#### **ORIGINAL PAPER**



# **Stress Relaxation of Comb Polymer Melts**

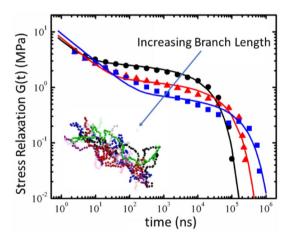
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#### **Abstract**

Branched polymers stress relaxation is at the center to their function as viscosity modifiers, though the fundamentals that underlie the correlation between the polymer topology and their impact on viscosity remains an open question. Here, the stress relaxation of short, branched polyethylene comb polymer melts is studied by molecular dynamics simulations. A coarse-grained model where four methylene groups constitute one bead is used, and the results are transposed to the atomistic level. For arms of length comparable to entanglement length  $n_e$  of the linear polymer, we show that while increasing the number of branches with the same arm length decreases the plateau modulus, the terminal diffusive time does not change significantly. Increasing the arm length decreases the plateau modulus and increases the terminal time. As arms shorter than  $n_e$  relax by the entanglement time, both the chain mobility and stress relaxation can be described by reptation of the backbone with an increased tube diameter and an increased friction coefficient; or in other words, the branches act as a solvent.

#### **Graphic Abstract**



Keywords Comb Polymers · Stress relaxation · Molecular dynamics simulations

### 1 Introduction

The addition of a small number of side branches to flexible, linear polymer chains have dramatic effects on the viscosity and shear response of melts [1–3]. Varying the number

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and length of the branches tethered to the backbone of these comb polymers provides a pathway to control their rheological properties. The immense impact of polymers topology on processing of macromolecules has driven numerous experimental studies, all demonstrating significant effects of the branches on the flow characteristics of the melts [4–15]. These studies show that the dependence of the stress relaxation and shear viscosity on the number and length of the branches is rather convoluted. For example, the storage and loss moduli for combs with short branches measured

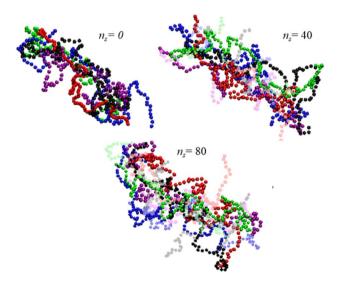


at different temperatures could be superimposed onto a universal curve with a simple shift, however, long-chain branching results in thermo-rheological complexity in which the data cannot be superimposed by a simple temperature shift [7, 9, 10].

On time scales longer than that for the branches to relax, the motion of well entangled comb polymers follow the reptation model [16, 17], in which the polymer chain confined in a tube of neighboring chains, moves cooperatively along the tube axis, commonly referred to as the primitive path. After the branches relax, the branches act as an effective solvent for the backbone while concurrently providing a drag on the motion of the backbone [1, 2, 18]. For comb polymers with short branches, comparable to the entanglement molecular weight of the linear backbone, the dominant contribution to the chain mobility and stress relaxation is from the entangled backbone. In this case, comb polymers behave as reptating chains with increased tube diameter  $d_T$ . For longer arms, the stress relaxation is more complex as the branch segments near the branch point relax on a time scale which depends exponentially on the arm length. This gives rise to an effective tube diameter which grows as the side branches relax. Only after the arms fully relax, does the backbone relax [6, 11, 18] and the backbone entanglements ultimately control the chain mobility and stress relaxation.

While there have been numerous experimental studies of stress relaxation of comb polymers, the local dynamics that governs the macroscopic motion of the polymers has not been realized. Limited number of numerical simulations have recently probed the local motion of comb polymers. Bačová et al. [19, 20] showed that in asymmetric stars and combs with two branches, the branch point movements can be described as the motion over a network of traps. Zhou and Larson [21] simulated asymmetric star polymers using a coarse grained model and showed that for time scales less than the short-arm relaxation time, the branch point remains anchored within a length scale of a tube diameter. Once the short arm relaxes, the branch point takes a random hop along the confining tube. Wijesinghe et al. [22] carried out molecular dynamics simulations for a coarse grained model for entangled polyethylene comb polymers. They showed that as the branch length increases, the combination of arm retraction and backbone mobility leads to a wide spectrum of arm relaxation times and that the motion of the polymer backbone is well described by repetition over intermediate time scales. They also showed that the tube diameter and the entanglement time, directly measured from the crossover from the early time Rouse to reptation regime, both increase linearly with the length of the branches. However, none of these numerical studies have been able to resolve the stress relaxation of comb polymers, critical to the understanding of the mechanism in which topology of polymers affect their stress release.

