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Strategic investment decisions in an oligopoly with a competitive fringe: An equilibrium problem with equilibrium constraints approach

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ABSTRACT

Modern wholesale electricity markets often have producers who exercise market power. The standard way to model market power in an oligopoly with a competitive fringe is by using Mixed Complementarity Problems (MCPs) and conjectural variations. However, such models can lead to myopic (sub-optimal) behaviour for oligopolists. We first build on existing literature to show that an oligopoly with a competitive fringe where all firms have investment decisions will also lead to myopic behaviour when modelled using MCPs with conjectural variations. To overcome this issue, we develop an Equilibrium Problem with Equilibrium Constraints (EPEC) to model such an electricity market structure. The EPEC models two types of players: price-making firms, who have market power, and price-taking firms, who do not. Our model is the first in the literature to consider an oligopoly with a competitive fringe where all firms have investment decisions. Our results indicate that an EPEC can model investment decisions in an oligopoly with a competitive fringe more credibly and thus overcome the myopic behaviour observed in MCPs. The EPEC found multiple equilibria for investment decisions and firms' profits. Despite this, market prices and consumer costs were found to remain relatively constant across the equilibria. A further contribution of the modelling approach is that it shows how it may be optimal for price-making firms to accept losses in some time periods in order to disincentivize price-taking firms from investing further into the market. We conclude our paper with a discussion of the computational limitations of our approach.

1. Introduction

Modelling electricity markets has attracted much attention in the recent operations research literature. Optimisation and equilibrium models, in particular, have been extensively used to represent the behaviour of electricity generators. Such tools provide insights from planning, operations, and regulatory perspectives. Regulators may use them to monitor market inefficiencies, profit-maximising generators may use them to devise trading strategies, while policymakers may use them to test the outcomes of different proposed policy mechanisms.

Since the 1980s, countries have been deregulating their electricity markets with the intention of splitting ownership of market activities (Pozo, Sauma, & Contreras, 2017). Governments' goals are to foster competition, increase market efficiencies and thus reduce consumer costs. As a result of the deregulation, individual market participants, also known as market players, have been seeking to independently maximise their profits (Facchinei & Pang, 2007).

Deregulation and liberalisation of electricity markets has resulted in evidence of market players exercising market power (Lee, 2016; Tangerås & Mauritzen, 2018). Market power is present when one (or more) seller(s) in the market strategically maximises their profits by influencing the selling price through the quantity they supply to the market. This is contrary to scenarios where sellers cannot influence prices, in which case we say the market is perfectly competitive. While accurately modelling market power is a challenging area of operations research, there have been many studies that have incorporated market power in electricity market models in different ways. For a comprehensive review of electricity market models that incorporate market power, we refer the reader to Pozo et al. (2017). More recent examples in the operations research literature include Devine and Bertsch (2018) who use a Mixed Complementarity Problem (MCP) to study different consumer-led load shedding strategies.

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In this paper, the solution to an MCP determines an equilibrium of multiple optimisation problems by finding a point that satisfies the Karush–Kuhn–Tucker (KKT) conditions of each optimisation simultaneously as a system of nonlinear equalities and inequalities (Gabriel, Conejo, Fuller, Hobbs, & Ruiz, 2012). Consequently, MCPs allow us to model players with market power as Cournot players. Other recent works in the energy market modelling literature have reformulated MCPs as convex optimisation problems (Ansari & Holz, 2019; Egging-Bratseth, Baltensperger, & Tomasgard, 2020) in order to solve them.

Fanzeres, Ahmed, and Street (2019) proposed a Mathematical Program with Equilibrium Constraints (MPEC) for strategic bidding in an electricity market. A MPEC can model a bilevel optimisation problem, which is a mathematical program where one or more optimisation problems are constrained by other optimisation problems. The outer optimisation is the upper-level optimisation while the inner optimisations, which are represented in the outer problem as constraints, are the lower-level optimisation problems. In an electricity market setting, MPECs can be used to model markets where a single leader has market power. The upper level represents the optimisation problem of the player who has market power while the lower-level problems represent the problems of players that do not have market power with respect to the leader. The lower-level players could still exercise market power with other competitors (Fanzeres et al., 2019; Pereira, Granville, Fampa, Dix, & Barroso, 2005; Ruiz & Conejo, 2009).

Steeger and Rebennack (2017) present a methodology that combines Lagrangian Relaxation and nested Benders decomposition to model a single hydro producer with market power. Similarly, Habibian, Downward, and Zakeri (2019) use an optimisation-driven heuristic approach to model a large electricity consumer with market power. In both of these works, only one market participant has a strategic advantage.

The papers in the previous paragraphs do not consider strategic behaviour when the overall market was characterised by an oligopoly with a competitive fringe. Such a market structure occurs when multiple generators (the oligopolists) have market power and at least one generator does not (the competitive fringe). Many modern electricity markets are characterised by an oligopoly with a competitive fringe (Bushnell, Mansur, & Saravia, 2008; Walsh, Malaguzzi Valeri, & Di Cosmo, 2016). However, when it comes to the energy market modelling literature, such markets structures are under-represented when compared to other structures with market power.

Exceptions include Huppmann (2013) and Ansari (2017), who use an MCP to model an oligopoly with a competitive fringe in an international oil market. Similarly, Devine and Bertsch (2019) develop an MCP model of an oligopoly with a competitive fringe to investigate the impact demand response has on market power in an electricity market. Huppmann (2013) shows that modelling an oligopoly with a competitive fringe using an MCP can in certain scenarios lead to myopic, counter-intuitive, and thus unrealistic optimal decisions from the oligopolists. In a standard MCP formulation, the model can account for each oligopolist optimising its own position in equilibrium with other oligopolists' actions, but cannot mathematically incorporate the optimal reaction of the competitive fringe to its own decision. Assuming an inverse relationship between price and demand, the oligopolists modelled with a standard MCP withhold their generation due to the incomplete information that the reduced supply will increase market price, and hence their overall profit. However, when the oligopolists withhold their generation, the competitive fringe increases their generation and fills the generation gap to satisfy the MCP conditions. This leads to lower prices than the oligopolists assume. Consequently, along with their reduced generation, the oligopolists have lower profits than they would have if the market was modelled as being perfectly competitive.

Like others in the literature, Huppmann (2013) proposes using conjectural variations to overcome the issue. Conjectural variations make assumptions about how the competitive fringe reacts to the oligopolists' reduction in quantity, and incorporates that change in the oligopolists problem formulation (Baltensperger, Füchslin, Krütli, & Lygeros, 2016; Egging, Gabriel, Holz, & Zhuang, 2008; Egging & Holz, 2016; Haftendorn & Holz, 2010; Huppmann & Egging, 2014). Thus, conjectural variations added to the standard MCP model allow the oligopolists to incorporate the reactions of the competitive fringe into their decision-making process (Ruiz, Conejo, & Arcos, 2010).

However, as Huppmann (2013), Kimbrough, Murphy, and Smeers (2014), and now our results demonstrate, conjectural variations incorporated in MCP models may lead to oligopolists making sub-optimal decisions. This is because conjectural variations are a modelling construct that are not necessarily grounded in economic theory, can often be interpreted to use circular reasoning in models, and in conjunction with linear demand curves can result in oversimplification of economic realities in models. While these issues with using conjectural variations have been documented in the modelling literature, we validate it by showing that they also show up when considering investment decisions in modelling markets with an oligopoly and competitive fringe.

As an example, neither Huppmann (2013), Ansari (2017) nor Devine and Bertsch (2019) consider investment decisions for new generation in their models. Consequently, in this work, we build on existing literature and validate it by showing that when investment decisions are incorporated into an MCP model of an oligopoly with a competitive fringe, conjectural variations still lead to myopic model output for oligopolists. We resolve this counter-intuitive modelling feature by developing an Equilibrium Problem with Equilibrium Constraints (EPEC) model of an oligopoly with a competitive fringe where both the oligopolists and the competitive fringe have investment decisions.

An EPEC consists of finding an equilibrium among multiple interconnected MPECs (Gabriel et al., 2012). In this work, each MPEC represents the optimisation problem of an individual oligopolist: an electricity generating firm which has market power, also known as a price-*making* firm. The price-making firms seek to maximise profits by selecting generation and investment decisions subject to capacity constraints. In addition, the equilibrium conditions representing the optimisation problems of the competitive fringe are embedded into each price-making firm's problem as constraints. Consequently, an EPEC approach can overcome the limiting assumptions associated with conjectural variations by allowing oligopolists to explicitly account for optimal investment and generation decisions of the competitive fringe. While anticipating investment decisions in traditional electricity markets might be unrealistic, recent works in the literature have suggested that, in markets with high renewable and distributed generation, investment decisions can be anticipated by price-makers (Andoni et al., 2021; Huang, Xu, & Courcoubetis, 2020; Michalski, 2017).

Compared with MCPs, EPECs are more computational and mathematically challenging to solve (Pozo et al., 2017; Wogrin, Centeno, & Barquin, 2013a; Wogrin, Hobbs, Ralph, Centeno, & Barquin, 2013b). Consequently, the computational complexity makes it difficult to capture all aspects of electricity markets, such as stochasticity and nonlinearities. Furthermore, there is a trade-off to be made when choosing the methodology to model strategic behaviour in electricity markets. Standard methods such as MCPs allow for a much larger number of model variables and thus make the inclusion of stochasticity and nonlinearities, for example, more tractable. However, for markets characterised by an oligopoly with a competitive fringe and investment decisions, this paper shows that EPECs model strategic behaviour more credibly than MCPs, under simplifying assumptions.

There have been many examples in the literature where EPECs have been used to model electricity markets. Many of the first EPEC models for electricity markets consider electricity generators at the upper level and an Independent System Operator (ISO) at the lower level (Hu & Ralph, 2007; Pozo & Contreras, 2011; Ruiz, Conejo, & Smeers, 2011). Building on these works, Wogrin, Barquín, and Centeno (2012) and Wogrin et al. (2013a) develop an EPEC model that incorporates both capacity expansion and generation decisions amongst electricity generators. At the upper level, the generators decide investment decisions whilst accounting for operational decisions at the lower level. Both Wogrin et al. (2013a) and Wogrin et al. (2013b) report that EPEC problems can model electricity markets more realistically, compared with open-loop equilibrium models. EPECs allow modelling both investment decisions and generation decisions in the same formulation, while allowing for leaders and followers in the market.

In the EPEC presented in Wogrin et al. (2013b), all generating firms have an upper-level problem where they make investment decisions subject to market equilibrium conditions but do not consider a competitive fringe. Building on this work, we model both the investment and generation decisions of the oligopolists at the upper level while the investment and generation decisions of the competitive fringe are modelled at the lower level. Modelling the oligopolists' generation decisions at the upper level allows them to explicitly anticipate the competitive fringes' reactions without the need for conjectural variations. We make this modelling advancement to incorporate the competitive fringe, which was not modelled by Wogrin et al. (2013b). Moreover, upper-level generation decisions have been considered in other published literature as well (Dai & Qiao, 2016; Ruiz & Conejo, 2009; Tsimopoulos & Georgiadis, 2019).

Pozo, Contreras, and Sauma (2013) and Pozo, Sauma, and Contreras (2012) developed a model similar to Wogrin et al. (2012) but add an extra level to the model; an ISO who makes transmission expansion decisions while accounting for generators' capacity investment and operational decisions. Jin and Ryan (2013) consider a similar EPEC model as well but, in contrast to Pozo et al. (2013) and Pozo et al. (2012), they model price-responsive demand and strategic interactions amongst the generators.

Kazempour, Conejo, and Ruiz (2013) and Kazempour & Zareipour (2013) use EPEC models where the upper-level problem determines the optimal investment for strategic producers while lower-level problems represent different market clearing scenarios. Similarly, Ye, Papadaskalopoulos, & Strbac (2017) use an EPEC model to investigate the impact consumer-led demand shifting has on market power and find that demand response can reduce the negative impacts of market power. The upper level again represents the producers' problems while the lower level represents the market clearing process, in addition to the consumers' decisions. An EPEC model is used in Huppmann & Egerer (2015) as part of a three-stage equilibrium model between a supra-national planner, zonal planners, and an ISO. Moiseeva, Wogrin, & Hesamzadeh (2017) develop an EPEC model that considers generators' operational decisions in the lower level and their ramping decisions in the upper level. More recently, Guo, Chen, Xia, and Kang (2019) introduce another EPEC model where the upper level maximises generators' decisions while the lower level represents an ISO. Interestingly, Guo et al. (2019) account for risk-averse decision making by incorporating Conditional Value at Risk (CVaR) into their model. The literature thus has extensive examples that show the appropriateness of using EPECs to model different market scenarios involving market power, investment decisions, and diverse market participants. Our paper advances the literature by using an EPEC to model a market with both price-making and price-taking firms, all of whom have investment decisions.

The closest work to the present paper is Zerrahn and Huppmann (2017). They propose a three-stage game to model transmission network expansion in an imperfectly competitive market where some generators have market power while others do not. They solve the model using backward induction. The third stage represents the problem of the ISO and the competitive fringe. The second stage represents the firms who have market power and can thus explicitly account for the decisions made by the ISO and competitive fringe. In the first stage, social welfare is maximised using network expansion decisions whilst accounting for the second and third stages. Significantly, our work advances the formulation by Zerrahn and Huppmann (2017) to include both generation *and* investment decisions for firms, as opposed to a centralized planner who makes investment decisions.

Despite the rich literature of EPECs and equilibrium models for electricity markets, none of the literature models a market characterised by an oligopoly with a competitive fringe where all generators have investment decisions. Consequently, this paper makes the following contributions.

First, we replicate the myopic and counter-intuitive behaviour (described above) by price-making firms when modelling market power in an oligopolistic market with a competitive fringe when using an MCP with conjectural variations. We advance the literature by showing that this behaviour shows up in models with investment decisions as well. We overcome this documented inadequacy of MCPs by demonstrating that an EPEC can model investment decisions in an oligopoly with a competitive fringe more credibly without such myopic and counter-intuitive output.

Second, this paper advances existing literature by providing a framework for including investment decisions when modelling an oligopoly with a competitive fringe. We can obtain results that highlight the importance of considering investment decisions. Consequently, we show that this market scenario results in multiple equilibria with varying investment and generation decisions. These results will be of interest to generating firms, particularly those with market power, since their profits can vary depending on the equilibrium the market ends up in.

Third, our results show that may it be optimal for generating firms with market power to occasionally operate some of their generating units at a short-term loss to obtain long-term gain. The price-taking firms' ability to invest further into the market motivates the price-making firms to depress prices at particular timesteps. This reduces the revenues for price-taking firms from new investments. Such strategic behaviour cannot be captured by modelling this scenario as an MCP or as a cost-minimisation optimisation problem. Consequently, this result further highlights the suitability of the EPEC modelling approach and the importance of including investment decisions in models of oligopolies with a competitive fringe.

Fourth, we show that carbon emissions vary across the multiple equilibria in this market scenario. This outcome will be of interest to policymakers who seek market equilibria that minimise carbon emissions. Finally, while consumer costs increase due to the presence of market power, we show the increase is not as large as in the scenario without considering investment decisions (Devine & Bertsch, 2019).

We apply the EPEC developed in this work to an electricity market representative of the Irish power system in 2025 using data from Lynch, Devine, and Bertsch (2019a) and Bertsch, Devine, Sweeney, and Parnell (2018). We solve the model numerically as it is too large to be solved in closed form. A closed-form solution is possible using standard techniques under assumptions of no corner solutions, but we combine two computational techniques from the literature to solve our problem. We use the standard

Gauss-Seidel algorithm to use diagonalization for solving the EPEC (Hori & Fukushima, 2019) and we solve each individual MPEC using disjunctive constraints (Fortuny-Amat & McCarl, 1981). In addition, to improve computational efficiency, we use the approach developed by Leyffer and Munson (2010) to provide an initial strong stationary point of the EPEC as a starting point for our diagonalization algorithm. We demonstrate that this method can help improve the computational efficiency of solving our EPEC.

