs, but that this fee affects each ISP's market shares of consumers and of content providers. The paper finds that if an ISP has market power, then the peering price depends not only on elasticities of demand and network externalities, but also on the ISP's relative market power. Furthermore, the ISP-chosen peering price does not maximize consumer surplus. Although there are some parallels between the results of Laffont et al. (2003) and the results of our paper, the issues and models are quite different, since Laffont et al. (2003) is concerned with interconnection fees between two competitive ISPs whereas we are concerned with interconnection fees between a monopoly ISP and content providers.

Wang et al. (2018) is concerned with how interconnection fees between an ISP and content providers affect ISP profit and consumer surplus. It proposes a two-sided model in which a monopoly ISP may provide content providers the choice between paid peering and settlement-free peering and in which the ISP charges consumers an amount proportional to their monthly usage. The ISP is assumed to choose both the peering price and the consumer per-unit usage price. The paper finds that when the ISP maximizes profit, it always offers paid peering, and it may or may not also offer settlement-free peering. In contrast, when prices are set to maximize consumer surplus, the ISP always offers settlement-free peering, and it may or may not also offer paid peering. Although both Wang et al. (2018) and our paper are concerned with the impact of interconnection fees on both ISP profit and consumer surplus, Wang et al. (2018) is focused primarily on the ISP decision of how much capacity to allocate to paid versus settlement-free peering, whereas we are focused primarily on the ISP decision of the peering price.

In addition to these three papers that use two-sided models to examine issues relating to interconnection, there is a much larger set of papers that use two-sided models to examine issues relating to net neutrality. Most of these papers are concerned with the impact of a paid prioritization prohibition on ISP profit, consumer surplus, and social welfare. We briefly discuss a few of these here. Musacchio et al. (2007) uses a two-sided model in which there are multiple ISPs, each of which has a monopoly over its subscribers. Content providers connect via an ISP. It develops and analyzes a game theoretic model to study how the ability (or lack thereof) of an ISP to charge non directly connected content providers affects user prices and ISP and content provider investments. It shows that whether charging non directly connected content providers maximizes social welfare depends on the advertising revenue model and on the amount of competition between ISPs for content providers. Weisman and Kulick (2010) attempts to apply generic concepts from the economic literature on two-sided markets and price discrimination to the issue of paid prioritization. It claims that a monopoly ISP will set prices according to a Lerner index. It argues that the economic literature should be understood to imply that paid prioritization with price discrimination would be presumptive social-welfare enhancing. However, the validity of its conclusions is limited by the lack of a model that reflects Internet architecture, network performance, or consumer utility. Economides and Tåg (2012) uses a two-sided model of a monopoly ISP to find that a paid prioritization prohibition may increase social welfare when a content provider values an additional consumer more than a consumer values an additional content provider. It also finds that the prioritization price that maximizes social welfare may be less than the associated marginal cost. Njoroge et al. (2014) uses a two-sided model of two ISPs that compete based on quality and price to explore the trade-off between consumer revenue and content provider revenue. It finds that paid prioritization may increase ISP investment. It also claims that this increased ISP investment results in increased content provider revenue due to increased quality, and that consumer surplus corresponding increases. However, this claim depends on several assumptions, including a pair of competitive ISPs and the lack of a model of Internet architecture that relates access network congestion to quality. Tang and Ma (2019) uses a two-sided model of a monopoly ISP to examine how a profit-maximizing ISP may allocate capacity between two classes of service. It finds that an ISP can maximize profit by allocating all capacity to the premium class of service, while social welfare is maximized by lower premium class prices and a more balanced capacity allocation. While this literature has useful insights about the effect of prices on one side of the two-sided model on ISP profit, consumer surplus, and social welfare, these insights relate to charging either content providers or consumers for prioritization of traffic, whereas we are focused on charging content providers for access to consumers.

Finally, there is an even larger set of papers that use two-sided models to analyze other aspects of various telecommunications markets. We briefly discuss a few of these here. Cortade (2006) surveys the economic literature on two-sided models of markets involving ISPs. It reiterates the use of a Lerner index for monopoly ISPs, and then discusses the effect of competition on ISP pricing. Ma et al. (2008) is concerned with the split of revenue between ISPs and transit providers. It considers both monopoly and competitive ISPs. It proposes the use of a Shapley value for the interconnection fee as a fair manner of splitting the revenue. Wu et al. (2011) are also concerned with the split of revenue between ISPs and transit providers. When the ISP and the transit provider don't collude, it analyzes the split as a Stackelberg game. It then shows that if the ISP and transit provide collude, they can increase their profits, and it proposes a split based on each's bargaining power. Wang et al. (2017) considers a two-sided model in which a monopoly ISP charges consumers and content providers based on the volume of traffic. It compares the prices that maximize ISP profit to those that maximize social welfare, and using a congestion model shows how these prices depend on capacity and congestion. It also shows that an ISP may be incentivized to shift from one-sided to two-sided pricing (charging content providers) as the percentage of Internet traffic that is video increases.

3. A Model of User Subscription to Broadband and to Video Streaming

Before we can analyze the effect of paid peering on broadband prices, we need a model of user subscription to broadband service tiers and to video streaming.

3.1. Service offerings

ISPs offer multiple tiers of broadband services, differentiated principally by download speed. ISPs typically market these broadband service tiers by recommending specific tiers to consumers who engage in specific types of online activities. For example, Comcast recommends a lower service tier to consumers who principally use their Internet connection for email and web browsing, but a higher service tier to consumers who use the Internet for video streaming. Much of the debate over paid peering concerns consumers who stream large volumes of video. Thus, we construct here a model that includes two broadband service tiers: a basic tier with a download speed intended for email, web browsing, and a limited amount of video streaming; and a premium tier (at a higher price) with a download speed intended for a substantial amount of video streaming. Although most often ISPs offer more than two tiers, the majority of customers subscribe to a subset of two tiers, and this two-tier model is sufficient to separately evaluate the effect of paid peering prices on consumers who utilize video streaming and on consumers who don't.

Specifically, we model a single monopoly ISP that offers a basic tier at a monthly price P^b and a premium tier at a monthly price $P^b + P^p$. We consider N consumers, each of whom may subscribe to the basic tier, the premium tier, or neither. We denote user i's utility per month from subscription to the basic tier by b_i, and user i's utility per month from subscription to the premium tier by $b_i + p_i$. We presume that a consumer who gains significant utility from video streaming subscribes to the premium tier.

To analyze the effect of paid peering prices on broadband prices, we focus on the aggregate of all video streaming providers that directly interconnect with the ISP and that may pay (or be paid) a fee for peering with the ISP. We model the aggregate of all plans offered by these video streaming providers, but to keep the model tractable we consider a single price of P^v per month for the aggregate. We denote user i's utility per month from subscription to video streaming providers by v_i. Consumer i's utility from all other content is included in $b_i + p_i$.

Consumers differ in the utilities they place on broadband service tiers and on video streaming. We assume that the number of consumers N is large, and we denote the joint probability density function of their utilities by $f_{B,P,V}(b, p, v)$.

3.2. Demand functions

Each consumer thus has four choices

$$X_i \triangleq \begin{cases} n, & \text{do not subscribe} \\ b, & \text{subscribe to the basic tier} \\ p, & \text{subscribe to the premium tier but not to a video streaming provider} \\ v, & \text{subscribe to the premium tier and to a video streaming provider.} \end{cases} \tag{1}$$

Consumer i's consumer surplus, defined as utility minus cost, under each choice is thus

$$CS_i(X_i; b_i, p_i, v_i) = \begin{cases} 0, & X_i = n \\ b_i - P^b, & X_i = b \\ b_i + p_i - P^b - P^p, & X_i = p \\ b_i + p_i + v_i - P^b - P^p - P^v, & X_i = v. \end{cases} \tag{2}$$

Each consumer is assumed to maximize consumer surplus. Thus, consumer i adopts the choice

$$X_i^*(b_i, p_i, v_i) \triangleq \arg\max_{X_i} CS_i(X_i; b_i, p_i, v_i), \tag{3}$$

and earns a corresponding consumer surplus $CS_i^* \triangleq CS_i(X_i^*)$.

Each of the N consumers makes an individual choice per (3). The consumers who choose to subscribe to the basic tier are those whose utility b_i from subscription to the basic tier exceeds its monthly price P^b, whose incremental utility p_i from subscription to the premium tier without subscribing to a video streaming provider falls below the incremental monthly price P^p, and whose incremental utility $p_i + v_i$ from subscription to the premium tier and to video streaming falls below the corresponding incremental monthly price $P^p + P^v$. Thus, the demand[3] for the basic tier is given by

$$N^b(P^b, P^p, P^v) = N \int_{-\infty}^{P^p} \int_{-\infty}^{P^p+P^v-p} \int_{P^b}^{\infty} f_{B,P,V}(b, p, v)\, db\, dv\, dp. \tag{4}$$

[3]Since we model a finite number N of consumers whose utilities are given by a joint probability density function, this equation, and other similar equations below, give the *average demand*. However, for simplicity of presentation, we use the term *demand*.

Similarly, the consumers who choose to subscribe to the premium tier but not to video streaming are those whose utility $b_i + p_i$ from subscription to the premium tier exceeds its monthly price $P^b + P^p$, whose incremental utility p_i from subscription to the premium tier without subscribing to video streaming exceeds the incremental monthly price P^p, and whose incremental utility v_i from subscription to video falls below the incremental monthly price P^v. Thus, the number of consumers who subscribe to the premium tier but who do not subscribe to video streaming is given by

$$N^p(P^b, P^p, P^v) = N \int_{-\infty}^{P^v} \int_{P^p}^{\infty} \int_{P^b+P^p-p}^{\infty} f_{B,P,V}(b,p,v)\; db\; dp\; dv. \tag{5}$$

Finally, the consumers who choose to subscribe to both the premium tier and video streaming are those whose utility $b_i + p_i + v_i$ from subscription to both services exceeds the combined cost $P^b + P^p + P^v$, whose incremental utility $p_i + v_i$ from subscription to only the basic tier exceeds the corresponding incremental price $P^p + P^v$, and whose incremental utility v_i from subscription to video streaming falls exceeds the incremental monthly price P^v. Thus, the demand for video streaming is given by

$$N^v(P^b, P^p, P^v) = N \int_{P^v}^{\infty} \int_{P^p+P^v-v}^{\infty} \int_{P^b+P^p+P^v-p-v}^{\infty} f_{B,P,V}(b,p,v)\; db\; dp\; dv. \tag{6}$$

The demand for the premium tier is $N^p + N^v$, the sum of the demands for the premium tier without and with a subscription to the streaming video provider.

3.3. Consumer surplus

The aggregate consumer surplus will be an important quantity to consider in our deliberations below. It can be easily determined for each set of consumers using the number of subscribers in each set (4-6) and the surplus of each consumer (2). Given a set of prices, the aggregate consumer surplus of subscribers to the basic tier is

$$CS^b(P^b, P^p, P^v) = N \int_{-\infty}^{P^p} \int_{-\infty}^{P^p+P^v-p} \int_{P^b}^{\infty} (b - P^b) f_{B,P,V}(b,p,v)\; db\; dv\; dp. \tag{7}$$

Similarly, the aggregate consumer surplus of consumers who subscribe to the premium tier but not to video streaming is

$$CS^p(P^b, P^p, P^v) = N \int_{-\infty}^{P^v} \int_{P^p}^{\infty} \int_{P^b+P^p-p}^{\infty} (b + p - P^b - P^p) f_{B,P,V}(b,p,v)\; db\; dp\; dv, \tag{8}$$

and the aggregate consumer surplus of consumers who subscribe to both the premium tier and video steaming is

$$CS^v(P^b, P^p, P^v) = N \int_{P^v}^{\infty} \int_{P^p+P^v-v}^{\infty} \int_{P^b+P^p+P^v-p-v}^{\infty} (b+p+v-P^b-P^p-P^v) f_{B,P,V}(b,p,v)\; db\; dp\; dv. \tag{9}$$

The aggregate consumer surplus over all consumers is defined as

$$CS(P^b, P^p, P^v) \triangleq CS^b(P^b, P^p, P^v) + CS^p(P^b, P^p, P^v) + CS^v(P^b, P^p, P^v) \tag{10}$$

3.4. Profits

We assume that the ISP incurs a monthly marginal cost C^b per basic tier subscriber. The ISP marginal profit per basic tier subscriber is thus $P^b - C^b$. We assume that the ISP incurs a monthly marginal cost $C^b + C^p$ per premium tier subscriber who does not also subscribe to video streaming. The ISP marginal profit per such broadband service tier subscriber is thus $P^b + P^p - C^b - C^p$. We also assume that the capacity is fixed in our model.