Here, we probe the effects of branching on the dynamics of melts of loosely branched comb polymers with branches of length comparable to the entanglement molecular weight  $M_a$  using molecular dynamics (MD) simulations. The branches are sparsely distributed along the backbone, so that they do not affect the end-to-end distance of the polymer. We study the stress relaxation of melts of entangled comb polymers with multiple short branches. MD simulations allow us to visualize and study the microscopic behavior that underlies macroscopic experimental observations. Given the broad time and length scales associated with the polymeric motion, we adopt a coarse grained (CG) model for polyethylene, where 4 methylene groups are represented by one bead, to capture the mobility and stress relaxation of loosely branched polyethylene melts on multiple time and length scales. This model captures well the diffusion, plateau modulus and viscosity of entangled, linear polyethylene melts [23, 24]. The results presented here add an important facet to our previous results on the mobility of comb polymers. The molecular motion of these branched polymers is captured through reptation model with a tube diameter that increases as the length of the branches increases, illustrated by the chain snapshots shown in Fig. 1.



**Fig. 1** Visualization of a coarse grained model of a polyethylene chain with backbone of n=480 carbons (120 CG beads) plotted five times (different colors), each separated by 62 ns. The chains have  $n_b=4$  randomly placed branches of length  $n_s=0$ , 40, and 80 carbon atoms (0, 10, 20 and 40 CG beads). Backbone is plotted in dark color while side chains are translucent



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# 2 Model and Methodology

A CG model for polyethylene with four methylene groups per CG bead (coarse graining degree  $\lambda_c = 4$ ) previously derived by Salerno et al. was used for the current study [23, 25, 26]. The interaction potential between CG beads was derived from an atomistic simulation for a linear chain melt of  $C_{96}H_{194}$  at density  $\rho = 0.76$  g/cm<sup>3</sup> and temperature T = 500 K using an iterative Boltzmann procedure. The atomistic simulations used the optimized potentials for liquid simulations (OPLS) force fields of Jorgensen et al. [27, 28] with modified dihedral coefficients to better reproduce the properties of long alkanes [29]. In the CG model nonbonded interactions are truncated at 1.0 nm. All simulations were run at constant volume using a velocity-Verlet integrator with a time step of 20 fs. A Langevin thermostat [30, 31] with damping time constant of 20 ps maintains the temperature at 500 K. Periodic boundary conditions were used in all three directions. All simulations use the large atomic molecular massive parallel simulator (LAMMPS) molecular dynamics code [32]. Detailed descriptions of the CG development is given by Salerno et al. [23, 25] and Peters et al. [26].

As coarse graining reduces the number of degrees of freedom, the free-energy landscape is much smoother compared to fully atomistic simulations. This results in an increase in the mobility of the chains in the CG models compared to the fully atomistic model [25]. For  $\lambda_c = 4$  at 500 K, Salerno et al. [25] showed that the mean squared displacement in the atomistic and CG model for melts of linear chains with n = 96 and 480 carbons overlapped if time in the CG model is scaled by a dynamic scaling factor  $\alpha = 6.2$ . We use this value of  $\alpha$  to scale time for the comb polymers.

The PE comb polymer melts were prepared as described in Wijesinghe et al. [22]. Our previous study was able to resolve mobility and viscosity for melts of chains with n = 480 backbone carbons (120 CG beads) and n = 1920 backbone carbons (480 CG beads) with  $n_b = 4$ to 16 randomly placed branches of length  $n_s = 0-160$  carbons (0-40 CG beads). Here we probe the stress relaxation of the longer chain melts with 800 chains of length n = 1920 backbone carbons with  $n_b = 16$  randomly placed branches of length  $n_s = 0$ , 40, and 80 and  $n_h = 16$ , 32 and 64 branches of length  $n_s = 40$ . For all values of  $n_s$  and  $n_h$ studied here, these chains are in the comb regime where the branches do not affect the polymer backbone rigidity, as reflected in a nearly constant mean squared radius of gyration  $\langle R_g^2 \rangle$  [22]. For linear polyethylene chains, this CG models reproduces quite well the single chain statistics including the mean squared end-to-end distance  $\langle R^2 \rangle$  $M_w = 1.29 \pm 0.02 \text{ Å}^2/(\text{g/mol})$  which is in good agreement

