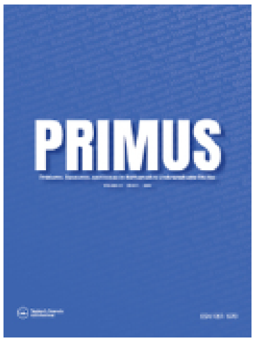

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Promote Students' Function Reasoning with Techtivities

Gary A. Olson and Heather Lynn Johnson

ABSTRACT
Students enrolled in introductory math courses, such as college algebra, deserve to do more than find answers and fix mistakes. We present one interactive digital activity, the Cannon Man "Techtivity," which we developed to provide opportunities for students to develop an understanding of function, beyond just applying a rule, such as the vertical line test. With an accompanying facilitation guide, we describe how we promote instructor moves to privilege student reasoning. We provide student data to support the effectiveness of this approach. We conclude with implications for weaving such opportunities into the fabric of college algebra.

KEYWORDS
Function; reasoning; college algebra; task design

1. INTRODUCTION

Student enrollment in college algebra has continued to increase [1], yet efforts to actuate reform in college algebra remain limited [17]. Researchers have called for change in college algebra, ranging from a complete overhaul of the curriculum [6], to adjustments in implementation, such as incorporating elements of active learning [2, 14]. With our effort, we take an incremental approach to change via layering new elements onto existing ones [11]. We work from within to address a pervasive problem in introductory courses such as college algebra: a focus on answer finding, rather than reasoning. If students experience participation in mathematics courses as an exercise in finding answers, they may view mathematics to be little more than a series of computations, and lack personal ownership over questions analyzed or ideas generated. To address this problem, we share a college algebra curricular intervention to promote students' function reasoning via dynamic computer activities (Techtivities).

We designed a set of Techtivities to provide opportunities for students to engage in mathematical reasoning with a focus on understanding key ideas of function. We describe the components of one such Techtivity, the Cannon Man, which we implemented after a lesson introducing students to the definition of function. To provide evidence of the viability of our approach, we report student data from two consecutive semesters of a single course section, taught by the same instructor. To expand the intervention, we developed facilitation guides to support instructors'

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implementation of the Techtivities to amplify students' voice and promote students' function reasoning. We conclude with implications for this work from within, to weave such opportunities into the fabric of the college algebra curriculum.

2. FUNCTIONS AND COLLEGE ALGEBRA

Function is a key idea to be threaded throughout a college algebra course, and it is important for college algebra students to have opportunities to understand and make meaning of functions [3]. A function denotes a specialized relationship between variables, such that each input variable is mapped to exactly one output variable. In textbook tasks, students may determine whether different relations are functions, find values for functions, and sketch graphs of functions. For each of these tasks, students can apply techniques to find answers. For example, given $f(x) = 3x^2 - 2x + 1$, graph $f(x)$ and find $f(-4)$. However, overemphasis on such exercises can constrain students' opportunities for understanding, because the answers, rather than the reasoning, become the focus [6].

The vertical line test is a typical technique students can use to determine if a relation is a function. The test relies on students having access to a graphical representation of a possible function $y = f(x)$, in a standard Cartesian coordinate system, such that the x and y variables are given on the horizontal and vertical axes, respectively. Such a graph represents a function if any vertical line drawn in the plane passes through at most one point. Applying the vertical line test, students can generate correct answers to skill-based questions about whether certain graphs represent functions. Consider a standard college algebra textbook question, in which students are given a graph, and asked: "Is this a function?" [15]. Implicit in this question is that the "function" specifies a relationship between variables such that y is a function of x. However, functions can specify other relationships between variables. For example, x could be a function of y. Therefore, if students only hear "is this a function?" they may miss opportunities to investigate relationships between variables that form the essence of function and also foster their sense-making of data in the world around them.

