A Discrete-Continuous Hybrid Approach t Periodic Routing of Waste Collection Vehicles With Recycling Operations

Chao Lei, Zhoutong Jiang, and Yanfeng Ouyang

***Abstract*— Waste management agencies need to plan their waste collection activities in an efficient way such that they not only provide high-quality and timely service to customers but also maximize their net profit from recyclable waste, i.e., the total recycling revenue minus total operating costs. This paper proposes a mixed-integer linear program model for the waste collection problem in urban areas while considering the recycling operation at the sorting facility. A hybrid solution approach with both discrete and continuous optimization components is developed to solve the problem. The continuous component builds upon asymptotic routing cost formulas from the continuous approximation literature, while it is integrated into the discrete optimization component via an iterative stochastic approximation procedure. A series of numerical experiments are conducted, and results show that the proposed model and hybrid solution approach significantly outperform state of the art benchmarks from the literature.**

***Index Terms*— Periodic vehicle routing, waste collection, continuum approximation, recycling.**

I. Introduction

DUE to increasing concerns over the environmental and ecological impacts of urban wastes, the waste collection problem has received considerable attention from the industry and the academic community in the last few decades, e.g., see [1]–[5]. Moreover, as the economic benefits of recyclables (such as glass, metal, plastics, or paper) become more prominent under the rapid development of recycling technologies, waste recycling has nowadays become an important revenue source for many waste management agencies. That is, instead of being directly dumped into landfills, the collected waste will first be delivered to a so-called sorting facility so that the recyclable materials in the waste are sorted out and packed for sale. The operations at such sorting facilities often complicate the waste collection decisions, and hence they should be considered jointly. However, only very few studies have jointly considered the waste collection and recycling operations [6].

In practice, a waste management agency usually operates a fleet of collection vehicles (often of different types for different

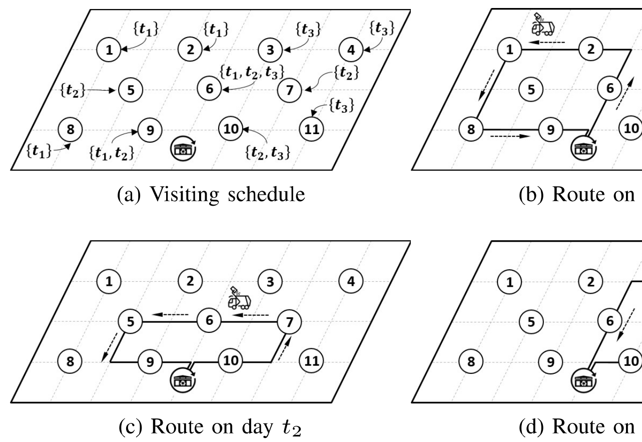

Fig. 1. Illustration of waste collection problem.

types of waste) and sends them out periodically or every other day) to visit various waste coll that are distributed over a spatial region, as s small example in Figure 1. Here we assume tha collection point is a dumpster (illustrated by num in Figure 1) which can be lifted and unloaded types of collection vehicles. Typically, most du not be visited every day, and therefore a plann several days has to be considered. As such, planning decisions consist of two main compone schedule that reveals which dumpsters are visited (Figure 1a), and (ii) a routing plan for collec day of the planning cycle (Figure 1b-1d). Co potential benefits of recycling, the objective of not merely to minimize the total operating costs i collection vehicles visiting dumpsters, but also the total revenue from selling recyclable materia in the aim of maintaining stable and smooth the sorting facility (such that the workers woul on certain days while being extremely busy on days), the agency often attempts to balance the the facility (i.e., indicated by the arrival of inbo over time.

The waste collection problem has been ma

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problem and has wide applications besides waste collection, e.g., raw materials pickup [7], product/good delivery [8], and on-site service planning [9], [10]. Surveys of the related variants, solution approaches and applications of PVRP can be found in [11] and [12], and more information of many other types of routing problems can be found in the book by Toth and Vigo [13]. Since the timing of customer visits is also part of the agency's decision, the closest literature to our problem might be that on PVRP with service choice (PVRP-SC) [14], which is an extension of the PVRP that also chooses each customer's visit frequency/schedule. This reference developed a solution approach based on Lagrangian relaxation and branch-and-bound which can solve instances of up to 50 customers within several hours of computation time. However, the interdependence between the scheduling and routing decisions, each posing as a hard combinatorial problem, imposes significant computational burdens for large-scale instances. The interested readers can refer to the monograph by Nemhauser and Wolsey [15] for more detailed explanations of the integer and combinatorial programming techniques. As alternatives, heuristic [16]–[18] and meta-heuristic [19]–[21] algorithms are often used to solve PVRP, and most of them decompose the problem into two sequential optimization phases: the first phase seeks an acceptable schedule that can be assigned to each customer and, the second phase solves a classic VRP to fulfill the service promise in each time period. However, these algorithms, due partly to the sequential nature, usually fail to consider the related travel costs while deciding the visiting schedule in the first phase. Such a disconnection often makes it difficult to verify the solution's effectiveness and efficiency. Moreover, since heuristic/meta-heuristic algorithms only produce numerical solutions, it is often challenging to obtain managerial insights.

In light of the challenges associated with discrete models, continuous approximation (CA) has often been used as an alternative approach to providing asymptotic estimates of routing costs in large-scale logistics systems [22]–[25]. One important advantage of the CA approach is that, by using continuous densities to replace discrete decision variables and formulating localized approximations, the model decouples spatial and temporal interdependence and yields an analytically tractable solution that is not only easy to optimize but also capable of offering valuable managerial insights. This modeling approach has been established as a useful tool for many related logistics problems; see [26] for a recent review. Francis and Smilowitz [27] proposed a CA formulation for PVRP-SC by modeling all the parameters and decisions as continuous functions of time and/or space. The continuous model facilitates strategic analysis of service choice options, but it cannot be used directly as implementable solutions in real-world practice — certain translation procedures would

the consideration of recycling operations, which
imizing the total profits of the system and b
workload of recycling operations at the sorting
time. To solve the model effectively, we develo
hybrid solution approach by (i) solving a discre
subproblem that integrates CA-based routing co
formulas and a group of independent vehicle rou
lems, and (ii) applying a stochastic approximatio
based on the *Adam* algorithm to iteratively corr
introduced by the asymptotic estimation. The pe
the proposed model and algorithm is tested thr
of numerical experiments. It is shown that the hy
approach can drastically outperform the benchma
current literature, and it holds the potential to pro
managerial insights to waste management agenc

This remainder of the paper is organized
Section II presents the notation and mathemat
tion of the problem. The hybrid approach is
Section III. Section IV presents numerical exp
test the performance of the proposed model
approach. Section V concludes the paper and dis
ble directions of future research.