The marginal cost to an ISP associated with video streaming is at the core of the debate over paid peering, and thus we must be careful in its formulation. Here, we have assumed that only premium tier subscribers engage in a substantial amount of video streaming, consistent with ISP marketing of their service tiers. We have further divided premium tier subscribers according to whether they also subscribe to video streaming services that have direct interconnection with the ISP.

For generality, we thus associate an ISP monthly marginal cost $C^b + C^P + C^d$ per video streaming subscriber, where the d denotes direct interconnection. The incremental ISP cost C^d per video streaming subscriber may be negative, zero, or positive. It is critical to note that this incremental cost is not that of the interconnection point itself between the ISP and each video streaming provider, as the cost of the interconnection point itself is negligible. However, there are several variables that may affect the incremental ISP cost per video streaming subscriber. First, video streaming subscribers receive substantially more traffic per month than premium tier subscribers who don't subscribe to video streaming. Second, when a content provider switches from indirect interconnection through a transit provider to an ISP to direct interconnection with the ISP, the location of the interconnection point may change. This change in the location of the interconnection point may result in either shorter or longer paths on the ISP's network from the interconnection point to the subscriber, and thus either a lower or higher incremental ISP cost per video streaming subscriber.

We also consider a peering price of P^d per video streaming subscriber for direct interconnection between the ISP and video streaming providers. This price may be positive if the ISP charges video streaming providers for direct interconnection, negative if the video streaming providers charge the ISP for direct interconnection, or zero if the peering is settlement-free.

The ISP marginal profit per video streaming subscriber is $P^b + P^p + P^d - C^b - C^p - C^d$. The total ISP profit (excluding fixed costs)[4] is thus

$$\pi^{ISP}(P^b, P^p, P^d, P^v) = (P^b - C^b)N^b + (P^b + P^p - C^b - C^p)N^p + (P^b + P^p + P^d - C^b - C^p - C^d)N^v. \tag{11}$$

We assume that the video streaming providers incur a monthly marginal cost C^v per subscriber. The aggregate video streaming provider marginal profit per subscriber is thus $P^v - C^v - P^d$, and their total profit (excluding fixed costs)[5] is

$$\pi^{VSP}(P^b, P^p, P^d, P^v) = (P^v - C^v - P^d)N^v. \tag{12}$$

[4]Throughout the paper, ISP profit excludes fixed costs.

[5]Throughout the paper, aggregate video streaming profit excludes fixed costs.

4. A Two-Sided Model for ISP Profit Maximization

The previous section presented a model for consumer demand for broadband and video streaming, resulting in the demand functions (4-6), the corresponding aggregate consumer surplus (7-9), and the corresponding ISP and video streaming provider profits (11-12). In this section, we formulate a two-sided model of how the prices are determined.

4.1. Analytical model

There are a number of options for modeling how the broadband service tier prices (P^b and P^p), the video streaming price (P^v), and the peering price (P^d) are determined.

Throughout the paper, we presume that the ISP has no significant competition for broadband service at acceptable speeds within the footprint of its service territory. Thus, we assume that the ISP determines its broadband service tier prices (P^b and P^p) to maximize its profit.

A key question, critical to this analysis, is how the peering price (P^d) is determined. Once a subscriber chooses an ISP, the ISP has a monopoly on the transport of traffic within the ISP's access network that the customer resides in. In contrast, there may be a competitive market for the transport of Internet traffic across core networks. In this section, we assume that the location of direct interconnection between the ISP and each video streaming provider is close enough to the consumers so that all of the transport from the interconnection point to the consumers falls within the ISP's access network. Correspondingly, we assume that the ISP has the market power to determine the peering price (P^d) and that it sets this price to maximize its profit.

The ISP thus chooses the broadband service tier prices (P^b and P^p) and the peering price (P^d) to maximize its profit, namely

$$(P^b_{ISP}, P^p_{ISP}, P^d_{ISP}) = \arg \max_{(P^b, P^v, P^d)} \pi^{ISP}(P^b, P^p, P^d, P^v) \tag{13}$$

In contrast, we assume that the market determines the aggregate video streaming price, excluding paid peering fees, when there is no regulation of prices. We denote the aggregate video streaming price, excluding paid peering fees, by P^v_0. We presume that an ISP charging peering prices would likely charge them to both directly interconnected content providers and directly interconnected transit providers. We further presume that transit providers would pass peering prices through to their customers. As a consequence, we foresee that peering prices would be paid by all large video service providers selling to the ISP's customers. An open question is whether the video streaming providers can pass through any peering price (P^d) to their customers by adding it to their video streaming prices. We denote the pass-through rate of the peering fee by $0 < \alpha \leq 1$:

$$P^v(P^d) = P^v_0 + \alpha P^d \tag{14}$$

Equations (13-14) set up a two-sided model in which the ISP earns revenue from both its customers and video service providers (if $P^d > 0$). The combination of the two equations captures the inter-dependencies between the ISP, the video services providers, and the consumers. The ISP-determined peering price (P^d), along with the pass-through rate (α), leads to an aggregate video streaming price (P^v). The ISP-determined broadband service tier prices (P^b and P^p), along with the aggregate video streaming price (P^v), lead to demands for each broadband service tier (N^b and $N^p + N^v$) and for video streaming services (N^v). These demands in turn affect how the ISP sets each of the prices.

Since the aggregate video service price (P^v) is solely determined by (14), we can represent the ISP's profit as a function of three variables rather than four:

$$(P^b_{ISP}, P^p_{ISP}, P^d_{ISP}) = \arg \max_{(P^b, P^v, P^d)} \pi^{ISP}(P^b, P^p, P^d, P^v_0 + \alpha P^d) \tag{15}$$

4.2. Numerical parameters

This two-sided model is somewhat amenable to closed-form analysis. However, we find it useful to also examine the model under a set of realistically chosen parameters. We set out those parameters in this subsection.

The joint probability density function of user utilities for the basic tier, the premium tier, and video streaming is represented by $f_{B,P,V}(b, p, v)$. For numerical evaluation, we assume that each utility is independent and has a Normal distribution: $B \sim \mathcal{N}(\mu_b, \sigma_b^2), P \sim \mathcal{N}(\mu_p, \sigma_p^2), V \sim \mathcal{N}(\mu_v, \sigma_v^2)$. We need to determine numerical values for the means and variances.

The ISP incurs a monthly marginal cost of C^b per subscriber, a monthly marginal cost of C^p per premium tier subscriber, and an incremental ISP cost C^d per video streaming subscriber. We need to determine numerical values for these three costs.

Unfortunately, direct information about user utilities and ISP costs is scarce. Instead, we choose numerical values for user utilities and ISP costs indirectly using available information about demand and prices in the United States.

There are several sets of publicly available statistics about broadband prices and subscriptions (Pew Research Center, 2021; The Wall Street Journal, 2019). While the set of statistics differ, they show that roughly 75% of households in the United States subscribe to fixed broadband service. Hence, we wish to choose numerical values for user utilities and ISP costs so that, at the ISP profit-maximizing prices, $(N^b + N^p + N^v)/N = 0.75$. For each ISP, the statistics show that subscribers predominately choose among two service tiers, which we map to the basic and premium tiers modeled above, with roughly 2/3 of subscribers choosing the premium tier. Hence, we wish to choose numerical values for user utilities and ISP costs so that, at the ISP profit-maximizing prices, $N^b/N = (0.75)(1/3) = 0.25$ and $(N^p + N^v)/N = (0.75)(2/3) = 0.50$. Moreover, the statistics also reveal that the price of the lower of the two popular tiers is roughly \$50 per month, and the price of higher of the two popular tiers is roughly \$70 per month. Hence, we wish to choose numerical values for user utilities and ISP costs so that the ISP profit-maximizing prices are $P^b = \$50.00$ and $P^p = \$20.00$.

According to Deloitte's TMT Center (2019), Americans subscribe to an average of three paid video streaming providers. There are also several sets of publicly available statistics about video streaming prices and subscriptions (Leichtman Research Group, 2020; Parks

Associates, 2020). While the set of statistics differ, they show that roughly 50% of households in the United States that subscribe to fixed broadband service also subscribe to at least two video streaming services. Hence, we wish to choose numerical values for user utilities and ISP costs so that, at the ISP profit-maximizing prices, $N^v/N = (0.75)(0.5) = 0.375$.

There is even less information about the variance of user utilities, or correspondingly about the elasticity of demand. We choose $\sigma_b = \mu_b/4$, $\sigma_p = \mu_p/4$, and $\sigma_v = \mu_v/4$, which results in reasonably wide distributions.[6]

From these statistics, we can generate targets for the ISP profit-maximizing broadband prices P^b and P^p, and for the demands N^b, N^p, and N^v at these prices. We cannot, however, use these statistics to generate a target for the ISP profit-maximizing peering fee P^d, since information about these fees is scarce. Instead, we estimate the incremental ISP cost C^d per video streaming subscriber. There are some statistics about the monthly usage of various sets of broadband subscribers. None of these are detailed enough to accurately estimate the monthly usage from video streaming. We use a very rough estimate of 400 GB per month of aggregate usage per subscriber, including 300 GB per month of aggregate video streaming per video streaming subscriber. We need to translate this estimate of usage to an estimate of ISP cost. Unfortunately, we know very little about ISP network costs. At a price of \$70/month for 400 GB, the price is \$0.175/GB. However, the marginal cost is much lower than this price, due to high fixed costs. Here we use \$0.01/GB, but we acknowledge this could be far off from the real value. Combining these two estimates, we obtain a target of $C^d = \$3.00$ per month per video subscriber. That said, later in this paper, we will consider a wide range of values of C^d.

This now gives us six target values (P^b, P^p, N^b, N^p, N^v, and C^d) to determine the six desired parameters (μ_b, μ_p, μ_v, C^b, C^p, and P^d). We can use the three equations for demand (4-6) and the ISP profit maximization equation (15) to determine these six desired parameters. The result is: $\mu_b \approx \$56.12$, $\mu_p \approx \$18.91$, $\mu_v \approx \$27.67$, $C^b \approx \$16.50$, $C^p \approx \$19.00$, and $P^d \approx \$4.59$. In addition, we assume the aggregate video streaming price $P_0^v = \$21.58$, based on the aggregate price of the three most popular video streaming services.[7] In addition, although we consider any pass-through rate of the paid peering fee ($0 < \alpha \leq 1$), in the numerical results below we use $\alpha = 1$.

We use these parameters in the remainder of the paper except as noted. In section 8, we will discuss the sensitivity of the numerical results to these numerical parameters.

4.3. Profit-maximizing prices

We now turn to the determination of the prices that an ISP chooses in order to maximize profit.

A common measure of a firm's market power is the Lerner index. When a firm has a single product, the Lerner index is defined by $L \triangleq \frac{P-MC}{P}$, where P is the price of the product

[6] The results below are not very sensitive to these choices.

[7] The sum of the advertised prices of the lowest price plans for Netflix, Hulu/Disney+, and HBO Max is \$26.17(Netflix, 2021; Hulu, 2021; Disney+, 2021; HBO Max, 2021). From this sum, we subtract the peering fee $P^d = \$4.59$.

and MC is the firm's marginal cost. When a firm is a monopoly, $L > 0$ and the Lerner Rule shows that the firm maximizes its profit when $-\varepsilon L = 1$, where ε is the price elasticity of demand.

In our model, however, there is more than one product. When a firm offers two products, say product x and product y, there are two price elasticities of demand. Denote the price elasticity of demand for product x by $\varepsilon_{N^x,P^x} \triangleq \frac{\partial N^x}{\partial P^x}\frac{P^x}{N^x}$, and denote the price elasticity of demand for product y by $\varepsilon_{N^y,P^y} \triangleq \frac{\partial N^y}{\partial P^y}\frac{P^y}{N^y}$, where N^j and P^j are the demand and price of product j respectively. There are also two cross elasticities of demand. Denote the cross-price elasticity of demand for product y with respect to the price of product x by $\varepsilon_{N^y,P^x} \triangleq \frac{\partial N^y}{\partial P^x}\frac{P^x}{N^y}$, and the cross-price elasticity of demand for product x with respect to the price of product y by $\varepsilon_{N^x,P^y} \triangleq \frac{\partial N^x}{\partial P^y}\frac{P^y}{N^x}$.