3. WHAT IS A TECHTIVITY? THE CANNON MAN

Cannon Man is one of a set of seven digital Techtivities designed to provide students with opportunities to reason about how graphs work, and in turn, promote students' function reasoning. Two key understandings serve as goals for the set of activities: first, that graphs represent relationships between variables, and second, that graphs can look different and still represent the same relationship between variables. We have developed the Techtivities in collaboration with Dan Meyer and the Desmos team. Each activity is freely available via the Desmos platform [4] and accessible from a computer, tablet, or mobile phone.

Each Techtivity contains four main components [8]. Figure 1 shows each component for the Cannon Man Techtivity. First, students view an animation for a

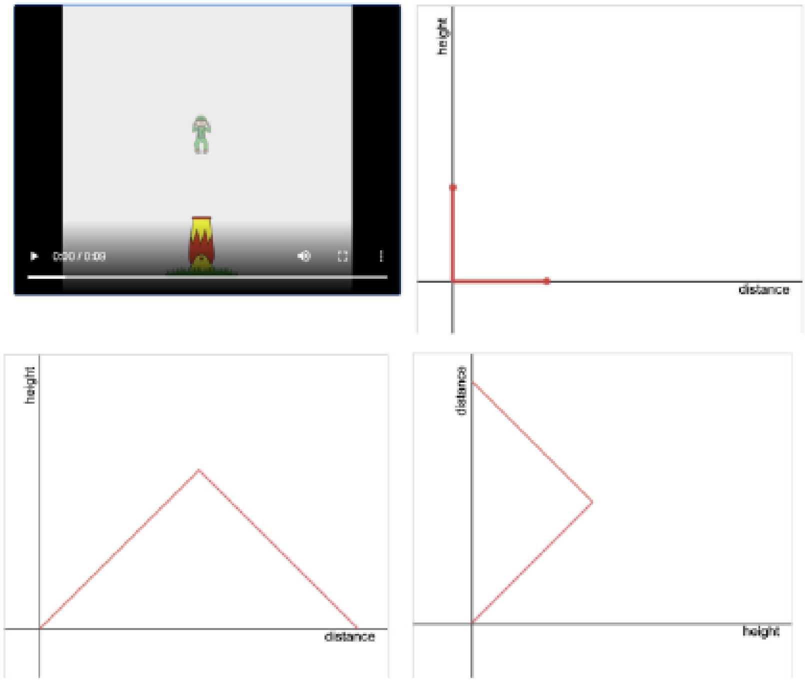

Figure 1. Components of Cannon Man.

situation involving changing attributes. In Cannon Man, the animation shows the flight of a circus performer who is shot out of a cannon straight up into the air and comes back down with the assistance of a parachute (Figure 1, upper left). Second, students use sliders to manipulate dynamic segments on axes of a Cartesian graph, to represent measures of attributes in the situation. Depending on the device, students may use a finger or mouse to manipulate the segments. In Cannon Man, the dynamic segments on the vertical and horizontal axis represent the circus performer's height from the ground and total distance traveled, respectively (Figure 1, upper right). Third, after representing each of the individual attributes, the user is to sketch a single Cartesian graph to represent the relationship between the two variables. In Cannon Man, students sketch a graph of height vs. total distance traveled (Figure 1, lower left). Fourth, students repeat the second and third components for a new Cartesian graph, in which the attributes are represented on different axes. In Cannon Man's second graph, height is on the horizontal axis and total distance traveled is on the vertical axis (Figure 1, lower right).

Each Techtivity includes reflection questions, in conjunction with the third and fourth components. After students have sketched a graph for each set of axes, they are shown a computer-generated graph. Students are to reflect on similarities and differences between their graph and the computer-generated one, noting any questions they have about the computer-generated one. A final reflection question is posed at the end of each Techtivity to elicit student reasoning about how their engagement with the Techtivity relates to course content. Students are to respond to