II. Mathematical Modeling

In this section, we present a mixed-integer
model for the waste collection and recycling
$\mathcal{V}$ denote the set of dumpsters distributed over
where each dumpster $i \in \mathcal{V}$ is associated w
capacity limit Q_i^{D}. The waste management age
task is to visit and clear all the dumpsters perio
multi-period cycle $\mathcal{T} = \{1, 2, \ldots, |\mathcal{T}|\}$; e.g., a c
week and each period can be a weekday such th
attempts to get a weekly schedule and a daily
More specifically, the agency needs to decide
to be assigned to each dumpster over the plan
i.e., the combination of days in which the dumps
and the visiting routes on each day $t \in \mathcal{T}$ so as
visiting schedule at each dumpster. We use $\mathcal{S}$
set of all possible schedules, indexed by s. The
between a schedule $s \in \mathcal{S}$ and a day $t \in \mathcal{T}$ is
a binary parameter $\alpha_{s,t}$, where $\alpha_{s,t} = 1$ if day
schedule s, or $\alpha_{s,t} = 0$ otherwise. A generic sc
be represented by a combination of binary par
across all $t \in \mathcal{T}$; i.e., $s = (\alpha_{s,1}, \alpha_{s,2}, \ldots, \alpha_{s,|\mathcal{T}|})$

Let $D_{i,t}$ denote the amount of waste generate
$i \in \mathcal{V}$ on day $t \in \mathcal{T}$. We assume that all the waste
at a dumpster up to the early morning of a d
cleared if the dumpster is visited by a vehicle in
and we also assume that partial collection is
as it often unnecessarily increases vehicle tra

demand pattern of $(1, 2, 3, 1, 2)$ from Monday to Friday. If the dumpster is visited according to either of the following two schedules, i.e., $s_1 = (1, 0, 1, 0, 1)$ and $s_2 = (1, 1, 0, 1, 0)$, the amounts of waste to be observed by the vehicle upon arrival on Monday to Friday mornings (dependent on the leftover from previous days) would be: $(2, 0, 3, 0, 4)$ and $(3, 1, 0, 5, 0)$, respectively. It should be noted that, since each dumpster can only hold a limited amount of waste, i.e., Q_i^{D}, certain schedules may not be feasible for certain dumpsters and should be excluded from $\mathcal{S}$. For example, if the capacity of the dumpster is 5 units, a third schedule option $s_3 = (0, 1, 0, 0, 1)$ leading to 6 units of waste at the dumpster on Friday morning shall be infeasible. Hence, we define the feasible schedule set S_i for each dumpster $i \in \mathcal{V}$ as

$$\mathcal{S}_i = \left\{ s \in \mathcal{S} \,\middle|\, W_{i,s,t} \leq Q_i^{\mathrm{D}}, \forall t \in \mathcal{T} \right\}, \quad \forall i \in \mathcal{V}.$$

The agency's scheduling decision, denoted by $y_{i,s}$, can be defined accordingly as

$$y_{i,s} = \begin{cases} 1 & \text{if dumpster } i \in \mathcal{V} \text{ is visited according to} \\ & \text{schedule } s \in \mathcal{S}_i; \\ 0 & \text{otherwise.} \end{cases}$$

Meanwhile, to fulfill the schedule promise, the agency needs to deploy a set of waste collection vehicles, denoted by $\mathcal{K}$, to visit a series of dumpsters according to a specific routing plan on each day. In this study, we assume that a vehicle can initiate multiple trips from a depot, which is indexed by node 0, within a single day. That means a vehicle $k \in \mathcal{K}$ can return to the depot before its on-board load reaches the fixed capacity of the vehicle Q^{V}, dump the waste, and then starts a new trip. Hence, the routing subproblem on each day resembles the so-called multi-trip vehicle routing problem (MTVRP). The readers can refer to [31] for more details of MTVRP. Let $\mathcal{N}_{k,t}$ denote the set of trips being conducted by vehicle $k \in \mathcal{K}$ on day $t \in \mathcal{T}$, we then define the routing decision variables $x_{t,k,n,i,j}$ on day $t \in \mathcal{T}$ as follows:

$$x_{t,k,n,i,j} = \begin{cases} 1 & \text{if vehicle } k \in \mathcal{K} \text{ travels from node} \\ & i \in \mathcal{V} \cup \{0\} \text{ to node } j \in \mathcal{V} \cup \{0\},\ j \neq i \text{ in} \\ & \text{trip } n \in \mathcal{N}_{k,t} \text{ on day } t \in \mathcal{T}; \\ 0 & \text{otherwise.} \end{cases}$$

In addition to the binary routing decision variables, we define continuous variables $u_{t,k,n,i}$ and $u'_{t,k,n,i}$ as the arrival and departure times of vehicle $k \in \mathcal{K}$ at node $i \in \mathcal{V} \cup \{0\}$ in trip $n \in \mathcal{N}_{k,t}$ of day $t \in \mathcal{T}$, respectively. These variables help keep track of the temporal order of the vehicles along their service trips. In practice, it usually takes a certain length of time S_i to

variables $v_{i,s,t,k,n}$ to link the scheduling and rout together

$$v_{i,s,t,k,n} = \begin{cases} 1 & \text{if dumpster } i \in \mathcal{V} \text{ is served accor} \\ & \text{schedule } s \in \mathcal{S}_i \text{ and meanwhile it} \\ & \text{vehicle } k \in \mathcal{K} \text{ in trip } n \in \mathcal{N}_{k,t} \text{ on} \\ 0 & \text{otherwise.} \end{cases}$$

As mentioned earlier, after a vehicle visits all t dumpsters on a trip, it needs to ship the load sorting facility before it starts the next trip. Her there is only one sorting facility and it is at the s as the depot.[1] Let $w_t \geq 0$ denote the amount o processed at the sorting facility on day $t \in \mathcal{T}$. E is ideal that all the collected waste can be proce usually an upper bound Q^{SF} being imposed on th w_t because of the limited working capacity a facility. If the total waste collected on a certain the sorting capacity, the surplus part would be di to the landfill, and the benefit of recycling is lo