There are two Lerner indices: $L^x \triangleq \frac{P^x - C^x}{P^x}$ and $L^y \triangleq \frac{P^y - C^y}{P^y}$, where C^x and C^y are the marginal costs for products x and y respectively. Whereas for a single product a firm maximizes its profit when $-\varepsilon L = 1$, when a firm offers two products there is a generalization of the Lerner Rule that the firm maximizes its profit when

$$-\begin{bmatrix} \varepsilon_{N^x,P^x} & \varepsilon_{N^x,P^y} \\ \varepsilon_{N^y,P^x} & \varepsilon_{N^y,P^y} \end{bmatrix} \begin{bmatrix} L^x \\ L^y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{16}$$

However, our model differs in two aspects from the general theory of profit maximization with multiple products. First, the two broadband tiers are related to each other, and thus we find it awkward to consider the prices of the two broadband tiers as independently chosen using (16). Second, we must also consider the ISP's choice of the peering price (P^d), even though it is not by itself the price of a third product. We consider these two challenges in turn.

First, we consider the issue that the two broadband tiers are related to each other. Rather than the ISP independently choosing the prices of each tier, it is more transparent to consider the ISP as choosing the price of the basic tier (P^b) and the incremental price (P^p) to upgrade from the basic tier to the premium tier. This is simply a change of basis. Consider product x to be the basic tier, and product y to be the premium tier without a video streaming subscription. It follows that:

$$\begin{bmatrix} P^x \\ P^y \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} P^b \\ P^p \end{bmatrix} \tag{17}$$

Denote the price elasticity of demand for the basic tier by $\varepsilon_{N^b,P^b} \triangleq \frac{\partial N^b}{\partial P^b}\frac{P^b}{N^b}$, and denote the price elasticity of demand for the premium tier without a video streaming subscription by $\varepsilon_{N^p,P^p} \triangleq \frac{\partial N^p}{\partial P^p}\frac{P^p}{N^p}$. Denote the cross price elasticity of demand for the premium tier without a video streaming subscription with respect to the price of the basic tier by $\varepsilon_{N^p,P^b} \triangleq \frac{\partial N^p}{\partial P^b}\frac{P^b}{N^p}$, and the cross price elasticity of demand for the basic tier with respect to the incremental price of the premium tier by $\varepsilon_{N^b,P^p} \triangleq \frac{\partial N^b}{\partial P^p}\frac{P^p}{N^b}$.

There are two Lerner indices: $L^b \triangleq \frac{P^b - C^b}{P^b}$ and $L^p \triangleq \frac{P^p - C^p}{P^p}$. If we ignore the third choice (subscription to both the premium tier and video streaming), we can derive a similar Lerner

Rule for the new basis[8]:

$$-\begin{bmatrix} \varepsilon_{N^b,P^b} & \varepsilon_{N^b,P^p} \\ \varepsilon_{N^p,P^b} & \varepsilon_{N^p,P^p} \end{bmatrix} \begin{bmatrix} L^b \\ L^p \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{18}$$

Second, we consider the issue of the ISP's choice of the peering price (P^d). It is not straightforward to incorporate this price into the Lerner Rule, since the peering price does not by itself represent the price of a third product.

We examine the situation from the perspective of the ISP. According to (15), the ISP is choosing three prices: (P^b, P^p, P^d). These three prices affect the demand for three products: b, p, and v. The profit-maximizing relationship between the prices P^b and P^p and their corresponding elasticities would be given by (18) in the absence of video streaming and a peering price.

Even though the peering price P^d does not directly represent a third product, the ISP perceives the peering price as determining the number of customers N^v who subscribe to both the premium tier and video streaming. Thus, mimicking the general approach for multiple products, we can define a price elasticity of demand for joint premium tier and video streaming subscription with respect to the paid peering price, denoted by $\varepsilon_{N^v,P^d} \triangleq \frac{\partial N^v}{\partial P^d}\frac{P^d}{N^v}$. We also define the other cross price elasticities of demand: $\varepsilon_{N^b,P^d} \triangleq \frac{\partial N^b}{\partial P^d}\frac{P^d}{N^b}$, $\varepsilon_{N^p,P^d} \triangleq \frac{\partial N^p}{\partial P^d}\frac{P^d}{N^p}$, $\varepsilon_{N^v,P^b} \triangleq \frac{\partial N^v}{\partial P^b}\frac{P^b}{N^v}$, and $\varepsilon_{N^v,P^p} \triangleq \frac{\partial N^v}{\partial P^p}\frac{P^p}{N^v}$. There is now a third Lerner index: $L^d \triangleq \frac{P^d - C^d}{P^d}$.

We assume that the joint probability density function $f_{B,P,V}(b,p,v)$ is continuous and bounded. We can now derive a Lerner Rule for the ISP profit maximization problem:

Theorem 1. *If there is a 100% pass-through rate of the paid peering fee, the profit-maximizing prices $(P^b_{ISP}, P^p_{ISP}, P^d_{ISP})$ in (15) satisfy:*

$$-\begin{bmatrix} \varepsilon_{N^b,P^b} & \varepsilon_{N^b,P^p} & \varepsilon_{N^b,P^d} \\ \varepsilon_{N^p,P^b} & \varepsilon_{N^p,P^p} & \varepsilon_{N^p,P^d} \\ \varepsilon_{N^v,P^b} & \varepsilon_{N^v,P^p} & \varepsilon_{N^v,P^d} \end{bmatrix} \begin{bmatrix} L^b \\ L^p \\ L^d \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \tag{19}$$

The proof can be found in Appendix A.

4.4. Elasticities of demand

Stakeholders disagree about the relationship between peering fees and demand. In this subsection, we first give the signs of elasticities in (19) and then investigate their numerical values using our numerical parameters.

Theorem 2. *At the profit-maximizing prices $(P^b_{ISP}, P^p_{ISP}, P^d_{ISP})$: The self price elasticities of demand are negative: $\varepsilon_{N^b,P^b} < 0$, $\varepsilon_{N^p,P^p} < 0$, and $\varepsilon_{N^v,P^d} < 0$. The cross price elasticity of demand for the basic tier with respect to the incremental price P^p is positive: $\varepsilon_{N^b,P^p} > 0$. The cross price elasticity of demand for the premium tier without video streaming with respect to the price of the basic tier is negative: $\varepsilon_{N^p,P^b} < 0$. The cross price elasticity of demand for*

[8]The proof can be found in Appendix E.

either tier without video streaming with respect to the peering price is positive: $\varepsilon_{N^b,P^d} > 0$, *and* $\varepsilon_{N^p,P^d} > 0$. *The cross price elasticity of demand for video streaming with respect to the price of either tier is negative:* $\varepsilon_{N^v,P^b} < 0$, *and* $\varepsilon_{N^v,P^p} < 0$.

The proof can be found in Appendix B.

Using the numerical parameters given in Section 4.2, the price elasticities of demand are:

$$\begin{bmatrix} \varepsilon_{N^b,P^b} & \varepsilon_{N^b,P^p} & \varepsilon_{N^b,P^d} \\ \varepsilon_{N^p,P^b} & \varepsilon_{N^p,P^p} & \varepsilon_{N^p,P^d} \\ \varepsilon_{N^v,P^b} & \varepsilon_{N^v,P^p} & \varepsilon_{N^v,P^d} \end{bmatrix} = \begin{bmatrix} -1.93 & 3.39 & 0.36 \\ -1.50 & -4.20 & 0.63 \\ -1.09 & -1.50 & -0.55 \end{bmatrix} \tag{20}$$

The signs of these elasticities match those given in Theorem 2. Note, however, that these price elasticities of demand are those at the ISP profit-maximizing prices and that they will vary substantially at different prices.

There are several academic papers that have estimated the price elasticity of demand for broadband. However, none of these papers consider multiple tiers of service, and hence all estimate only the price elasticity of demand for broadband service over all tiers. The price elasticity of demand for broadband estimated in these papers ranges from -0.18 to -3.76 (Glass and Stefanova, 2010; Rosston et al., 2010; Goolsbee, 2006; Rappoport et al., 2003; Varian, 2001; Duffy-Deno, 2000).

Recall that the demand for broadband service is $N^b + N^p + N^v$. Using the numerical parameters given in Section 4.2, the price elasticities of demand for broadband service with respect to the basic tier price P^b and with respect to the incremental premium tier price P^p, respectively, are:

$$\begin{aligned} \varepsilon_{(N^b+N^p+N^v),P^b} &= \frac{N^b\varepsilon_{N^b,P^b} + N^p\varepsilon_{N^p,P^b} + N^v\varepsilon_{N^v,P^b}}{N^b+N^p+N^v} = -1.44 \\ \varepsilon_{(N^b+N^p+N^v),P^p} &= \frac{N^b\varepsilon_{N^b,P^p} + N^p\varepsilon_{N^p,P^p} + N^v\varepsilon_{N^v,P^p}}{N^b+N^p+N^v} = -0.32 \end{aligned} \tag{21}$$

The range of price elasticities of demand for broadband estimated in the literature thus encapsulates our estimates for the price elasticities of demand for broadband service with respect to the basic tier price P^b and with respect to the incremental premium tier price P^p.

The self price elasticities of demand are negative, meaning that neither broadband service nor video streaming is a Giffen good. The three demand functions in (4-6), along with the relationship in (14) between the peering price and the aggregate video streaming price, can be used to determine the impact of changes in the three prices P^b, P^p, and P^d, and thus of the signs of the cross-price elasticities of demand.

If the ISP increases the price of the premium tier by increasing the incremental price P^p (but leaves P^b and P^d unchanged), then some subscribers to the premium tier downgrade to the basic tier, resulting in a decrease in the demand $N^p + N^v$ for the premium tier and an increase in the demand N^b for the basic tier. The increase in the price of the premium tier also decreases the demand N^v for video streaming. Thus $\varepsilon_{N^b,P^p} > 0$ and $\varepsilon_{N^v,P^p} < 0$. Using our numerical parameters given in section 4.2, $\varepsilon_{N^b,P^p} = 3.39$, $\varepsilon_{N^p,P^p} = -4.20$, and $\varepsilon_{N^v,P^p} = -1.50$.

If the ISP increases the prices of both broadband service tiers by increasing P^b (but leaves P^p and P^d unchanged), then demand for both broadband tiers decrease. The increase in the price of the premium tier also decreases the demand N^v for video streaming. Thus $\varepsilon_{N^p,P^b} < 0$ and $\varepsilon_{N^v,P^b} < 0$. Using our numerical parameters, $\varepsilon_{N^b,P^b} = -1.93$, $\varepsilon_{N^p,P^b} = -1.50$ and $\varepsilon_{N^v,P^b} = -1.09$.

In contrast, if the ISP increases the price P^b for the basic tier but keeps unchanged the price $P^b + P^p$ for the premium tier (and leaves P^d unchanged), then some subscribers to the basic tier upgrade to the premium tier, and some stop subscribing to broadband Internet access altogether, resulting in a decrease in the demand N^b for the basic tier and an increase in the demand $N^p + N^v$ for the premium tier. The increase in demand for the premium tier also increases the demand N^v.

If the ISP increases the peering price P^d (but leaves P^b and P^p unchanged), then this causes the video streaming providers to increase the aggregate price P^v. The effect is that some subscribers to the video streaming service discontinue their video streaming subscription but retain enough incremental utility to remain subscribers to the premium tier. Other subscribers to video streaming discontinue their video streaming subscriptions and, having lost the associated utility, now downgrade to the basic tier. As a result, the demand N^v for the video streaming service decreases, the demand N^p for the premium tier without video streaming increases, and the demand N^b for the basic tier increases. Thus, $\varepsilon_{N^b,P^d} > 0$ and $\varepsilon_{N^p,P^d} > 0$. Using our numerical parameters, $\varepsilon_{N^b,P^d} = 0.36$, $\varepsilon_{N^p,P^d} = 0.63$, and $\varepsilon_{N^v,P^d} = -0.55$. In addition, the demand $N^p + N^v$ for the premium tier decreases.

Theorem 1 shows that the price for consumers is proportional to the marginal cost of providing services to each tier. However, the marginal rate of return may be dramatically different for the different tiers. Using our numerical parameters, the marginal rate of return on the basic tier is very large, with $(P^b - C^b)/C^b \approx 200\%$. The marginal rate of return on the incremental price from the basic tier to the premium tier may be small; using our numerical parameters, we have $(P^p - C^p)/C^p \approx 5\%$. There may also be a substantial marginal rate of return on the ISP's chosen peering fee; using our numerical parameters $(P^d - C^d)/C^d \approx 50\%$. However, these marginal rates of return do not consider fixed costs, which are likely to dominate the total cost of providing broadband service, since our model has no need to consider fixed costs.

5. The Effect of Paid Peering on Prices

We now consider the effect of paid peering on broadband prices. ISPs assert that paid peering revenue is offset by lower broadband prices, and that ISP profits remain unchanged. Content providers assert that peering prices do not result in lower broadband prices, but simply increase ISP profits. The goal is this section is to evaluate these assertions.