with the experimental value of 1.21 Å<sup>2</sup>/(g/mol) obtained from small-angle neutron scattering at 443 K and packing length  $p = (\rho_{\text{chain}} < R^2 >) = 1.8$  Å compared to the experimental value of p = 1.69 Å at 413 K [33]. For reference, the experimentally [34, 35] measured entanglement molecular weight  $M_e$  of linear polyethylene without branches is 1.1-1.2 kg/mol or about  $n_e = 80$  (20 CG beads). Hence, the comb polymers studied here are well entangled with  $Z = n/n_e = 24$  while the length of the side branches are relatively short,  $n_s = 0$ ,  $n_e/2$  and  $n_e$ . Here  $n_e$  is the entanglement strand length for linear polymers, the effective  $n_e$  for the backbone of the comb polymers is larger as suggested by the lower plateau modulus.

The stress relaxation modulus was measured for each system using the Green–Kubo relation where  $\sigma_{\alpha\beta}(t)$  are the off-diagonal components xy, xz and yz of the stress, V is the volume of the system and t is time. Results for G(t) presented here are an average of the three components. We also measured the normal stress decay after deforming polymer chains in a melt by a small step strain [36, 37]. This was done by applying a uniaxial elongation to deform the simulation cell in the x-direction  $L_x = \lambda L$  while shrinking the simulation cell in the other two directions  $L_y = L_Z = L/\sqrt{\lambda}$  to keep the density of the system constant. Using the stress–strain description for classical rubber elasticity [38], the stress relaxation modulus is given by

$$G(t) = \frac{\sigma_{xx}(t) - \frac{1}{2} \left( \sigma_{yy}(t) + \sigma_{zz}(t) \right)}{\lambda^2 - 1/\lambda} \tag{1}$$

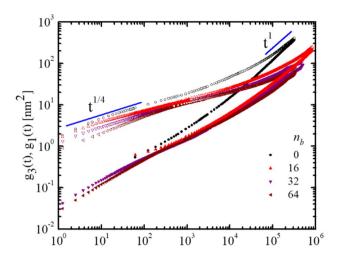
Here, we use  $\lambda = 1.1$  and 1.2 following Hsu and Kremer [36] who applied this method to determine G(t) for the standard bead spring model [39].

#### 3 Results

To distinguish the relevant time scales and different scaling regimes, we first consider the effect adding multiple branches on the chain mobility. Previously we considered the effect of increasing the branch length  $n_s$ , here we present new results for increasing number of branches  $n_h$ . The mean squared displacement (MSD) of chains in equilibrated melts was calculated for the center six beads,  $g_1(t) = \langle (r_i(t)) \rangle$  $-r_i(0)$ )<sup>2</sup> > to suppress the fluctuations caused by chain ends and for the center of mass (cm) of the chains,  $g_3(t) = \langle (r_{cm}) \rangle$  $(t)-r_{cm}(0))^2$  >, where  $r_i(t)$  is the position of atom i at time t, and  $r_{cm}(t)$  is the position of the center of mass of the chains. Figure 2 shows results for the MSD of chains of n = 1920carbon atoms (480 CG beads) with a different number of branches  $n_b$  each with branch length  $n_s = 40$ . These data show that the number of branches has only a small effect on chain mobility in the comb regime. They also show the



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**Fig. 2** Mean-squared displacement (MSD) of the center six CG beads  $g_1(t)$  (open symbols) and of the center of mass  $g_3(t)$  (full symbols) for chains of n = 1920 carbon atoms for the indicated number of branches  $n_b$  of lengths  $n_s = 40$ . The solid lines represent the scaling predictions  $t^l$  for the diffusive regime and  $t^{1/4}$  for the reptation regime

distinctive  $t^{1/4}$  scaling regime in  $g_1(t)$  at intermediate times, which arises from the relaxation dynamics of the backbone confined in the tube. At later times the MSD increases as  $t^1$  in the diffusive regime. The diffusive time  $\tau_d$ , determined from the time that the inner beads have moved a distance comparable to the size of the chain  $g_1(\tau_d) = 3 < R_{gb}^2 >$ , increases only slightly with increasing  $n_b$ . Here,  $< R_{gb}^2 >$  is the mean squared radius of gyration of the backbone.