The remainder of this paper is structured as follows. In Section 2, we describe the data inputs. Sections 3 and 4 describe the MCP and EPEC model formulations, respectively. Following these, in Section 5, we present the results from the MCP and EPEC formulations. In Sections 6 and 7, we provide some discussion and conclusions, respectively. Finally, in Appendix A and Appendix B, we provide additional material related to the case study.

2. Modelling assumptions and data

In this section, we introduce the market under consideration and describe the data inputs for the models we use in Sections 3 and 4. The electricity market we consider consists of two types of players: price-making firms and price-taking firms. Price-making firms may exert market power by using generation decisions to influence the market price. Price-taking firms do not have such ability.

Each firm makes its forward market generation decisions to maximise its profits. Each firm may also hold multiple generating units with the technologies considered being baseload, mid-merit, and peak-load. The firms are distinguished by their price-making ability and their initial generation portfolios. However, each firm may also invest in new generation capacity in any of the technologies.[1]

We consider an electricity market that consists of four generating firms; firms $l=1$ and $l=2$ are price-making firms and while firms $f=3$ and $f=4$ are price-taking firms. We consider $|T|=6$ generating technologies: existing baseload, existing mid-merit, existing peaking, new baseload, new mid-merit, and new peaking. Each of the four firms hold different initial generating capacities. Firms $l=2$, $f=3$ and $f=4$ are, initially, specialised baseload, mid-merit, and peaking firms, respectively. In contrast, firm $l=1$ is an integrated firm initially holding capacity across each of the existing technologies. Because of their sizes, the integrated firm and the specialised baseload firm are modelled as the price-making firms while the specialised mid-merit and peaking firms are the price-taking firms. Initially, each firm only holds 'existing' technologies but, through their respective optimisation problems, may invest in any of the 'new' technologies. Following Devine, Nolan, Lynch, and O'Malley (2019) and Lynch, Nolan, Devine, and O'Malley (2019b) in order to keep a stylized version of the model, wind is incorporated into the model via the (net) demand intercept (see Market Clearing Condition (13)). We assume wind is not owned by any generation firm and its sole function is to reduce net demand. This is because wind has a marginal generation cost of zero and furthermore can only be dispatched downwards. Hence, given an exogenous level of wind capacity, wind generation itself is unlikely to ever be strategically withheld by a generation firm, a fact that is replicated in our model (Devine et al., 2019).

The initial portfolios of each firm are displayed in Table 4. These capacities follow from Lynch et al. (2019a) and Bertsch et al. (2018) and are broadly based on EirGrid (2016).

The different characteristics associated with the technologies are displayed in Table 5. The emissions factors (tonne of carbon dioxide per megawatt hour of electricity, unit: t CO_2/MWh_{el}) were calculated using the following formula:

$$E_t = \frac{E_t^{\mathrm{TH}}}{\mathrm{Efficiency}_t}, \tag{1}$$

where E_t^{TH} is the tonne of carbon dioxide per megawatt hour of thermal energy associated with technology t (unit: t CO_2/MWh_{th}) while $\mathrm{Efficiency}_t$ is the efficiency of technology t (unit: %). We assume the baseload technologies (both new and existing) are powered by hard coal and thus have an emissions factor of $E_t^{\mathrm{TH}} = 0.35$ t CO_2/MWh_{th}. Similarly, we assume the mid-merit and peaking (both new and existing) are powered by natural gas and thus have an emissions factor of $E_t^{\mathrm{TH}} = 0.18$ t CO_2/MWh_{th}. We assume an efficiency of 30% for existing baseload units and 45% for new baseload units. Similarly, we assume that the mid-merit units are Combined Cycle Gas Turbines and thus have efficiencies of 50% for existing units and 60% for new units. Moreover, we assume that

Table 1
Indices and sets.

$f \in F$	Generating firms
$t \in T$	Generating technologies
$p \in P$	Time periods
PT	Price-taking firms
PM	Price-making firms

Table 2
Variables.

Price-taking firms' primal variables	
$gen^{\mathrm{PT}}_{f,t,p}$	Forward generation from price-taking firm f with technology t in period p (MWh)
$inv^{\mathrm{PT}}_{f,t}$	Investment in new generation capacity (technology t) for price-taking firm f (MW)
Price-making firms' primal variables	
$gen^{\mathrm{PM}}_{l,t,p}$	Forward generation from price-making firm l with technology t in period p (MWh)
$inv^{\mathrm{PM}}_{l,t}$	Investment in new generation capacity (technology t) for price-making firm l (MW)
Dual variables	
γ_p	System price for time period p (€ /MWh)
$\lambda^{\mathrm{PT}}_{f,t,p}$	Lagrange multiplier associated with price-taking firm f's capacity constraint for technology t and timestep p (€ /MWh)
$\lambda^{\mathrm{PM}}_{l,t,p}$	Lagrange multiplier associated with price-making firm l's capacity constraint for technology t and timestep p (€ /MWh)

Table 3
Parameters.

$CAP^{\mathrm{PT}}_{f,t}$	Initial hourly generating capacity for price-taking firm f with technology t (MW)
$CAP^{\mathrm{PM}}_{f,t}$	Initial hourly generating capacity for price-making firm l with technology t (MW)
A_p	Demand curve intercept for timestep p (€ /MWh)
B	Demand curve slope (€ /MWh^2)
C^{GEN}_t	Marginal generation cost for technology t (€ /MWh)
IC^{GEN}_t	Investment in generating technology t cost (€ /MW y)
W_p	Weighting (number of hours) associated with forward period p
CV_l	Conjectural variation associated with firm l
E_t	Emissions factor level for technology t (t CO_2/MWh_{el})

Table 4
Initial power generation capacity (hourly) by firm ($CAP_{f,t}$).

Technology	firm 1 price-making	firm 2 price-making	firm 3 price-taking	firm 4 price-taking
Existing baseload (MW)	1947	1940	-	-
Existing mid-merit (MW)	512	-	404	-
Existing peak-load (MW)	270	-	-	234

[1] While firms are free to invest in any of the new technologies, when both the MCP and EPEC modelling frameworks are employed, the generating firms only invest in one technology (new mid-merit).

Table 5
Summary of techno-economic input data of considered supply side technologies.

Technology	Investment cost (IC_t^{GEN}) (€ /MW)	Marginal gen. costs (C_t^{GEN}) (€ /MWh$_{el}$)	Spec. CO_2 emissions (E_t) (t CO_2/MWh$_{el}$)
Existing baseload	-	48.87	1.17
Existing mid-merit	-	41.10	0.36
Existing peak-load	-	63.38	0.56
New baseload	110,769	31.58	0.78
New mid-merit	67,268	34.00	0.30
New peak-load	40,363	50.50	0.45

Table 6
Demand curve intercept (A_p) values.

Time Period (p)	1	2	3	4	5
	25175.993	26768.307	30429.701	34302.196	37465.783

the peaking units are open cycle gas turbines and thus have efficiencies of 32% for existing units and 40% for new units. These values are aligned with the Irish power system.

Both the marginal generation and investment costs in Table 5 follow from Lynch et al. (2019a) and Bertsch et al. (2018). The marginal investment costs represent annualised investment costs, annualised over the lifespan of the unit. Lifespans for baselaod, mid-merit and peaking units are assumed to be 40, 30, and 20 years, respectively. We consider $|P| = 5$ forward time periods. Thus, the prices modelled throughout this paper are forward prices. Considering forward prices allows us to model a smaller number of time periods, making our model computationally tractable. Moreover, forward prices are less likely to be affected by system features such as transmission constraints and start-up costs. Modelling such features often requires integer primal variables and thus prohibit us from deriving Karush-Kuhn-Tucker conditions. Furthermore, we believe investment decisions are more likely to be made based on forward prices.

Table 6 displays the demand curve intercept values which correspond to average hourly values for each time period. However, each time period p is assigned a weight $W_p = \frac{8760}{5}$. Thus, the test case in this work represents one year. Following Lynch and Devine (2017), the five time periods represent summer low demand, summer high demand, winter low demand, winter high demand and winter peak demand. The demand curve slope is $B = 9.091$. This parameter choice follows from Devine et al. (2019) and Di Cosmo and Hyland (2013).

3. Modelling electricity market as a mixed complementarity problem

In this section we describe the Mixed Complementarity Problem (MCP) framework. It represents an electricity market with two types of players: price-making firms and price-taking firms. The difference between price-making and price-taking firms is that we assume that price-making firms have market power and thus behave strategically to maximise their profits. They do so through both their investment and forward generation decisions. For instance, price-making firms may at times increase/decrease their generation to increase/decrease overall market supply and thus decrease/increase market prices. They could also invest in more capacity to increase their generation.

In contrast, we assume price-taking firms do not behave strategically. Instead, we assume that they behave as they would in a fully competitive perfect market setting. That is, price-taking firms assume that their decisions do not affect the market price. They accept whatever the equilibrium price may be, as determined by the overall market.

Each firm chooses its forward market generation decision to maximise its profits. Each firm may also hold multiple generating units with the technologies considered being baseload, mid-merit, and peak-load. The firms are distinguished by their price-making ability and by their initial generation portfolios. However, each firm may also invest in new generation capacity in any of the technologies. In Section 5 we demonstrate the naivety of using an MCP to model an oligopoly with a competitive fringe where both price-making and price-taking firms have investment decisions.

Using a suitable algorithm such as the PATH solver, an MCP determines an equilibrium of multiple optimisation problems by finding a point that satisfies the KKT conditions of each optimisation simultaneously as a system of nonlinear equations (Gabriel, Zhuang, & Egging, 2009). MCPs have been used to model many energy markets (Egging, 2013; Huppmann, 2013). However, in an MCP modelling framework, a price-making firm's optimisation problem does not contain the optimal reactions of price-taking firms as constraints. As Section 5.1 shows, this omission leads to myopic and counter-intuitive outcomes. This is in contrast to our proposed EPEC results described in Section 5.2.

Section 3.1 describes the MCP problem. The MCP consists of the KKT conditions for all price-making and price-taking firms and the market clearing condition.

Throughout this paper the following conventions are used: lowercase Roman letters indicate indices or primal variables, uppercase Roman letters represent parameters, while Greek letters indicate prices or dual variables. The variables in parentheses alongside each constraint in this section are the Lagrange multipliers associated with those constraints. Tables 1–3 explain the indices, variables, and parameters, respectively, associated with both the price-making and price-taking firms' optimisation problems.

3.1. Price-making firm l's problem (in an MCP context)

Price-making firm l seeks to maximise its profits (revenue less costs) by choosing investments in new capacity ($inv_{l,t}^{PM}$) and by choosing the amount of electricity to generate from each technology at each time period ($gen_{l,t,p}^{PM}$). We assume each time period represents a forward time period. Thus $gen_{L,t,p}^{PM}$ represents forward market generation decisions. Firm l's costs include the per unit investment cost (IC_t^{GEN}) and the marginal cost of generation (C_t^{GEN}), while its revenues comes from the forward market price γ_p.

When the problem is presented as an MCP, price-making firm l's optimisation problem takes the following form:

$$\max_{gen_{l,t,p}^{PM}\geq 0, inv_{l,t}^{PM}\geq 0} \sum_{t,p} W_p \times gen_{l,t,p}^{PM} \times \left(\gamma_p - C_t^{GEN}\right) - \sum_{t} IC_t^{GEN} \times inv_{l,t}^{PM}. \quad (2)$$

subject to:

$$gen_{l,t,p}^{PM} \leq CAP_{l,t}^{PM} + inv_{l,t}^{PM}, \quad \forall t, p, \quad \left(\lambda_{l,t,p}^{PM}\right), \quad (3)$$

where the parameter W_p is the weight associated with forward period p. Constraint (3) ensures, for each generating technology in each timestep, firm l cannot generate more than its initial capacity ($CAP_{l,t}^{PM}$) plus any new investments. The variable alongside constraint (3) ($\lambda_{l,t,p}^{PM}$) is the Lagrange multiplier associated with this constraint. In addition, each of firm l's generation and investment decisions are constrained to be non-negative.

The KKT conditions associated with firm l's optimisation problem are

$$0 \leq gen_{l,t,p}^{PM} \perp -W_p \times \left(\gamma_p + \frac{\partial \gamma_p}{\partial gen_{l,t,p}^{PM}} \times gen_{l,t,p}^{PM} - C_t^{GEN}\right) + \lambda_{l,t,p}^{PM} \geq 0, \quad \forall t, p, \quad (4)$$

$$0 \le inv^{\mathrm{PM}}_{l,t} \perp IC^{\mathrm{GEN}}_{t} - \sum_{p} \lambda^{\mathrm{PM}}_{l,t,p} \ge 0, \quad \forall t, \tag{5}$$

$$0 \le \lambda^{\mathrm{PM},1}_{l,t,p} \perp -gen^{\mathrm{PM}}_{l,t,p} + CAP^{\mathrm{PM}}_{l,t} + inv^{\mathrm{PM}}_{l,t} \ge 0, \quad \forall t, p, \tag{6}$$

where

$$\frac{\partial \gamma_p}{\partial gen^{\mathrm{PM}}_{l,t,p}} = -CV_l \times B, \quad \forall l, t, p, \tag{7}$$

is determined via market clearing condition (13). Furthermore, the parameter $CV_l \in [0, 1]$ represents the Conjectural Variation (CV) associated with firm l. CVs have been used in MCP models (Egging et al., 2008; Egging & Holz, 2016; Haftendorn & Holz, 2010; Huppmann & Egging, 2014) as a way for price-making firms to myopically account for the optimal reactions of competitors. CVs take a value in the range [0,1]. Huppmann (2013) proposes a methodology to determine CVs that can be used to overcome myopic behaviour in models of an oligopoly with a competitive fringe. We advance the work of Huppmann (2013) by incorporating investment decisions in an MCP framework.

Firm l's problem described by Eqs. (2) and (3) is a convex optimisation problem and its KKT conditions (4)–(6) are both necessary and sufficient for optimality (Gabriel et al., 2012).

In Section 5.1, we demonstrate that modelling price-making firms via Eqs. (2)–(7) leads to counter-intuitive and unrealistic results. Regardless of the data, Eq. (7) means that whenever $CV_l > 0$, price-making firm l does not correctly account for the optimal reactions of the competitive fringe in an MCP. Consequently, when price-making firm l decreases its generation, the price-taking firms increase their generation to make up for the decrease in the price-making firm l's generation. Thus, the forward market price in the MCP approach does not increase as much the price-making firm l anticipates if it increases at all. The expanded opportunity for price-taking firms to generate enables them to invest (rationally) into new generation if the marginal benefit of generation and investing is greater than the marginal cost.

Furthermore, it is important to note that when $gen^{\mathrm{PM}}_{l,t,p} > 0$, then condition (4) is only satisfied if $\gamma_p \ge C^{\mathrm{GEN}}_{t}$. Thus, when the above MCP is used in Section 5.1, not one generating unit will operate at below marginal cost in the short term, even if such a strategy is profitable in the long run. This is in contrast to the EPEC analysis in Section 5.2 and highlights a further limitation of the MCP modelling approach.

3.2. Price-taking firm f's problem

Price-taking firm f seeks to maximise its profits (revenue less costs) by choosing investments in new capacity ($inv^{\mathrm{PT}}_{f,t}$) and by choosing the amount of electricity to generate from each technology at each forward period ($gen^{\mathrm{PT}}_{f,t,p}$). Firm f's costs include the per unit investment cost (IC^{GEN}_{t}) and the marginal cost of generation (C^{GEN}_{t}) while its revenues comes from the forward market price γ_p.