With an understanding of how the ISP sets the prices $(P^b_{ISP}, P^p_{ISP}, P^d_{ISP})$, we can now evaluate the impact of the peering price P^d upon the broadband prices P^b and P^p.

As we did in the previous section, we assume that the video streaming price P^v is set by (14). However, whereas in (15) the ISP sets the peering price P^d to maximize profit, in

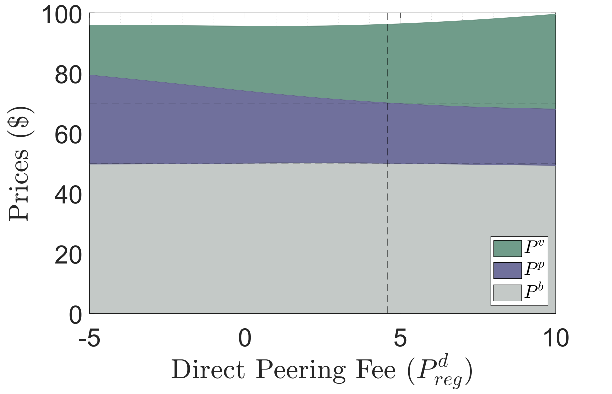


Figure 2: Effect of Peering Fee on Broadband Prices and the Aggregate Video Streaming Price

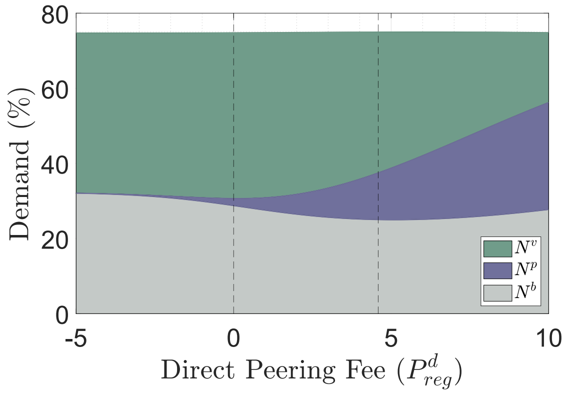


Figure 3: Effect of Peering Fee on Demand (Percentage to Total)

this section we make the peering price P^d an independent variable so that we can judge its impact on other prices.

Given a specified peering price P^d_{reg}, the ISP is assumed to choose the tier prices P^b and P^p so as to maximize profit, namely

$$(P^b_{reg}, P^p_{reg}) = \arg \max_{(P^b, P^p)} \pi^{ISP}(P^b, P^p, P^d_{reg}, P^v_0 + \alpha P^d_{reg}). \tag{22}$$

The ISP chosen prices (P^b_{reg}, P^p_{reg}) are a function of the independently set price P^d_{reg}. The video streaming price P^v is also a function of P^d_{reg}.

Figure 2 shows the prices of both broadband tiers and the aggregate video streaming price as a function of the independently chosen peering fee P^d_{reg}.

We initially compare prices and profits in the case in which the ISP chooses the peering price to maximize profit ($P^d = \$4.59$) to the case in which settlement-free peering is used (i.e., $P^d = \$0$). We start at the profit-maximizing peering price $P^d = \$4.59$ and consider a small decrease. If the ISP did not change the prices for the broadband tiers (which it will), then a small decrease in the peering price would result in a small decrease in demand for the basic tier (because $\varepsilon_{N^b,P^d} = 0.36$), a small decrease in demand for the premium tier without video streaming (because $\varepsilon_{N^p,P^d} = 0.63$), and a small increase in demand for the premium tier with video streaming (because $\varepsilon_{N^v,P^d} = -0.55$).

However, the ISP now has the motivation to modify the broadband tier prices. The decrease in the peering price results in a decrease in the aggregate price of video streaming. As a consequence, the ISP will recoup most of the decreased peering price by increasing the incremental price for the premium tier P^p. It does not, however, change the basic tier price P^b by much at all, since increasing the premium tier price results in some users downgrading to the basic tier, which more than offsets those who would otherwise upgrade from the basic tier to the premium tier to take advantage of lower video streaming prices. The signs of these trade-offs remain the same in the entire range from $P^d = \$4.59$ to $P^d = \$0$.

Figure 3 shows the corresponding demands for each broadband tier and for video streaming. Again, we start at the profit-maximizing peering price $P^d = \$4.59$ and consider a small

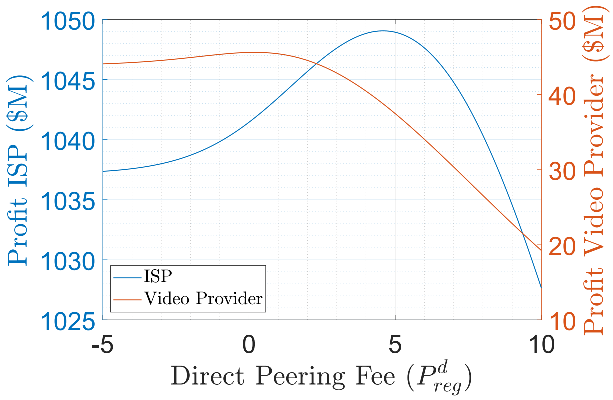


Figure 4: Effect of Peering Fee on Profit of ISP and Video Streaming Provider

decrease. The ISP's increase in the premium tier price drives some consumers who subscribe to the premium tier but not to video streaming to downgrade to the basic tier. However, the total price for the premium tier and video streaming, $P^b + P^p + P^v$, decreases, and thus some consumers who subscribe to the premium tier but not to video streaming now choose to start subscribing to video streaming.

Figure 4 shows the corresponding ISP profit and aggregate video streaming provider profit. Again, we start at the profit-maximizing peering price $P^d = \$4.59$ and consider a small decrease. The ISP's profit from the video streaming subscribers increases because the demand N^v increases and the price per subscriber $P^b + P^p$ increases. The ISP's profit from premium tier subscribers without video streaming decreases, because the demand N^p decreases more than the price $P^b + P^p$ increases. Finally, the ISP's profit from basic tier subscribers increases, because the demand N^b increases while the price P^b remains virtually unchanged.

We can now evaluate the stakeholder claims about the effect of paid peering on broadband prices and ISP profits. Recall that ISPs assert that paid peering revenue is offset by lower broadband prices, whereas content providers assert that peering prices do not result in lower broadband prices. We find that the basic tier price P^b is almost the same in the case in which the ISP chooses the peering price to maximize profit ($P^d = \$4.59$) as in the case in which settlement-free peering is used ($P^d = \$0$). We also find that the premium tier price $P^b + P^p$ decreases by \$3.98 (from \$73.98 to \$70.00) if we change from settlement-free peering ($P^d = \$0$) to paid peering ($P^d = \$4.59$), but the aggregate video streaming price increases by \$4.60 (from \$21.59 to \$26.19). Thus, to the extent that ISPs assert that paid peering reduces the price of the basic tier, we disagree. Paid peering should be expected to reduce the price of the premium tier, but this reduction in broadband price is more than offset by an increase in video streaming prices.

Recall that ISPs assert that their profits are unaffected by peering fees, whereas content providers assert that peering fees increase ISP profits. We find that the ISP profit increases by 0.8% if we change from settlement-free peering ($P^d = \$0$) to paid peering ($P^d = \$4.59$).

However, the larger effect is on aggregate video streaming profit, which decreases by 18%.

6. The Effect of Paid Peering on Aggregate Consumer Surplus

In the previous section, we analyzed the effect of paid peering on broadband prices. In this section, we turn to the impact of paid peering on consumer surplus. ISPs assert that paid peering fees increase aggregate consumer surplus because they eliminate an inherent subsidy of consumers with high video streaming use by consumers without such use. Content providers assert that paid peering fees decrease aggregate consumer surplus because they are passed onto consumers through higher video streaming prices without a corresponding reduction in broadband prices.

A portion of these assertions was addressed in the previous section. We now know that when an ISP sets peering prices so as to maximize profit, it sets those prices to be positive. Compared to settlement-free peering, positive peering prices result in reduced premium tier prices. Directly connected video streaming providers increase their prices to compensate. However, the ISP only passes onto its customers a portion of the paid peering revenue.

However, this leaves unanswered the question of the impact on aggregate consumer surplus. It also leaves unanswered the question of what value of peering price maximizes aggregate consumer surplus. We attempt to answer those questions now.

We consider the peering price P^d to be an independent variable set by a regulator. The aggregate consumer surplus $CS(P^b_{CS_{reg}}, P^p_{CS_{reg}}, P^v)$ is a function of P^d. The regulator is presumed to set the peering price P^d so that it maximizes aggregate consumer surplus:

$$\begin{array}{c} (P^b_{CS_{reg}}, P^p_{CS_{reg}}) = \arg\max_{(P^b,P^p)} \pi^{ISP}(P^b, P^p, P^d_{CS_{reg}}, P^v_0 + \alpha P^d_{CS_{reg}}) \\ P^d_{CS_{reg}} = \arg\max_{P^d} CS(P^b_{CS_{reg}}(P^d), P^p_{CS_{reg}}(P^d), P^d, P^v_0 + \alpha P^d). \end{array} \tag{23}$$

Equation (23) determines the resulting aggregate consumer surplus maximizing value of the peering price P^d, as well as the resulting broadband prices P^b and P^p and video streaming price P^v.

Another optimization metric commonly used is aggregate social welfare, which we define here as

$$W(P^b, P^p, P^d, P^v) \triangleq CS(P^b, P^p, P^v) + \pi^{ISP}(P^b, P^p, P^d, P^v) + \pi^{VSP}(P^b, P^p, P^d, P^v). \tag{24}$$

Note that whereas the ISP profit maximization problem (15) considers only ISP profit, and the aggregate consumer surplus maximization problem (23) considers only consumer surplus, social welfare includes both ISP profit and consumer surplus, as well as aggregate video streaming provider profit.

Although we believe that a regulator should attempt to maximize consumer surplus not aggregate social welfare, the social welfare maximization problem is:

$$\begin{array}{c} (P^b_{W_{reg}}, P^p_{W_{reg}}) = \arg\max_{(P^b,P^b)} \pi^{ISP}(P^b, P^p, P^d_{W_{reg}}, P^v_0 + \alpha P^d_{W_{reg}}) \\ P^d_{W_{reg}} = \arg\max_{P^d} W(P^b_{W_{reg}}, P^p_{W_{reg}}, P^d, P^v_0 + \alpha P^d). \end{array} \tag{25}$$

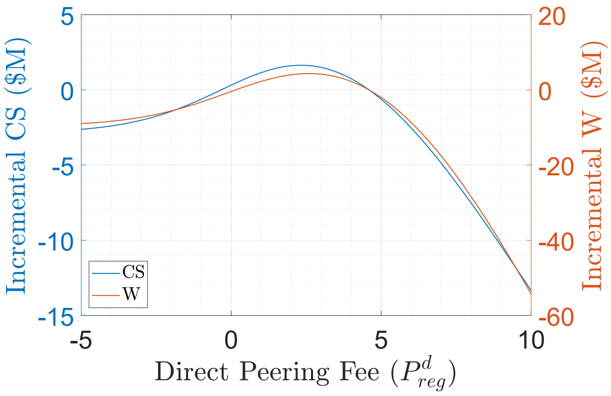


Figure 5: Effect of Peering Fee on Incremental Consumer Surplus and Incremental Social Welfare

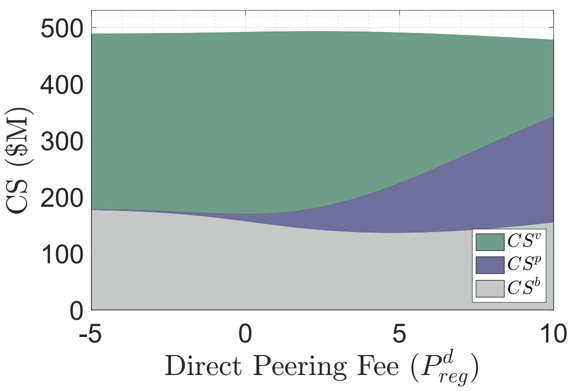


Figure 6: Effect of Peering Fee on Consumer Surplus with Different Services

However, the optimization problem is no longer analytically tractable. Thus, we will turn back to our numerical evaluation. Figure 5 shows the incremental consumer surplus as a function of the regulator chosen peering price P^d. The incremental consumer surplus is defined as the difference between the aggregate consumer surplus at the regulator chosen peering price P^d and at the peering price that maximizes ISP profit (P^d_{ISP}).

