Adaptive Control for Cooperative Aerial Transportation Using Catenary Robots

Gustavo A. Cardona, Diego S. D'Antonio, Rafael Fierro, and David Saldaña

Abstract— **We present a method for cooperative transportation using two catenary robots. Each catenary robot is composed of two quadrotors connected by a hanging cable. Unlike other methods in the literature for aerial transportation using cables, we do not assume that the cables are attached to the object. Instead, the quadrotors wrap cables around the object and pull. Since the cable is not attached to the object, the quadrotors need to avoid slipping by maintaining friction between the cable and the object. In this work, we focus on manipulating objects with cuboid shapes or boxes. We use two catenary robots to pull the box from two opposite edges. Once the robots are in contact with the box, they do not know the contact points between the cable and the object. We propose an adaptive controller to track a reference trajectory without information about the box's contact points, mass, and inertia tensor. We validate our approach through simulations.**

I. Introduction

In recent years, aerial transportation using micro aerial vehicles has become popular in industry and academia. Aerial vehicles offer a fast and low-cost solution for object transportation in urban and rural scenarios [1], [2]. However, one of the main problems of aerial vehicles is their lack of versatility. For instance, in object transportation, the robot depends on the weight and dimensions of the object. On the one hand, large vehicles can carry many objects in contrast to small vehicles carrying light-weight payloads only; however, large vehicles have motion limitations in cluttered environments. On the other hand, Multiple micro aerial vehicles working cooperatively can get over payload restrictions while grasping and manipulating objects. A sub-problem of object transportation is box transportation, specifically for package transportation and drone delivery [3]. In this way, we focus our work on transporting cuboid objects without requiring any additional customization.

Recently, quadrotors using cables and tethers have been shown as a versatile solution to manipulate objects [4], [5], [6]. Multiple approaches [7], [8], [9], [10] model the cable as a rigid link that is previously attached to the object. However, attaching the cable to the object requires human

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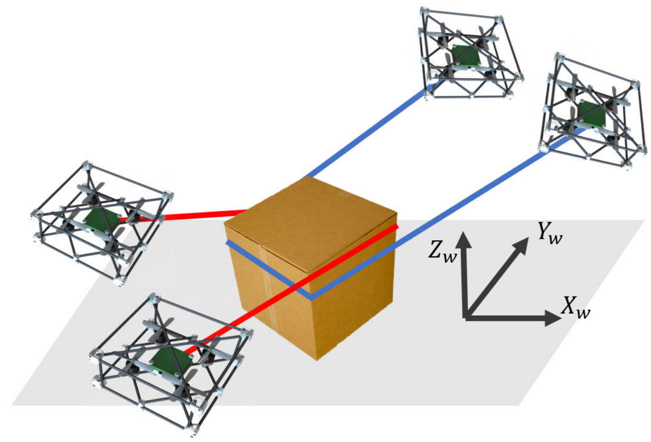

Fig. 1: Cooperative aerial transportation using two catenary robots. https://youtu.be/MeEGL_PUBx4

intervention that slows down a process that can be fully automated. In our previous work [11], we showed that a catenary robot, composed of two quadrotors connected with a cable, can be used to transport objects with a hook-shaped, e.g., an umbrella or a bicycle. Then, in the extended work, we showed how to transport an object that does not require hook-shaped objects like a box by dragging or rolling in a planar surface [12]. Here, there are additional factors to take into account in the transportation, such as slipping conditions, type of contacts, contact forces, and friction [13]. This work also considers a box that does not have any hook shape. Manipulation only depends on the friction between the box and cables.

It is worth mentioning that cable-suspended loads [14], [15], [16], quadrotors attached by a rod [17], are not the only approaches for aerial manipulation. For example, The aerial snake robot manipulates the object surrounding it with the body while changing the direction of its propellers [18]. In [19] multiple quadrotors are used pushing an object from different locations and generating opposing forces that collaboratively grasp and allows the transport of the object. Also, articulated aerial-type manipulators [20], [21] that can be modified to lift objects, a robot arm, or gripper [22] are effective when manipulating objects, but the size and shape of the objects they can manipulate depends directly on the capacity of the end effector. Furthermore, these systems' increase in weight decreasing the payload capacity while spending a higher amount of energy and requiring more space to operate. In contrast, the catenary robot uses a cable that adapts to any object's shape while maintaining the payload capacity.

Additionally, when considering more realistic scenarios while manipulating an object, the knowledge of the object that will be transported is partially or entirely unknown, causing many parameters in the quadrotors or objects to be uncertain. Adaptive control of a quadrotor to compensate for changes in the center of gravity is proposed in [23], also for unknown cooperative forces and external disturbances in [24]. Authors in [25] proposed an energy-based adaptive control design for a quadrotor transporting a suspended load, assuming that the length of the cable connecting the payload and the UAV is unknown. Furthermore, the authors in [26] propose a decentralized Model Reference Adaptive Control MRAC to manipulate an object on a planar surface. However, all of these works consider the object physically attached to the quadrotors or knowing apriori the contact points in which the forces will be applied. In contrast, this work does not consider knowing the location of contact points and some box parameters while tracking a trajectory.

The main contribution of this paper is threefold. First, we propose a versatile, cooperative, and autonomous algorithm for two catenary robots to manipulate a box and track the desired trajectory without human intervention in the previous stage or during the manipulation. Second, we design an algorithm that considers the restrictions of the cable, meaning cables that cannot have arbitrary angles. We constraint cables to stay within a designed threshold to guarantee that the cable will not slip with the box. Finally, we design an adaptive controller that is able to track a trajectory while ignoring not only some box parameters but also the contact point positions needed to compute control forces for the box.

II. Problem Statement

The goal of this work is to manipulate objects using two *catenary robots*. We define the *catenary robot* as follows.