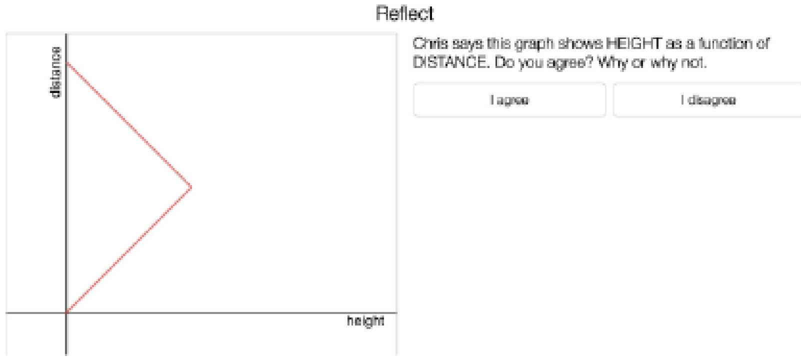

Figure 2. Cannon Man reflection question.

another student's claim and explain their reasoning. Figure 2 shows the final reflection question for Cannon Man. With this question, we intend for students to go beyond applying a rule, to consider relationships between variables represented in the graph.

Our inspiration for the Techtivities draws from multiple sources. With the sliders, we enact Thompson's [16] recommendation that students use their fingers as tools to explore change in variables. By incorporating two different graphs to represent the same relationship between variables, we make a move compatible with Moore and colleagues [12], who have broken conventions with graphs to promote students' understanding of mathematical ideas. For example, Cannon Man's second graph is unconventional because it fails the vertical line test but still represents height as a function of distance. By having students respond to another student's claim, we intend to make the question feel more like a conversation rather than a judgment. Yet, we know that students can think that gender identity plays a role in mathematical ability [10]. Hence, we have incorporated informal, less gendered names, such as Chris, positioning the "student" in the task as making a claim worth considering [9].

4. IS HEIGHT A FUNCTION OF DISTANCE? A CHANGE IN STUDENT RESPONSES

We implemented the Cannon Man Techtivity in one section of college algebra during consecutive semesters, taught by the same, experienced instructor. In both iterations, the instructor devoted time for students to construct community norms and built a positive classroom culture and welcoming environment to foster students' engagement with mathematics. In the first iteration, students interacted with Cannon Man as an "add on," to promote active learning, with all other aspects of class remaining business as usual. In the second iteration, the instructor made adaptations to the facilitation of the Techtivity and to the lesson to introduce function. Across iterations, we address differences in student responses to the reflection question posed in Figure 2, and we discuss how we account for those differences.

4.1. The First Iteration: 3 Agree, 23 Disagree

For the first iteration, all aspects of class remained consistent with previous semesters, with the addition of the Cannon Man Techtivity. The instructor first gave a 30 min lesson which included the definition of function as a relationship between two variables such that each input variable mapped to exactly one output variable. He then introduced the vertical line test as a tool for determining when a graph of two variables represents a function. Working in table groups, students analyzed different relationships in the form of sets of ordered pairs, equations, and graphs and then discussed whether they represented a function $y = f(x)$.

After the lesson, students worked on the Cannon Man Techtivity. When responding to the reflection question, only three students agreed with Chris, that the graph in Figure 2 represents height as a function of distance; 23 students disagreed. Table 1 shows sample responses students provided to justify their choice of agree/disagree. The sample responses represent the scope of responses from all students.

The responses of students who agreed with Chris differed from those who disagreed. All three students who agreed with Chris mentioned at least one of the attributes of the problem, height and distance. Student 1-A1's response described what total distance means in the Cannon Man situation (the accumulation of height, both up and down), and Student 1-A2's response described how the distance is changing in the situation (always increasing). Student 1-A3's response addressed the unidirectional nature of the function relationship (height is a function of distance). Only 6 of the 23 students who disagreed with Chris mentioned height or distance. Student 1-D1's mentioned height and distance, yet maintained the assumption that y must be a function of x. Another fourteen of the 23 students who disagreed mentioned the vertical line test. As suggested in the responses from student 1-D2, when students mentioned the vertical line test, they spoke in general terms (e.g., "it"), or they talked about "the graph." Two other students who disagreed mentioned inputs and outputs as did student 1-D3, with the assumption that inputs would be on the horizontal axis and outputs on the vertical. One student simply said "It is not a function."