The effect of increasing the length of the branches is much stronger than that of increasing the number of branches. We previously showed [22] that the extent of the  $t^{1/4}$  scaling regime increases with increasing branch length  $n_s$ . The tube diameter  $d_T$  and entanglement time  $\tau_e$ , extracted from the crossover from the early time  $t^{1/2}$  Rouse regime to the  $t^{1/4}$  scaling regime in  $g_I(t)$  both increase linearly with increasing branch length  $n_s$ , while the diffusive time  $\tau_d$ , increased exponentially with increasing branch length [22]. Figure 1 illustrates this increase in  $d_T$  with increasing chain length of the branches  $n_s$ . For illustration we show results for n = 480, though all the results in this paper are longer chains with n = 1920 backbone carbons.

In the reptation mode, the polymer chain confined in a tube moves coherently along the primitive path, after all internal dynamic modes have been relaxed. The primitive path length scales with the entanglement length as  $L_{pp} \sim {}^1/n_e^{1/2}$ . The friction coefficient  $\zeta$  of the chain scales linearly with the number of monomers in the chain. The reptation time  $\tau_d$  is the time for the chain to diffuse a distance  $L_{pp}$ , and scales as  $\tau_d \sim L_{pp}^2 \zeta \sim \zeta/n_e$ . In the approximation of a comb polymer with short branches as an effective linear polymer, both  $\zeta$  and  $n_e$  are increased by the addition of side branches. Since short branches undergo Rouse dynamics,

the overall chain friction coefficient  $\zeta$  is nevertheless proportional to the number of monomers, which is increased by the additional monomers of the branches.

The relaxation time of the branches compared to the backbone has a strong effect on both the chain mobility and stress relaxation. We measured the autocorrelation function of the end-to-end vector of the branches and extracted the branch relaxation times  $\tau_a$  [22]. For  $n_s$  = 40, the time for the arms to relax  $\tau_a$  ~ 13 ns, is much shorter than the time  $\tau_e$  ~ 50 ns to reach the reptation regime, while for  $n_s$  = 80,  $\tau_a$  ~ 100 ns, which is comparable to  $\tau_e$  [22] for  $n_s$  = 80. That the arm relaxation times are comparable to or shorter than  $\tau_e$  supports our finding that these comb polymers with short arms behave as reptating chains with increased tube diameter.

The stress response function after a small perturbation G(t) is an important experimental measure of polymer rheology. For long entangled linear polymers, at short times G(t)decays as the chains locally relax in response to the perturbation like any fluid. However, for intermediate times G(t) plateaus at  $G_N^o = \frac{4}{5}\rho RT/M_e$  This plateau region in G(t) occurs for intermediate times where the chains are assumed to move in a tube due to entanglements from the other chains. Only after the chains have reached the diffusive regime, does G(t) relax to zero at a time  $\tau_d$ . For short branches, the relaxation time is shorter than the time  $\tau_d$  for the backbone to feel the effect of entanglements from other chains. In this case, the branches only affect the very early decay of G(t). Short branches increase the effective tube diameter  $d_T$  and entanglement time  $\tau_d$ , and thus allow more internal dynamics modes to relax without being affected by the tube. The dominant contribution to the stress relaxation in this case of short branches is from the entangled backbone. For branches longer than  $n_e$ , the stress relaxation is more complex as the branch segments near the branch point relax on time scale which depends exponentially on the branch length. This gives rise to an effective tube diameter which grows as the side branches relax. Only after the branches fully relax, does the backbone relax [6, 11, 18].

The relaxation modulus was measured from equilibrium stress autocorrelations and from stress relaxation after small strain. Figure 3a shows G(t) measured from the stress autocorrelation function for comb polymers  $n_b = 16$ , 32 and 64 branches of length  $n_s = 40$  and Fig. 3b shows G(t) for branch lengths  $n_s = 0$ , 40, and 80 with  $n_b = 16$  branches. As the number of branches  $n_b$  increases the plateau modulus  $G_N^o$  decreases while terminal time  $\tau_d$  is approximately unchanged. Increasing the length of the side branches  $n_s$  leads to a decrease in  $G_N^o$  and an increase in  $\tau_d$ . To quantify the changes in the plateau modulus, G(t) for the short branched comb polymers studied here was fit to a modified Likhtman-McLeish (LM) expression [40], which combines self-consistent theories for contour length fluctuations and constraint release with reptation theory,