Definition 1 (**Catenary robot**, [11]). *A catenary robot is composed of two quadrotors connected by a cable. The cable is non-stretchable with neglected mass, and it is attached to the center of mass of each quadrotor.*

We use the force of the quadrotors to pull objects. One cable with two quadrotors can generate two forces to pull the object, but that is not enough to control the six degrees of freedom of the rigid object. In the case of attached cables, the minimum is three, as described in [4]. Therefore, we need at least two catenary robots to manipulate the object. In Fig. 2, we illustrate two catenary robots pulling a box from two opposite edges with a blue cable and red cable. We focus on manipulating objects with cuboid shapes.

Definition 2 (Box)**.** *A box is an object with cuboid shape, mass $m_b \in \mathbb{R}_{>0}$, and inertia around the x-axis $J_b \in \mathbb{R}$. Its length, width and high are $w, l, h \in \mathbb{R}_{>0}$ respectively.*

We first move the cable of each catenary robot and place it in the desired location on the box, creating an initial contact

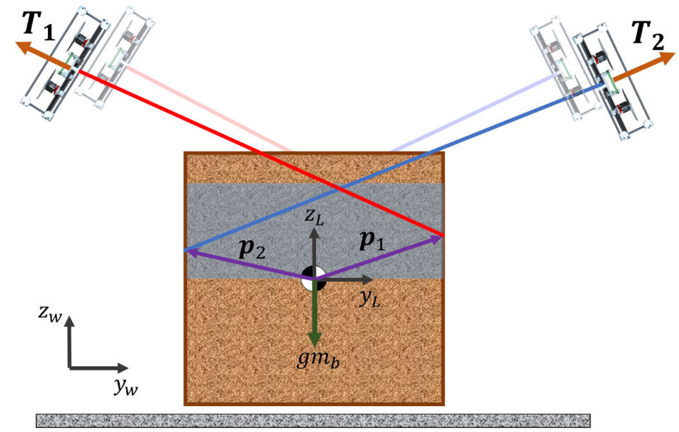

Fig. 2: Side view of a box being transported by forces generated by two catenary robots. Note that faded quadrotors and cables in the image are just a representation that each catenary robot has two quadrotors pulling the box.

between the box and each cable. Then, the quadrotors move around the box to surround it with the cable.

In our previous work [11], we designed a navigation controller for the catenary robot without considering interaction with external objects. Then we extended to use one catenary robot to transport one box by performing dragging, and rolling actions on planar surface [12]. However, this work focuses on transporting objects using two catenary robots, adding the possibility to lift the box. We study the two-dimensional case assuming that both quadrotors in the same catenary robot maintain the same pose as illustrated in Fig. 2. Additionally, we try to avoid slipping conditions by using the friction between the cable and the box, restricting the cable's angle.

Each quadrotor has mass and inertia tensor denoted by $m \in \mathbb{R}_{>0}$ and $J_{xx} \in \mathbb{R}$ respectively. Each cable that connects a pair of quadrotors has length ℓ. The position of each quadrotor in the inertial reference frame, $\{\mathcal{W}\}$, is $\mathbf{r}_i = [y_i, z_i]^\top \in \mathbb{R}^2$ with $i = 1, ..., 4$. The orientation and angular velocity are denoted by $\theta_i \in \mathbb{R}$, and $\omega_i \in \mathbb{R}$. Each quadrotor can use its propellers to generate a force magnitude $f_i \in \mathbb{R}$ (direction of the force is given by the pose of quadrotors since rotors are fixed) and a moment $\tau_i \in \mathbb{R}$.

The position of the box in $\{\mathcal{W}\}$ is denoted by $\mathbf{r}_b = [y_b, z_b]^\top \in \mathbb{R}^2$, the orientation is θ_b, and the angular velocity is ω_b. The contact points between the box and the cables in $\{\mathcal{B}\}$ are $\mathbf{p}_1$ and $\mathbf{p}_2$. The rotation matrix from $\{\mathcal{B}\}$ to $\{\mathcal{W}\}$ is ${}^w\mathbf{R}_b \in \mathrm{SO}(2)$.

A. Box Dynamics

Each quadrotor $i = 1, ..., 4$ generates a tension $\mathbf{T}_i$ with respect to $\{\mathcal{B}\}$. In this work, we study the planar case, assuming that the box is aligned with the yz-plane, and the two quadrotors in the back mimic the behavior of the quadrotors on the front, i.e., $\mathbf{T}_3 = \mathbf{T}_1$ and $\mathbf{T}_4 = \mathbf{T}_2$, see Fig. 2. We describe the dynamics of the box using the

Newton-Euler equations

$$
\begin{aligned}
m_b \ddot{\mathbf{r}}_b &= -m_b g \mathbf{e}_z + 2\sum_{i=1}^{2} {}^{w}\mathbf{R}_b \mathbf{T}_i, \\
J_b \dot{\omega}_b &= \tau_b,
\end{aligned} \tag{1}
$$

where g is the gravity constant, and $\mathbf{e}_y = [1, 0]^\top$ and $\mathbf{e}_z = [0, 1]^\top$ are unitarian vectors along the y- and z-axis respectively. To simplify the notation, we will consider $\mathbf{T}_1$ and $\mathbf{T}_2$ containing the forces of $\mathbf{T}_3$ and $\mathbf{T}_4$ respectively, instead of writing $2\mathbf{T}_1$ and $2\mathbf{T}_2$. Then we can expand the dynamics as,

$$
\begin{aligned}
m_b \ddot{y}_b &= T_2^y - T_1^y, \\
m_b \ddot{z}_b &= T_1^z + T_2^z - m_b g, \\
J_b \dot{\omega}_b &= \tau_b,
\end{aligned} \tag{2}
$$

where ${}^{w}\mathbf{R}_b\mathbf{T}_1 = [T_1^y, T_1^z]$, ${}^{w}\mathbf{R}_b\mathbf{T}_2 = [T_2^y, T_2^z]$ are the tension components that the cables exert on the box in $\{\mathcal{W}\}$, and τ_b is the torque generated by the two cables in the box which is defined by

$$
\begin{aligned}
\tau_b &= \mathbf{p_1} \times \mathbf{T_1} + \mathbf{p_2} \times \mathbf{T_2}, \\
&= (p_1^y T_1^z - p_1^z T_1^y) + (p_2^y T_2^z - p_2^z T_2^y).
\end{aligned}
$$