Price-taking firm f's optimisation problem is as follows:

$$\max_{gen^{\mathrm{PT}}_{f,t,p} \ge 0, inv^{\mathrm{PT}}_{f,t} \ge 0} \quad \mathrm{Profit}^{\mathrm{PT}}_{f} = \sum_{t,p} W_p \times gen^{\mathrm{PT}}_{f,t,p} \times \left(\gamma_p - C^{\mathrm{GEN}}_{t}\right) - \sum_{t} IC^{\mathrm{GEN}}_{t} \times inv^{\mathrm{PT}}_{f,t}, \tag{8}$$

subject to:

$$gen^{\mathrm{PT}}_{f,t,p} \le CAP^{\mathrm{PT}}_{f,t} + inv^{\mathrm{PT}}_{f,t}, \quad \forall t, p, \quad \left(\lambda^{\mathrm{PT}}_{f,t,p}\right). \tag{9}$$

Constraint (9) ensures that, for each generating technology in each timestep, firm f cannot generate more than its initial capacity ($CAP^{\mathrm{PT}}_{f,t}$) plus any new investments. The variable alongside constraint (9) ($\lambda^{\mathrm{PT}}_{f,t,p}$) is the Lagrange multiplier associated with this constraint. In addition, each of firm f's generation and investment decisions are constrained to be non-negative.

The KKT conditions associated with firm f's optimisation problem are

$$0 \le gen^{\mathrm{PT}}_{f,t,p} \perp -W_p \times \left(\gamma_p - C^{\mathrm{GEN}}_{t}\right) + \lambda^{\mathrm{PT}}_{f,t,p} \ge 0, \quad \forall f, t, p, \tag{10}$$

$$0 \le inv^{\mathrm{PT}}_{f,t} \perp IC^{\mathrm{GEN}}_{t} - \sum_{p} \lambda^{\mathrm{PT}}_{f,t,p} \ge 0, \quad \forall f, t, \tag{11}$$

$$0 \le \lambda^{\mathrm{PT},1}_{f,t,p} \perp -gen^{\mathrm{PT}}_{f,t,p} + CAP^{\mathrm{PT}}_{f,t} + inv^{\mathrm{PT}}_{f,t} \ge 0, \quad \forall f, t, p. \tag{12}$$

As firm f is a price-taker, we assume it accepts whatever forward price it receives and thus, the variable γ_p is exogenous to firm f's problem. Consequently, when determining firm f's Karush-Kuhn-Tucker (KKT) conditions, it is assumed $\frac{\partial \gamma_p}{\partial gen^{\mathrm{PT}}_{f,t,p}} = 0$. This is in contrast to the previous subsection. Moreover, firm f cannot see and hence account for the optimal decisions of other price-taking firms in addition to those of price-making firms. As firm f's problem is linear, solving its associated KKT conditions ensures its problem is optimised (Gabriel et al., 2012).

3.3. Market clearing conditions

The forward market price for each time period is determined from the following market clearing condition:

$$\gamma_p = A_p - B \times \left(\sum_{ll,tt} gen^{\mathrm{PM}}_{ll,tt,p} + \sum_{ff,tt} gen^{\mathrm{PT}}_{ff,tt,p} \right), \quad \forall p, \tag{13}$$

where A_p represents the demand curve intercept for each time period while B is the time independent demand curve slope. Condition (13) represents a linear demand curve and allows the market price to increase as the total market generation decreases and vice-versa. Nonlinear demand curves have been used is the literature (Aneiros, Vilar, Cao, & San Roque, 2013; Conejo, Contreras, Arroyo, & De la Torre, 2002; Kelman, Barroso, & Pereira, 2001) and provide a better representation of real electricity markets. We use a linear demand curve because, as Huppmann (2013) explains, a linear demand curve is amenable when using conjectural variations to model market power in MCPs, thus making our assumption reasonable when comparing the MCP and EPEC modelling approaches. This is yet another advantage of the EPEC approach that it can incorporate nonlinear demand curves much easier than the MCP approach.

When the overall market problem is presented as an MCP, the problem consists of the KKT conditions for all price-making firms (Eqs. (4)–(6)), the price-taking firms' KKT conditions (Eqs. (10)–(12)) and the market clearing condition (13).

4. Equilibrium problem with equilibrium constraints

In this section we describe the Equilibrium Problem with Equilibrium Constraints (EPEC) framework. As before, it represents an electricity market with two types of players: price-making firms and price-taking firms. Price-making firms may exert market power by using generation decisions to influence the market price. Price-taking firms do not have such ability.

As in the previous section, each firm chooses its forward market generation decision to maximise its profits. Each firm may also hold multiple generating units with the technologies considered being baseload, mid-merit, and peak-load. The firms are distinguished by their price-making ability and by their initial generation portfolio. However, each firm may also invest in new generation capacity in any of the technologies.

In contrast to the previous section, the optimisation problems of each price-taking firm are embedded into the optimisation problem of each price-making firm. Thus, each price-making firm's problem is a bilevel optimisation problem and can be described as a Mathematical Program with Equilibrium Constraints (MPEC); the equilibrium constraints are the optimality conditions of the price-taking firms. This problem formulation enables each price-making firm to influence the market price through its decision variables, account for the optimal reactions of price-taking firms and maximise their own profits. In the EPEC framework, price-taking firm f's problem remains the same as in the previous section, as described in Section 3.2. Similarly, the Market Clearing Conditions remain the same, as described in Section 3.3.

The overall EPEC problem is to find an equilibrium among the MPEC problem of each price-making firm, which represents a Nash Equilibrium. Each MPEC problem can be represented as a Mixed Integer Nonlinear Problem (MINLP), making it computationally difficult to find Nash Equilibria. To do so, we employ the Gauss-Seidel algorithm (Hori & Fukushima, 2019). Furthermore, to obtain an initial starting solution for this algorithm we use the approach taken by Leyffer and Munson (2010) for solving EPEC problems (henceforth known as the Leyffer-Munson approach). Both the Guass-Seidel algorithm and the Leyffer-Munson approach are described in detail in this section.

4.1. Price-maker l's MPEC

Similar to Section 3.1, price-making firm l's optimisation problem is to maximise profits (revenues less cost) by choosing its investment ($inv^{PM}_{l,t}$) and forward market generation ($gen^{PM}_{l,t,p}$) decisions. As before, firm l's revenues come from the forward market price while its costs include marginal generation and investment costs. In contrast to Section 3.1 however, price-making firm l also accounts for the optimal reactions of the price-taking firms. Thus firm l's bilevel optimisation problem takes the form:

$$\max_{\substack{gen^{PM}_{l,t,p},inv^{PM}_{l,t}\\ gen^{PT}_{f,t,p},inv^{PT}_{f,t}\\ \gamma_p,\lambda^{PT}_{f,t,p}}} \quad \text{Profit}^{PM}_l, \tag{14}$$

subject to:

$$\text{Constraints}^{PM}_l \le 0, \tag{15}$$

$$gen^{PT}_{f,t,p}, inv^{PT}_{f,t}, \lambda^{PT}_{f,t,p} \in \text{argmax}\left\{\text{Profit}^{PT}_f : \text{Constraints}^{PT}_f, f\in F\right\}, \tag{16}$$

where Profit^{PM}_l and Profit^{PT}_f represent price-making firm l's and price-taking firm f's profits, respectively, while $\text{Constraints}^{PM}_l$ and $\text{Constraints}^{PT}_f$ represent their set of constraints, respectively.

In detail, firm l's objective function is

$$\max_{\substack{gen^{PM}_{l,t,p}\ge 0,inv^{PM}_{l,t}\ge 0\\ gen^{PT}_{f,t,p}\ge 0,inv^{PT}_{f,t}\ge 0\\ \lambda^{PT}_{f,t,p}\ge 0,\gamma_p}} \text{Profit}^{PM}_l = \sum_{t,p} W_p \times gen^{PM}_{l,t,p} \times\left(\gamma_p - C^{GEN}_t\right) - \sum_t IC^{GEN}_t \times inv^{PM}_{l,t}. \tag{17}$$

As firm l can influence the market price through its generation decisions, we re-write objective function (17) using market clearing condition (13) as follows:

$$\max_{\substack{gen^{PM}_{l,t,p}\ge 0,inv^{PM}_{l,t}\ge 0\\ gen^{PT}_{f,t,p}\ge 0,inv^{PT}_{f,t}\ge 0\\ \lambda^{PT}_{f,t,p}\ge 0,\gamma_p}} \sum_{t,p} W_p \times \left(A_p - B\times\left(\sum_{ll,tt} gen^{PM}_{ll,tt,p} + \sum_{ff,tt} gen^{PT}_{ff,tt,p}\right) - C^{GEN}_t\right)\times gen^{PM}_{l,t,p} - \sum_t IC^{GEN}_t \times inv^{PM}_{l,t}. \tag{18}$$

The constraints of price-making firm l's problem are

$$gen^{PM}_{l,t,p} \le CAP^{PM}_{l,t} + inv^{PM}_{l,t}, \quad \forall t,p, \quad \left(\lambda^{PM}_{l,t,p}\right), \tag{19}$$

where each of firm l's generation and investment decisions are also constrained to be non-negative. As with the price-taking firms, constraint (19) ensures, for each technology at each time period, firm l cannot generate more electricity than its initial capacity plus any new investments. In addition to constraint (19), firm l's constraints also include the KKT conditions of each price-taking firm:

$$0 \le gen^{PT}_{f,t,p} \perp -W_p \times \left(\gamma_p - C^{GEN}_t\right) + \lambda^{PT}_{f,t,p} \ge 0, \quad \forall f,t,p, \tag{20}$$

$$0 \le inv^{PT}_{f,t} \perp IC^{GEN}_t - \sum_p \lambda^{PT}_{f,t,p} \ge 0, \quad \forall f,t, \tag{21}$$

$$0 \le \lambda^{PT,1}_{f,t,p} \perp -gen^{PT}_{f,t,p} + CAP^{PT}_{f,t} + inv^{PT}_{f,t} \ge 0, \quad \forall f,t,p. \tag{22}$$

Using market clearing condition (13) leads to condition (20) being re-written as follows:

$$0 \le gen^{PT}_{f,t,p} \perp -W_p \times \left(A_p - B\times\left(\sum_{ll,tt} gen^{PM}_{ll,tt,p} + \sum_{ff,tt} gen^{PT}_{ff,tt,p}\right) - C^{GEN}_t\right) + \lambda^{PT}_{f,t,p} \ge 0, \quad \forall f,t,p. \tag{23}$$

Constraints (21)–(23) represent the optimal reactions of each price-taking firm. As firm f's problem (Eqs. (8) and (9)) is a linear optimisation problem, the KKT conditions are both necessary and sufficient for optimality for the price-taking firms (Gabriel et al., 2012).

Incorporating these conditions as constraints ensures firm l correctly anticipates how each price-taking firm will react to its decisions. Thus, this allows firm l to adjust its decisions accordingly when maximising profits.

Firm l's optimisation problem is affected by the generation decisions of all other price-making firms as shown in objective function (18) and constraint (23). However, we assume the decisions of all other price-making firms are fixed and exogenous to firm l's problem. Sections 4.2–4.4 describe how the optimisation problems of all price-making firms are solved simultaneously such that solutions to the EPEC represent Nash equilibria.

As the KKT conditions (21)–(23) represent the equilibrium constraints (optimal reactions) of the price-taking firms, firm l's problem is a Mathematical Program with Equilibrium Constraints. We denote this problem as MPEC_l, which is a nonlinear mathematical program because of the bilinear terms in objective function (18). Following the approach presented in Fortuny-Amat and McCarl (1981), we can remove the nonlinearities resulting from complementarity conditions by using disjunctive constraints and big M notation. Consequently, this leads to constraints (21)–(23) being re-written as follows:

$$0 \le gen^{PT}_{f,t,p} \le M \times r^1_{f,t,p}, \quad \forall f,t,p, \tag{24}$$

$$0 \leq -W_p \times \left(A_p - B \times \left(\sum_{ll,tt} gen^{\text{PM}}_{ll,tt,p} + \sum_{ff,tt} gen^{\text{PT}}_{ff,tt,p} \right) - C^{\text{GEN}}_t \right) + \lambda^{\text{PT}}_{f,t,p} \leq M \times \left(1 - r^1_{f,t,p}\right), \quad \forall f, t, p, \tag{25}$$

$$0 \leq inv^{\text{PT}}_{f,t} \leq M \times r^2_{f,t}, \quad \forall f, t, \tag{26}$$

$$0 \leq IC^{\text{GEN}}_t - \sum_p \lambda^{\text{PT}}_{f,t,p} \leq M \times \left(1 - r^2_{f,t}\right), \quad \forall f, t, \tag{27}$$

$$0 \leq \lambda^{\text{PT},1}_{f,t,p} \leq M \times r^3_{f,t,p}, \quad \forall f, t, p. \tag{28}$$

$$0 \leq -gen^{\text{PT}}_{f,t,p} + CAP^{\text{PT}}_{f,t} + inv^{\text{PT}}_{f,t} \leq M \times \left(1 - r^3_{f,t,p}\right), \quad \forall f, t, p. \tag{29}$$

where $r^1_{f,t,p}$, $r^2_{f,t}$ and $r^3_{f,t,p}$ all represent binary 0–1 variables.

Note that the big-M values are chosen so that they do not interfere with any of the solutions. Consequently, in all our numerical results, no upper-bound constraints with the big-M values were binding in any solution. When solving the overall EPEC problem using the Gauss-Seidel algorithm (see Section 4.2), MPEC$_l$ is characterized by objective function (18), subject to constraint (19) and constraints (24)–(29). Consequently price-making firm l's optimisation problem is a Mixed Integer Nonlinear Problem. In Section 5.2, we use the DICOPT solver in GAMS to solve it.

4.2. Overall EPEC

The overall EPEC can be expressed as the problem of finding Nash equilibria among the price-makers l:

$$\text{Find:} \quad \Big\{ inv^{\text{PM}}_{l=1,t}, \ldots, inv^{\text{PM}}_{l=L,t}, gen^{\text{PM}}_{l=1,t,p}, \ldots, gen^{\text{PM}}_{l=L,t,p}, inv^{\text{PT}}_{f=1,t}, \ldots, inv^{\text{PT}}_{f=F,t}, gen^{\text{PT}}_{f=1,t,p}, \ldots, gen^{\text{PT}}_{f=F,t,p}, \lambda^{\text{PT}}_{f=1,t,p}, \ldots, \lambda^{\text{PT}}_{f=F,t,p}, \gamma_p \Big\} \text{ that solve: MPEC}_l \text{ for each } l = 1, \ldots, L.$$

To find the such equilibria, we implement the following Gauss-Seidel (Gabriel et al., 2012) algorithm. The algorithm iteratively solves each price-making firm's MPEC problem by fixing all other price-making firms' decisions, until it converges to a point where neither leader has an optimal deviation. where TOL and K represent a pre-defined convergence tolerance and a maximum number of allowable iterations, respectively. The vector $x_{l,t}$ represents the vector of all MPEC$_l$'s primal variables at iteration k. Thus, the investment decisions and generation mix for the competitive fringe are included in our convergence checks, ensuring we do not end up with inconsistent convergence points.

Note that Algorithm 1 is based on Gauss-Seidel diagonalization, which is standard in the literature (Gabriel et al., 2012; Steffensen & Bittner, 2014; Su, 2004). As described in the literature, if Algorithm 1 converges, the point it converges to is guaranteed to be a Nash Equilibrium if each MPEC is solved to optimality in each iteration. Our formulation of disjunctive constraints in Section 4.1 guarantees that if each MPEC$_l$ solves to optimality and the Gauss-Seidel iteration converges, we are at an equilibrium point for the EPEC (Gabriel et al., 2012).