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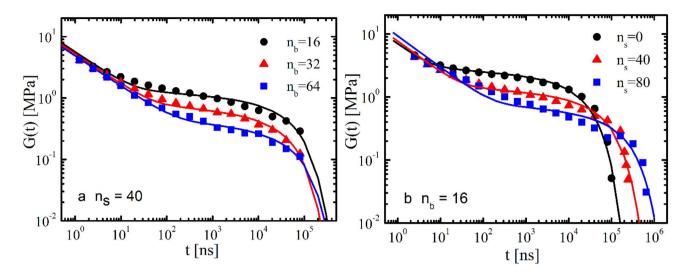


Fig. 3 Stress relaxation function G(t) for comb polymers with backbone of n = 1920 carbons for melts with a)  $n_b = 16$ , 32 and 64 branches of length  $n_s = 40$  and b) branch lengths  $n_s = 0$ , 40, and 80 with  $n_b = 16$  branches. Solid lines are fit to eqs. 2 and 3

$$G(t) = \frac{k_B T}{nv} \left[ \frac{1}{5} \sum_{p=1}^{Z} \left( 4\mu(t) R(t) + e^{-tp^2/\tau_R} \right) + \sum_{p=Z+1}^{N} e^{-2tp^2/\tau_R} \right]$$
(2)

Here,  $\mu(t)$  and R(t) account for single- and multi-chain relaxation processes of the tube model and n is the total number of CH<sub>2</sub> monomers in the system. The key quantity in this expression is the single-chain memory function  $\mu(t)$  for the fraction of the primitive chain which has not escaped from its original tube after a time t. For  $\mu(t)$ , we use the Doi-Edwards reptation stress relaxation function [17]

$$\mu(t) = \frac{8}{\pi^2} \sum_{q=1}^{\infty} \frac{1}{q^2} \exp\left(-\frac{q^2 t}{\tau_d}\right)$$
 (3)

For R(t), we use the double-reptation expression for constraint release  $R(t) = \mu(t)$  [41, 42]. The three fitting parameters are the number of entanglements per chain Z, the terminal relaxation time  $\tau_{\rm d}$ , and the Rouse time  $\tau_{\rm R}$ . As Z is related to the mode index and has to be an integer. In the fitting, Z is varied over integer values to obtain the best fit. The volume of a CH<sub>2</sub> monomer is v = 0.031 nm<sup>3</sup> for  $\rho = 0.76$  g/cm<sup>3</sup>. Equations 2 and 3 are appropriate for the systems studied here since the arms relax on a time scale shorter than that for the backbone of feel mutual entanglements from other chains. For branches longer than  $n_e$ , one has to include the fact that the tube diameter  $d_T$  increases and the effective entanglement length  $n_e$  increases as the branches relax so that the total primitive path length becomes dependent on the current extent of relaxation of the branches [6, 11, 18].

As the number of branches increase from 16 to 64, the plateau modulus  $G_N^o$  decreases approximately linearly,  $G_N^o$ =

1.13 MPa, 0.66 MPa and 0.38 MPa for  $n_b = 16$ , 32 and 64, respectively. For comparison,  $G_N^o = 2.45$ MPa for linear polymers. The entanglement time  $\tau_e$  increases with increasing  $n_b$ ,  $\tau_e = 40$  ns, 101 ns and 265 ns for  $n_b = 16$ , 32 and 64, respectively. From the fit to eqs. 2 and 3, the longest relaxation times  $\tau_d \sim 1.5 \times 10^5$  ns are independent of the number of branches within the error of our measurements. The diffusive time  $\tau_d \sim 5 \times 10^5$  ns extracted from the MSD of the center backbone beads (Fig. 2) agrees very well with that found from the stress relaxation. These results show that as the number of branches increases, the ratio  $\zeta/n_e$  and thus  $\tau_d$  remains almost unchanged.