Let us define a pose vector as $\mathbf{q} = [y_b, z_b, \theta_b]^\top$. We can represent the system in (2) using the Lagrangian for robot motion [27], [28], also called the "manipulator equation",

$$
\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g} = \mathbf{w}, \tag{3}
$$

where $\mathbf{M}(\mathbf{q}) \in \mathbb{R}^{3\times 3}$ is the matrix that collects the mass and the inertia tensor,

$$
\mathbf{M} = \begin{bmatrix} m_b & 0 & 0 \\ 0 & m_b & 0 \\ 0 & 0 & J_b \end{bmatrix},
$$

$\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{0}_{3\times 3}$ is the Coriolis matrix that does not appear in the planar case, $\mathbf{g} = [0, -m_b g, 0]^\top$ is a gravitational vector, and $\mathbf{w} = [{}^{w}f_b^y, {}^{w}f_b^z, \tau_b]^\top$ is the wrench, a combination of the forces and torques applied by the catenary robots to the box. Based on the contact points, we can compute the total wrench as follows,

$$
\underbrace{\begin{bmatrix} {}^{w}f_b^y \\ {}^{w}f_b^z \\ \tau_b \end{bmatrix}}_{\mathbf{w}} = \underbrace{\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ -p_1^z & p_1^y & -p_2^z & p_2^y \end{bmatrix}}_{\mathbf{D}} \underbrace{\begin{bmatrix} T_1^y \\ T_1^z \\ T_2^y \\ T_2^z \end{bmatrix}}_{\mathbf{T}}. \tag{4}
$$

Based on the dynamics in (3), our transportation problem is stated as follows.

Problem 1. *Given a desired trajectory $\mathbf{q}_b^d(t)$ for a box with with known mass, inertia, contact points, design a control policy for ($\mathbf{T}_1, \mathbf{T}_2$) such that the box asymptotically tracks $\mathbf{q}_b^d(t)$, $\|\mathbf{q}_b(t) - \mathbf{q}_b^d(t)\| \to 0 \, as \, t \to \infty$.*

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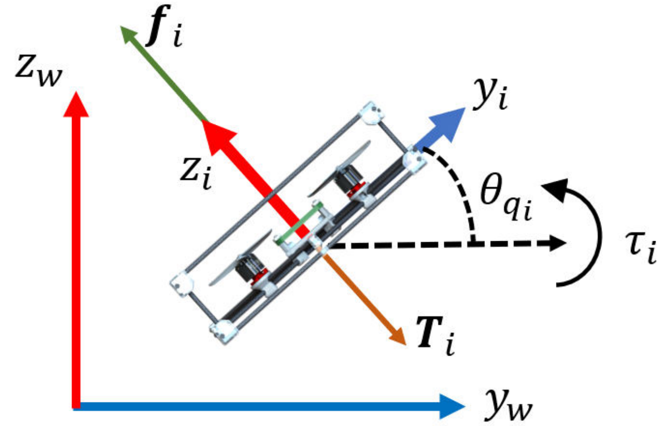


Fig. 3: Planar quadrotor modeling.

B. Planar Quadrotor Modeling

Consider a planar quadrotor (illustrated in Fig. 3) with state vector $\mathbf{q}_i = [y_i, z_i, \theta_i]^\top$ with $i = 1, 2$. The quadrotor dynamics is described by Newton-Euler equations

$$
\begin{bmatrix} \ddot{y}_i \\ \ddot{z}_i \\ \dot{\omega}_i \end{bmatrix} = \begin{bmatrix} \frac{T_i^y}{m} \\ \frac{T_i^z}{m} - g \\ 0 \end{bmatrix} + \begin{bmatrix} -\frac{1}{m}\sin\theta_i & 0 \\ \frac{1}{m}\cos\theta_i & 0 \\ 0 & \frac{1}{J_{xx}} \end{bmatrix} \begin{bmatrix} f_i \\ \tau_i \end{bmatrix}. \tag{5}
$$

Then, our problem is finding the control inputs f_i, τ_i to manipulate a box with unknown information.

Problem 2. *Given two catenary robots attached to a box with unknown mass m_b, unknown inertia J_b, unknown dimensions (l, h), design a control policy for the quadrotors $\mathbf{u} = (f_1, f_2, \tau_1, \tau_2)$, such that without knowing the contact points $(\mathbf{p}_1, \mathbf{p}_2)$, the tension in the cables $(\mathbf{T}_1, \mathbf{T}_2)$ lead the box to asymptotically track a desired trajectory $\mathbf{q}_b^d(t)$, that is $\|\mathbf{q}_b(t) - \mathbf{q}_b^d(t)\| \to 0 \, as \, t \to \infty$.*

Although the contact points $\mathbf{p}_i$ are unknown, the range of possible values is bounded. In Fig. 2, it is shown in blue the threshold in which the contact points can be located. Note that the position of the contact points affects the box dynamics.

For the control design, we consider Problem 1 and Problem 2, respectively. First, we find the wrench necessary for the box to track a trajectory. Here we assume we know all the parameters in the model. This wrench is then translated into the quadrotor dynamics to generate the desired wrench. Finally, we assume that the contact points between the box and the cables, m_b and J_b, are unknown, and using an adaptive controller the box can still track asymptotically the desired trajectory.