The agency may bear multiple objectives while waste collection service. For example, it may wan the travel costs of the fleet as vehicles require fu to operate. We assume that the travel cost betwe $i, j \in \mathcal{V} \cup \{0\}, i \neq j$ is proportional to the ass time $T_{i,j}$, with a cost factor of C^{T} per time unit. may also want to maximize the total revenue recyclable materials. It is assumed that such re $t \in \mathcal{T}$ is linearly proportional to the amount is processed at sorting facility on that day, i.e benefit factor C^{R} per unit of waste. To achieve balancing the workload at the sorting facility, a C^{B} is imposed on each unit of load differences facility across days, i.e., $|w_t - w_{t'}|, \forall t, t' \in \mathcal{T}, t$

To this end, the waste collection problem can b as the following MILP:

$$\max \sum_{t \in \mathcal{T}} C^{\mathrm{R}} \cdot w_t - \sum_{t \in \mathcal{T}} \sum_{\substack{t' \in \mathcal{T} \\ t' > t}} C^{\mathrm{B}} \cdot |w_t - w_{t'}|$$

$$- \sum_{t \in \mathcal{T}} \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}_{k,t}} \sum_{\substack{i,j \in \mathcal{V} \cup \{0\} \\ j \neq i}} C^{\mathrm{T}} \cdot T_{i,j} \cdot x_{t,k,n,\ldots}$$

$$\text{s.t.} \sum_{s \in \mathcal{S}_i} y_{i,s} = 1, \quad \forall i \in \mathcal{V}$$

$$\sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}_{k,t}} v_{i,s,t,k,n} = \alpha_{s,t} \cdot y_{i,s},$$

$$\forall t \in \mathcal{T}, i \in \mathcal{V}, s \in \mathcal{S}_i$$

$$w_t \leq Q^{\mathrm{SF}}, \quad \forall t \in \mathcal{T}$$

$$x_{t,k,n,0,i} + \sum_{j \in \mathcal{V}} x_{t,k,n,j,i} = \sum_{s \in \mathcal{S}_i} v_{i,s,t,k,n}, \quad \forall t \in \mathcal{T}, k \in \mathcal{K}, n \in \mathcal{N}_{k,t}, i \in \mathcal{V} \tag{6}$$

$$x_{t,k,n,i,0} + \sum_{j \in \mathcal{V}} x_{t,k,n,i,j} = \sum_{j \in \mathcal{V}} x_{t,k,n,j,i} + x_{t,k,n,0,i}, \quad \forall t \in \mathcal{T}, k \in \mathcal{K}, n \in \mathcal{N}_{k,t}, i \in \mathcal{V} \tag{7}$$

$$\sum_{j \in \mathcal{V}} x_{t,k,n,0,j} = \sum_{j \in \mathcal{V}} x_{t,k,n,j,0} \le 1, \quad \forall t \in \mathcal{T}, k \in \mathcal{K}, n \in \mathcal{N}_{k,t} \tag{8}$$

$$\sum_{i \in \mathcal{V}} \sum_{s \in \mathcal{S}_i} W_{i,t,s} \cdot v_{i,s,t,k,n} \le Q^{\mathrm{V}}, \quad \forall t \in \mathcal{T}, k \in \mathcal{K}, n \in \mathcal{N}_{k,t} \tag{9}$$

$$u_{t,k,n,i} + S_i + T_{i,j} \le u_{t,k,n,j} + M\left(1 - x_{t,k,n,i,j}\right), \quad \forall t \in \mathcal{T}, k \in \mathcal{K}, n \in \mathcal{N}_{k,t}, i \in \mathcal{V}, j \in \mathcal{V} \cup \{0\} \tag{10}$$

$$u_{t,k,n,i} + S_i + T_{i,j} \ge u_{t,k,n,j} - M\left(1 - x_{t,k,n,i,j}\right), \quad \forall t \in \mathcal{T}, k \in \mathcal{K}, n \in \mathcal{N}_{k,t}, i \in \mathcal{V}, j \in \mathcal{V} \cup \{0\} \tag{11}$$

$$u'_{t,k,n,0} + T_{0,i} \le u_{t,k,n,i} + M\left(1 - x_{t,k,n,0,i}\right), \quad \forall t \in \mathcal{T}, k \in \mathcal{K}, n \in \mathcal{N}_{k,t}, i \in \mathcal{V} \tag{12}$$

$$u'_{t,k,n,0} + T_{0,i} \ge u_{t,k,n,i} - M\left(1 - x_{t,k,n,0,i}\right), \quad \forall t \in \mathcal{T}, k \in \mathcal{K}, n \in \mathcal{N}_{k,t}, i \in \mathcal{V} \tag{13}$$

$$u'_{t,k,n,0} \ge u_{t,k,n-1,0} + S_0 - M\left(2 - \sum_{j \in \mathcal{V}} x_{t,k,n,0,j} - \sum_{j \in \mathcal{V}} x_{t,k,n-1,j,0}\right), \quad \forall t \in \mathcal{T}, k \in \mathcal{K}, n \in \mathcal{N}_{k,t} \backslash \{1\} \tag{14}$$

$$y_{i,s} \in \{0,1\}, \quad \forall i \in \mathcal{V}, s \in \mathcal{S}_i \tag{15}$$

$$v_{i,s,t,k,n} \in \{0,1\}, \quad \forall i \in \mathcal{V}, s \in \mathcal{S}_i, t \in \mathcal{T}, k \in \mathcal{K}, n \in \mathcal{N}_{k,t} \tag{16}$$

$$x_{t,k,n,i,j} \in \{0,1\}, \quad \forall t \in \mathcal{T}, k \in \mathcal{K}, n \in \mathcal{N}_{k,t}, i, j \in \mathcal{V} \cup \{0\} \tag{17}$$

$$w_t \ge 0, \quad \forall t \in \mathcal{T} \tag{18}$$

$$u_{t,k,n,i}, u'_{t,k,n,i} \ge 0, \quad \forall t \in \mathcal{T}, k \in \mathcal{K}, n \in \mathcal{N}_{k,t}, i \in \mathcal{V} \tag{19}$$

The objective function (1) maximizes the total profit of the system throughout the entire planning cycle, i.e., the difference between the total revenue from selling recycled materials and the overall costs for providing the waste collection service. The three terms in (1) represent the total sales revenue, the total penalty for unbalanced load at the sort facility, and the total travel cost, respectively. Constraints (2) ensure that each dumpster is assigned to a specific schedule. Constraints (3) connect the scheduling and routing related decisions. Constraints (4)

capacity limit along a trip. Constraints (10) - (
relationships between the vehicle trips and the
at dumpsters and the depot, which also serve
of eliminating subtours. Note that M denotes t
duration of the entire planning horizon. Constrain
define binary and nonnegative variables.