Algorithm 1: Gauss-Seidel algorithm where $x_{l,t}$ represents the vector of all MPEC$_l$'s primal variables at iteration k.

```
while Σ_l |x_{l,k} − x_{l,k−1}| > TOL and k < K do
    for l = 1, ..., L do
        Assume price maker_l's decision variables are fixed;
        Solve MPEC_l;
    end
end
```

4.3. Obtaining an initial solution

To improve computational efficiency, we utilise the approach to obtaining a strong stationary point for EPECs, as described in Leyffer and Munson (2010). We then use the stationary point obtained as a starting point to the Gauss-Seidel algorithm. In this subsection, we describe the Leyffer-Munson method as applied to the EPEC presented in this work.

First, we re-write price-making firm l's problem, as defined by Eqs. (18)–(23), using slack variables. We do this by converting firm l's inequality constraints into equality constraints as follows:

$$\max \sum_{t,p} W_p \times \left(A_p - B \times \left(\sum_{ll,tt} gen^{\text{PM}}_{ll,tt,p} + \sum_{ff,tt} gen^{\text{PT}}_{ff,tt,p} \right) - C^{\text{GEN}}_t \right) \times gen^{\text{PM}}_{l,t,p} - \sum_t IC^{\text{GEN}}_t \times inv^{\text{PM}}_{l,t}, \tag{30}$$

subject to:

$$CAP^{\text{PM}}_{l,t} + inv^{\text{PM}}_{l,t} - gen^{\text{PM}}_{l,t,p} - s^{\text{CON\_LR}}_{l,t,p} = 0, \quad \forall t, p, \quad \left(\lambda^{PM}_{l,t,p}\right), \tag{31}$$

$$gen^{\text{PM}}_{l,t,p} \geq 0, \quad \forall t, p, \quad \left(\chi^{\text{GEN}}_{l,t,p}\right), \tag{32}$$

$$inv^{\text{PM}}_{l,t} \geq 0, \quad \forall t, \quad \left(\chi^{\text{INV}}_{l,t}\right), \tag{33}$$

$$s^{\text{CON\_LR}}_{l,t,p} \geq 0, \quad \forall t, p, \quad \left(\mu^{\text{CON\_LR}}_{l,t,p}\right), \tag{34}$$

$$-W_p \times \left(A_p - B \times \left(\sum_{ll,tt} gen^{\text{PM}}_{ll,tt,p} + \sum_{ff,tt} gen^{\text{PT}}_{ff,tt,p} \right) - C^{\text{GEN}}_t \right) + \lambda^{\text{PT}}_{f,t,p} - s^{\text{KKT\_GEN}}_{f,t,p} = 0, \quad \forall f, t, p, \quad (\alpha^{\text{KKT\_GEN}}_{l,f,t,p}), \tag{35}$$

$$gen^{\text{PT}}_{f,t,p} \geq 0, \quad \forall f, t, p, \quad \left(\mu^{\text{KKT\_GEN}}_{l,f,t,p}\right), \tag{36}$$

$$s^{\text{KKT\_GEN}}_{f,t,p} \geq 0, \quad \forall f, t, p, \quad \left(\mu^{\text{s\_KKT\_GEN}}_{l,f,t,p}\right), \tag{37}$$

$$gen^{\text{PT}}_{f,t,p} \times s^{\text{KKT\_GEN}}_{f,t,p} = 0, \forall f, t, p, \quad \left(\mu^{\text{GEN\_s\_KKT\_GEN}}_{l,f,t,p}\right), \tag{38}$$

$$IC^{\text{GEN}}_t - \sum_p \lambda^{\text{PT}}_{f,t,p} - s^{\text{KKT\_INV}}_{f,t} = 0, \quad \forall f, t, \quad \left(\alpha^{\text{KKT\_INV}}_{l,f,t}\right), \tag{39}$$

$$inv^{\text{PT}}_{f,t} \geq 0, \quad \forall f, t, \quad \left(\mu^{\text{KKT\_INV}}_{l,f,t}\right), \tag{40}$$

$$s^{\text{KKT\_INV}}_{f,t} \geq 0, \quad \forall f, t, \quad \left(\mu^{\text{KKT\_s\_INV}}_{l,f,t}\right), \tag{41}$$

$$inv^{PT}_{f,t} \times s^{KKT\_INV}_{f,t} = 0, \quad \forall f,t, \quad \left(\mu^{INV\_s\_KKT\_INV}_{l,f,t}\right), \tag{42}$$

$$-gen^{PT}_{f,t,p} + CAP^{PT}_{f,t} + inv^{PT}_{f,t} - s^{CON\_FR}_{f,t,p} = 0 \quad \forall f,t,p, \quad \left(\alpha^{CON}_{f,t,p}\right), \tag{43}$$

$$\lambda^{PT}_{f,t,p} \geq 0, \quad \forall f,t,p, \quad \left(\mu^{CON}_{l,f,t,p}\right), \tag{44}$$

$$s^{CON\_FR}_{f,t,p} \geq 0, \quad \forall f,t,p, \quad \left(\mu^{s\_CON}_{l,f,t,p}\right), \tag{45}$$

$$\lambda^{PT}_{f,t,p} \times s^{CON\_FR}_{f,t,p} = 0, \quad \forall f,t,p, \quad \left(\mu^{CON\_s\_CON}_{l,f,t,p}\right). \tag{46}$$

The variables in brackets alongside each of these constraints are the Lagrange multipliers associated with those constraints. Each multiplier has a subscript l associated with it showing how there are unique multipliers for each price-making firm.

Secondly, we find the stationary KKT conditions of the optimisation problem (30)–(46). Let $\mathcal{L}_l$ be the Lagrangian associated with that problem.

$$\frac{\partial \mathcal{L}_l}{\partial gen^{PM}_{l,t,p}} : -W_p \times \left(A_p - B \times \left(\sum_{ll,t} gen^{PM}_{ll,tt,p} + \sum_{ff,tt} gen^{PT}_{ff,tt,p}\right) - B \times gen^{PM}_{l,t,p} - C^{GEN}_t\right) + \lambda^{PT}_{f,t,p} + \sum_{ff,tt} W_p \times B \times \alpha^{KKT\_GEN}_{l,ff,tt,p} - \chi^{GEN}_{l,t,p} = 0, \quad \forall t,p, \tag{47}$$

$$\frac{\partial \mathcal{L}_l}{\partial inv^{PM}_{l,t}} : IC^{GEN}_t - \sum_p \lambda^{PM}_{l,t,p} - \chi^{INV}_{l,t} = 0, \quad \forall t, \tag{48}$$

$$\frac{\partial \mathcal{L}_l}{\partial gen^{PT}_{f,t,p}} : \sum_{tt} W_p \times B \times gen^{PM}_{l,tt,p} + \sum_{ff,tt} W_p \times B \times \alpha^{KKT\_GEN}_{l,ff,tt,p} - \mu^{KKT\_GEN}_{l,f,t,p} + s^{KKT\_GEN}_{f,t,p} \times \mu^{GEN\_s\_KKT\_GEN}_{l,f,t,p} - \alpha^{CON}_{f,t,p} = 0, \quad \forall f,t,p, \tag{49}$$

$$\frac{\partial \mathcal{L}_l}{\partial inv^{PT}_{f,t}} : -\mu^{KKT\_INV}_{l,f,t} + s^{KKT\_INV}_{f,t} \times \mu^{INV\_s\_KKT\_INV}_{l,f,t} - \sum_p \alpha^{CON}_{f,t,p} = 0, \quad \forall f,t, \tag{50}$$

$$\frac{\partial \mathcal{L}_l}{\partial \lambda^{PT}_{f,t,p}} : -\alpha^{KKT\_GEN}_{l,f,t,p} + \alpha^{KKT\_INV}_{l,f,t} - \mu^{CON}_{l,f,t,p} + s^{CON\_FR}_{f,t,p} \times \mu^{CON\_s\_CON}_{l,f,t,p} = 0, \quad \forall f,t,p, \tag{51}$$

$$\frac{\partial \mathcal{L}_l}{\partial s^{CON\_LR}_{l,t,p}} : \lambda^{PM}_{l,t,p} - \mu^{CON\_LR}_{l,t,p} = 0, \quad \forall t,p, \tag{52}$$

$$\frac{\partial \mathcal{L}_l}{\partial s^{KKT\_GEN}_{f,t,p}} : -\alpha^{KKT\_GEN}_{l,f,t,p} - \mu^{s\_KKT\_GEN}_{l,f,t,p} + gen^{PT}_{f,t,p} \times \mu^{GEN\_s\_KKT\_GEN}_{l,f,t,p} = 0, \quad \forall f,t,p, \tag{53}$$

$$\frac{\partial \mathcal{L}_l}{\partial s^{KKT\_INV}_{f,t}} : -\alpha^{KKT\_INV}_{l,f,t} - \mu^{KKT\_s\_INV}_{l,f,t} + inv^{PT}_{f,t} \times \mu^{INV\_s\_KKT\_INV}_{l,f,t} = 0, \quad \forall f,t, \tag{54}$$

$$\frac{\partial \mathcal{L}_l}{\partial s^{CON\_FR}_{f,t,p}} : -\alpha^{CON}_{f,t,p} - \mu^{s\_CON}_{l,f,t,p} + \lambda^{PT}_{f,t,p} \times \mu^{CON\_s\_CON}_{l,f,t,p} = 0, \quad \forall f,t,p. \tag{55}$$

In addition, each of the Lagrange multipliers associated with inequality constraints in (30)–(46) is constrained to be non-negative.

Following this, we find the complementary KKT conditions of the optimisation problem (30)–(46) as follows:

$$gen^{PM}_{l,t,p} \times \chi^{GEN}_{l,t,p} = 0, \quad \forall t,p, \tag{56}$$

$$inv^{PM}_{l,t} \times \chi^{INV}_{l,t} = 0, \quad \forall t, \tag{57}$$

$$s^{CON\_LR}_{l,t,p} \times \mu^{CON\_LR}_{l,t,p} = 0, \quad \forall t,p, \tag{58}$$

$$gen^{PT}_{f,t,p} \times \mu^{KKT\_GEN}_{l,f,t,p} = 0, \quad \forall f,t,p, \tag{59}$$

$$s^{KKT\_GEN}_{f,t,p} \times \mu^{s\_KKT\_GEN}_{l,f,t,p} = 0, \quad \forall f,t,p, \tag{60}$$

$$inv^{PT}_{f,t} \times \mu^{KKT\_INV}_{l,f,t} = 0, \quad \forall f,t, \tag{61}$$

$$s^{KKT\_INV}_{f,t} \times \mu^{KKT\_s\_INV}_{l,f,t} = 0, \quad \forall f,t, \tag{62}$$

$$\lambda^{PT}_{f,t,p} \times \mu^{CON}_{l,f,t,p} = 0, \quad \forall f,t,p, \tag{63}$$

$$s^{CON\_FR}_{f,t,p} \times \mu^{s\_CON}_{l,f,t,p} = 0, \quad \forall f,t,p. \tag{64}$$

The Leyffer-Munson method obtains a solution set that satisfies conditions (31)–(64) of each price-making firm l simultaneously[2] To do this, each of the KKT conditions with bilinear terms (Eqs. (38), (42), (46) and (56)–(64)) are removed as constraints and are summed together to create the following objective function:

$$\min \sum_{f,t,p} gen^{PT}_{f,t,p} \times s^{KKT\_GEN}_{f,t,p} + \sum_{f,t} inv^{PT}_{f,t} \times s^{KKT\_INV}_{f,t} + \sum_{f,t,p} \lambda^{PT}_{f,t,p} \times s^{CON\_FR}_{f,t,p} + \sum_{l,t,p} gen^{PM}_{l,t,p} \times \chi^{GEN}_{l,t,p} + \sum_{l,t} inv^{PM}_{l,t} \times \chi^{INV}_{l,t} + \sum_{l,t,p} s^{CON\_LR}_{l,t,p} \times \mu^{CON\_LR}_{l,t,p} + \sum_{f,t,p} gen^{PT}_{f,t,p} \times \mu^{KKT\_GEN}_{l,f,t,p} + \sum_{f,t,p} s^{KKT\_GEN}_{f,t,p} \times \mu^{s\_KKT\_GEN}_{l,f,t,p} + \sum_{f,t} inv^{PT}_{f,t} \times \mu^{KKT\_INV}_{l,f,t}$$

[2] Note, that this is not the same as solving an MCP representing all firms' KKT conditions simultaneously because the conditions above were derived by taking first order conditions of each leader's problem as opposed to the standard MCP representation where conditions are derived for each player without consideration of leaders or followers.

$$+\sum_{f,t} s^{\text{KKT\_INV}}_{f,t} \times \mu^{\text{KKT\_s\_INV}}_{l,f,t}$$
$$+\sum_{f,t,p} \lambda^{\text{PT}}_{f,t,p} \times \mu^{\text{CON}}_{l,f,t,p} + \sum_{f,t,p} s^{\text{CON\_FR}}_{f,t,p} \times \mu^{\text{s\_CON}}_{l,f,t,p}. \quad (65)$$

Thus, the Leyffer-Munson optimisation problem is to minimise Eq. (65) subject to constraints (31)–(37), (39)–(41), (43)–(45) and (47)–(55). In addition, each of the Lagrange multipliers associated with inequality constraints in (30)–(46) are constrained to be non-negative. The Leyffer-Munson optimisation problem is a nonlinear Program (NLP) and, in Section 5.2, we use the CONOPT solver in GAMS to solve it. When the objective function value output of this optimization problem is zero, it implies that the solution is a feasible point to the EPEC.

4.4. Overall algorithm

Algorithm 2 describes the overall algorithm for finding Nash equilibria from the EPEC problem. For iteration i, we first provide a random feasible point from the search space and use it as an initial starting point for the Leyffer-Munson approach. As the Leyffer-Munson approach is a nonlinear optimisation problem, the CONOPT solver does not always find a local minimum. If the Leyffer-Munson method does not converge to a locally optimal solution, then iteration i is deemed unsuccessful and the algorithm skips ahead to iteration $i+1$. If the Leyffer-Munson approach does converge however, the locally optimal solution is then used as starting point for the Gauss-Seidel algorithm. If the Gauss-Seidel algorithm does (not) converge to a Nash equilibrium solution, then iteration i is (not) deemed successful. This process is repeated for I iterations.

Algorithm 2: Overall algorithm for finding Nash Equilibria.

```
for i = 1, ..., I do
    Provide random initial solutions;
    Solve Leyffer-Munson optimisation problem;
    if Solution from LM is locally optimal then
        Solve EPEC using Gauss-Seidel algorithm using
        solutions from LM as starting point ;
        if Gauss-Seidel algorithm converges then
            Save solution;
        end
    end
end
```

An initial starting point in Algorithm 2 provides a big computational boost in our efforts to solve the EPEC. However, there is recent research that provides algorithms for EPECs that could potentially increase the computational tractability of our method (e.g., Fanzeres, Street, & Pozo, 2020 and Jara-Moroni, Pang, & Wächter, 2018). We believe some of these methods would improve the computational tractability of Algorithm 1 as well.

5. Results

In this section, we present the results of the paper. First, we present the results when the data presented in Section 2 are applied to the MCP model described in Section 3. This analysis shows that an MCP is inappropriate for modelling an oligopoly with a competitive fringe and investment decisions as it leads to myopic and counter-intuitive behaviour by the price-making firms.

Following this, we present the results when the data presented in Section 2 is applied to the EPEC model described in Section 4. This analysis shows how an EPEC can more credibly model strategic behaviour in markets characterised by an oligopoly with a competitive fringe and investment decisions.