III. Wrench Based Control

Let $\mathbf{e}_b = \mathbf{r}_b^d(t) - \mathbf{r}_b(t)$ and $\dot{\mathbf{e}}_b = \dot{\mathbf{r}}_b^d - \dot{\mathbf{r}}_b$ be the position and velocity errors regarding the desired trajectory. We use the errors to find the tensions on the box that track the desired trajectory as a feedforward term,

$$\mathbf{f}_w^d = \mathbf{K}_p \mathbf{e}_p + \mathbf{K}_v \dot{\mathbf{e}}_p + m_b g \mathbf{e}_z, \tag{6}$$

where $\mathbf{K}_p$ and $\mathbf{K}_v$ are gain matrices of a PD controller with appropriate dimensions. We design an attitude controller by defining errors in orientation and angular velocity as $e_\theta = \theta_b^d - \theta_b$ and $e_\omega = \dot{\theta}_b^d - \dot{\theta}_b$, and then based on this attitude errors, the PD controller is defined by

$$\tau_b^d = k_\theta e_\theta + k_\omega e_\omega, \tag{7}$$

where k_θ and k_ω are positive constants. Then, based on the desired force in $\{\mathcal{W}\}$, we transform it to the frame $\{\mathcal{B}\}$,

$$\mathbf{f}_b^d = {}^w\mathbf{Rot}_x(\theta_b)_b^\top \mathbf{f}_w^d, \tag{8}$$

Therefore, the desired wrench on the box is $\mathbf{w} = [\mathbf{f}_b^d, \tau_b^d]^\top$. To find the desired tension in the cables, we compute $\mathbf{T}$ the pseudo-inverse of the matrix $\mathbf{D}$ and multiply it by $\mathbf{w}$. However, our system has some constraints, and not all desired forces in the box can be generated. The angles of the tension are constrained by $\theta_{i,\min}$ and $\theta_{i,\max}$. Hence, this problem can be modeled as a least-square minimization problem with linear constraints,

$$\begin{aligned} \min \quad & \|\mathbf{T}\| \\ \text{s.t.} \quad & \mathbf{w} = \mathbf{D}\mathbf{T}, \\ & T_1^z \geq k_1 T_1^y, \\ & T_1^z \leq k_2 T_1^y, \\ & T_2^z \geq k_3 T_2^y, \\ & T_2^z \leq k_4 T_2^y, \end{aligned} \tag{9}$$

where $k_1 = \tan(\theta_{1,\min})$, $k_2 = \tan(\theta_{1,\max})$, $k_3 = \tan(\theta_{2,\min})$, and $k_4 = \tan(\theta_{2,\max})$.

IV. Adaptive Controller with Unknown Contact Points

In the previous section, we assumed that all system parameters are known, including the contact points. However, the desired contact point is not always the actual point. As it can be seen in Fig. 4, there is an error in the desired contact points and the actual contact points due to the natural shape of the cable and the displacement of the lowest point when the cable is transitioning from being slack to tightening the box. We denote the displacement in the contact points by $\Delta\mathbf{p}_i$. This displacement causes additional error in the actuation during the trajectory tracking. Note that we do not have any active sensor in the cable or box that can provide any feedback on the contact points.

The main problem is that (4) cannot be correctly computed since the contact points are inaccurate. Additionally, m_b and J_b are unknown in (3). However, we overcome this issue by similarly proposing an adaptive controller as in [27], [29].

Based on (3), when all the information is known, we have

$$\ddot{\mathbf{q}} = \mathbf{M}^{-1}(\mathbf{q})(\mathbf{w} - \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} - \mathbf{g}),$$

which can be linearized as $\ddot{\mathbf{q}} = \mathbf{a}$, by taking

$$\mathbf{a} = \ddot{\mathbf{q}}^d - \mathbf{K}_v \dot{\tilde{\mathbf{q}}} - \mathbf{K}_p \tilde{\mathbf{q}},$$

where $\dot{\tilde{\mathbf{q}}} = \dot{\mathbf{q}} - \dot{\mathbf{q}}_d$, $\tilde{\mathbf{q}} = \mathbf{q} - \mathbf{q}_d$. However, in order to apply an adaptive control algorithm, we assume that $p_1^y = -p_2^y = p_y$, due to the geometry of the box, and $p_1^z = p_2^z = p_z$. Therefore, by considering $v_1 = T_1^y + T_2^y$, $v_2 = T_1^z + T_2^z$, and $v_3 = T_1^z - T_2^z$ in (4), we obtain

$$\mathbf{w} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -p_z & 0 & p_y \end{bmatrix}}_{\mathbf{N}} \underbrace{\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}}_{\mathbf{V}}. \tag{10}$$

Now we compute premultiply both sides of (3) by $\mathbf{N}^{-1}$ to obtain $\hat{\mathbf{M}}\ddot{\mathbf{q}} + \hat{\mathbf{g}} = \mathbf{v}$, where $\hat{\mathbf{M}} = \mathbf{N}^{-1}\mathbf{M}$, $\hat{\mathbf{g}} = \mathbf{N}^{-1}\mathbf{g}$. We define an equivalent system as

$$\hat{\mathbf{M}}(\mathbf{q})\ddot{\mathbf{q}} + \hat{\mathbf{g}}(\mathbf{q}) = \mathbf{Y}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})\boldsymbol{\gamma},$$

where $\mathbf{Y}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})$ contains all parameters we know from the system and $\boldsymbol{\gamma}$ is the vector with the parameters to be adapted using the method described in [27]. Let us define $\mathbf{x} = [\mathbf{e}_b, \dot{\mathbf{e}}_b]$ and compute the error dynamics,

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\boldsymbol{\Phi}\tilde{\boldsymbol{\gamma}},$$

where

$$\mathbf{A} = \begin{bmatrix} \mathbf{0}_{3\times3} & \mathbf{I}_{3\times3} \\ -\mathbf{K}_p & -\mathbf{K}_v \end{bmatrix},$$

contains controller parameters,

$$\mathbf{B} = \begin{bmatrix} \mathbf{0}_{3\times3} \\ \mathbf{I}_{3\times3} \end{bmatrix},$$

the regressor matrix $\boldsymbol{\Phi}$ is given by

$$\boldsymbol{\Phi} = \hat{\mathbf{M}}^{-1}(\mathbf{q})\mathbf{Y}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}),$$

and $\tilde{\boldsymbol{\gamma}} = \hat{\boldsymbol{\gamma}} - \boldsymbol{\gamma}$ is the error of the unknown parameters. Let us define the following Lyapunov function

$$\mathbf{V}(\mathbf{x}, \tilde{\boldsymbol{\gamma}}) = \mathbf{x}^\top \mathbf{P}\mathbf{x} + \tilde{\boldsymbol{\gamma}}^\top \boldsymbol{\Gamma}^{-1}\tilde{\boldsymbol{\gamma}}, \tag{11}$$

where $\mathbf{P} = \mathbf{P}^\top > 0$ is the unique symmetric positive definite solution of the linear Lyapunov equation $\mathbf{A}^\top\mathbf{P} + \mathbf{P}\mathbf{A} = -\mathbf{Q}$, and $\boldsymbol{\Gamma} = \boldsymbol{\Gamma}^\top > 0$ therefore, (11) is positive. By checking if the