Aggregate consumer surplus is a uni-modal function of the peering price. We find that the peering price that maximizes consumer surplus is $P^d_{CS_{reg}} = \$2.34$. This is substantially less than the peering price that maximizes ISP profit (P^d_{ISP}=\$4.59). At peering prices lower than \$2.34, aggregate consumer surplus decreases principally because the premium tier price is too high, and this decreases the surplus of premium tier subscribers. At peering prices higher than \$2.34, aggregate consumer surplus decreases principally because the price of video streaming is too high, and this decreases the surplus of video streaming subscribers.

To understand why, we need to revisit the impact of the peering price on broadband tier prices and demand, and how these changes in price and demand affect aggregate consumer surplus. We compare prices and demands in the case in which the ISP chooses the peering price to maximize profit ($P^d_{ISP} = \$4.59$) to the case in which the regulator chooses the peering price to maximize aggregate consumer surplus ($P^d_{CS_{reg}} = \$2.34$).

As we discussed in the previous section, a reduction in the peering price below that which maximizes ISP profit results in lower aggregate video streaming prices and increased premium tier prices. However, the amount of the increase in the premium tier price is less than the amount of the decrease in the aggregate video streaming price. Thus, the price of the premium tier with video streaming ($P^b + P^p + P^v$) decreases. These changes in prices cause some premium tier subscribers without video streaming to downgrade to the basic tier, and some to start subscribing to video streaming.

These changes in prices and demand affect aggregate consumer surplus. Figure 6 shows the aggregate consumer surplus of all subscribers to the basic tier, to the premium tier without video streaming, and to the premium tier with video streaming. A reduction in the peering price below that which maximizes ISP profit results in increased demand for the basic tier, but with basic tier prices virtually unchanged. The result is that the aggregate consumer surplus of basic tier subscribers increases. A reduction in the peering price also results in increased premium tier prices and decreased demand for the premium tier without video streaming. The result is that the aggregate consumer surplus of premium tier subscribers without video streaming decreases. Finally, a reduction in the peering price results in decreased prices of the premium tier with video streaming and increased demand. The result is that the aggregate consumer surplus of premium tier subscribers with video streaming increases. The aggregate consumer surplus is the sum of these three. As the peering price decreases from the price that maximizes ISP profit (P^d_{ISP}=\$4.59) to the price that maximizes consumer surplus ($P^d_{CS_{reg}} = \$2.34$), the increase in the aggregate consumer surplus of basic tier subscribers and premium tier subscribers with video streaming dominates the decrease in the aggregate consumer surplus of premium tier subscribers without video streaming. However, at peering prices below the price that maximizes consumer surplus ($P^d_{CS_{reg}} = \$2.34$), the opposite is true.

Finally, we consider the difference if a regulator would choose the peering price that maximizes aggregate social welfare (25) instead of maximizing aggregate consumer surplus (23). Figure 5 shows the incremental social welfare as a function of the regulator chosen peering price P^d. The incremental social welfare is defined as the difference between the aggregate social welfare at the regulator chosen peering price P^d and at the peering price that maximizes ISP profit (P^d_{ISP}).

Recall that the aggregate social welfare is the sum of ISP profit, aggregate video streaming provider profit, and aggregate consumer surplus. As discussed in the previous section, the ISP profit decreases as the peering price decreases below \$4.59, because the ISP's increase in the premium tier price is less than the decrease in the peering price. However, the aggregate video streaming provider profit increases, because demand for video streaming increases and the profit per video streaming subscriber remains constant. Finally, as discussed above, the aggregate consumer surplus increases until the peering price goes below \$2.34.

Aggregate social welfare is a uni-modal function of the peering price. We find that the peering price that maximizes social welfare is $P^d_{W_{reg}} = \$2.61$. This is substantially less than the peering price that maximizes ISP profit ($P^d_{ISP} = \$4.59$), but higher than the peering price that maximizes aggregate consumer surplus ($P^d_{CS_{reg}} = \$2.34$). As the peering price decreases from the price that maximizes ISP profit ($P^d_{ISP} = \$4.59$) to the price that maximizes social welfare ($P^d_{W_{reg}} = \$2.61$), the sum of the increase in the aggregate video streaming provider profit and increase in the aggregate consumer surplus dominates the decrease in ISP profit. At peering prices below \$2.61 but above \$2.34, the opposite is true.

We can now evaluate the stakeholder claims about the effect of paid peering on consumer surplus. Recall that ISPs assert that paid peering fees increase aggregate consumer surplus whereas content providers assert that they decrease aggregate consumer surplus. The peering price that maximizes aggregate consumer surplus is below the price that maximizes ISP profit. Using our numerical parameters, we found that the peering price that maximizes aggregate consumer surplus is $P^d_{CS_{reg}} = \$2.34$, whereas if unregulated the ISP would choose $P^d_{ISP} = \$4.59$. Furthermore, we found that aggregate consumer surplus is \$1.65M higher at the peering price that maximizes aggregate consumer surplus than at the peering price that maximizes ISP profit. However, we also found that when the incremental ISP cost per video streaming subscriber is $C^d = \$3.00$, aggregate consumer surplus is \$1.33M higher at the peering price that maximizes aggregate consumer surplus than at settlement-free peering ($P^d = \$0$). Thus, neither settlement-free peering nor paid peering with an ISP-determined price maximizes consumer surplus.

7. The Effect of the Incremental ISP Cost C^d Per Video Streaming Subscriber

The peering price that maximizes aggregate consumer surplus depends critically on the incremental ISP cost C^d per video streaming subscriber. Without knowledge of this cost, we cannot say whether the peering price that maximizes aggregate consumer surplus is negative, zero, or positive. In this section, we consider how the incremental ISP cost C^d per video streaming subscriber affects the results in this paper. For each value of C^d, we determine the numerical parameters (μ_b, μ_p, μ_v, C^b, C^p, and P^d) using the method discussed in Section 4.2. This analysis is thus a study of the impact of the unknown value of C^d, given fixed values for the observed known parameters.

Figure 7 shows the peering prices that maximize ISP profit, aggregate consumer surplus, and aggregate social welfare as a function of the incremental ISP cost C^d per video streaming subscriber.Thus, not only does C^d direct affect the peering prices, it also indirectly affects all prices and demands. The peering price that maximizes aggregate consumer surplus, $P^d_{CS_{reg}}$, increases nearly linearly, from -\$1.80 to \$2.34 as C^d increases from -\$1.12 to \$3.00. Notably, it is positive when $C^d > \$0.68$, but negative at lower values of C^d. Recall that the incremental ISP cost C^d per video streaming subscriber depends on both the incremental Internet usage of video streaming subscribers over non-subscribers and the length of the path on the ISP's network. As video content providers interconnect with the ISP closer to consumers, the incremental ISP cost C^d per video streaming subscriber decreases, and may be negative if the interconnection point is close enough to the consumer. In contrast, if the interconnection point is far from the consumer, then the incremental Internet usage may dominate and C^d may be positive.

If a regulator were to set the peering price to maximize social welfare, then $P^d_{W_{reg}}$ similarly increases from -\$1.85 to \$2.61 as C^d increases from \$1.12 to \$3.00. This closely tracks $P^d_{CS_{reg}}$, with the difference diminishing at lower costs. The peering price that maximizes ISP profit, P^d_{ISP}, also increases nearly linearly with the incremental ISP cost C^d per video streaming subscriber, from \$0.00 to \$4.59 as C^d increases from \$1.12 to \$3.00. Notably, the incremental ISP profit $P^d_{ISP} - C^d$ per video streaming subscriber remains positive at all values above $C^d = -\$1.12$, and indeed increases with higher values of C^d.

The effect on consumers is qualitatively similar, but different in magnitude. When $C^d =$ \$3.00, premium tier subscribers without video streaming would pay \$70.00 at the ISP chosen peering price ($P^d = \$4.59$) but \$71.69 if the regulator sets the peering price to maximize consumer surplus ($P^d = \$2.34$), and premium tier subscribers with video streaming would pay \$96.19 at the ISP chosen peering price but \$95.61 at the regulator chosen peering price. Thus, regulation of the peering price results in premium tier subscribers without video

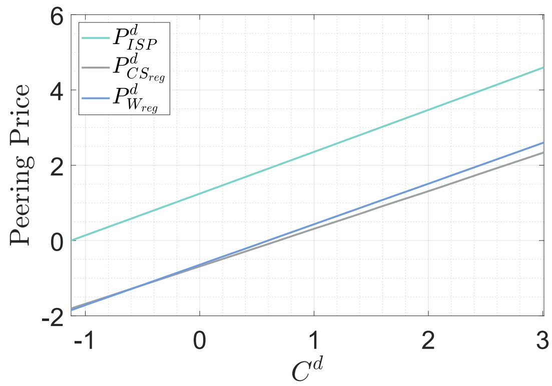


Figure 7: Effect of the Incremental ISP Cost Per Video Streaming Subscriber on the Peering Price

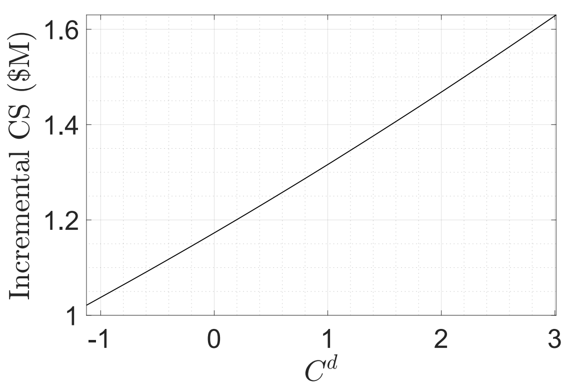


Figure 8: Effect of the Incremental ISP Cost Per Video Streaming Subscriber on the Incremental Consumer Surplus

streaming paying \$1.69 more and in premium tier subscribers with video streaming paying \$0.58 less; however the regulated peering price also increases demand for video streaming from 37.5% to 42.6%.

When $C^d = -\$1.12$, premium tier subscribers without video streaming would pay \$70.00 at the ISP chosen peering price but \$71.37 at the regulator chosen peering price, and premium tier subscribers with video streaming would pay \$91.59 at the ISP chosen peering price but \$91.15 at the regulator chosen peering price. Thus, regulation of the peering price results in premium tier subscribers without video streaming paying \$1.37 more and in premium tier subscribers with video streaming paying \$0.44 less; however the regulated peering price also increases demand for video streaming from 37.5% to 42.3%.

Finally, we revisit our evaluation of stakeholder claims about broadband prices, ISP profit, and consumer surplus, under different values of the incremental ISP cost C^d per video streaming subscriber. If $C^d = \$3.00$, we found that paid peering should be expected to reduce the price of the premium tier, but this reduction in broadband price is more than offset by an increase in video streaming prices. At lower values of C^d, paid peering still should be expected to reduce the price of the premium tier, but less so. Similarly, neither the change in ISP profit nor the change in video streaming profit is very sensitive to C^d.

If $C^d = \$3.00$, we found that aggregate consumer surplus is \$1.65M higher at the peering price that maximizes aggregate consumer surplus than at the peering price that maximizes ISP profit, but that aggregate consumer surplus is also \$1.33M higher at the peering price that maximizes aggregate consumer surplus than at settlement-free peering ($P^d = \$0$). Figure 8 shows the incremental consumer surplus, which is the difference between the aggregate consumer surplus at ISP-chosen peering price and that at the peering price that maximizes consumer surplus, for various values of C^d. We observe that the incremental consumer surplus is significant at all values of C^d, rising from \$1.02M to \$1.63M as C^d increases from \$1.12 to \$3.00.

The incremental ISP cost C^d per video streaming subscriber, however, does have a large impact on the optimal peering price. The peering price that maximizes consumer surplus is strongly correlated with C^d. At values of $C^d > \$0.68$, settlement-free peering is too aggressive. and the regulator should limit the peering price to at least \$2.00 less than the ISP-chosen peering price. At negative values of C^d, settlement-free peering is too timid, and the ISP should pay content providers for paid peering at locations so close to the consumers. At small positive values of C^d ($0 < C^d < \$0.68$), the ISP bears a cost, but the peering price that maximizes consumer surplus is negative; we turn to this issue next.

So far, we found that the peering price that maximizes aggregate consumer surplus ($P^d_{CS_{reg}}$) is less than the incremental network cost C^d for video streaming. This result seems counter-intuitive, since we generally expect that consumer surplus is maximized when prices reflect costs. In order to explain this result, we formulate an optimization problem in which a regulator maximizes aggregate consumer surplus by choosing not only the peering price P^d but also the broadband prices P^b and P^p and the aggregate video streaming price P^v. We are not proposing that a regulator control all of these prices, but it will serve as an informative comparison.