5.1. Results from MCP model

The MCP is solved eleven times. Each time with a different conjectural variation (CV) for the two price-making firms; both firms have the same CV in each case. When $CV_{l=1} = CV_{l=2} = 0$, both price-making firms lose their price-making ability and thus the market outcome corresponds to perfect competition. The remaining cases correspond to an oligopoly with a competitive fringe modelled through conjectural variations.

Fig. 1 describes the profits of the price-making firm $l=1$ for the MCP cases. It shows that firm $l=1$ makes less profits in the cases with an oligopoly and a competitive fringe, compared with the perfect competition case. Clearly, if firms have price-making ability then they should be able to use that ability, at the very least, to make the same profits as they would have in a perfect competition setting.

The result can be explained by Fig. 2 which shows the investment in new mid-merit generation for perfect competition case and $CV_{l=1} = CV_{l=2} = 1$ case (similar results are observed for the $0 < CV_l < 1$ cases). In the perfect competition case, all firms invest 713MW into of new mid-merit generation. However, in the oligopoly with a competitive fringe case, the two price-making firms do not invest in any new technology while the two price-taking firms each increase their investment in new mid-merit generation to 1736MW. Note: in both cases, there are zero investments in new baseload or new peaking generation by either the price-making or price-taking firms. As Eqs. (4) and (7) show, price-making firms $l=1$ and $l=2$ assume $\frac{\partial \gamma_p}{\partial gen^{\text{PM}}_{l,t,p}} = -CV_l \times B$ in the oligopoly with a competitive fringe case. These means that these two firms assume that if they decrease the amount of electricity they generate by one MW, then the equilibrium forward market price will increase by € $CV_l \times B$. In seeking to increase profits, price-making firms $l=1$ and $l=2$ decrease their generation in this way and hence do no invest in any new technology, as there is no point if they are not going to fully use that new generation.

However, in the MCP with CV setting, price-making firms do not correctly account for the optimal reactions of the competitive fringe. Consequently, when price-making firms decrease their generation, the price-taking firms increase their generation and replace the price-making firms' generation. Thus, the forward market price does not increase as much as the price-making firms anticipate if it increases at all. The expanded opportunity for price-taking firms to generate enables them to invest further into new mid-merit generation, as evidenced by Fig. 2.

Moreover, Fig. 2 and Table 4 show that the total capacities of the price-taking firms are similar to those of the price-making firms in the oligopoly with a competitive fringe case. This further highlights that an MCP is an inappropriate framework for modelling investment decisions in an oligopoly with a competitive fringe because it leads to market outcomes that are not appropriately described by an oligopoly with a competitive fringe.

To quantify the effects of the oligopolists not anticipating the competitive fringe's investment decisions, we consider two analyses:

- First, we consider a scenario where the oligopolists do not anticipate both the investment and generation decisions of the competitive fringe by solving the MCP model with $CV_{l=1} = CV_{l=2} = 1$.
- Second, we consider a scenario where the oligopolists do not anticipate the investment decisions of the competitive fringe but do anticipate the fringe's generation decisions. We do this

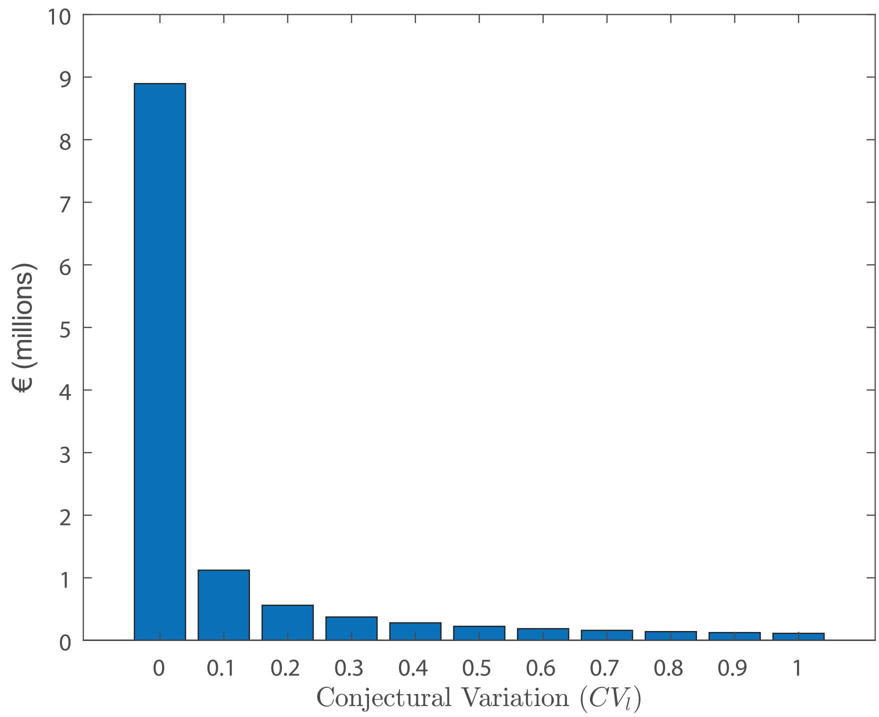


Fig. 1. Price-making firm $l = 1$'s profits using MCP modelling approach.

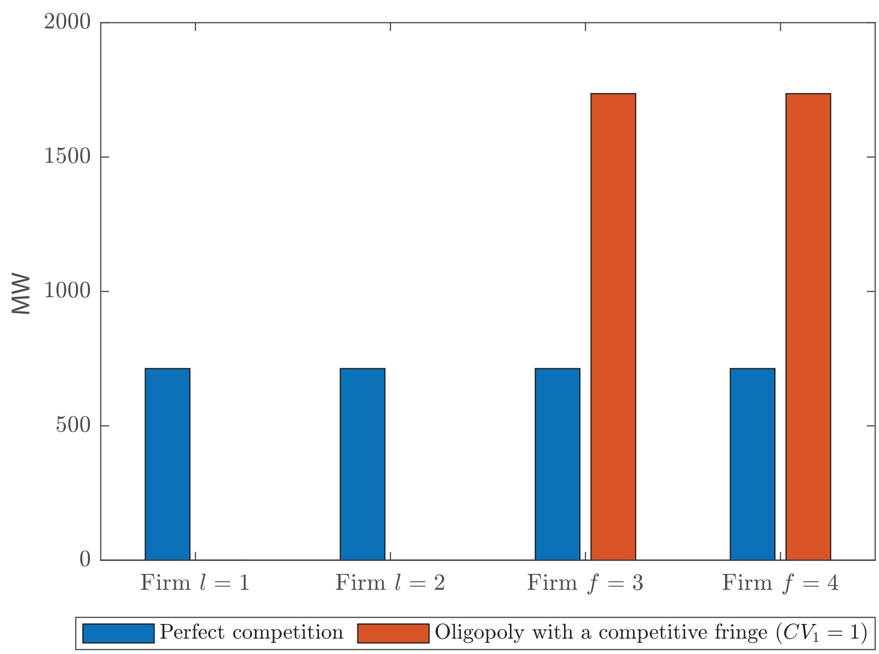


Fig. 2. Price-making firms' investment into new mid-merit generation under MCP modelling approach.

by solving the EPEC model with the price-taking firms' investment decisions fixed at the values observed from the MCP model, i.e., both firm $f = 1$'s and $f = 2$'s investment in new mid-merit generation is fixed at 1736MW.

Table 7 displays the differences in profits, for both price-making firms firms $l = 1$ and $l = 2$, between their highest levels observed from the full EPEC model (Figs. 3 and 4) and the analyses. Not anticipating both the investment and generation decisions of the competitive fringe costs € 12.838M and € 12.896M, for price-making firms firms $l = 1$ and $l = 2$, respectively. When only the investment decisions are not anticipated, the differences are lower at € 12.798M and € 12.894M, respectively.

Table 7
Differences in profits (€ millions) when the oligopolists do not anticipate the competitive fringe's investment and generation decisions (Scenario 1) and when they only anticipate generation decisions (Scenario 2).

	Scenario 1	Scenario 2
Firm $l = 1$	12.838	12.798
Firm $l = 2$	12.896	12.894

The assumption that $\frac{\partial \gamma_p}{\partial gen^{PM}_{l,t,p}} = -CV_l \times B$ in the oligopoly with a competitive fringe case is not valid in an MCP setting where investment decisions are also incorporated. However, this assump-

Fig. 3. Profits for price-making firm $l = 1$ for each successful iteration.

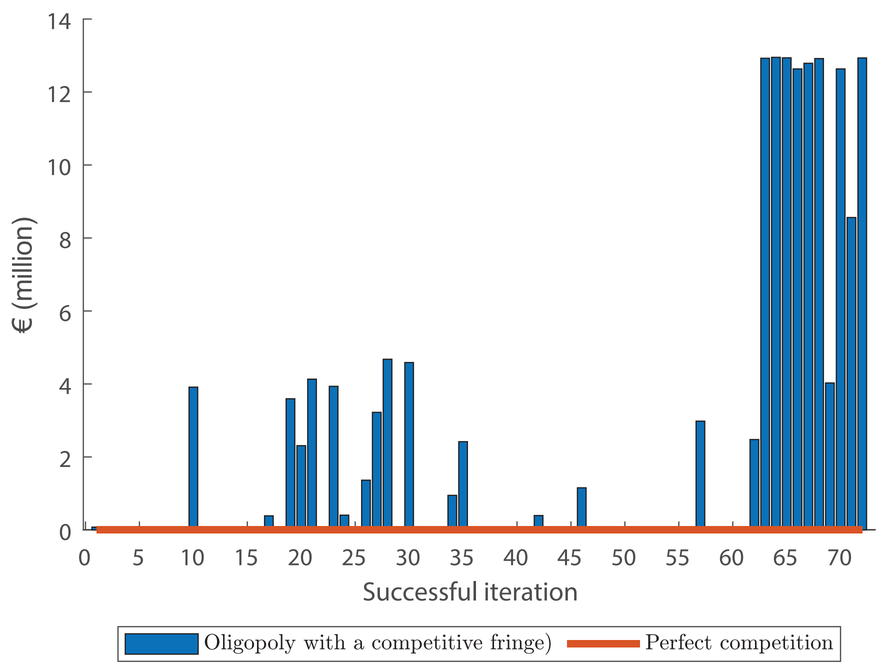


Fig. 4. Profits for price-making firm $l = 2$ for each successful iteration.

tion is valid in an MCP setting if all firms are price-making firms and hence behave in the same manner. In other words, when one firms seeks to increase the market price by decreasing its generation, so does the rest of the firms and no one firm replaces the decreased generation from any other firm. Appendix B describes the results when all the four generating firms considered in this section are modelled as price-makers. This section demonstrates how the MCP modelling approach is unsuited to modelling an oligopoly with a competitive fringe when investment decisions are also accounted for. Moreover, conjectural variations cannot overcome this modelling issue. Placing constraints on the investment levels or on the financial capability of firms are ways to overcome these issues. However, determining these constraints would not necessarily be reflective of reality and the parameters of such constraints would be arbitrary. In the following section, we show how the EPEC modelling approach overcomes the short-sighted/myopic behaviour observed in this section.

5.2. Results from EPEC model

In this section, we present the results when the data presented in Section 2 are applied to the EPEC model described in Section 4. We focus on the firms' profits and investment decisions, forward market prices, carbon emissions, consumer costs and social welfare. To obtain these results we utilise the algorithm described in Section 4.4 for $I = 2000$ iterations. For the first 1000 iterations firm

$l = 1$'s MPEC problem converged before firm $l = 2$'s MPEC problem. For the subsequent 1000 iterations the opposite applies and firm $l = 2$'s MPEC problem converged before firm $l = 1$'s MPEC problem.

The algorithm did not always find a Nash Equilibrium (NE) solution. In fact, in the results to follow, only 72 of the 2000 iterations successfully found a NE solution, henceforth known as successful iterations. Of these, 62 iterations occurred when firm $l = 1$'s MPEC problem converged before firm $l = 2$'s MPEC problem while the remaining 10 successful iterations occurred when firm $l = 2$'s MPEC problem converged first. For unsuccessful iterations, the algorithm failed to find a NE solution for one of two reasons:

1. For the random initial solution provided, the Leyffer-Munson approach was found to be locally infeasible by the CONOPT solver.
2. For the Gauss-Seidel algorithm, the convergence tolerance remained greater than $TOL = 10^{-3}$ after $K = 100$ iterations.

For each of the 72 successful iterations, the Leyffer-Munson approach converged to a locally optimal solution which implied that it is not necessarily a feasible solution to the EPEC. For 43% of these successful iterations, the objective function for the Leyffer-Munson approach (Eq. (65)) converged to zero. For the remaining 57% of successful iterations, the objective function converged to a strictly positive objective function value. When the Leyffer-Munson approach gives a non-zero objective function value, the solutions cannot be guaranteed to be a feasible point for the overall EPEC. However, the results in this section show that, despite this, such solutions can still provide good starting point solutions to the Gauss-Seidel algorithm.

5.2.1. Equilibrium profit and investment levels

Figs. 3 and 4 display price-making firms $l = 1$ and $l = 2$ profits, respectively, for each of the successful iterations. The horizontal lines in each figure represent the profits each firm would make from the perfect competition case in the Section 5.1. Figs. 3 and 4 both show that the algorithm found multiple NE solutions. Firm $l = 1$'s profits varies from € 0 to € 12.98M while firm $l = 2$'s profits ranged from € 50,000 to € 6.8M. For most equilibria, both firms made profits greater than they would in a perfect framework. This occurred in 66.7% and 100% of the successful iterations for firms $l = 1$ and $l = 2$ profits, respectively. There was no NE found where both firms' profits were below their perfect competition equivalent.

Fig. 5 displays the combined profits of the two price-making firms. It shows that the combined profits varied across equilibria, suggesting that there was not a zero-sum game between the price-making firms on how profits were split between them. In Appendix A, we describe why there are multiple equilibria and why, in some equilibria, one of the price-making firms makes a profit less than it would in a perfectly competitive market.

Fig. 6 displays the combined investments into new mid-merit generation for the two price-making firms for each successful iteration. In comparison to Section 5.1, both of these firms did not invest in any baseload or peaking generation at any equilibrium point.

For most equilibria (80%), the combined investments of the two price-making firms were 2941MW. But, for some equilibria, the combined investments were slightly higher with maximum combined investment reaching 2967MW at one equilibrium point while the lowest combined investment was 2831MW. The first 62 successful iterations in Figs. 3–5 show when firm $l = 1$'s MPEC problem was solved before firm $l = 2$'s. At these equilibria, firm $l = 1$ and $l = 2$'s investments in new mid-merit generation averaged at 2807MW and 496MW, respectively. The final 10 successful iterations occurred when firm $l = 2$'s MPEC problem was solved first. At these equilibria, firm $l = 1$ average investments in new generation decreased to 1467MW while firm $l = 2$'s increased to 2618MW.

In contrast to Section 5.1, in all but one equilibrium, the two price-taking firms did not invest in any new generation technology. The exception was at the 30th successful iteration, where firm $f = 3$ and $f = 4$ invested 2766MW and 86MW into new mid-merit generation, respectively. They did not invest in new baseload nor new peaking generation. Furthermore, as Fig. 6 shows, neither of the price-making firms invested in any new technology at the same equilibrium point.

The results show that, in some of the equilibria, firm $l = 1$ makes a smaller profit compared with the perfectly competitive market. This is because we model two price-making firms. When price-making firm l commits to a large amount of forward generation, it can leave firm $\hat{l} \neq l$ with a reduced opportunity to generate and hence reduced profits. We explain this in further detail in Section 5.2.2 and in Appendix A. Interestingly, in each of the successful iterations where firm $l = 2$ MPEC converged first, firm $l = 1$'s profits are significantly below their perfect competition result while firm $l = 2$ are significantly higher; this result highlights the importance of the order in which the MPEC problems of an EPEC converge, when searching for equilibria.