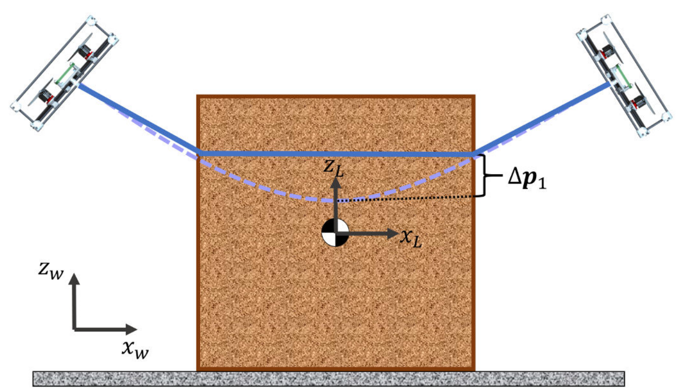


Fig. 4: Side view of a catenary robot controlling the lowest point in the cable to define a desired contact point in the box.

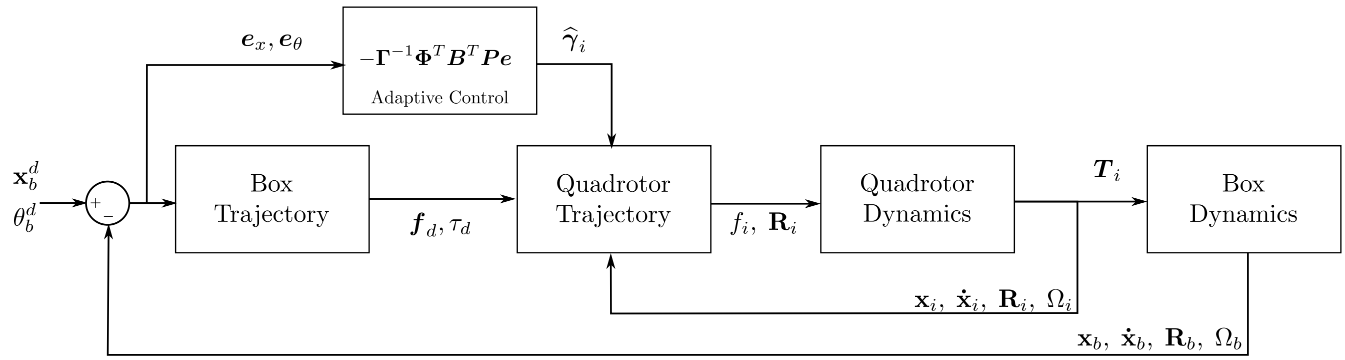


Fig. 5: Control diagram.

time derivative of the Lyapunov function is negative we can verify that the error of the system is asymptotically stable

$$\begin{aligned}\dot{\mathbf{V}}(\mathbf{x},\tilde{\gamma}) &= \dot{\mathbf{x}}^\top \mathbf{P}\mathbf{x} + \mathbf{x}^\top \mathbf{P}\dot{\mathbf{x}} + 2\tilde{\gamma}^\top \mathbf{\Gamma}^{-1}\dot{\tilde{\gamma}}, \\ &= -\mathbf{x}^\top Q\mathbf{x} + 2\tilde{\gamma}^\top(\mathbf{B}^\top \mathbf{\Phi}^\top \mathbf{P}\mathbf{x} + \mathbf{\Gamma}^{-1}\dot{\tilde{\gamma}}).\end{aligned} \tag{12}$$

Assuming $\dot{\tilde{\gamma}} = 0$, then we can define the adaptation law as

$$\dot{\tilde{\gamma}} = -\mathbf{\Gamma}\mathbf{\Phi}^\top \mathbf{B}^\top \mathbf{P}\mathbf{x}, \tag{13}$$

then plugging in (12). The time derivative of Lyapunov function becomes

$$\dot{\mathbf{V}}(\mathbf{x},\tilde{\gamma}) = -\mathbf{x}^\top \mathbf{Q}\mathbf{x} \leq 0, \tag{14}$$

since $\mathbf{Q} = \mathbf{Q}^\top > 0$. However, $\dot{\mathbf{V}}(\mathbf{x},\tilde{\gamma})$ is negative semidefinite. Since for $\mathbf{x} = 0$, $\dot{\mathbf{V}}(\mathbf{x},\tilde{\gamma}) = 0$ then by taking the second time derivative, we can show that $\ddot{\mathbf{V}}(\mathbf{x},\tilde{\gamma})$ is bounded. By applying Barbalat's Lemma [30], we can conclude that $\dot{\mathbf{V}}(\mathbf{x},\tilde{\gamma})$ is uniformly continuous and therefore $\mathbf{x}$ converges asymptotically to zero and $\tilde{\gamma}$ is bounded.