Given a set of prices, the aggregate consumer surplus was given in (10). An overly simplistic approach to maximizing aggregate consumer surplus would be to choose the consumer-facing prices:

$$
\begin{aligned}
(P^b_{CS}, P^p_{CS}, P^v_{CS}) = \arg \max_{(P^b, P^p, P^v)} \; & CS(P^b, P^p, P^v) \\
\text{s.t.} \; & \\
& P^b_{CS} \geq 0 \\
& P^p_{CS} \geq 0 \\
& P^v_{CS} \geq 0.
\end{aligned}
\tag{26}
$$

However, we can show that the marginal aggregate consumer surplus with respect to the tier prices and the peering price are all negative:

Theorem 3.

$$
\begin{aligned}
\frac{\partial CS}{\partial P^b} &= -N^b - N^p - N^v < 0 \\
\frac{\partial CS}{\partial P^p} &= -N^p - N^v < 0 \\
\frac{\partial CS}{\partial P^v} &= -N^v < 0.
\end{aligned}
\tag{27}
$$

The proof can be found in Appendix C.

It follows that the solution to (26) is at $P^b = P^p = P^v = 0$. It is common to guarantee that a business's rate of return does not fall below a specified minimum. The rate of return for the ISP is

$$
r^{ISP}(P^b, P^p, P^d, P^v) \triangleq \frac{\pi^{ISP}(P^b, P^p, P^d, P^v)}{(C^b)N^b + (C^b + C^p)N^p + (C^b + C^p + C^d)N^v}. \tag{28}
$$

The rate of return of the video streaming providers is

$$r^{VSP}(P^b, P^p, P^d, P^v) \triangleq \frac{\pi^{VSP}(P^b, P^p, P^d, P^v)}{C^v N^v}. \tag{29}$$

Maximization of aggregate consumer surplus, subject to rate of return constraints for both the ISP and the video streaming providers, is:

$$\begin{aligned}(P^b_{CS}, P^p_{CS}, P^d_{CS}, P^v_{CS}) = \arg \max_{(P^b, P^v, P^d, P^v)} & \; CS(P^b, P^p, P^v) \\ \text{s.t.} & \\ & r^{ISP}(P^b, P^p, P^d, P^v) \geq r^{ISP}_{\min} \\ & r^{VSP}(P^b, P^p, P^d, P^v) \geq r^{VSP}_{\min}, \end{aligned} \tag{30}$$

where $r^{ISP}_{\min}$ is the specified minimum rate of return for the ISP, and $r^{VSP}_{\min}$ is the specified minimum rate of return for the video streaming providers. Although aggregate consumer surplus is not directly a function of the peering price P^d, the peering price P^d is present in the rate of return constraints.

It follows that the prices that maximize aggregate consumer surplus are those at which the rate of return constraints are binding:

Theorem 4. *The prices $(P^b_{CS}, P^p_{CS}, P^d_{CS}, P^v_{CS})$ that maximize aggregate consumer surplus in (30) are:*

$$\begin{aligned} P^b_{CS} &= (r^{ISP}_{\min} + 1)C^b \\ P^p_{CS} &= (r^{ISP}_{\min} + 1)C^p \\ P^d_{CS} &= (r^{ISP}_{\min} + 1)C^d \\ P^v_{CS} &= (r^{VSP}_{\min} + 1)C^v + P^d_{CS}. \end{aligned} \tag{31}$$

The proof can be found in Appendix D.

The prices that maximize aggregate consumer surplus are thus those that allow the ISP to cover the costs of each tier and the cost of paid peering (if any), plus the specified rate of return.

We wish to compare the solution to this maximization problem to the solution of the ISP profit maximization problem (15) and to the solution of the aggregate consumer surplus maximization problem with a regulator-chosen peering price (23). For numerical purposes, we set $r^{ISP}_{\min} = r^{VSP}_{\min} = 13.6\%$.[9] The solution to (30) is: $P^b_{CS} = \$18.74$, $P^p_{CS} = \$21.58$, $P^d_{CS} = \$3.41$, and $P^v_{CS} = \$25.00$.

The peering price that maximizes aggregate consumer surplus under rate of return constraints, $P^d_{CS} = \$3.41$, is equal to the incremental network cost $C^d = \$3.00$ for video streaming plus the desired minimal rate of return. This is less than the peering price that maximizes ISP profit, $P^d_{ISP} = \$4.59$, but greater than the peering price that maximizes aggregate consumer surplus, $P^d_{CS_{reg}} = \$2.34$.

[9]This rate of return is taken from Netflix (2019), the 2019 Netflix 10-K report.

If the regulator instead chooses all of the prices, then it will choose all prices equal to cost plus the desired rate of return, according to Theorem 4. However, when the regulator can set the peering price, but allows the ISP to set the broadband prices, the regulator chooses a price $P^d_{CS_{reg}} = \$2.34 < C^d = \$3.00$. The reason for setting the price below cost is that the regulator cannot directly control the broadband prices, P^b and $P^b + P^p$, both of which substantially exceed cost plus the desired rate of return. In the absence of such control, aggregate consumer surplus is maximized by a peering price less than cost.

8. Conclusion

ISPs and content providers disagree about the effect of paid peering on broadband prices. ISPs assert that the revenue they generate from paid peering fees is used to lower broadband prices, whereas content providers assert that paid peering fees increase ISP profit but do not affect broadband prices.

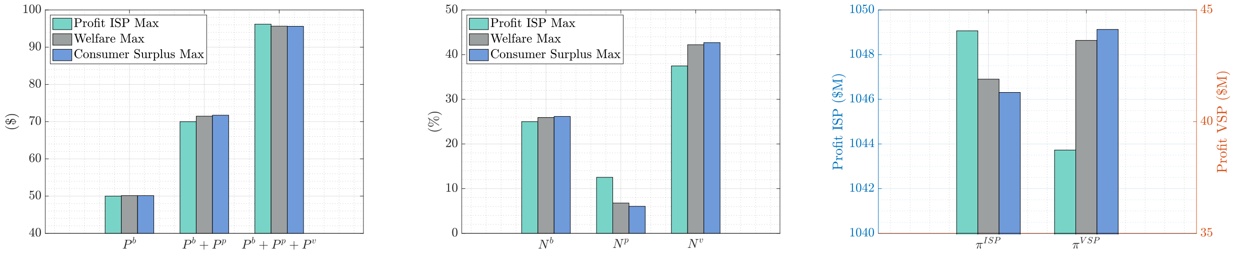
To address this debate, we modeled a monopoly ISP offering two tiers of service. Consumers decide whether to subscribe to broadband and if so to which tier, and whether to subscribe to video streaming services. We modeled demand for the broadband tiers and video streaming services based on these consumer choices, and evaluate the resulting ISP profit, video streaming profit, and consumer surplus.

To focus on the effect of peering fees, we considered a two-sided model in which a profit-maximizing ISP determines broadband prices and the peering price and in which video streaming providers choose their price based on the peering price. Numerical parameters were chosen based on public information about broadband and video streaming prices and subscription. We proved that a profit-maximizing ISP's chosen broadband and peering prices satisfy a generalization of the well-known Lerner rule, which specifies how these prices are related to a matrix of elasticities and cross-elasticities of demand.

We also determined the peering fees that maximize either consumer surplus or social welfare, such as a regulator may set. We compared the effect of an ISP-chosen peering fee with a regulator-chosen peering fee. Figure 9 summarizes our results. We find when that a regulator sets the peering price to maximize consumer surplus, it chooses a lower peering price than does the ISP. As a result, video streaming prices drop to reflect the lower video streaming costs. However, the ISP then increases the price of the premium tier, recouping most of its loss from the lower peering price and regaining some of the increased consumer surplus from lower video streaming prices.

These changes in prices affect the demand for broadband and for video streaming. When the regulator steps in, the reduction in the price of video streaming, combined with the increase in the price of the premium tier, creates two shifts. First, some premium tier subscribers with moderate utility from video streaming will start subscribing, due to the reduced sum of the premium tier price and the video streaming price. Second, some premium tier subscribers with low utility from video streaming will downgrade to the basic tier, due to the increased premium tier price.

Our results show that the claims of the ISPs and of the content providers are both incorrect. When an ISP chooses peering prices, some of the revenue from these fees is used



(a) Prices (b) Demand (c) Profit

Figure 9: Comparison between different policies

to decrease the price of the premium tier, but some of the revenue increases ISP profit. In contrast, when a regulator sets peering prices to maximize consumer surplus, the lower price stimulates significant additional demand for video streaming. If a regulator sets peering prices to maximize social welfare and if video streaming providers pass through 100% of the peering fee, then prices and demands are in between the ISP profit-maximizing and consumer surplus maximizing cases, but closer to the consumer surplus maximization.[10]

ISPs and content providers also disagree about the effect of paid peering on consumer surplus, and ultimately about whether peering prices should be regulated. ISPs assert that paid peering increases consumer surplus because it eliminates an inherent subsidy of consumers with high video streaming use by consumers without, whereas content providers assert that paid peering decreases consumer surplus because paid peering fees are passed onto consumers through higher video streaming prices and because there is no corresponding reduction in broadband prices. As a result, ISPs argue that the market should determine peering prices, while content providers argue that they should be entitled to settlement-free peering if they interconnect with the ISP close enough to consumers.

Our results show that the peering price that maximizes consumer surplus is lower than the peering price an ISP would choose. Although an ISP-chosen peering price does eliminate an inherent subsidy of video streaming (if there is a positive incremental ISP cost per video streaming subscriber), the ISP-chosen peering price substantially exceeds this incremental cost. As a result, the ISP-chosen peering price reduces consumer surplus, largely because it reduces demand for video streaming.

However, it does not follow that settlement-free peering is always the policy that maximizes consumer surplus. When there is a moderate incremental ISP cost per video streaming subscriber, the peering price that maximizes consumer surplus is positive, but lower than the ISP-chosen price. This positive price is beneficial for consumers because the incremental

[10]However, if the pass-through rate is less than 100%, then the welfare-maximizing prices and demands may no longer be in between the ISP profit-maximizing and the consumer surplus-maximizing prices and demands. Using our numerical parameters, when $\alpha \leq 0.93$, the pass-through rate lessens the impact of the peering fee upon consumer surplus enough such that the increase in ISP profit dominates the reduction in consumer surplus. As a consequence, the social welfare-maximizing peering price is no longer in between the ISP profit-maximizing and consumer surplus maximizing cases. However, all other qualitative results in this paper remain true for pass-through rates less than 100%.

ISP cost for video streaming is paid by video streaming subscribers. In contrast, if content providers bring the content closer to consumers, there may be a negative incremental ISP cost per video streaming subscriber, in which case the peering price that maximizes consumer surplus is negative. In this situation, the content provider should be entitled to settlement-free peering, or even to be paid by the ISP.

Although the theorems presented here hold for all values of parameters (except that Theorem 1 requires a 100% pass-through rate), the numerical results depend on the numerical values chosen for these parameters. To judge whether the results presented here are robust to the values chosen for these parameters, we repeated all numerical analyses in this paper with a wide range of values of the following parameters, each of which was changed one at a time:

- the ISP profit-maximizing basic tier price (P^b),
- the ISP profit-maximizing incremental premium tier price (P^p),
- the standard deviations of user utilities ($\sigma_b^2, \sigma_p^2, \sigma_v^2$),
- the percentage of the population that subscribes to broadband ($(N^b + N^p + N^v)/N$),
- the percentage of broadband subscribers who subscribe to the basic tier ($N^b/(N^b + N^p + N^v)$),
- the percentage of premium tier subscribers who subscribe to video streaming services ($N^v/(N^p + N^v)$),
- the aggregate video streaming price with settlement-free peering (P_0^v), and
- the pass-through rate (α).

Although the numerical results depend on the numerical values of these parameters, all of the qualitative results presented in this paper remain true for all parameter ranges that we analyzed (except for the qualitative result about welfare-maximization, as discussed above). We briefly discuss here the sensitivity of our results to these parameter choices.

The ISP profit-maximizing paid peering fee is relatively insensitive to the ISP profit-maximizing basic tier price (P^b) and to the incremental premium tier price (P^p). However, it is positively correlated with the standard deviations of user utilities; higher standard deviations increase the willingness-to-pay of video streaming subscribers (who have above average utilities), which the ISP can then leverage through higher peering fees.