III. Solution Approach

The MILP model proposed in the previous
variant of PVRP, which is known to be extrem
to solve. State-of-the-art algorithms for solvin
[19], [20] typically decompose the solution pro
sequential phases. The first phase determines a s
for each customer by solving a scheduling subp
as follows:

$$\text{(SP1)} \ \max \sum_{t \in \mathcal{T}} C^{\mathrm{R}} \cdot w_t - \sum_{t \in \mathcal{T}} \sum_{\substack{t' \in \mathcal{T} \\ t' > t}} C^{\mathrm{B}} \cdot |w$$
$$\text{s.t. (2), (4), (5), (15) and (18).}$$

Then, given the scheduling solution $\overline{y}_{i,s}$, the s
determines a routing plan for each day by solv
subproblem SP2-t:

$$\text{(SP2-}t\text{)} \ \min \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}_{k,t}} \sum_{\substack{i,j \in \mathcal{V} \cup \{0\} \\ j \neq i}} C^{\mathrm{T}} \cdot T_{i,j} \cdot x_{t,k,}$$
$$\text{s.t. (6)} - \text{(14), (16), (17), (19) and}$$
$$\sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}_{k,t}} v_{i,s,t,k,n} = \alpha_{s,t} \cdot \overline{y}_{i,s}, \quad \forall$$

Since the scheduling and routing decisions are
dependent, the sequential nature of the two-pha
(i.e., no spatial routing information is used to
temporal scheduling of visits) may likely lea
solutions. In hopes of overcoming this shortcom
pose an improved solution approach that incorpo
cost estimates into the scheduling decision-ma
Specifically, we use asymptotic routing cost f
the CA literature to obtain a parametric approxi
travel cost on each day, and incorporate it into
of scheduling subproblem SP1.

Following results in [23], [32], the total trave
large-scale planar vehicle routing problems can b
a line-haul part (i.e., travel from the depot to t
the customers) and a local detour part (i.e., trav
customers). We assume that the region can be
into a set of disjoint subregions, denoted by $\mathcal{Z}$
tomer point and demand distributions are dens
homogeneous inside each subregion $z \in \mathcal{Z}$.
$\Omega_{t,z}$ denote the asymptotic line-haul and local

depot, i.e., $T_{0,i} + T_{i,0}$. Meanwhile, the local detour travel time for each dumpster visit is proportional to the average separation between two neighboring points in that zone [23], i.e., $\hat{k} \cdot \nu \cdot \left(\frac{\sum_{i \in \mathcal{V}_z} \sum_{s \in \mathcal{S}_i} \alpha_{s,t} \cdot y_{i,s}}{A_z} \right)^{-\frac{1}{2}}$, where $\hat{k}$ is a dimensionless constant that depends only on the distance metric, ν is the inverse of vehicle speed, and $\frac{\sum_{i \in \mathcal{V}_z} \sum_{s \in \mathcal{S}_i} \alpha_{s,t} \cdot y_{i,s}}{A_z}$ is the spatial density of dumpster points. In summary, the total travel costs $\Pi_{t,z}$ and $\Omega_{t,z}$ can be formulated in closed form as follows:

$$\Pi_{t,z} = C^{\mathrm{T}} \cdot \frac{\sum_{i \in \mathcal{V}_z} \sum_{s \in \mathcal{S}_i} W_{i,t,s} \cdot (T_{0,i} + T_{i,0}) \cdot y_{i,s}}{Q^{\mathrm{V}}}, \quad \forall t \in \mathcal{T}, z \in \mathcal{Z}, \tag{20}$$

$$\Omega_{t,z} = C^{\mathrm{T}} \cdot \hat{k} \cdot \nu \cdot \left(A_z \cdot \sum_{i \in \mathcal{V}_z} \sum_{s \in \mathcal{S}_i} \alpha_{s,t} \cdot y_{i,s} \right)^{\frac{1}{2}}, \quad \forall t \in \mathcal{T}, z \in \mathcal{Z}. \tag{21}$$

Since $y_{i,s}$ is a binary variable, we know that $y_{i,s} = (y_{i,s})^2, \forall i \in \mathcal{V}, s \in \mathcal{S}_i$, holds. Thus, by taking square on both sides of (21) and replacing $y_{i,s}$ by $(y_{i,s})^2$, the equations above can be transformed into the following second order cone constraints [34]:

$$(\Omega_{t,z})^2 = \left(C^{\mathrm{T}} \cdot \hat{k} \cdot \nu \right)^2 \cdot \left[A_z \cdot \sum_{i \in \mathcal{V}_z} \sum_{s \in \mathcal{S}_i} \alpha_{s,t} \cdot (y_{i,s})^2 \right], \quad \forall t \in \mathcal{T}, z \in \mathcal{Z}. \tag{22}$$

Now, we are ready to reformulate the scheduling subproblem, denoted by SP1$'$, as the following mixed-integer quadratically constrained program (MIQCP):

$$(\mathrm{SP1}') \ \max \sum_{t \in \mathcal{T}} C^{\mathrm{R}} \cdot w_t - \sum_{t \in \mathcal{T}} \sum_{\substack{t' \in \mathcal{T} \\ t' > t}} C^{\mathrm{B}} \cdot |w_t - w_{t'}| - \sum_{t \in \mathcal{T}} \sum_{z \in \mathcal{Z}} \left(\lambda_{t,z} \cdot \Pi_{t,z} + \theta_{t,z} \cdot \Omega_{t,z} \right)$$
$$\text{s.t. (2), (4), (5), (15), (18), (20) and (22).} \tag{23}$$

In the above formulation, we have also introduced two sets of coefficients $\{\lambda_{t,z}\}$ and $\{\theta_{t,z}\}$ in the objective function (23) to adjust the approximated line-haul and local detour travel costs, respectively. Their values should both be asymptotically equal to 1 if the customer points and demand are densely and homogeneously distributed over the subregions. Allowing their values to deviate from 1 for each subregion gives us the tolerance for moderate violations to the asymptoticity assumptions in real-world situations (e.g., the number of customers being less than infinite, and their spatial distribution being slightly nonhomogeneous).