V. Catenary Quadrotor Controller

Once we find the desired tension in the cables to be applied in the box to track a desired trajectory, we can then find the thrust so that the quadrotors generate the desired tensions. Also, note that due to the system constraints

$$\mathbf{r}_i = \mathbf{r}_b + \frac{\ell - w}{2}\boldsymbol{\eta}_i, \qquad \boldsymbol{\eta} = \frac{{}^w\mathbf{R}_b\mathbf{T}_i}{\|{}^w\mathbf{R}_b\mathbf{T}_i\|},$$

the following condition holds

$$\ddot{\mathbf{r}}_i^d = \ddot{\mathbf{r}}_b^d + \frac{\ell - w}{2}\ddot{\boldsymbol{\eta}}_i. \tag{15}$$

By defining the quadrotor position and velocity errors as $\mathbf{e}_i = \mathbf{r}_i^d - \mathbf{r}_i$, and $\dot{\mathbf{e}}_i = \dot{\mathbf{r}}_i^d - \dot{\mathbf{r}}_i$, then thrust controller becomes

$$f_i = ({}^w\mathbf{R}_b^\top \mathbf{T}_i + mg\mathbf{e}_z + \mathbf{K}_{p_i}\mathbf{e}_i + \mathbf{K}_{v_i}\dot{\mathbf{e}} + \ddot{\mathbf{r}}_i^d)\cdot \mathbf{e}_z, \tag{16}$$

where $\mathbf{K}_{p_i}$ and $\mathbf{K}_{v_i}$ are well-defined positive definite matrices. In the same way, we need to know the desired angles for each quadrotor since, in the previous step, we just found the magnitudes of the thrusts

$$\theta_i^d = \arctan2(T_i^y, T_i^z). \tag{17}$$

We can define angle position and velocity errors as $e_{\theta_i} = \theta_i^d - \theta_i$, and $\dot{e}_{\theta_i} = \dot{\theta}_i^d - \dot{\theta}_i$, then the torque controller is given by

$$\tau_i = J_{xx}(k_\tau e_{\theta_i} + k_{d\tau}\dot{e}_{\theta_i}), \tag{18}$$

where k_τ and $k_{d\tau}$ are positive constants.

VI. Simulation and Results

To validate the controller, we perform five different simulation cases. The parameter we use are $m_b = 0.1\,\text{kg}$, $J_b = 0.1\,\text{kg/m}^2$, width of the box $w = 1\text{m}$, height of the box $h = 1.5\text{m}$, cable length $\ell = 2.5\,\text{m}$, gravity $g = 9.8\,\text{m/s}^2$, desired trajectory $y_b^d = \cos(t)$, $z_b^d = \sin(t)$, $\mathbf{p}_1 = [0.5, 0.9]^\top$, $\mathbf{p}_2 = [-0.5, 0.9]^\top$, $\mathbf{K}_p = 4\cdot\mathbf{I}$, $\mathbf{K}_v = 2\cdot\mathbf{I}$, $k_\theta = 1$, $k_\omega = 0.1$.

In all simulations, we plot the coordinates of the y and z-axis, and the angle in the world frame regarding time. The Blue dashed line is the desired trajectory for the box, the solid red line is the actual trajectory of the box, the solid yellow line is the evolution of catenary robot 1, and the solid purple line is the response of catenary robot 2. Finally, at the bottom, a simple animation of two red boxes representing the quadrotors, one light brown box representing the box, and black lines are the cables.

A. Experiment 1: Tracking Trajectory without Constraints

In the first experiment, we compute the pseudo-inverse of matrix $\mathbf{D}$ in (4) in order to get the tension values. As it can be seen in Fig. 6, the pseudo-inverse finds a solution to drive the position errors to zero, and the box can reach the location while oscillating in a sinusoidal form. However, this solution considers tensions in any direction, which is a more realistic scenario that will not be possible. Since friction between the cables and the box is lacking, the contact points will slide or lose contact with the box entirely. However, in order to show that the system is able to track trajectories, we show how the box can perfectly track a circular trajectory in Fig. 7. For both cases, we consider we know all of the parameters of the box needed for the simulation.

B. Experiment 2: Tracking a Trajectory with Angle Constraints

Here we get closer to solve the problem where the cables have angle constraints, meaning the forces cannot be

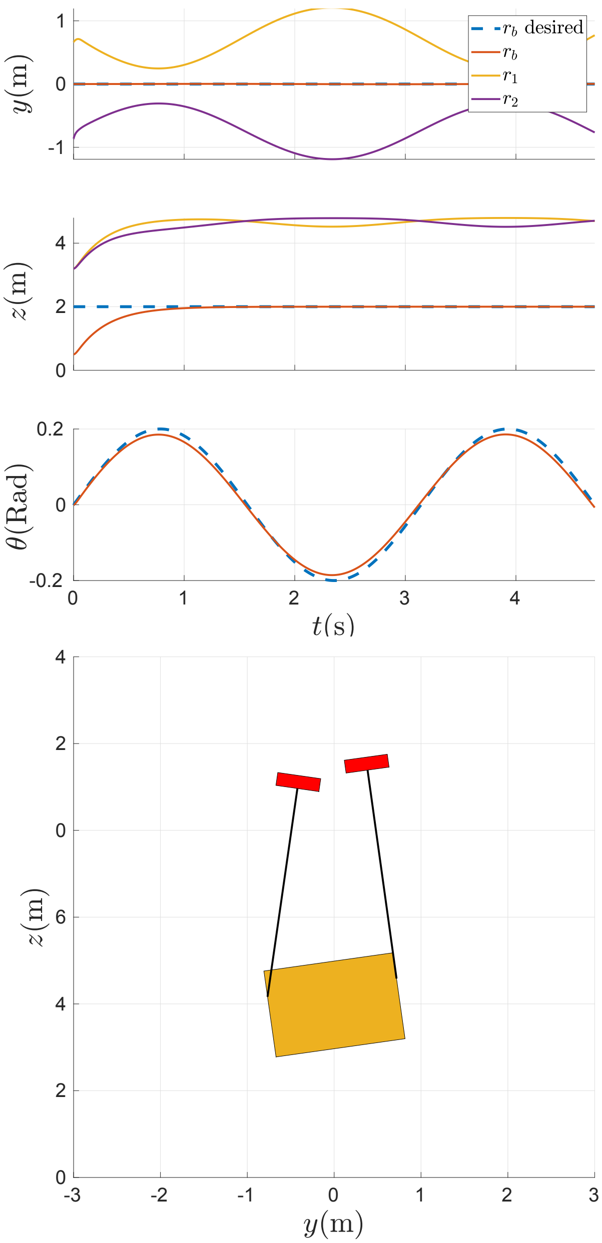


Fig. 6: Box transported by two catenary robots to a fixed location with variations in angle without considering angles constraints in the cables.