Table 8
Generation mix for the first successful iteration (MWh).

	Time period (p)				
	1	2	3	4	5
Firm $f = 4$'s existing peaking	-	-	-	-	234
Firm $f = 3$'s existing mid-merit	-	-	-	-	404
Firm $l = 2$'s new mid-merit	1	176	176	176	176
Firm $l = 2$'s existing baseload	-	-	-	-	2
Firm $l = 1$'s new mid-merit	2766	2766	2766	2766	2766
Firm $l = 1$'s existing mid-merit	-	-	403	512	512
Firm $l = 1$'s existing baseload	-	-	-	316	21

Table 9
Revenue (€ millions) earned by firm $l = 1$ for the first successful iteration.

	Time period (p)				
	1	2	3	4	5
New mid-merit	0.00	0.00	0.00	34.49	151.49
Existing mid-merit	-	-	- 5.02	0.00	21.67
Existing baseload	-	-	-	- 4.31	0.60

5.2.2. Equilibrium forward prices

Despite Figs. 3–6 presenting the multiple equilibria, the forward prices rested at one of three price time series. Time series one and two were observed in 81.9% and 16.7% of the equilibria found while the third series was only observed at one of the equilibrium points found. Fig. 7 displays these three price time series along with the prices from the perfect competition case of Section 5.1. For time periods $p = 2, 3$, the forward market prices in the oligopoly with a competitive fringe case are lower than those from the perfect competition case. This is despite half the firms having price-making ability. However, the market prices in the oligopoly with a competitive fringe case are higher at later time steps. Note: both the first and second equilibrium price time series were found when both firm $l = 1$ and firm $l = 2$ MPEC's converged first while the only instance of the third equilibrium price time series occurred when firm $l = 2$'s MPEC converged first.

The forward prices in Fig. 7 can be explained by Tables 8–10. Table 8 shows the forward generation mix for the first successful iteration. The results in Table 8 do not correspond to a least-cost merit order curve where generating units are brought online in as-

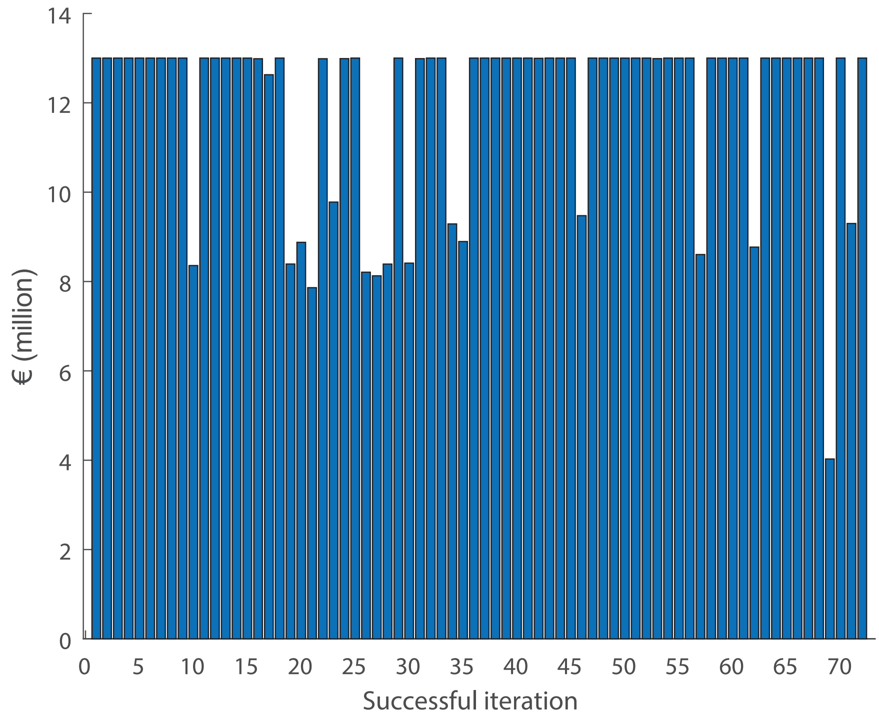


Fig. 5. Combined profits for price-making firms for each successful iteration.

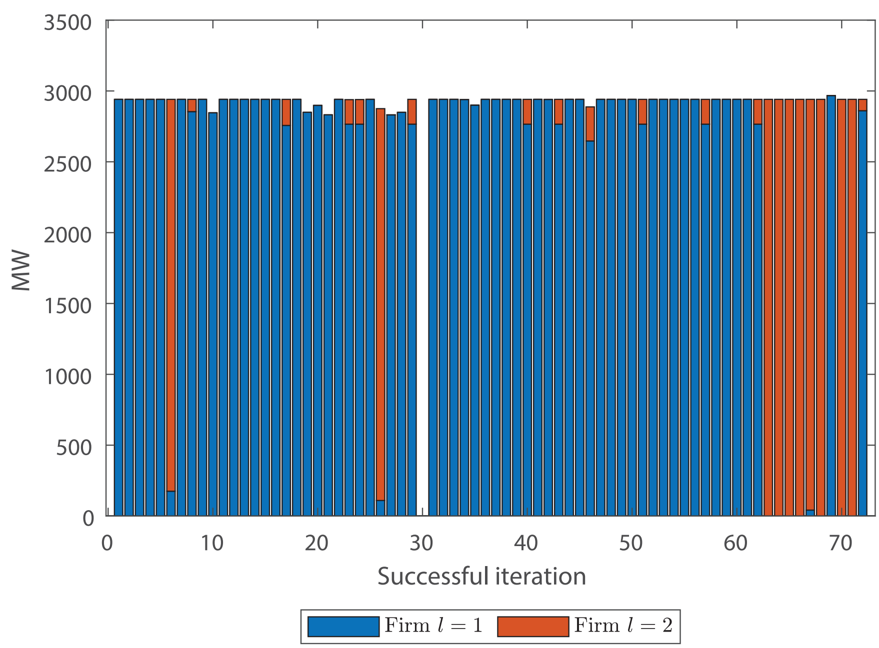


Fig. 6. Combined investment in new mid-merit for price-making firms for each successful iteration.

cending order according to marginal costs.[3] However, they do correspond to a merit-order curve where the price-making firms alter their capacity offers (i.e., generation levels) to strategically maximise their profits. Furthermore, the generation levels in Table 8 do not represent dispatched levels of generation but rather forward generation values.[4]

Table 10
Revenue (€ millions) earned by firm $l = 2$ for the first successful iteration.

	Time period (p)				
	1	2	3	4	5
New mid-merit	0.00	0.00	0.00	2.20	9.64
Existing baseload	-	-	-	-	0.06

[3] In the perfect competition cases, the generation levels result in a least-cost merit order curve.

[4] Dispatched generation represents actual physical levels of electricity generated. In contrast, forward generation does not represent actual physical levels of electricity generated but rather electricity that a firm commits to selling at some point in the future. The timescales of dispatched generation are typically sub-hourly while the timescales of forward generation are typically months or seasons.

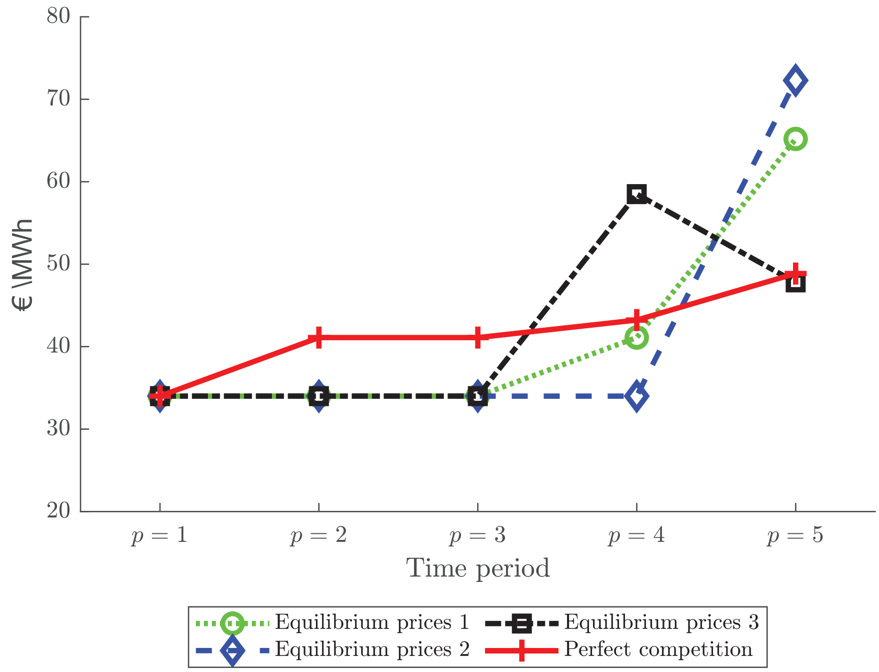


Fig. 7. Equilibrium forward market prices (γ_p) for EPEC model versus perfect competition.

Tables 9 and 10 display the revenues earned/lost by firms $l = 1$ and $l = 2$, respectively, in each time period for the same iteration. At this equilibrium point, forward prices converged to first time series in Fig. 7 and firm $l = 1$ and firm $l = 2$ invested 2765MW and 176MW into new mid-merit generation, respectively. In time periods $p = 1$ and $p = 2$, only firm $l = 1$ and $l = 2$'s new mid-merit units are generating leading to forward prices of $\gamma_{p=1} = \gamma_{p=2} = 34$, the marginal cost of new mid-merit. Consequently, neither price-making firm earns, nor loses, revenue at these two time periods.

The forward price is the same for $p = 3$ but since the demand curve intercept is higher (see Table 6), more generation is needed to meet demand. The increased demand is primarily met by firm $l = 2$'s new mid-merit unit. In addition, firm $l = 1$'s existing mid-merit unit generates 403MWh. This is despite existing mid-merit having a marginal cost of 41.1. Thus, as Table 9 outlines, firm $l = 1$ loses revenue at this timepoint. Firm $l = 2$ does not earn revenue, nor does it lose revenue, at $p = 3$.

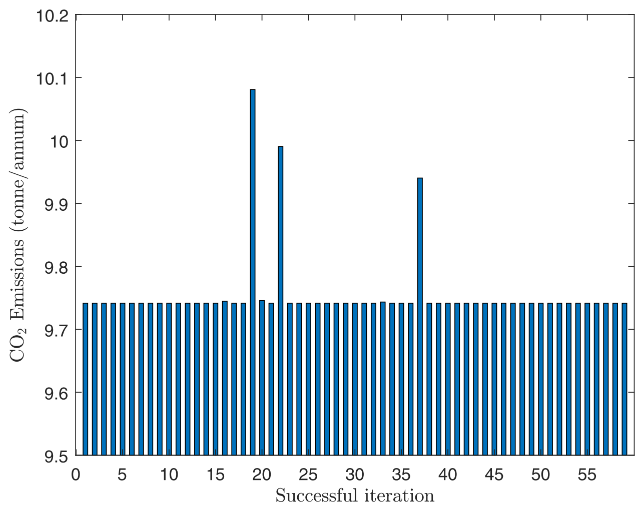
In time period $p = 4$, the forward price is 41.1 which is the marginal cost of an existing mid-merit unit. Consequently, all mid-merit units, for firm $l = 1$, $l = 2$ and $f = 3$ are utilised. In addition, firm $l = 1$ also utilises its existing baseload despite the marginal cost of existing baseload being 48.87. The forward price is $\gamma_{p=5} = 65.19$ at timestep $p = 5$. Because this price is higher than the marginal cost of existing baseload, both price-making firms utilise their existing baseload units and make a profit from doing so. The two price-making firms use their generation to set $\gamma_{p=5} = 65.19$ and hence maximise their respective profits. This forward price allows the two price-making firms to partially recover the investment cost associated with investing in new mid-merit generation. Because they both do not earn any revenue from new mid-merit in timesteps $p = 1, 2, 3$, the remaining investment costs are recovered in timestep $p = 4$ where the forward market price of $\gamma_{p=4} = 41.1$ allows both firms $l = 1$ and $l = 2$ to earn enough revenue from their new mid-merit units to break even on their investments. If either price-making firm adjusted its generation to set a price higher than 41.1 in $p = 4$ or higher than 65.19 in $p = 5$, then the two price-taking firms would invest in new mid-merit generation. The price-making firms prevent this because investment from the price-taking fringe would erode the substantial revenues they earn in timestep $p = 5$.

Similarly, it is beneficial for long-term profit maximising for firm $l = 1$ to generate using its existing mid-merit unit, at below marginal cost in time period $p = 3$. If firm $l = 1$ did not do this, the remaining demand would be met by firm $f = 3$'s existing mid-merit unit, which would drive up the market price and thus, make investing in a new mid-merit a profitable option for both price-taking firms. Again, it is optimal for firm $l = 1$ to take the small losses in time period $p = 3$ so as to prevent the fringe from eroding its large profits in timestep $p = 5$. As Table 9 shows, firm $l = 1$'s revenues from $p = 4$ and $p = 5$ far exceed its losses from $p = 3$. Additionally, in time periods $p = 1, 2$, it is optimal for firm $l = 1$ to ensure the market price is $\gamma_{p=1} = 34$. However, in these timesteps, firm $l = 1$ does not need its existing mid-merit unit to maintain the price at this level.

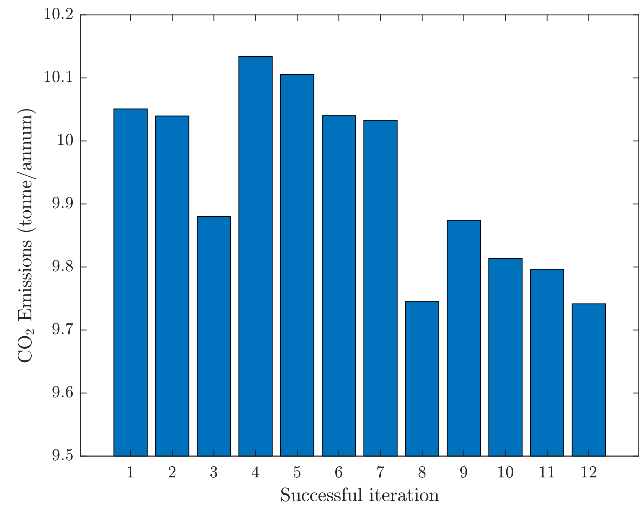
These results are in contrast to the prices observed in the perfect competition case in Fig. 7. In the perfect competition setting, firms only utilise a generating unit if the market price is at or above the marginal cost of that unit. Consequently, the market price is set by the marginal cost of the most expensive unit that is generating. Hence, there is no below marginal cost operation of units in time periods $p = 3$ and $p = 4$, which leads to higher forward prices compared with the oligopoly with a competitive fringe case. Similarly, in time period $p = 5$, the forward price in the perfect competition case is set by the most expensive unit that is generating: existing baseload. In contrast, in the oligopoly with a competitive fringe case, it is optimal for the price-making firms to adjust their generation to ensure the forward price is higher than the perfect competition case.

Similar results to those in Tables 8 and 9 can be seen in the rest of the successful iterations. The exact level of revenue earned or lost in each timestep, for both price-making firms, varies in a similar manner to Figs. 3–6.

Section 3.1 and Eq. (4) show that when using an MCP model, it is never optimal for a generator to operate one of its units at below marginal cost. In contrast, when the EPEC approach of this work is utilised, Eq. (47) shows that when $gen^{\mathrm{PM}}_{l,t,p} > 0$, γ_p can be



(a) Emission levels for equilibria with first price time series.