The ISP profit-maximizing paid peering fee is moderately positively correlated with the percentage of the population that subscribes to broadband ($(N^b + N^p + N^v)/N$), because higher broadband demand increases premium tier demand and thus compensates for the decreased demand that would be caused by a higher peering fee. It is relatively moderately negatively correlated with the percentage of broadband subscribers who subscribe to the basic tier ($N^b/(N^b + N^p + N^v)$), because higher relative demand for the basic tier decreases demand for the premium tier, which causes the ISP to decrease the peering fee. The ISP profit-maximizing paid peering fee is strongly positively correlated with the percentage of premium tier subscribers who subscribe to video streaming services ($N^v/(N^p+N^v)$), because the ISP can take advantage of the higher relative demand for video streaming.

The ISP profit-maximizing paid peering fee is moderately positively correlated with the aggregate video streaming price under settlement-free peering (P_0^v), because the ISP can increase its paid peering fee to take advantage of higher streaming providers' revenue.

Finally, the ISP profit-maximizing paid peering fee is strongly inversely correlated with the pass-through rate of paid peering. When the pass-through rate decreases below 100%, the ISP can increase its paid peering fee to take advantage of the lower sensitivity of premium tier subscribers to the paid peering fee. The ISP will also moderately decrease the incremental premium tier price to increase premium tier demand. The ISP's decrease in revenue from premium tier subscribers is more than offset by its increase in revenue from paid peering. The impact upon video streaming providers' profit increases as the pass-through rate decreases. With lower pass-through rates, video streaming providers have less ability to recoup paid peering fees, and the gap between the profit they earn under ISP profit-maximizing paid peering fees and the profit they would earn under paid peering fees that maximize consumer surplus grows.

These results are not the end of the story. In this paper, we only considered direct interconnection between content providers and an ISP. However, despite the reduction in the percentage of Internet traffic passing through a transit provider, it would be useful to examine the decision of a content provider choosing between direct interconnection with an ISP and transit service from a transit provider. Large ISPs assert that direct interconnection is a competitive alternative to indirect connection through transit, whereas content providers assert that ISPs retain a terminating monopoly on both. Further research is also warranted to examine the incremental ISP cost for video streaming. An ISP's costs for transporting Internet traffic depend on whether the traffic is carried across the ISP's core and middle-mile networks, as well as the ISP's access network. Research could consider how routing and interconnection affect ISP costs, and in particular, the incremental ISP cost per video streaming subscriber. Finally, policymakers could benefit from further research on the impact of peering fees on the video marketplace. Although we found that ISP-determined peering prices likely exceed related costs and do not maximize consumer surplus, we found that they likely affect video streaming demand more than they affect either consumer surplus or ISP profits. A model that evaluates the impact on competition between an ISP's video streaming products and competing video streaming products may be insightful.

Appendix A. Proof of Theorem 1

The profit-maximizing prices (P^b, P^p, P^d) in (15) satisfy these first order conditions:

$$\frac{\partial \pi^{ISP}}{\partial P^b} = (P^b - C^b)\frac{\partial N^b}{\partial P^b} + (P^b + P^p - C^b - C^p)\frac{\partial N^p}{\partial P^b} + (P^b + P^p + P^d - C^b - C^p - C^d)\frac{\partial N^v}{\partial P^b} + N^b + N^p + N^v = 0. \tag{A.1}$$

$$\frac{\partial \pi^{ISP}}{\partial P^p} = (P^b - C^b)\frac{\partial N^b}{\partial P^p} + (P^b + P^p - C^b - C^p)\frac{\partial N^p}{\partial P^p} + (P^b + P^p + P^d - C^b - C^p - C^d)\frac{\partial N^v}{\partial P^p} + N^p + N^v = 0. \tag{A.2}$$

$$\frac{\partial \pi^{ISP}}{\partial P^d} = (P^b - C^b)\frac{\partial N^b}{\partial P^d} + (P^b + P^p - C^b - C^p)\frac{\partial N^p}{\partial P^d} + (P^b + P^p + P^d - C^b - C^p - C^d)\frac{\partial N^v}{\partial P^d} + N^v = 0. \tag{A.3}$$

We can rearrange these three equations in order to isolate the dependence on N^b, N^p, and N^v:

$$\begin{aligned}(P^b - C^b)(\frac{\partial N^b}{\partial P^b} + \frac{\partial N^p}{\partial P^b} + \frac{\partial N^v}{\partial P^b} - \frac{\partial N^b}{\partial P^p} - \frac{\partial N^p}{\partial P^p} - \frac{\partial N^v}{\partial P^p}) + (P^p - C^p)(\frac{\partial N^p}{\partial P^b} + \frac{\partial N^v}{\partial P^b} - \frac{\partial N^p}{\partial P^p} - \frac{\partial N^v}{\partial P^p}) \\ + (P^d - C^d)(\frac{\partial N^v}{\partial P^b} - \frac{\partial N^v}{\partial P^p}) + N^b = 0\end{aligned} \tag{A.4}$$

$$\begin{aligned}(P^b - C^b)(\frac{\partial N^b}{\partial P^p} + \frac{\partial N^p}{\partial P^p} + \frac{\partial N^v}{\partial P^p} - \frac{\partial N^b}{\partial P^d} - \frac{\partial N^p}{\partial P^d} - \frac{\partial N^v}{\partial P^d}) + (P^p - C^p)(\frac{\partial N^p}{\partial P^p} + \frac{\partial N^v}{\partial P^p} - \frac{\partial N^p}{\partial P^d} - \frac{\partial N^v}{\partial P^d}) \\ + (P^d - C^d)(\frac{\partial N^v}{\partial P^p} - \frac{\partial N^v}{\partial P^d}) + N^p = 0\end{aligned} \tag{A.5}$$

$$(P^b - C^b)(\frac{\partial N^b}{\partial P^d} + \frac{\partial N^p}{\partial P^d} + \frac{\partial N^v}{\partial P^d}) + (P^p - C^p)(\frac{\partial N^p}{\partial P^d} + \frac{\partial N^v}{\partial P^d}) + (P^d - C^d)(\frac{\partial N^v}{\partial P^d}) + N^v = 0 \tag{A.6}$$

Applying the Leibniz Integral Rule to (4-6), we can show that the partial derivatives of demands with respect to prices are:

$$\begin{aligned}&\frac{\partial N^b}{\partial P^b} = -I4, \frac{\partial N^p}{\partial P^b} = -I6, \frac{\partial N^v}{\partial P^b} = -I1 \\ &\frac{\partial N^b}{\partial P^p} = I2 + I5, \frac{\partial N^p}{\partial P^p} = -I5 - I6, \frac{\partial N^v}{\partial P^p} = -I1 - I2 \\ &\frac{\partial N^b}{\partial P^d} = I2, \frac{\partial N^p}{\partial P^d} = I3, \frac{\partial N^v}{\partial P^d} = -I1 - I2 - I3\end{aligned} \tag{A.7}$$

where

$$\begin{aligned}
I1 &= N\int_{P^v}^{\infty}\int_{P^p+P^v-v}^{\infty} f_{B,P,V}(P^b+P^p+P^v-p-v,p,v)\,dp\,dv\\
I2 &= N\int_{P^v}^{\infty}\int_{P^b}^{\infty} f_{B,P,V}(b,P^p+P^v-v,v)\,db\,dv\\
I3 &= N\int_{P^p}^{\infty}\int_{P^b+P^p-p}^{\infty} f_{B,P,V}(b,p,P^v)\,db\,dp\\
I4 &= N\int_{-\infty}^{P^p}\int_{-\infty}^{P^p+P^v-p} f_{B,P,V}(P^b,p,v)\,dv\,dp\\
I5 &= N\int_{-\infty}^{P^v}\int_{P^b}^{\infty} f_{B,P,V}(b,P^p,v)\,db\,dv\\
I6 &= N\int_{-\infty}^{P^v}\int_{P^p}^{\infty} f_{B,P,V}(P^b+P^p-p,p,v)\,dp\,dv.
\end{aligned} \tag{A.8}$$

In (A.4-A.6), particular combinations of partial derivatives of demand with respect to price occur. It can be shown using (A.7) that:

$$\frac{\partial N^p}{\partial P^d}+\frac{\partial N^v}{\partial P^d}=\frac{\partial N^v}{\partial P^p} \tag{A.9}$$

$$\frac{\partial N^b}{\partial P^d}+\frac{\partial N^p}{\partial P^d}+\frac{\partial N^v}{\partial P^d}=\frac{\partial N^v}{\partial P^b} \tag{A.10}$$

$$\frac{\partial N^b}{\partial P^p}-\frac{\partial N^b}{\partial P^d}+\frac{\partial N^p}{\partial P^p}-\frac{\partial N^p}{\partial P^d}+\frac{\partial N^v}{\partial P^p}-\frac{\partial N^v}{\partial P^d}=\frac{\partial N^p}{\partial P^b} \tag{A.11}$$

$$\frac{\partial N^v}{\partial P^b}-\frac{\partial N^v}{\partial P^p}=\frac{\partial N^b}{\partial P^d} \tag{A.12}$$

$$\frac{\partial N^p}{\partial P^b}+\frac{\partial N^v}{\partial P^b}-\frac{\partial N^p}{\partial P^p}-\frac{\partial N^v}{\partial P^p}=\frac{\partial N^b}{\partial P^p} \tag{A.13}$$

We now substitute A.12 and A.13 into A.4, A.9 and A.11 into A.5, and A.9 and A.10 into A.6, to obtain:

$$\begin{aligned}
(P^b-C^b)\frac{\partial N^b}{\partial P^b}+(P^p-C^p)\frac{\partial N^b}{\partial P^p}+(P^d-C^d)\frac{\partial N^b}{\partial P^d}+N^b=0\\
(P^b-C^b)\frac{\partial N^p}{\partial P^b}+(P^p-C^p)\frac{\partial N^p}{\partial P^p}+(P^d-C^d)\frac{\partial N^p}{\partial P^d}+N^p=0\\
(P^b-C^b)\frac{\partial N^v}{\partial P^b}+(P^p-C^p)\frac{\partial N^v}{\partial P^p}+(P^d-C^d)\frac{\partial N^v}{\partial P^d}+N^v=0
\end{aligned} \tag{A.14}$$

By rearranging (A.14), we can show that:

$$
\begin{aligned}
\frac{P^b - C^b}{P^b}\frac{\partial N^b}{\partial P^b}\frac{P^b}{N^b} + \frac{P^p - C^p}{P^p}\frac{\partial N^b}{\partial P^p}\frac{P^p}{N^b} + \frac{P^d - C^d}{P^d}\frac{\partial N^b}{\partial P^d}\frac{P^d}{N^b} &= -1 \\
\frac{P^b - C^b}{P^b}\frac{\partial N^p}{\partial P^b}\frac{P^b}{N^p} + \frac{P^p - C^p}{P^p}\frac{\partial N^p}{\partial P^p}\frac{P^p}{N^p} + \frac{P^d - C^d}{P^d}\frac{\partial N^p}{\partial P^d}\frac{P^d}{N^p} &= -1 \\
\frac{P^b - C^b}{P^b}\frac{\partial N^v}{\partial P^b}\frac{P^b}{N^v} + \frac{P^p - C^p}{P^p}\frac{\partial N^v}{\partial P^p}\frac{P^p}{N^v} + \frac{P^d - C^d}{P^d}\frac{\partial N^v}{\partial P^d}\frac{P^d}{N^v} &= -1
\end{aligned}
\tag{A.15}
$$

By using the definitions of the price elasticity of demand and the Lerner indices, it follows that the profit-maximizing prices (P^b, P^p, P^d) in (15) satisfy:

$$
-\begin{bmatrix} \varepsilon_{N^b,P^b} & \varepsilon_{N^b,P^p} & \varepsilon_{N^b,P^d} \\ \varepsilon_{N^p,P^b} & \varepsilon_{N^p,P^p} & \varepsilon_{N^p,P^d} \\ \varepsilon_{N^v,P^b} & \varepsilon_{N^v,P^p} & \varepsilon_{N^v,P^d} \end{bmatrix} \begin{bmatrix} L^b \\ L^p \\ L^d \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}
\tag{A.16}
$$