As the length of the branches increases, the plateau modulus  $G_N^o$  also decreases linearly with increasing  $n_s$ ,  $G_N^o = 2.45$  MPa, 1.22 MPa and 0.66 MPa for branch length  $n_s = 0$ , 40 and 80, respectively. As seen in Fig. 4, the relaxation times describing the stress relaxation are in excellent agreement with those extracted from the MSD, supporting the idea that for these short comb polymers can be treated as linear chains with a larger tube diameter. The Rouse time does not change much with increasing branch length, as it is the relaxation time of the internal dynamic modes along the backbone, which are not affected by the branches.

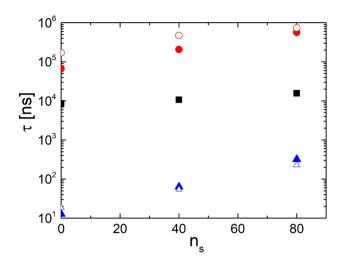
Fetters et al. [33, 34] have shown that for a wide range of flexible linear Gaussian chains, the entanglement molecular weight  $M_e$  can be expressed as a universal power law of the packing length p,

$$M_e = n_t^2 N_A \rho p^3 \tag{4}$$

where  $N_A$  is the Avogadro number. Fetters et al. [34] found that the coefficient  $n_t$  is insensitive to temperature and equal to  $19.1 \pm 1.4$  (assuming  $M_e = \frac{4}{5} \rho RT/G_N^o$ ). As seen in Fig. 5, the backbone entanglement length  $M_e$  scales with  $p^3$  as



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**Fig. 4** Entanglement time  $\tau_e$  (blue triangles) Rouse time,  $\tau_R$  (black square) and diffusive time  $\tau_d$  (red circles) as a function of length of the branches extracted from the MSD (open) and from G(t) (filled). Length of the backbone n=1920 carbons with  $n_b=16$  branches. Error bars are size of symbols

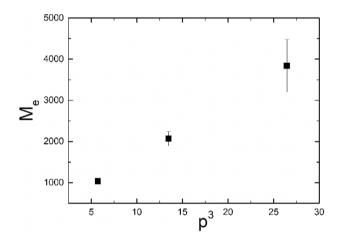
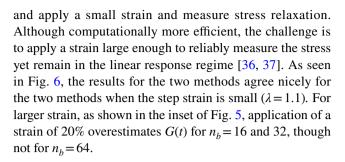


Fig. 5 Entanglement molecular weight  $M_e$  of the backbone as a function of the packing length p

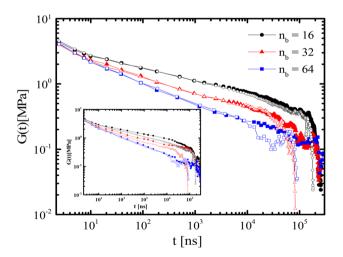
predicted for flexible linear chains. The best fit to the coefficient  $n_t = 16.4 \pm 0.1$ . Combs with  $n_b = 32$ ,  $n_s = 16$  and  $n_b = 16$ ,  $n_s = 40$  have the same packing length p and the same plateau modulus from the best fits to G(t) as predicted by Eq. 4. This further supports the idea that the combs with short chains behave as reptating chains with increased tube diameter.

Determination of the stress relaxation function G(t) from fluctuations in the off-diagonal components of the stress tensor requires long simulations, usually several times longer than the longest relaxation time, to obtain reliable results. An alternative method is to follow the experimental procedure



#### 4 Conclusions

The effect of the number of branches and branch length on the chain mobility and stress relaxation of short-branched polyethylene comb polymer melts with short branches has been studied. We find that the presence of sparsely placed branches greatly reduces the chain mobility of comb polymers compared to their linear analogs. The short branches act as an effective solvent for the backbone and increase the tube diameter and the entanglement time. While increasing the number of branches has only a small effect on the chain mobility and the terminal relaxation time, the plateau modulus decreases linearly with the number of branches. Increasing the branch length significantly reduces the chain mobility and plateau modulus. The entanglement time  $\tau_a$ and terminal relaxation times  $t_d$  extracted from the chain mobility and from the stress relaxation are in very good agreement.



**Fig. 6** Comparison of stress relaxation function G(t) measured from the Green–Kubo relationship (filled symbols) and from step strain with  $\lambda = 1.1$  (Eq. 1) (open symbols) for comb polymers with  $n_b = 16$ , 32 and 64 branches of length  $n_s = 40$ . Inset compares results for  $\lambda = 1.1$  (open symbols) and 1.2 (filled symbols)



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