The values of coefficients $\lambda_{t,z}$ and $\theta_{t,z}$ should be

update $\theta_{t,z}$ and $\lambda_{t,z}$, $\forall t \in \mathcal{T}, \forall z \in \mathcal{Z}$, as fo generic iteration n, with the current coefficien SP1$'$ to obtain the scheduling decisions (as we distribution), the resulting asymptotic cost estim $\Omega^n_{t,z}$ from (20)-(21), as well as the approximate cost $\overline{V}^n_{t,z} = \lambda^n_{t,z} \cdot \Pi^n_{t,z} + \theta^n_{t,z} \cdot \Omega^n_{t,z}, \forall t, z$. Then, SP to obtain the actual routing cost $\hat{v}^n_{t,z}, \forall t, z$, and we adaptive stepsize that will be used to update the In so doing, we note that the gradients of the e $\frac{1}{2}\left[\sum_t \sum_z (\overline{V}^n_{t,z} - \hat{v}^n_{t,z})\right]^2$ with respect to $(\lambda_{t,z},$ written as

$$\left(g^n_{\lambda_{t,z}}, g^n_{\theta_{t,z}}\right) = \Bigg(\Pi^n_{t,z} \cdot \sum_t \sum_z (\overline{V}^n_{t,z} - \hat{v}^n_{t,z}),$$
$$\Omega^n_{t,z} \cdot \sum_t \sum_z (\overline{V}$$

The estimates of the first and second moment the gradients are then computed as follows, with zero and exponential decaying factors $\beta_1, \beta_2 \in$

$$\left(m^n_{\lambda_{t,z}}, m^n_{\theta_{t,z}}\right) = \beta_1 \cdot \left(m^{n-1}_{\lambda_{t,z}}, m^{n-1}_{\theta_{t,z}}\right) + (1 - \beta_1) \cdot \left(g^n_{\lambda_{t,z}}, g^n_{\theta_{t,z}}\right),$$
$$\left(q^n_{\lambda_{t,z}}, q^n_{\theta_{t,z}}\right) = \beta_2 \cdot \left(q^{n-1}_{\lambda_{t,z}}, q^{n-1}_{\theta_{t,z}}\right) + (1 - \beta_2) \cdot \left[\left(g^n_{\lambda_{t,z}}\right)^2, (g$$

To account for the bias of moment estimate moments are corrected as

$$\left(\hat{m}^n_{\lambda_{t,z}}, \hat{m}^n_{\theta_{t,z}}, \hat{q}^n_{\lambda_{t,z}}, \hat{q}^n_{\theta_{t,z}}\right) = \Bigg(\frac{m^n_{\lambda_{t,z}}}{1 - \beta^n_1}, \frac{m^n_{\theta_{t,z}}}{1 - \beta^n_1}, \frac{q^n_{\lambda_{t,z}}}{1 - \beta^n_2}$$

Finally, we use the corrected $\left(\hat{m}^n_{\lambda_{t,z}}, \hat{m}^n_{\theta_{t,z}}, \hat{q}^n_{\lambda_{t,z}}, \hat{q}^n_{\theta_{t,z}}\right)$ and a predetermine γ to update $\lambda_{t,z}$ and $\theta_{t,z}$ as follows:

$$\left(\lambda^n_{t,z}, \theta^n_{t,z}\right) = \left(\lambda^{n-1}_{t,z}, \theta^{n-1}_{t,z}\right) - \gamma \cdot \Bigg(\frac{\hat{m}^n_{\lambda_{t,z}}}{\sqrt{\hat{q}^n_{\lambda_{t,z}}} + \epsilon},$$

where ϵ is a very small number to prevent any zero. Note that one main advantage of the *Adam* that the magnitudes of parameter updates are in rescaling of the gradient.

As a final remark, it should be noted that t

TABLE I

COMPARISON RESULTS BETWEEN THE HYBRID AND BENCHMARK SOLUTION APPROACHES IN SECTION IV-A

Case	$\lvert\mathcal{V}\rvert$	$\lvert\mathcal{K}\rvert$	Balance cost ($)		Recycle revenue ($)		Travel cost ($)		Total profit ($)		
			Hybrid	BM	Hybrid	BM	Hybrid	BM	Hybrid	BM	Diff.
1	45	3	0.0	0.9	44.4	44.9	41.7	44.7	2.7	-0.6	521.9%
2	45	4	0.0	0.3	45.4	45.7	42.9	44.2	2.5	1.1	127.3%
3	45	5	0.0	1.1	46.2	44.0	41.4	43.4	4.8	-0.4	1242.9%
4	60	3	0.0	0.7	58.2	63.7	53.2	59.4	5.1	3.5	45.7%
5	60	4	0.0	0.7	58.7	60.3	54.4	57.7	4.3	1.8	138.9%
6	60	5	0.0	0.8	59.4	63.5	56.7	60.6	2.7	2.1	28.6%
7	75	3	0.0	0.7	75.5	76.1	68.2	72.8	7.3	2.6	180.8%
8	75	4	0.0	1.1	75.7	79.2	65.4	73.9	10.3	4.2	145.2%
9	75	5	0.0	0.2	75.3	76.2	68.4	70.0	6.9	6.0	15.0%

can be easily estimated as

$$\Delta_{j,z,s} \approx C^{\mathrm{T}} \cdot \Bigg[\frac{\left(T_{0,j} + T_{j,0}\right) \cdot \sum_{t \in \mathcal{T}} \overline{\lambda}_{t,z} \cdot W_{j,t,s}}{Q^{\mathrm{V}}} + \hat{k} \cdot \nu \cdot \sqrt{A_z} \cdot \sum_{t \in \mathcal{T}} \frac{\alpha_{s,t} \cdot \overline{\theta}_{t,z}}{2\sqrt{\overline{n}_{t,z}}} \Bigg], \quad (24)$$

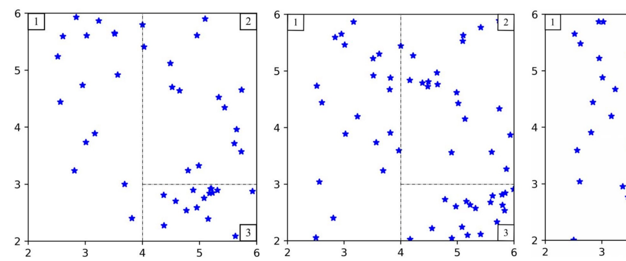
where $\overline{n}_{t,z}$ denotes the number of dumpsters in zone z that are currently visited on day t, and $\overline{\lambda}_{t,z}$ and $\overline{\theta}_{t,z}$ are the converged values of coefficients $\lambda_{t,z}$ and $\theta_{t,z}$ from the hybrid approach.