Appendix B. Proof of Theorem 2

Each elasticity in Theorem 2 is, by definition, the same sign as the related partial derivative. All integrals in A.8 are positive, since the joint density function $f_{B,P,V}$ is positive. By using A.7, it follows that:

$$
\begin{aligned}
\varepsilon_{N^b,P^b} &\triangleq \frac{\partial N^b}{\partial P^b}\frac{P^b}{N^b} = -(I4)\frac{P^b}{N^b} < 0 \\
\varepsilon_{N^b,P^p} &\triangleq \frac{\partial N^b}{\partial P^p}\frac{P^p}{N^b} = (I2 + I5)\frac{P^p}{N^b} > 0 \\
\varepsilon_{N^b,P^d} &\triangleq \frac{\partial N^b}{\partial P^d}\frac{P^d}{N^b} = (I2)\frac{P^d}{N^b} > 0 \\
\varepsilon_{N^p,P^b} &\triangleq \frac{\partial N^p}{\partial P^b}\frac{P^b}{N^p} = -(I6)\frac{P^b}{N^p} < 0 \\
\varepsilon_{N^p,P^p} &\triangleq \frac{\partial N^p}{\partial P^p}\frac{P^p}{N^p} = -(I5 + I6)\frac{P^p}{N^p} < 0 \\
\varepsilon_{N^p,P^d} &\triangleq \frac{\partial N^p}{\partial P^d}\frac{P^d}{N^p} = (I3)\frac{P^d}{N^p} > 0 \\
\varepsilon_{N^v,P^b} &\triangleq \frac{\partial N^v}{\partial P^b}\frac{P^b}{N^v} = -(I1)\frac{P^b}{N^v} < 0 \\
\varepsilon_{N^v,P^p} &\triangleq \frac{\partial N^v}{\partial P^p}\frac{P^p}{N^v} = -(I1 + I2)\frac{P^p}{N^v} < 0 \\
\varepsilon_{N^v,P^d} &\triangleq \frac{\partial N^v}{\partial P^d}\frac{P^d}{N^v} = -(I1 + I2 + I3)\frac{P^d}{N^v} < 0.
\end{aligned}
\tag{B.1}
$$

Appendix C. Proof of Theorem 3

Applying the Leibniz Integral Rule to (7-9), we can show that the partial derivatives of each consumer surplus with respect to prices are:

$$\begin{aligned}
&\frac{\partial CS^b}{\partial P^b} = -N^b, \frac{\partial CS^p}{\partial P^b} = -N^p, \frac{\partial CS^v}{\partial P^b} = -N^v \\
&\frac{\partial CS^b}{\partial P^p} = I9 + I7, \frac{\partial CS^p}{\partial P^p} = -I9 - N^p, \frac{\partial CS^v}{\partial P^p} = -I7 - N^v \\
&\frac{\partial CS^b}{\partial P^v} = I7, \frac{\partial CS^p}{\partial P^v} = I8, \frac{\partial CS^v}{\partial P^v} = -I7 - I8 - N^v
\end{aligned} \tag{C.1}$$

where

$$\begin{aligned}
I7 &= N \int_{-\infty}^{P^p} \int_{P^b}^{\infty} (b - P^b) f_{B,P,V}(b, p, P^p + P^v - p)\; db\; dp \\
I8 &= N \int_{P^p}^{\infty} \int_{P^b+P^p-p}^{\infty} (b + p - P^b - P^p) f_{B,P,V}(b, p, P^v)\; db\; dp \\
I9 &= N \int_{-\infty}^{P^v} \int_{P^b}^{\infty} (b - P^b) f_{B,P,V}(b, P^p, v)\; db\; dv
\end{aligned} \tag{C.2}$$

Using (10) and (C.1), we can calculate the partial derivative of total consumer surplus respect to each price:

$$\begin{aligned}
\frac{\partial CS}{\partial P^b} &= \frac{\partial CS^b}{\partial P^b} + \frac{\partial CS^p}{\partial P^b} + \frac{\partial CS^v}{\partial P^b} = -N^b - N^p - N^v \\
\frac{\partial CS}{\partial P^p} &= \frac{\partial CS^b}{\partial P^p} + \frac{\partial CS^p}{\partial P^p} + \frac{\partial CS^v}{\partial P^p} = -N^p - N^v \\
\frac{\partial CS}{\partial P^v} &= \frac{\partial CS^b}{\partial P^v} + \frac{\partial CS^p}{\partial P^v} + \frac{\partial CS^v}{\partial P^v} = -N^v
\end{aligned} \tag{C.3}$$

Equation (27) follows.

Appendix D. Proof of Theorem 4

To solve the nonlinear optimization problem with constraints, we use the Karush–Kuhn–Tucker (KKT) theorem. The Lagrangian function of (30) is:

$$\begin{aligned}
\mathcal{L} = {} & CS^b + CS^p + CS^v + \lambda_1[P^v - P^d - (r_{\min}^{VSP} + 1)C^v] \\
& + \lambda_2[(P^b - (r_{\min}^{ISP} + 1)C^b)N^b + (P^b + P^p - (r_{\min}^{ISP} + 1)(C^b + C^p))N^p \\
& + (P^b + P^p + P^d - (r_{\min}^{ISP} + 1)(C^b + C^p + C^d))N^v]
\end{aligned} \tag{D.1}$$

There exist KKT multipliers (λ_1 and λ_2), such that the following conditions hold.

Stationarity:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial P^b} &= 0\\ \frac{\partial \mathcal{L}}{\partial P^p} &= 0\\ \frac{\partial \mathcal{L}}{\partial P^d} &= 0\\ \frac{\partial \mathcal{L}}{\partial P^v} &= 0\end{aligned} \tag{D.2}$$

Complementary slackness:

$$\begin{aligned}&\lambda_1[P^v - P^d - (r_{\min}^{VSP} + 1)C^v] = 0\\ &\lambda_2[(P^b - (r_{\min}^{ISP} + 1)C^b)N^b + (P^b + P^p - (r_{\min}^{ISP} + 1)(C^b + C^p))N^p\\ &+ (P^b + P^p + P^d - (r_{\min}^{ISP} + 1)(C^b + C^p + C^d))N^v] = 0\end{aligned} \tag{D.3}$$

Feasibility:

$$\begin{aligned}\lambda_1 &\geq 0\\ \lambda_2 &\geq 0\end{aligned} \tag{D.4}$$

Since there are 2 constraints in (D.3), there are 4 different cases depending on which of the constraints are binding. After investigation, we concluded that both constraints are binding. It follows that:

$$\begin{aligned}&P^v = (r_{\min}^{VSP} + 1)C^v + P^d\\ &(P^b - (r_{\min}^{ISP} + 1)C^b)N^b + (P^b + P^p - (r_{\min}^{ISP} + 1)(C^b + C^p))N^p\\ &+ (P^b + P^p + P^d - (r_{\min}^{ISP} + 1)(C^b + C^p + C^d))N^v = 0\end{aligned} \tag{D.5}$$

Using (D.5) and (27), we can calculate the partial derivative of the Lagrangian with respect to each price:

$$\begin{aligned}&\frac{\partial \mathcal{L}}{\partial P^b} = \lambda_2[(P^b - (r_{\min}^{ISP} + 1)C^b)\frac{\partial N^b}{\partial P^b} + (P^b + P^p - (r_{\min}^{ISP} + 1)(C^b + C^p))\frac{\partial N^p}{\partial P^b}\\ &+ (P^b + P^p + P^d - (r_{\min}^{ISP} + 1)(C^b + C^p + C^d))\frac{\partial N^v}{\partial P^b} + N^b + N^P + N^v] - N^b - N^P - N^v = 0\\ &\frac{\partial \mathcal{L}}{\partial P^p} = \lambda_2[(P^b - (r_{\min}^{ISP} + 1)C^b)\frac{\partial N^b}{\partial P^p} + (P^b + P^p - (r_{\min}^{ISP} + 1)(C^b + C^p))\frac{\partial N^p}{\partial P^p}\\ &+ (P^b + P^p + P^d - (r_{\min}^{ISP} + 1)(C^b + C^p + C^d))\frac{\partial N^v}{\partial P^p} + N^P + N^v] - N^P - N^v = 0\\ &\frac{\partial \mathcal{L}}{\partial P^d} = \lambda_2[(P^b - (r_{\min}^{ISP} + 1)C^b)\frac{\partial N^b}{\partial P^d} + (P^b + P^p - (r_{\min}^{ISP} + 1)(C^b + C^p))\frac{\partial N^p}{\partial P^d}\\ &+ (P^b + P^p + P^d - (r_{\min}^{ISP} + 1)(C^b + C^p + C^d))\frac{\partial N^v}{\partial P^d} + N^v] - N^v = 0\end{aligned} \tag{D.6}$$

The prices in (31) satisfy both (D.5) and (D.6). The theorem follows.

Appendix E.

By using the definitions of the Lerner indices in (16), we have:

$$-\begin{bmatrix} \varepsilon_{N^x,P^x} & \varepsilon_{N^x,P^y} \\ \varepsilon_{N^y,P^x} & \varepsilon_{N^y,P^y} \end{bmatrix} \begin{bmatrix} \frac{P^x - C^x}{P^x} \\ \frac{P^y - C^y}{P^y} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{E.1}$$

By using the definition of the price elasticity of demand, we have:

$$-\begin{bmatrix} \frac{\partial N^x}{\partial P^x} & \frac{\partial N^x}{\partial P^y} \\ \frac{\partial N^y}{\partial P^x} & \frac{\partial N^y}{\partial P^y} \end{bmatrix} \begin{bmatrix} P^x - C^x \\ P^y - C^y \end{bmatrix} = \begin{bmatrix} N^x \\ N^y \end{bmatrix} \tag{E.2}$$

By rearranging (E.2), we can show that:

$$-\begin{bmatrix} \frac{\partial N^x}{\partial P^x} + \frac{\partial N^x}{\partial P^y} & \frac{\partial N^x}{\partial P^y} \\ \frac{\partial N^y}{\partial P^x} + \frac{\partial N^y}{\partial P^y} & \frac{\partial N^y}{\partial P^y} \end{bmatrix} \begin{bmatrix} P^x - C^x \\ (P^y - C^y) - (P^x - C^x) \end{bmatrix} = \begin{bmatrix} N^x \\ N^y \end{bmatrix} \tag{E.3}$$

By using the fact that $N^b = N^x$, $N^p = N^y$, $C^x = C^b$, and $C^y = C^b + C^p$, we can rewrite (E.3) as:

$$-\begin{bmatrix} \frac{\partial N^b}{\partial P^x} + \frac{\partial N^b}{\partial P^y} & \frac{\partial N^b}{\partial P^y} \\ \frac{\partial N^p}{\partial P^x} + \frac{\partial N^p}{\partial P^y} & \frac{\partial N^p}{\partial P^y} \end{bmatrix} \begin{bmatrix} P^b - C^b \\ P^p - C^p \end{bmatrix} = \begin{bmatrix} N^b \\ N^p \end{bmatrix} \tag{E.4}$$

By using (17), we can define demand as a function of P^x and P^y:

$$N^b(P^x, P^y) = N \int_{-\infty}^{P^y - P^x} \int_{P^x}^{\infty} f_{B,P}(b,p)\; db\; dp. \tag{E.5}$$

$$N^p(P^x, P^y) = N \int_{P^y - P^x}^{\infty} \int_{P^y - p}^{\infty} f_{B,P}(b,p)\; db\; dp. \tag{E.6}$$

Applying the Leibniz Integral Rule to (E.5) and (E.6), we can show that the partial derivatives of demands with respect to prices are:

$$\begin{aligned} \frac{\partial N^b}{\partial P^x} &= \frac{\partial N^b}{\partial P^b} - \frac{\partial N^b}{\partial P^p} \\ \frac{\partial N^b}{\partial P^y} &= \frac{\partial N^b}{\partial P^p} \\ \frac{\partial N^p}{\partial P^x} &= \frac{\partial N^b}{\partial P^p} \\ \frac{\partial N^p}{\partial P^y} &= \frac{\partial N^p}{\partial P^p} \\ \frac{\partial N^p}{\partial P^b} &= \frac{\partial N^p}{\partial P^p} + \frac{\partial N^b}{\partial P^p} \end{aligned} \tag{E.7}$$

We now substitute (E.7) into (E.4) to obtain:

$$-\begin{bmatrix} \frac{\partial N^b}{\partial P^b} & \frac{\partial N^b}{\partial P^p} \\ \frac{\partial N^p}{\partial P^b} & \frac{\partial N^p}{\partial P^p} \end{bmatrix} \begin{bmatrix} P^b - C^b \\ P^p - C^p \end{bmatrix} = \begin{bmatrix} N^b \\ N^p \end{bmatrix} \tag{E.8}$$

By rearranging (E.8):

$$-\begin{bmatrix} \frac{\partial N^b}{\partial P^b}\frac{P^b}{N^b} & \frac{\partial N^b}{\partial P^p}\frac{P^p}{N^b} \\ \frac{\partial N^p}{\partial P^b}\frac{P^b}{N^p} & \frac{\partial N^p}{\partial P^p}\frac{P^p}{N^p} \end{bmatrix} \begin{bmatrix} \frac{P^b-C^b}{P^b} \\ \frac{P^p-C^p}{P^p} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{E.9}$$

By using the definitions of the price elasticity of demand and the Lerner indices, equation (18) follows.

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