(b) Emission levels for equilibria with second price time series.

Fig. 8. CO_2 emission levels.

less than C_t^{GEN}. This is because of the additional $\alpha^{KKT\_GEN}_{l,ff,tt,p}$ that is in Eq. (47) but not in Eq. (4). Moreover, this further highlights the benefit of the EPEC approach and the limitations of the MCP approach when modelling an oligopoly with a competitive fringe and investment decisions.

The generation levels of price-taking firms $f=3$ and $f=4$ were similar for all equilibria that converged to the first two equilibrium price series. Both price-taking firms utilised their existing mid-merit and peaking units, respectively, to maximum capacity in time period $p=5$ as this was the only time period where the price was high enough for them to make profits. As the equilibrium forward prices converged to one of only three series, the price-taking firms' profits similarly converged to one of three levels. For equilibria that converged on the first price time series in Fig. 7, the profits were € 17.1M and € 0.7M for firms $f=3$ and $f=4$, respectively, while for equilibria with the second time series, the profits were € 22.14M and € 3.6M, respectively. At the equilibrium point where the third price time series was observed, firm $f=3$ also utilised its existing mid-merit at time period $p=4$ in addition to $p=5$. At this equilibrium point, firm $f=4$ did not generate any electricity as the price was never high enough for them to so. Consequently, firm $f=4$ made zero profits while firm $f=3$ made a profit of € 17.1M.

5.2.3. Carbon dioxide emissions

Figs. 8a and b display the carbon dioxide emissions level for equilibria that converged at the first and second set of price time series, respectively. These represent the amount of carbon dioxide emissions that would result from the firms' forward generation, summed over all firms, technologies, and time periods. Eq. (66) describes how we calculate them:

$$\text{CO}_2\text{ emission levels} = \sum_{p,t}\left(W_p \times E_t \times \left(\sum_{ll} gen^{PM}_{ll,t,p} + \sum_{ff} gen^{PT}_{ff,p}\right)\right), \tag{66}$$

where the parameter E_t gives the emissions factor level for technology t, as displayed in Table 5. Fig. 8 show that, despite equilibrium prices remaining constant across subsets of the equilibria, the emissions levels varied across the equilibria. This is particularly evident in Fig. 8b for equilibria with the second price time series. The reasons behind these results are explained in Appendix A.

5.2.4. Consumer and producer surplus, social welfare, and consumer costs

Fig. 9a displays consumer surplus, as defined by the following equation:

$$\text{Consumer Surplus} = \sum_p\left(\frac{1}{2}\times W_p \times (A_p - \gamma_p) \times\left(\sum_{ll,tt} gen^{PM}_{ll,tt,p} + \sum_{ff,tt} gen^{PT}_{ff,tt,p}\right)\right). \tag{67}$$

Because the equilibrium prices landed at one of three price time series, consumer surplus also converged to one of three levels. This is because the amount of energy consumed has a fixed relationship with market prices; see market clearing condition (13). Fig. 9a shows how the consumer surplus decreased by 0.028%, 0.032%, 0.016% for equilibria that converged to first, second and third time series of forward prices, respectively.

Fig. 9b displays producer surplus for the three different equilibrium price time series, where producer surplus is defined as follows:

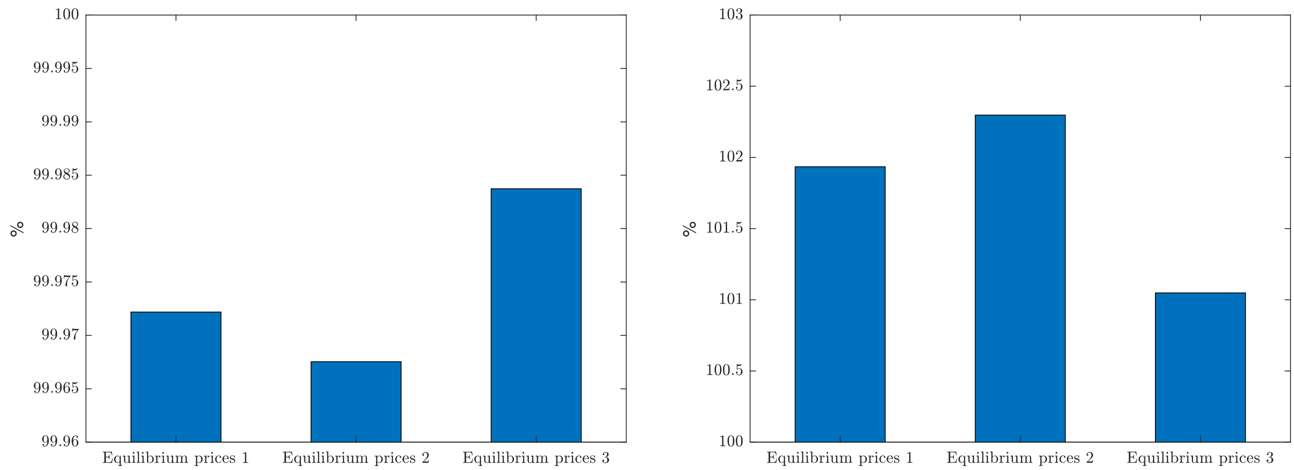
$$\text{Producer Surplus} = \sum_p\left(\frac{1}{2}\times W_p \times \gamma_p \times\left(\sum_{ll,tt} gen^{PM}_{ll,tt,p} + \sum_{ff,tt} gen^{PT}_{ff,tt,p}\right)\right) - \sum_{l,t} IC^{GEN}_t \times inv^{PM}_{l,t} - \sum_{f,t} IC^{GEN}_t \times inv^{PT}_{f,t}. \tag{68}$$

Fig. 9b shows that producer surplus increased, on average, by 1.9%, 2.3% and 1% for the equilibria that converged to the first, second and third time series of forward prices, respectively. The differences in producer surplus levels across the equilibria that converged to the first time series of forward prices were minimal ($\leq 0.1\%$). Likewise, a similar result was observed for the equilibria that converged to the second time series of forward prices.

Fig. 10a displays the consumer costs, as defined by the following equation:

$$\sum_p\left(W_p \times \gamma_p \times\left(\sum_{ll,tt} gen^{PM}_{ll,tt,p} + \sum_{ff,tt} gen^{PT}_{ff,tt,p}\right)\right). \tag{69}$$

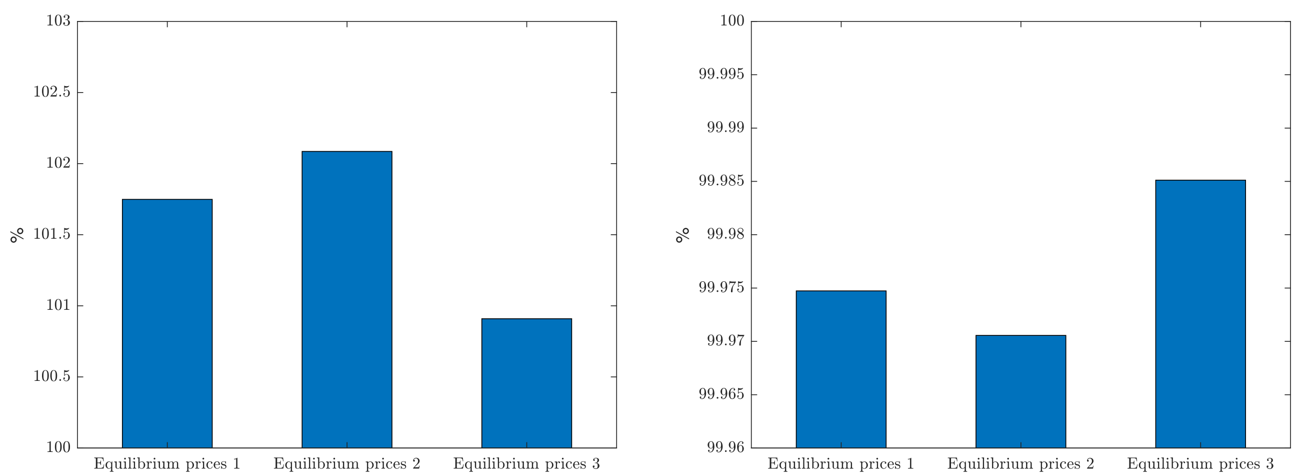
As above, because the equilibrium prices landed at one of three price time series, consumer costs also converged to one of three



(a) Consumer surplus as % of perfect competition case.

(b) Producer surplus as % of perfect competition case.

Fig. 9. Consumer and producer surplus.



(a) Consumer costs as % of perfect competition case.

(b) Social welfare as % of perfect competition case.

Fig. 10. Consumer costs and social welfare.

levels. Fig. 10a shows how the consumer costs increase by 1.7%, 2.1%, 0.9% for equilibria that converged to first, second and third time series of forward prices, respectively.

In previous works that use similar data, the presence of price-making behaviour was found to lead to a larger increase in consumer costs (Devine & Bertsch, 2018). However, the ability of the price-taking fringe to invest in new generation motivates the price-making firms to reduce forward market prices in some time periods. While the market prices increase again in subsequent time periods, these consumer cost results show how the presence of a competitive fringe helps mitigate the negative effects of market power.

Fig. 10b displays social welfare levels for the three different equilibrium price levels. Social welfare is defined as the sum of consumer surplus (Eq. (67)) and producer surplus (Eq. (68)).

Following on from Fig. 10a, b shows that social welfare decreased, on average, by 0.025%, 0.029% and 0.015% for the equilibria that converged to the first, second and third time series of forward prices, respectively. The differences in social welfare levels across the equilibria that converged to the first time series of forward prices were minimal ($\leq 0.0001\%$). Likewise, a similar result was observed for the equilibria that converged to the second time series of forward prices.

6. Discussion

The following summarises the five main findings of our research. First, an Equilibrium Problem with Equilibrium Constraints (EPEC) is a prudent model choice when modelling an oligopoly with a competitive fringe and investments. As outlined in Section 5.1, when investment decisions are included in the model, using a Mixed Complementarity Problem (MCP) can lead to myopic model behaviour and thus counter-intuitive results. Our analysis shows that an EPEC model can overcome this issue and does not require the limiting assumption of conjectural variations.

Second, the analysis in Section 5.2 found multiple market equilibria. This led to varied investment decisions and profits for the price-making firms. These results will be of interest to generating firms, particularly those with market power. Figs. 3–6 highlight the benefit of making investment decisions before other competing price-making firms do so. In fact, our results indicate that if firms do not expand their generation portfolios, then they may face profits lower than they would if the market was perfectly competitive.

Third, our results show that may it be optimal for generating firms with market power to occasionally operate some of their generating units at a loss in the short term in order to make profit in the long term. The driving factor behind this outcome is the ability of both price-making and price-taking firms to make investment decisions. The ability of price-taking firms to invest further into the market motivates the price-making firms to depress prices in earlier timepoints. This reduces the revenues price-taking firms could make from new investments and thus prevents them from making such investments. Such behaviour would not be captured by MCP or cost-minimisation unit commitment models. Consequently, this result again highlights the suitability of the EPEC modelling approach and the importance of including investment decisions in models of oligopolies with competitive fringes.

Fourth, the multiple equilibria also indicate that generation from existing baseload may be higher in some equilibria compared with others. Such market outcomes will be of interest to energy policymakers who are concerned about carbon emission levels. Older baseload generators tend to be coal-based and thus emit higher levels of carbon. Consequently, while the market may be indifferent to where the electricity comes from, policymakers may seek to put measures in places to encourage the equilibrium outcomes where existing baseload generation is reduced.

Fifth, Fig. 10a showed that the presence of market power increases consumer costs by 1% – 2%. While this outcome is not surprising, the level is relatively small compared with the literature. For instance, using similar data, Devine and Bertsch (2019) estimate market power in an oligopoly with a competitive fringe context could double consumer costs compared with a perfectly competitive market. However, Devine and Bertsch (2019) do not include investment decisions in their model. Thus, this result again highlights the impact of including investment decisions in models of oligopolies with a competitive fringe. It also highlights the importance for policymakers to encourage new entrants into electricity markets and, moreover, the benefits of encouraging smaller generating firms to expand their portfolios, or at least threaten to.

As the literature details (Pozo et al., 2017), solving EPEC problems can be computationally challenging. In this work we utilised the method outlined in Leyffer and Munson (2010) to obtain an initial starting point to our algorithm. Using this approach our algorithm successfully found an equilibrium from 72 of the 200 iterations attempted. When instead we used a random initial starting point solution, we found an equilibrium from only 2 of the 2000 iterations attempted.

A potential improvement to our EPEC formulation is possible through using the strong-duality condition (Ruiz & Conejo, 2009) of the price-taker's problem as well as writing the problem as a Mathematical Program with Primal Dual Constraints (MPPDC). This requires non-trivial reformulations and advances that provide an avenue for improvement to our formulation. In general, EPECs are computationally challenging, and our algorithm does not necessarily work for larger systems due to these computational hurdles. However, there is recent research that provides algorithms for EPECs that could potentially increase the computational tractability of our method (e.g., Fanzeres et al., 2020 and Jara-Moroni et al., 2018). The goal of our paper is to show that EPECs are more appropriate to model market power when looking at investment decisions with a competitive fringe. We leave the expansion of our approach to more computationally tractable methods for future work and are encouraged by the recent advances in this space.

Critically reflecting on our approach, we wish to acknowledge some limitations. First, because EPEC problems are challenging to solve, we choose the relatively small number of five timesteps. These represented hours in summer low demand, summer high demand, winter low demand, winter high demand and winter peak demand. Thus, the net demand intercept values represent average values for these timesteps. In reality, particularly in systems with a large amount of renewables, these intercept values will fluctuate from hour to hour. As a result, the average values may over- or under-estimate the total profits each generating firm could make in each time period. This would impact investments decisions and consumer costs.

Second, we did not model several physical features of electricity systems, for example, ramp and start-up costs in addition to transmission constraints. To do so would require integer decision (primal) variables and nonlinear functions, thus preventing us from deriving equivalent KKT conditions for the lower-level problems and hence prohibiting us from using an EPEC approach. However, we note that Tangerås and Mauritzen (2018) found empirical evidence of market power in the Nord Pool market and suggest that this is, at least partially, due to transmission constraints. We leave this for future work that can build upon the framework we have outlined in this paper.

Third, we did not account for any stochasticity in the model. Due to the intermittent and uncertain nature of wind energy, stochasticity is a feature of many electricity market models. Rintamäki, Siddiqui, and Salo (2020) suggest that the presence of renewables gives more flexible firms further leverage to exert market power. Such stochasticity is typically introduced by making generation capacity scenario-dependent (Lynch et al., 2019a). Deterministic capacity values may also over- or under-estimate the profits each firm may make in each timestep. However, we do not anticipate that further timesteps and stochastic capacity values will affect the qualitative findings discussed above.

As discussed in the introduction, there is a modelling trade off to be made when choosing the methodology to represent market power in electricity markets. MCPs and mixed integer programs allow for a much larger number of model variables and thus make the inclusion of stochasticity and nonlinear effects, for example, more tractable. However, for markets characterised by an oligopoly with a competitive fringe and investment decisions, this paper shows that EPECs model strategic investment behaviour more credibly. Due to the computational difficulty in modelling EPECs with integer variables and nonlinear functions, we leave the advancement of our model to future work.

Finally, we did not consider a capacity market as part of the market modelled in this work. Capacity payments exist when firms get paid for simply owning generation units and making them available to the grid. Capacity payments do not depend on the extent that the unit(s) are utilised. Regulators and policymakers include such payments to ensure security of supply (Lynch & Devine, 2017). The market we considered was an 'energy-only' market, where the generating firms only get paid on the basis of how much they generate. A capacity market can affect the level of investment into new generation. Future research activities can address each of these modelling limitations.