IV. NUMERICAL STUDY

In this section, a series of numerical experiments are conducted to test the applicability and effectiveness of the proposed CA-based hybrid solution approach. This hybrid solution approach is implemented as follows: in Section IV-A, we apply a standard MIQCP solver to solve both the scheduling and routing subproblems for medium-size test instances; in Section IV-B, we implement a variable neighborhood search (VNS) based algorithm to solve the routing subproblems for larger instances. In both sections, the performance of the proposed hybrid solution approach is compared with that of the conventional sequential optimization approach. All these numerical tests are performed on a personal computer with 3.4 GHz CPU and 16 GB RAM.

A. Medium-Size Instances

We first consider a $[2,6] \times [2,6]$ service region and a 5-weekday planning cycle. A total of 9 test cases with different number of dumpsters, $\lvert\mathcal{V}\rvert \in \{45, 60, 75\}$, and different number of vehicles, $\lvert\mathcal{K}\rvert \in \{3,4,5\}$ are constructed. The geographic distributions of dumpsters and the region partition are illustrated in Figure 2. The vehicle depot and the sorting facility are both located at $(0,0)$. The amount of waste generated at dumpster i on day t, $D_{i,t}$, follows a normal distribution with



(a) 45 dumpsters (b) 60 dumpsters (c) 7

Fig. 2. Spatial distribution of dumpsters for test cases in S

For these medium-size test instances, we in proposed hybrid approach by solving both SP1′ and SP2 using the standard MIQCP solver An improved two-phase heuristic algorithm is benchmark for comparison. This benchmark app upon what is described in Section III, i.e., it first tial scheduling/routing solution by solving SP1 a the Gurobi solver. Then it further improves the series of iterations; in each of these iterations, t searches for a better solution by randomly p scheduling decision and solve the routing decision For each test case, both the proposed "hybrid" mark (BM) approaches are tested for five random instances, and the CPU time limit for each test i to be 3600 seconds. The average output, includin cost, the recycling revenue, the total travel cost, profit, are presented in Table I.

From the last three columns of Table I, we that, given the same computation time, our propo drastically outperforms the benchmark approach generating more profits for all 9 cases. Moreove approach can significantly reduce the total trav maintaining a similar recycling revenue. Since bot use the same routing solver, such performance from the proposed hybrid approach seems to re incorporation of travel cost estimation into th

TABLE II

COMPARISON RESULTS BETWEEN THE HYBRID AND BENCHMARK SOLUTION APPROACHES

Case	$\|\mathcal{V}\|$	C^{B}	C^{R}	C^{T}	Balance cost (\$)		Travel cost (\$)		Recycle revenue (\$)		Total profit (\$)		
					Hybrid	BM	Hybrid	BM	Hybrid	BM	Hybrid	BM	Diff.
1	45	1.0	0.1	0.5	0.0	6.2	15.5	19.0	44.1	44.4	28.7	19.2	49.6%
2	45	1.0	0.1	1.0	0.0	4.6	30.7	36.8	43.9	44.4	13.2	3.0	331.9%
3	45	1.0	0.5	0.5	0.0	6.6	15.5	19.0	221.4	222.1	205.9	196.5	4.8%
4	45	1.0	0.5	1.0	0.0	4.6	31.0	36.8	221.1	222.1	190.1	180.8	5.2%
5	45	5.0	0.1	0.5	0.0	23.5	15.4	19.3	44.2	44.4	28.8	1.7	1625.8%
6	45	5.0	0.1	1.0	0.0	25.5	30.6	37.2	44.1	44.4	13.5	-18.3	173.4%
7	45	5.0	0.5	0.5	0.0	23.5	15.4	19.3	221.4	222.1	206.0	179.4	14.8%
8	45	5.0	0.5	1.0	0.0	25.5	30.7	37.2	221.1	222.1	190.4	159.4	19.4%
9	90	1.0	0.1	0.5	0.0	8.8	28.7	35.6	89.0	89.4	60.4	45.0	34.0%
10	90	1.0	0.1	1.0	0.0	10.7	57.4	71.9	89.0	89.4	31.6	6.8	365.0%
11	90	1.0	0.5	0.5	0.0	9.5	28.7	35.8	445.8	447.0	417.1	401.7	3.9%
12	90	1.0	0.5	1.0	0.0	10.7	57.1	72.4	445.0	447.0	387.9	363.9	6.6%
13	90	5.0	0.1	0.5	0.0	32.7	28.5	34.7	88.9	89.4	60.4	22.0	174.6%
14	90	5.0	0.1	1.0	0.0	27.4	57.2	68.8	88.9	89.4	31.7	-6.8	564.1%
15	90	5.0	0.5	0.5	0.0	33.8	29.0	34.7	446.1	447.0	417.1	378.5	10.2%
16	90	5.0	0.5	1.0	0.0	21.9	57.5	69.1	445.2	447.0	387.7	356.0	8.9%
17	150	1.0	0.1	0.5	0.0	4.9	46.6	56.7	148.9	149.1	102.2	87.6	16.8%
18	150	1.0	0.1	1.0	0.0	7.2	93.1	112.4	149.0	149.1	55.8	29.6	88.7%
19	150	1.0	0.5	0.5	0.0	6.3	46.6	56.3	745.0	745.7	698.4	683.1	2.2%
20	150	1.0	0.5	1.0	0.0	6.9	93.4	113.2	745.0	745.7	651.6	625.6	4.2%
21	150	5.0	0.1	0.5	0.0	15.1	46.7	53.2	148.9	149.1	102.2	80.8	26.4%
22	150	5.0	0.1	1.0	0.0	15.1	93.5	106.3	148.8	149.1	55.3	27.7	99.5%
23	150	5.0	0.5	0.5	0.0	15.1	46.5	53.2	745.1	745.7	698.7	677.4	3.1%
24	150	5.0	0.5	1.0	0.0	15.1	93.2	106.4	745.3	745.7	652.1	624.2	4.5%

take the following values: $|\mathcal{V}| \in \{45, 90, 150\}$, $C^{\mathrm{B}} \in \{1.0, 5.0\}$, $C^{\mathrm{R}} \in \{0.1, 0.5\}$ and $C^{\mathrm{T}} \in \{0.5, 1.0\}$. We set $|\mathcal{K}| = 5$, and all the other parameters are the same as those in Section IV-A.