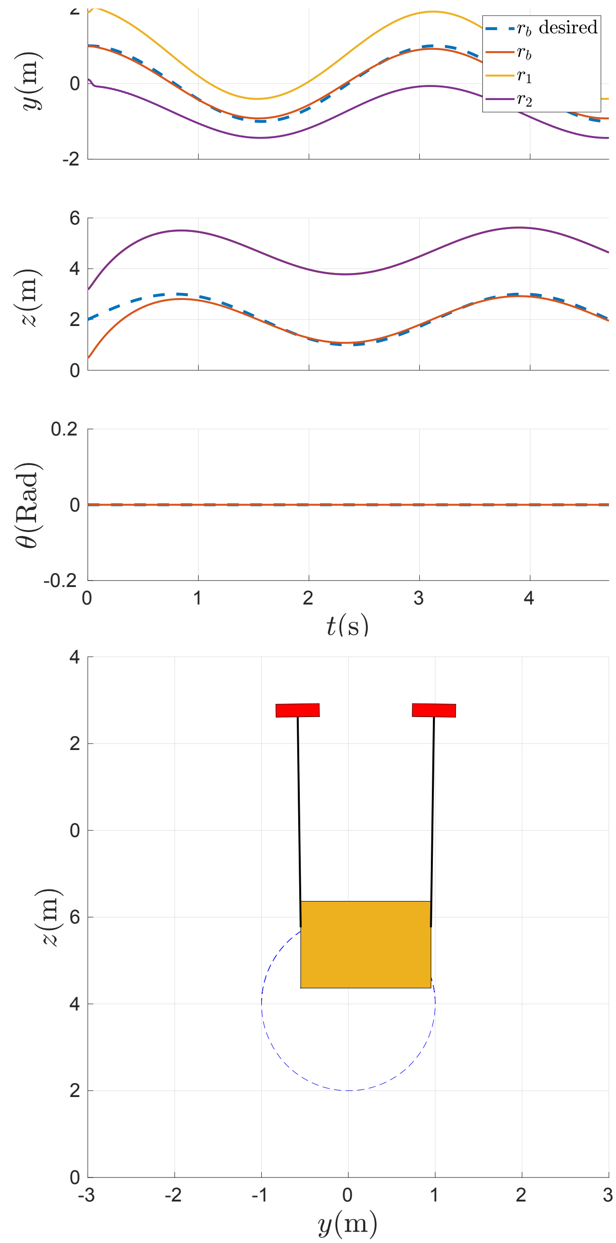


Fig. 7: Box transported by two catenary robots tracking a circular trajectory with static angle in the box and without considering angle constraints in the cables.

generated in an arbitrary direction. Since the force direction is opposite the box, the cable will slip and lose contact with it. Here $\theta_{1,\min} = 115°$, $\theta_{1,\max} = 165°$, $\theta_{2,\min} = 25°$, and $\theta_{2,\max} = 75°$. In Fig. 8, we can see how the cables remain pointing to the opposite direction of the contact point while maintaining the desired cable angle enough to have friction between the cable and the box. Note that for keeping the cable in a constant orientation, one force must compensate the other for canceling out torques.

C. Experiment 3: Tracking a Trajectory with Unknown Parameters: Adaptive Case

In the third experiment, we consider the case where the location of the contact points $\mathbf{p}_i$, with $i \in \{1, 2\}$, the mass of the box m_b, and the inertia matrix J_b are unknown. Then, by using the procedure outlined in Section IV, the box can track a circular trajectory with tracking errors equal to zero and considering the angle constraints in the cables as shown in Fig. 9.

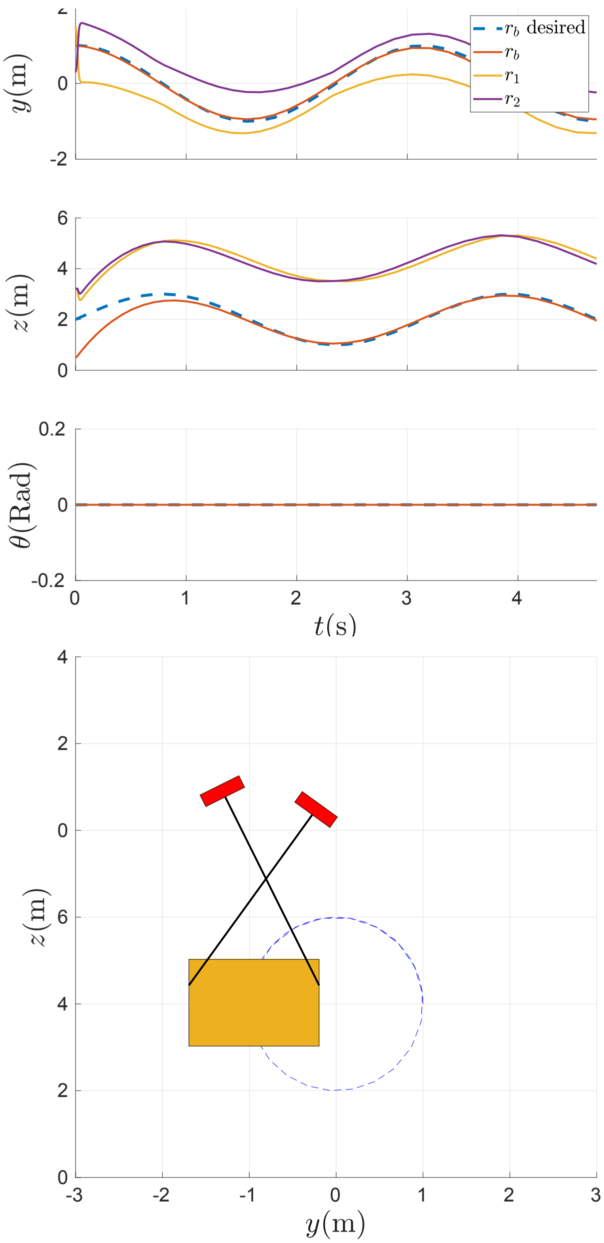


Fig. 8: Box transported by two catenary robots tracking a circular trajectory with angle angle constraints in the cables.