We conjectured that the positioning of the Techtivity as an "add-on" to classroom instruction accounted for the results, at least in part. Even though students had an opportunity to interact with an innovative activity, it was not enough to promote the kind of reasoning we intended, because it felt separate from students' other

Table 1. Iteration 1 – sample student responses and justifications.

Agree with Chris (3 total)	Disagree with Chris (23 total)
1-A1 Yes because we are measuring the height up and down to get the overall distance	1-D1 This graph shows distance (y) as a function of height (x)
1-A2 Because if you are going up or down the distance is always increasing.	1-D2 The graph does not pass the vertical line test, which means it is not a function
1-A3 The height is a function of the distance but the distance it is not a function of height	1-D3 Each input has multiple outputs

work in the course. With textbook tasks, a vertical line test was sufficient to determine whether a graph represented a function. Hence, students lacked opportunities to examine the utility of the vertical line test for a broader range of functions. Cannon Man was different from the textbook tasks because students were to examine function relationships beyond just y as a function of x. Hence, aspects of classroom instruction needed to shift to provide better tools for students to broaden their notions of function, so that they could depend less on rules such as the vertical line test, and in turn, engage in reasoning about attributes involved in function relationships.

4.2. The Second Iteration: 18 Agree, 3 Disagree

During the second iteration, the instructor made changes to the lesson preceding the Cannon Man Techtivity. First, he used more specific and precise language when introducing the concept of function by always referencing the names of both variables involved and the direction of the function relationship. For example, instead of showing students a graph and asking, "does this graph represent a function?" the instructor would ask "does this graph represent y as a function of x?" In another example, when analyzing equations, graphs or sets of ordered pairs the instructor posed questions about both directions of the relationship, asking "is A a function of B?" and "is B a function of A?". Second, he expanded the range of functions students encountered, including examples from everyday situations, rather than just symbolic rules with x and y. For example, students considered relationships representing members of the class and number of pets owned (e.g., Is the number of pets owned a function of the student members of this class? and likewise, Are the student members of this class a function of the number of pets owned?). Only after this had taken place, did he present the vertical line test. In doing so, he emphasized the limitations inherent in the rule, primarily that it only addresses the question of whether a graph represents y as a function of x, where y represents the variable on the vertical axis and x the variable on the horizontal axis.

We observed a dramatic shift in student responses to the reflection question from the first iteration to the second. In the second iteration, 18 students agreed with Chris that height is a function of distance; only three disagreed. Table 2 shows sample responses students used to justify their reasoning. Sample responses are representative of the scope of student responses.

Not only was there a shift in students' agreement with Chris, there was a shift in the nature of their responses. None of the three students who disagreed with Chris mentioned the vertical line test. Instead they talked about attributes or variables in the situation. Student 2-D1's response indicated a different relationship, distance as a function of height. Student 2-D2 indicated that the graph in Figure 2 was not typical, because of the independent variable's placement on the coordinate axes. Student 2-D3 talked about a relationship between inputs and outputs, identifying the inputs as values on the horizontal axis. Again, we found that students who agreed mentioned specific attributes in their responses, with 17 of the 18 students describing

Table 2. Iteration 2 – sample student responses and justifications.

Agree with Chris (18 total)	Disagree with Chris (3 total)
2-A1 Because for each distance there is one unique height	2-D1 We think that distance is a function of height instead
2-A2 True-for any amount of distance that Cannon Man has traveled, there is exactly one corresponding height he has reached	2-D2 Typically the independent variable is on the horizontal axis, i.e., $y = f(x)$
2-A3 Yes, because for every input of distance, there is one output of height	2-D3 There is [*sic*] two different outputs for one single input, making the graph not a function

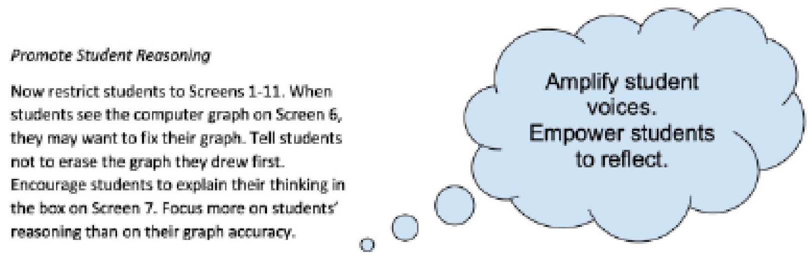

Figure 3. Excerpt from Cannon Man facilitation guide.

a connection between height and distance. Even though distance was represented on the vertical axis, students who agreed overwhelmingly recognized a relationship between the distance and height. That is, each value of distance corresponded to a unique value of height, and hence they agreed with Chris's claim.