For each test case, we generate five random instances and embed a state-of-the-art VNS algorithm [6], [20] inside the hybrid approach to solve the routing subproblem SP2, while still using the Gurobi solver for only the scheduling subproblem SP1′. The VNS algorithm mainly consists of a constructive heuristic, and an iterative algorithm with embedded local search subroutines. The original pseudocode of the VNS algorithm is presented in Appendix B – although, for our hybrid approach, any operations that are related to the scheduling solution are skipped. For comparison, this VNS algorithm is also used to solve the sequential subproblems SP1 and SP2, as a benchmark. For each computation instance, we set the CPU time limit to be 1800 seconds.

Table II presents the average balance cost, recycling revenue, travel cost, and profit for all 24 test cases. We can observe that the proposed hybrid approach significantly outperforms the state-of-the-art sequential solution approach (even with the same embedded VNS algorithm) in terms of generating profits for all 24 cases, and can always achieve a much lower travel cost as compared to the benchmark (up to 21.1% reduction). Such results are consistent with our observations in Section IV-A. Since we use the same algorithms to solve the routing subproblems for both approaches, it can be inferred that such significant improvements are primarily due

TABLE III

NUMERICAL RESULTS FOR TEST CASE 1

Day	$\overline{\lambda}$			$\overline{\theta}$		
	z_1	z_2	z_3	z_1	z_2	z_3
1	0.297	0.292	0.277	1.669	1.623	1.675
2	0.269	0.282	0.247	1.677	1.683	1.667
3	0.249	0.262	0.307	1.674	1.668	1.614
4	0.242	0.250	0.301	1.673	1.672	1.665
5	0.249	0.258	0.237	1.677	1.671	1.669

solution approach is more likely to achieve a bett decision than the benchmark sequential approach scheduling decision lies at the center of the en process, and their solution quality would have influence on the system performance.

Furthermore, as we mentioned at the end of the hybrid approach provides a convenient way the marginal cost/profit of adding an extra dum one test instance of case 1 as an example, values for $\overline{n}_{t,z}$, $\overline{\lambda}_{t,z}$ and $\overline{\theta}_{t,z}$ are presented in Ta extra dumpster at location (5, 5) with a demar $(1, 2, 3, 1, 2)$ will be added to the problem, th gives the approximate marginal cost for serving according to schedules $(1, 0, 1, 0, 1)$ and $(1, 1,$ and \$6.1, respectively. As such, the decision m able to use the estimated coefficients $\lambda_{t,z}$ and hybrid approach to assess the impacts of various

TABLE IV
RESULTS FOR TEST CASES IN THE LITERATURE

Case	$\|\mathcal{V}\|$	$\|\mathcal{T}\|$	Q^{V}	Balance cost ($)		Recycle revenue ($)		Travel cost ($)		Total profit ($)		
				Hybrid	BM	Hybrid	BM	Hybrid	BM	Hybrid	BM	Diff.
1	48	4	200	0.0	6.0	2,620.0	2,626.0	277.7	391.4	2,342.3	2,228.6	5.1%
2	96	4	195	0.0	27.0	4,880.0	4,871.0	370.1	564.8	4,509.9	4,279.2	5.4%
3	144	4	190	0.0	3.0	7,152.0	7,151.0	706.8	891.3	6,445.2	6,256.7	3.0%
4	192	4	185	0.0	9.0	9,908.0	9,903.0	765.9	944.7	9,142.1	8,949.3	2.2%
5	240	4	180	0.0	39.0	13,400.0	13,391.0	966.9	1,128.5	12,433.1	12,223.5	1.7%
6	288	4	175	0.0	13.0	14,680.0	14,675.0	1,159.6	1,377.6	13,520.4	13,284.4	1.8%
7	72	6	200	0.0	88.0	5,682.0	5,664.0	623.3	755.6	5,058.7	4,820.4	4.9%
8	144	6	190	0.0	140.0	12,030.0	11,940.0	1,116.0	1,378.3	10,914.0	10,421.7	4.7%
9	216	6	180	0.0	451.0	16,416.0	16,227.0	1,399.6	1,686.0	15,016.4	14,090.0	6.6%
10	288	6	170	0.0	59.0	23,088.0	23,085.0	1,931.1	2,220.6	21,156.9	20,805.4	1.7%

number of time periods $|\mathcal{T}|$, and different vehicle capacity Q^{V}. It should be noted that, the original test cases assume preset frequencies/schedules at customers (i.e., dumpsters in this study) and unlimited capacity at the depot (i.e., sorting facility in this study). To accommodate the problem settings in this study, a few adjustments are made: (i) the assumption on preset frequencies and schedules is relaxed, and (ii) explicit limits on the customer storage levels and the throughput at the depot are enforced. Specifically, the storage capacity for each customer is calculated as $Q_i^{\mathrm{D}} = \frac{D_i \cdot |\mathcal{T}|}{F_i}, \forall i \in \mathcal{V}$, where D_i and F_i are the daily demand and the preset frequency at customer $i \in \mathcal{V}$, respectively; and the capacity at the sorting facility Q^{SF} is calculated as the sum of the daily demand at all customers, i.e, $Q^{\mathrm{SF}} = \sum_{i \in \mathcal{V}} D_i$. The number of vehicles $|\mathcal{K}| = 5$ for all cases and the maximum number of trips conducted by each vehicle per day is $|\mathcal{N}_{k,t}| = 10, \forall k \in \mathcal{K}, t \in \mathcal{T}$. In each case, the whole area is partitioned into four zones according to the Cartesian quadrants. The cost factors are set as $C^{\mathrm{B}} = C^{\mathrm{R}} = C^{\mathrm{T}} = 1$, and the algorithm parameters are the same as those in Section IV-A. Table IV shows that the proposed hybrid approach outperforms the benchmark approach for all 10 test cases. This is consistent with the findings in the previous two sections.