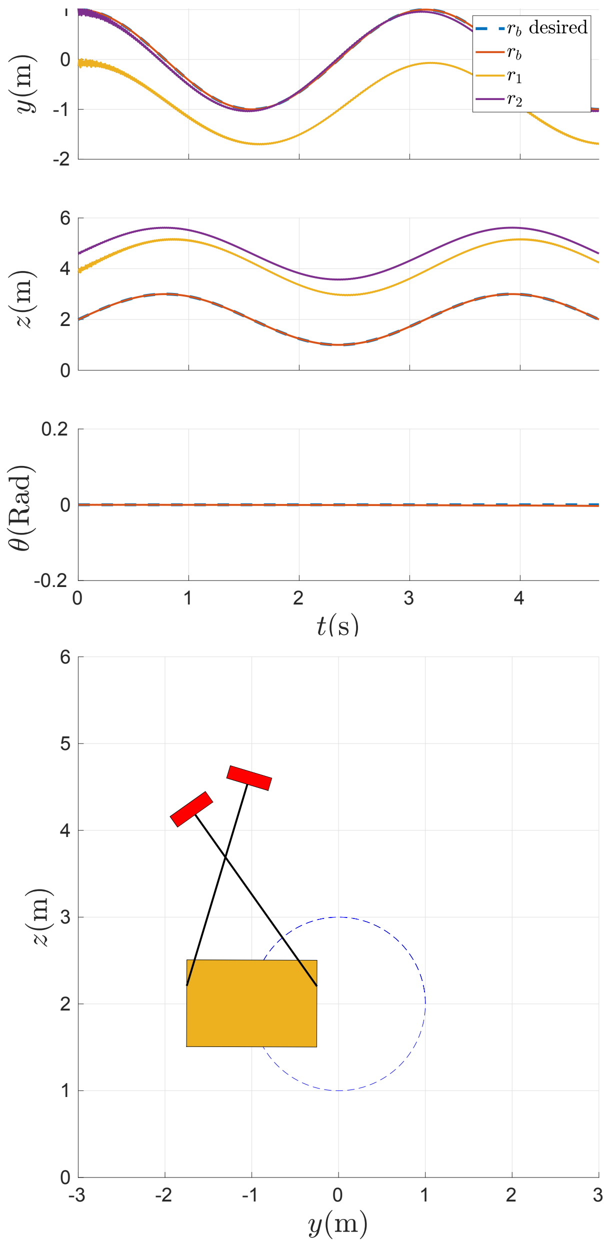


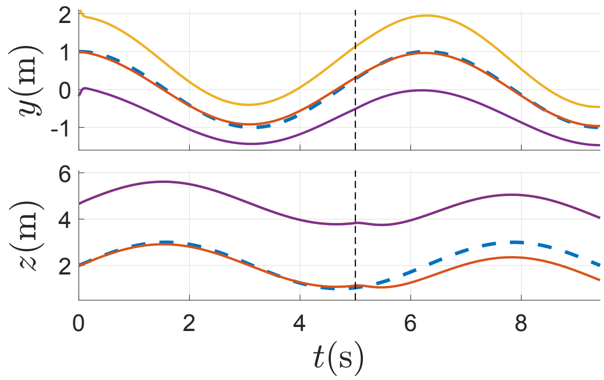
Fig. 9: Box transported by two catenary robots tracking a circular trajectory considering angle constraints in the cables and m_b, J_b, $\mathbf{p}_1$, and $\mathbf{p}_2$ unknown parameters in the simulation.

D. Experiment 4: Varying box mass in Experiment 1 and Experiment 3

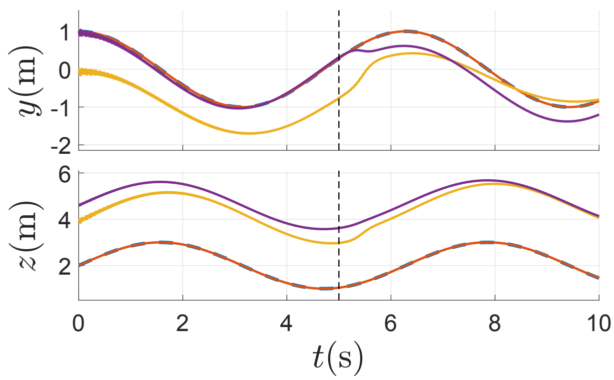
Finally, the last experiment shows the comparison of the traditional controller discussed in Section III and the adaptive controller presented in Section IV when varying the mass of the box. Fig. 10a it is shown how the traditional controller is unable to track the desired trajectory from t = 5 s onwards due to the addition of mass in the box. On the contrary, Fig. 10b illustrates how the adaptive controller, despite the change of mass at t = 5 s, is still able to track the desired trajectory.

VII. Conclusion

In this work, we use two catenary robots to transport a box; first, we consider the case where the cables can apply tensions in any direction. This is the standard case where the cables are attached to the box. Second, by converting to the problem in an optimization framework, we can consider the cable's angle constraints so that the system does not lose friction while allowing us to manipulate the box without previous human intervention and track the desired trajectory. Finally, we show an adaptive controller that overcomes the challenges of unknown parameters in the box as the



(a) Varying box mass using unconstrained controller.



(b) Varying box mass using adaptive controller.

Fig. 10: Varying box mass.

mass, inertia tensor, and contact points. At the same time, still asymptotically track the desired trajectory, making the position error zero and the estimated parameters bounded.

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