5. PROMOTING REASONING WITH FACILITATION GUIDES

We were inspired by the success of the second iteration and wanted to do more than hope such results could continue. Furthermore, we aimed to expand implementation of the Techtivities beyond a single instructor's course section. Hence, we developed facilitation guides to support college algebra instructors and maximize learning opportunities for students [13].

We incorporate three main types of supports in the guides. First, we include tips for the use of available features in Desmos such as pausing, graph overlays, anonymization, and real-time student feedback. Second, we provide implementation suggestions and specific instructor questions to focus the activity toward centering student reasoning rather than answer finding and mistake fixing. Third, we include thought bubbles to emphasize the need for instructors to make room to amplify student voices, so that they may empower students to reflect. Figure 3 shows a portion of the Cannon Man implementation guide [13].

We encourage instructors to use the Desmos pacing feature to prevent students from racing through the activity. To make pacing meaningful, we suggest pausing at key junctures, such as just before students receive computer feedback on an individual graph. To quickly display all student graphs, we recommend instructors use

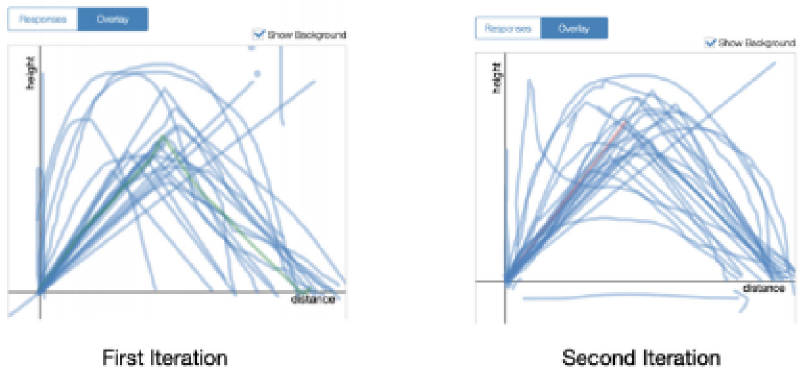

Figure 4. Graph overlays for height vs. distance student graphs.

a graph overlay. Figure 4 shows a graph overlay of students' first graphs for the first (left) and second (right) iterations.

While displaying a graph overlay, instructors can ask questions to promote students' reasoning. Two such questions include: "Discuss with your table why most people chose to start their graph at the origin. What does that represent in the animation?" or "I see a general trend in the overall shapes of these graphs, why did you choose to start your graph at the bottom and then go up in the middle and come back down?" With these questions, instructors can encourage students to connect the graph to the attributes in the situation. Only after students have had a chance to discuss do we recommend instructors remove pacing restrictions so that students can proceed through the activity.

When students see a computer-generated graph they often want to go back and fix their original graph. We recommend instructors place less emphasis on graph accuracy, so that students may reflect on relationships that the graphs represent (see Figure 3). Rather than penalizing students for incorrect graphs, we suggest that instructors tell students to leave their graphs as they are, and reflect on the differences they see in the computer-generated graph.

The fourth component of the Cannon Man Techtivity repeats earlier components with the same variables represented on different axes. With this move we intend for students to have opportunities to learn that the same relationship between variables can be graphed in different ways [7]. Some students may not notice a change in the labeling of the axes, and expect the second graph to be the same as the first. We recommend instructors again use the pacing and graph overlay features, so students may discuss their thinking related to the graphs. Figure 5 shows a graph overlay of students' second graphs for the first (left) and second (right) iterations.