V. CONCLUSION

This paper proposes an MILP model for the periodic waste collection planning problem to help waste management agencies determine both scheduling and routing decisions to maximize the total profits from recyclable materials. The load balance of recycling operations at the sorting facility is also considered. An iterative hybrid solution framework including discrete and continuous components is developed to enhance the performance of the solution approach. More specifically, a scheduling subproblem that integrates asymptotic travel cost estimation formulas from the continuum approximation literature and a group of vehicle routing subproblems are solved iteratively, and a stochastic approximation subroutine based on the *Adam* algorithm is embedded along the solution

types of recyclable waste may have their uniqu
requirements (e.g., paper waste typically needs to
before food waste so as to prevent contamination
ing such additional requirements will further co
problem. If there are multiple sorting facilities
limited capacity, the load assignment decision
should be considered as well. In certain cases, i
of limited capacities of vehicles as well as dum
collection may become helpful although it incr
travel distances. Finally, we are also interested in
other types of approximation methods, e.g., ne
for the travel cost estimation while solving th
subproblem.

APPENDIX A
NOTATION LIST

$\mathcal{R}$	The entire region being studied
$\mathcal{V}$	Set of dumpsters, indexed by i
$\mathcal{K}$	Set of waste collection vehicles, index
$\mathcal{T}$	Set of time periods, indexed by t
$\mathcal{S}$	Set of all possible schedules, indexed
$\mathcal{S}_i$	Feasible schedule set for dumpster $i \in$
$\alpha_{s,t}$	Binary parameter, where $\alpha_{s,t} = 1$ if part of schedule s, or $\alpha_{s,t} = 0$ otherw
Q_i^{D}	Storage capacity limit for dumpster i
Q^{V}	Load capacity of a waste collection ve
Q^{SF}	Processing capacity at the sorting facil
$D_{i,t}$	Amount of waste generated at dumpst on day $t \in \mathcal{T}$
$W_{i,s,t}$	Amount of waste at dumpster $i \in \mathcal{V}$ by of day t, if this dumpster is visited ac to schedule $s \in \mathcal{S}$
$\mathcal{N}_{k,t}$	Set of trips being conducted by vehicl on day $t \in \mathcal{T}$, indexed by n
S_i	Service time at dumpster $i \in \mathcal{V}$
$T_{i,j}$	Travel time between two nodes $i, j \in$
C^{T}	Travel cost per time unit

A_z	Area size of subregion $z \in \mathcal{Z}$
$\Pi_{t,z}$	Asymptotic line-haul travel costs for visiting zone $z \in \mathcal{Z}$ on day $t \in \mathcal{T}$
$\Omega_{t,z}$	Asymptotic local detour travel costs for visiting zone $z \in \mathcal{Z}$ on day $t \in \mathcal{T}$
$\lambda_{t,z}$	Parameter that adjust the approximated line-haul travel costs for visiting zone $z \in \mathcal{Z}$ on day $t \in \mathcal{T}$
$\theta_{t,z}$	Parameter that adjust the approximated local detour travel costs for visiting zone $z \in \mathcal{Z}$ on day $t \in \mathcal{T}$
$x_{t,k,n,i,j}$	Routing decision variable, whose value equals 1 if vehicle $k \in \mathcal{K}$ travels from node $i \in \mathcal{V} \cup \{0\}$ to node $j \in \mathcal{V} \cup \{0\}$, $j \neq i$ in trip $n \in \mathcal{N}_{k,t}$ on day $t \in \mathcal{T}$, or 0 otherwise
$y_{i,s}$	Scheduling decision variable, whose value equals to 1 if dumpster $i \in \mathcal{V}$ is visited according to schedule $s \in \mathcal{S}_i$, or 0 otherwise
$v_{i,s,t,k,n}$	Binary decision variable, whose value equals to 1 if dumpster $i \in \mathcal{V}$ is served according to schedule $s \in \mathcal{S}_i$ and meanwhile it is visited by vehicle $k \in \mathcal{K}$ in trip $n \in \mathcal{N}_{k,t}$ on day $t \in \mathcal{T}$, or 0 otherwise
$u_{t,k,n,i}$	Arrival time of vehicle $k \in \mathcal{K}$ at node $i \in \mathcal{V} \cup \{0\}$ in trip $n \in \mathcal{N}_{k,t}$ of day $t \in \mathcal{T}$

Algorithm State of the Art VNS Algorithm to Solve the Waste Collection Problem

1: **Initialization:**
  (a) Select the set of neighborhood structure $N_\kappa, \kappa = 1, \ldots, \kappa_{\max}$, where $\kappa_{\max}$ is the maximum number of neighborhood structures;
  (b) Find an initial solution x: (i) for scheduling decision, each customer is assigned a visiting schedule randomly; (ii) for routing decisions, solving a vehicle routing problem for each day using the Clarke and Wright savings algorithm [40].
  (c) Set the stopping criteria: the total computation time limit $T_{\max}$;
2: **while** elapsed time $< T_{\max}$ **do**
3:   Set $\kappa \leftarrow 1$
4:   **while** $\kappa \leq \kappa_{\max}$ **do**
5:     (a) *Shaking:* Randomly generate a point from κ^{th} neighborhood of x (i.e., $x' \in N_\kappa(x)$);
6:     (b) *Local search:* Apply the 3-opt local search algorithm with x' as initial solution, obtain a local optimum x''. If x'' is feasible, continue; otherwise, set $\kappa \leftarrow \kappa + 1$ and go back to step 5;

$u'_{t,k,n,i}$	Departure time of vehicle $k \in \mathcal{K}$ at $\mathcal{V} \cup \{0\}$ in trip $n \in \mathcal{N}_{k,t}$ of day $t \in \mathcal{T}$
w_t	Amount of waste being processed at th facility on day $t \in \mathcal{T}$.

Appendix B
Pseudocode for the Benchmark So Approach in Section IV-B

(See Algorithm in the left column)

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