7. Conclusion

In this paper, we developed a novel mathematical model of an imperfect electricity market, one that is characterised by an oligopoly with a competitive fringe. We modelled two types of generating firms; price-making firms, who have market power, and price-taking firms who do not. All firms had both investment and forward generation decisions. The model took the form of an Equilibrium Problem with Equilibrium Constraints (EPEC), which finds an equilibrium of multiple bilevel optimisation problems. The bilevel formulation allowed the optimisation problems of the price-taking firms to be embedded into the optimisation problems of the price-making firms. This enabled the price-making firms to correctly anticipate the optimal reactions of the price-taking firms into their decisions. We applied the model to data representative of the Irish power system for 2025.

To solve the EPEC problem, we utilised the Gauss-Seidel algorithm. Furthermore, we found the computational efficiency of the algorithm was improved when the algorithm's starting point was provided by the approach detailed in Leyffer and Munson (2010). Overall, we found that an EPEC problem is a prudent model choice when modelling investment decisions in an oligopoly with a competitive fringe. This is because it overcomes modelling issues previously found in the literature and requires fewer limiting assumptions. However, we note that EPEC problems can be computationally challenging to solve in comparison to Mixed Complementarity Problems (MCPs).

The model found multiple equilibria. This was due to the market's indifference to which price-making firm generates electricity. Although consumer costs were found to be relatively constant across the equilibria, this result is important to policymakers who wish to avoid equilibrium outcomes that lead to higher carbon emission levels.

We also observed that it may be strategically optimal for price-making firms to occasionally to generate at a price that is lower than their marginal cost in the short term in order to make long term profits. This is because we incorporated investment decisions into the optimisation problems of both types of generating firms. Consequently, the price-making firms seek to depress prices occasionally to discourage the fringe from investing further into the market. Furthermore, we found that consumer costs only decreased by 1% – 2% when market power was removed from the model.

While the results of this work are applied to an electricity market setting, the modelling framework and results are relevant to any market characterised by an oligopoly with a competitive fringe.

In future research, we will study the effects of increasing the number of timesteps in the model. Moreover, we will explore the impact stochasticity, particularly from wind generation, would have. In addition, future research will analyse how the introduction of a capacity market would affect equilibrium outcomes.

Acknowledgments

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Appendix A. Reasons for multiple equilibria

Figs. 3–6 show that there are multiple equilibria of the EPEC presented in this work. In this appendix, we explore the reasons behind this finding. First, we look at two equilibria where the forward prices were the same, i.e., the first equilibrium time series from Fig. 7.

The multiple equilibria are driven by the market's indifference to what firm is providing electricity when firms are generating at the same price. For example, Tables 8 and A.1 display the generation mixes for the first two successful iterations, respectively. In the first successful iteration firms $l=1$ and $l=2$ invest 2765MW and 176MW into new mid-merit generation, respectively. In contrast, in the second successful iteration, they invest 2941MW and 0MW into new mid-merit generation, respectively.

At time period $p=5$ in the first iteration, firm $l=1$ uses its new and existing mid-merit units at maximum capacity while also generating 21MW from its baseload unit. At the same iteration, firm $l=2$ uses its new mid-merit unit to full capacity and also generates 2MW from its baseload unit. In the second successful iteration, firm $l=1$ decreases its baseload generation at $p=5$ from 21MW to 4MW but increases its generation from new mid-merit from 2765MW to 2941MW. This allows firm $l=1$ to make less profits in Table A.1; firms break even on their new mid-merit investments but make profits from existing baseload generation. Firm $l=2$ increases its baseload generation from 2MW to 19MW but decreases generation from new mid-merit from 176MW to 0MW. This allows firm $l=1$ to make more profits in Table A.1.

Table A.1
Generation mix (MWh) for the second successful iteration.

	Time period (p)				
	1	2	3	4	5
Firm $f=4$'s existing peaking	-	-	-	-	234
Firm $f=3$'s existing mid-merit	-	-	-	-	404
Firm $l=2$'s new mid-merit	-	-	-	-	-
Firm $l=2$'s existing baseload	-	-	-	17	19
Firm $l=1$'s new mid-merit	2766	2941	2941	2941	2941
Firm $l=1$'s existing mid-merit	-	-	403	512	512
Firm $l=1$'s existing baseload	-	-	-	299	4

Because the market prices are the same across both equilibria considered, the market is indifferent to whether the electricity comes firm $l=1$'s baseload or mid-merit or from firm $l=2$'s baseload or mid-merit units. Once firm l commits to forward generation decisions, firm $\hat{l}\neq l$ is not willing to adjust its generation levels so as to either increase or decrease forward market price of $\gamma_{p=5}=65.19$. If either price-making firm increased any of the forward prices, then the price-taking firms would invest in new mid-merit generation, as explained in the previous subsection. It is also not profit-maximising for firm l to undercut firm $\hat{l}\neq l$ at a price lower than 65.19. To do so, would mean firm l would make a loss on its new mid-merit investment. Furthermore, if firm l adjusted its generation so as to decrease $\gamma_{p=5}$ by € 1, then it would only be able to, at most, increase its generation from existing baseload $\frac{1}{B}=0.11$MW (see market clearing condition (13)). This is because it would continue to be profitable for firm $\hat{l}\neq l$ to utilise its existing baseload at the reduced price. The small increase in generation opportunity would not make up for the decreased revenues resulting from the reduced price. This paragraph explains why in some of the equilibria found, firm $l=1$ makes less profits than in the perfect competition case.

Similar market in-differences are also observed in time periods $p=2-4$ and in the other 57 successful iterations that converge to the same price time series, thus explaining the multiple equilibria displayed in Figs. 3–6. In some of other equilibria found, both price-making firms generate significant amounts from their baseload units in $p=5$, thus preventing each other from generating and investing in new mid-merit generation. Consequently, both firms do not make as large a profit as they otherwise could. Such equilibria are also evident in Figs. 3–6. This particular result highlights the absence of collusion between the two price-making firms modelled in this work.

We now examine the differences between two equilibria that converged to different forward price time series. Table A.2 displays the generation mix for the first successful iteration where the forward prices converged to the second time series in Fig. 7 while Tables A.3 and A.4 show the revenues for firms $l=1$ and $l=2$, respectively, for the same iteration. At the equilibrium point, firms $l=1$ and $l=2$ invest 2831MW and 1MW into new mid-merit generation, respectively, and make profits of € 4.92M and € 3.21M, respectively.

In Table 8, firm $l=2$ generated 17MW from its existing baseload unit at time period $p=4$. In contrast, in Table A.2, firm $l=2$ increased its baseload generation to 93MW at time period $p=4$. Following from market clearing condition (13), this leads to the forward price decreasing from $\gamma_{p=4}=41.1$ to $\gamma_{p=4}=31$ be-

Table A.2
Generation mix (MW) for the first successful iteration that results in the second time series for forward prices.

	Time period (p)				
	1	2	3	4	5
Firm $f = 4$'s existing peaking	-	-	-	-	234
Firm $f = 3$'s existing mid-merit	-	-	-	-	404
Firm $l = 2$'s new mid-merit	-	-	1	1	1
Firm $l = 2$'s existing baseload	-	-	-	93	137
Firm $l = 1$'s new mid-merit	2766	2831	2831	2831	2831
Firm $l = 1$'s existing mid-merit	-	110	512	512	506
Firm $l = 1$'s existing baseload	-	-	-	333	-

Table A.3
Revenue (€ millions) earned by firm $l = 1$ for the first successful iteration that results in the second time series for forward prices.

	Time period (p)				
	1	2	3	4	5
New mid-merit	0.00	0.00	0.00	0.00	190.42
Existing mid-merit	-	- 1.37	- 6.39	- 6.39	27.75
Existing baseload	-	-	-	- 8.69	-

Table A.4
Revenue (€ millions) earned by firm $l = 2$ for the first successful iteration that results in the second time series for forward prices.

	Time period (p)				
	1	2	3	4	5
New mid-merit	0.00	0.00	0.00	0.00	0.05
Existing baseload	-	-	-	- 2.44	5.65

tween the two time series. This decrease in forward price meant that firm $l = 1$ needed to decrease its overall generation in $p = 5$ from 3456MW in Table 8 to 3337MW in Table A.2. This resulted in a higher forward price in $p = 5$ for in the second time series and thus allowed both price-making firms to recover its investment capital costs, despite the decreased price in $p = 4$.

Firm $l = 2$ cannot make a profit from its existing baseload unit in time period $p = 4$ at either $\gamma_{p=4} = 41.1$ or $\gamma_{p=4} = 31$. Consequently, firm $l = 2$ prefers a higher forward price in $p = 5$ as this allows it to maximise its profits on its existing baseload unit; see Table 10 compared with Table A.4. In contrast, firm $l = 1$ prefers the first forward price time series, i.e., a higher price in $p = 4$ and a slightly lower price in $p = 5$. In time period $p = 4$, firm $l = 1$ can earn positive revenues from its new mid-merit unit and not make a loss from its existing mid-merit unit in $p = 4$, if the forward price is 41.1. In contrast however, firm $l = 2$ does not own an existing mid-merit unit and, in Tables A.2–A.4, only invests in 1MW of new mid-merit generation. Consequently, firm $l = 2$ prefers a lower forward price in $p = 4$ and a higher price in $p = 5$ as this allows firm $l = 2$ to maximise its profits from its existing baseload unit. For both forward price time series, the forward price is not high enough for existing baseload units to earn positive revenues in $p = 4$.

In general, the equilibria resulting from the second time series represent equilibria where firm $l = 2$ has invested in a relatively small amount of new mid-merit generation, if any at all, but where firm $l = 2$ has also made forward generation decisions before firm $l = 1$. When firm $l = 2$ commits to a large amount of generation in $p = 4$, firm $l = 1$ must reduce its generation in $p = 5$ in order to allow the forward price increase and hence break even on its and firm $l = 2$'s new mid-merit investments.

Interestingly, there is one equilibrium point where firm $l = 1$ does not invest in any new technology and consequently, commits to a large amount of generation in $p = 4$. This leads to the second equilibrium price time series from Fig. 7. This also forces firm $l = 2$ to reduce its generation in $p = 5$ and motivates it to not make any investment decisions either. As a result, this is the only equilibrium point where the followers make investment decisions; firm $f = 3$ invests 2766MW into a new mid-merit facility while firm $f = 4$ invest 86MW into the same technology. Because of the generation commitments of the price-making firms set the equilibrium prices, both price-taking firms break exactly even on these investments. Thus, in the model, the price-making firms are indifferent to whether they do the investment at this equilibrium point or the price-taking firms do. However, this indifference may not reflect reality. In the real-world, price-making firms may fear losing their price-making ability if they allow the competitive fringe to invest. The EPEC model presented in this work does not account for this as the price-making/price-taking characteristics of all firms remain unchanged throughout the model.

Finally, as mentioned above, there was one equilibrium point found where the prices converged to the third equilibrium time series in Fig. 7. In comparison with the second equilibrium time series, firm $l = 2$ commits to investing in new mid-merit generation before $l = 1$. However, at this equilibrium point, firm $l = 2$ invests in less than 1 MW of new mid-merit generation while firm $l = 1$ invests in 2831 MW of new mid-merit of generation. As a result, firm $l = 1$ commits to a large amount of generation in $p = 5$ which leads to a reduced price of $\gamma_{p=5} = 47.79$, from which its existing mid-merit unit profits from. Consequently, in order for firm $l = 2$ to break even on its new mid-merit investment, firm $l = 2$ is forced to ensure its generation in $p = 4$ is low enough to allow $\gamma_{p=4} = 58.59$.

Appendix B. Modelling all firms as price-makers

In this appendix, we model all four firms considered in this paper as price-making firms and hence we do not consider a price-taking competitive fringe. We do this using the same data as those described in Section 2. When there is no competitive fringe, there is no lower problem in the price-making firms' MPECs, i.e., Eqs. (20)–(29) become redundant. Consequently, the results from the EPEC and MCP modelling frameworks are the same.

When all four generating firms are modelled as price-makers there is one market equilibrium. They all invest in 'new mid-merit' generation and do not invest in any other technology. Table B.1 displays the investment levels. All four firms also use their generation strategically so as to maximise profits. They do this by withholding some of their generation in order to increase the forward market prices via market clearing condition (13). When all firms withhold generation capacity in this manner, no firm is short-sighted to any firm behaving as if they were in a perfectly competitive market, in contrast to the results presented in Section 5.1.

Fig. B.1 and Table B.1 display the firms' generation and profit levels, respectively. These results show that considering all firms as price-makers is not suitable for modelling an oligopoly with a competitive fringe. All firms commit to similar levels of generation, i.e., there is no competitive fringe.

Table B.2 shows that when all four firms are modelled as price-makers, then the forward prices are substantially higher (roughly

Table B.1
Investments in new mid-merit generation (MW) and profits (€) when all firms are modelled as price-makers.

	Investment	Profits
Firm $f = 4$	754	3.6×10^9
Firm $f = 3$	420	3.6×10^9
Firm $l = 2$	312	3.6×10^9
Firm $l = 1$	668	3.6×10^9

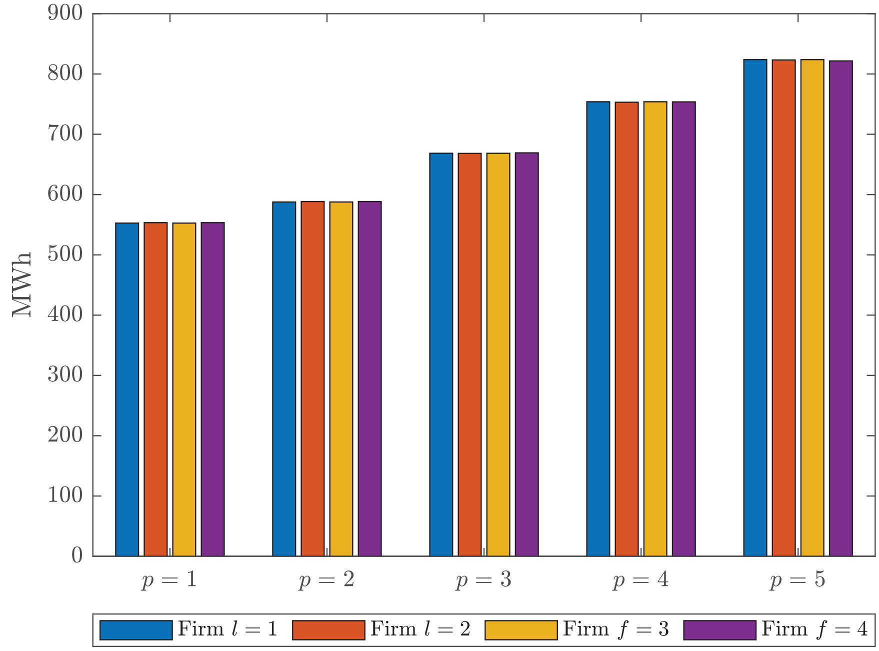

Fig. B.1. Generation mix $\left(\sum_t gen_{l,f,t,p}\right)$ when all firms are modelled as price-makers (MW).

Table B.2
Forward market prices (γ_p) when are firms are modelled as price-makers (€ /MWh).

Time period (p)				
1	2	3	4	5
5065	5383	6118	6895	7533

150 times) than those observed in the perfect competition case and in each of the equilibria found in the oligopoly with a competitive fringe case - see Fig. 7. This is because there is no competitive fringe to suppress the forward prices. Consequently, we do not believe this market structure is reflective of reality. If these forward prices were observed in reality, then a fifth (or more) firms would enter the market. These additional firms would become the competitive fringe and we would observe results similar to those presented in Section 5.2.

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