We encourage instructors to make use of the anonymization and real-time feedback features. We have found that anonymization allows students to feel less concern about having their responses displayed to the class and can generate opportunities for instructors to amplify student voices that may be left out of classroom discourse. As students agree or disagree with Chris (see Figure 2), they see each other's responses in real time. To culminate the activity, we suggest instructors

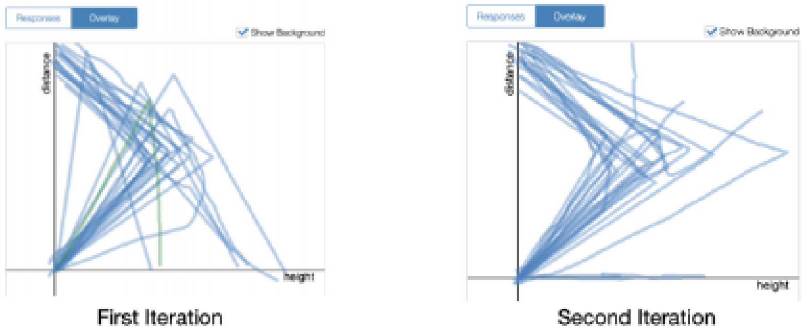

Figure 5. Graph overlay for distance vs. height student graphs.

contrast Chris's statement with a more typical question, "Is this a function?" that generally brings with it an implicit assumption that y is a function of x. With this suggestion, we intend to promote students' consideration of variables represented in the graph.

6. DISCUSSION/IMPLICATIONS

Why are activities such as Cannon Man important for college algebra? First, Cannon Man engenders students' focus on reasoning and sense making rather than rote answer finding. Dougherty et al. [5] advocated for high school mathematics teachers to engage in instruction that avoids privileging "rules that expire." This applies to early undergraduate math courses as well. The vertical line test is a rule that expires. While it is useful for making quick determinations as to whether a Cartesian graph represents y as a function of x, it is not at all useful for other univariate functions (e.g., $f(y)$) or multivariate functions. Second, Cannon Man promotes students' focus on attributes involved in function relationships (e.g., Is height a function of distance?). A focus on attributes is important not only for analyzing functions in college algebra, it is useful for critical thinking as an educated citizen. When students focus on attributes, they are better equipped to analyze graphs in the media, because they know to extend beyond appearances and examine the variables in the situation.

Why are activities such as Cannon Man necessary, but not sufficient? As evidenced by our results, the way instructors position activities such as Cannon Man impacts students' experience. When Cannon Man was merely an "add on" to the status quo, students relied on "business as usual" rules, such as the vertical line test, to interpret the activity. In contrast, when the instructor adapted instruction on function to include a focus on attributes, the activity became more meaningful for students. Furthermore, when implementing activities such as Cannon Man, it is important for instructors to attend to issues of power and identity in the mathematics classroom. For instance, if instructors communicate (tacitly or explicitly) that answers are the only things that matter, students may reflect less on relationships and more on fixing their graphs. Both accuracy and exploration play important roles in mathematics. With activities such as Cannon Man, instructors can make room for investigation and promote students' conceptual understanding.

Students deserve introductory mathematics courses such as College algebra to be meaningful learning experiences in and of themselves, not merely preparation for another course they may or may not take. College algebra is not high school algebra, just twice as fast. It is a place where students can develop a deep understanding of function, which can impact not only their mathematical knowing, but also beyond.

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BIOGRAPHICAL SKETCHES

Gary Olson serves as Director of General Education Mathematics at the University of Colorado Denver. He has 15 years of experience teaching undergraduate mathematics and statistics courses and oversees the professional development of graduate teaching assistants. He is a passionate advocate for active learning and the use of TACTivities to promote reasoning and connections throughout the undergraduate curriculum.

Heather Lynn Johnson is an associate professor of mathematics education at the University of Colorado Denver. She designs interactive, online tasks to provide students opportunities to expand their mathematical reasoning, and she teaches teachers to grow their students' mathematical reasoning. Johnson shares her research and reflections on Twitter @HthrLynnJ and on the website HthrLynnJ.com.