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Refuel Scheduling for Multirobot Charging-on-Demand

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***Abstract*— In this paper, we consider the refuel scheduling problem for a team of ground robots deployed in "aisle-like" environments wherein the robots are constrained to move along rows. In order to maintain a minimum service rate or throughput for the ground robots, we investigate the problem of scheduling a team of mobile charging stations deployed to replace the batteries on-board the ground robots without any interruption in their task. We propose two scheduling schemes for the mobile chargers to serve the ground robots for long-term service, and derive the parameters associated with the system required for persistent uninterrupted operation.**

I. Introduction

In the last two decades, there has been a growing interest in designing and developing multi-robot systems for societal needs [1], [2]. Some features of multi-robot systems that make them more effective compared to a single robot are time-efficiency demonstrated in complex task implementation, less susceptibility to single-points of failure, and demonstration of multiple capabilities [3]. Multi-robot systems have been successfully deployed in large-scale surveillance tasks related to environmental monitoring [4], long-duration geographic mapping [5], and agricultural data collection [6]. These applications usually require robots to operate in open spaces and cover a large area wherein persistence and long-term autonomy are key requirements. Access to a reliable external power source is an absolute necessity in such scenarios. In this work, we address a scheduling problem that arises when a team of mobile chargers is deployed to replace the on-board battery of a team of mobile robots.

Existing literature on recharging robots deployed in the field requires them to pause their tasks and either get recharged in the field or exit the workspace and visit the charging docks. In [7], authors propose a scheduling strategy for long-term persistent operations that allows aerial robots to return to their docking stations for recharging. In [8], authors introduce an energy-aware control policy that combines a robot's mission objective with its desire to reach a charging dock to recharge. In [9], authors consider the charging dock itself as an autonomous robot that attempts to incrementally improve recharging efficiency. [10] proposes a method that replaces the discharged robots with fully charged robots to eliminate interruption and round-trip cost (fuel and time) associated with visiting a charger. However, such a technique significantly increases the number of robots. In [11], authors formulate the problem of assigning chargers to robots as a non-cooperative game. In [12], authors propose a graph-based approach for recharging multi-robot systems deployed in persistent tasks. The aforementioned approaches either require the robots to travel to the charging station or stop their task to recharge for long-term operations.

Autonomous battery charging can be substituted by autonomous battery swapping [13]. The discharged battery cells on a robot can be replaced by fully charged batteries without halting the operation [14]. Autonomous battery swapping mechanisms for UAVs and robots have already been proposed and developed in [15], [16]. Our current work explores the scheduling problems that arise out of a similar set up being developed in our lab to replace batteries on-board ground robots deployed in agricultural applications [6].

Scheduling problems in robotic systems represent special cases of the general scheduling problem which often refers to the problem of arranging a set of tasks on a set of processors [17]. Collision avoidance and deadlock prevention are important issues in scheduling [18]. In [19], we studied a refuel scheduling problem for a team of autonomous grain carts that serves a team of combines without any interruption. The objective was to find the relation between the capacities of the autonomous ground vehicles for persistent operation without any collaboration between the vehicles. Our current work is in a similar vein. In this work, we investigate the extent to which the server (aerial chargers) and clients (ground robots) need to collaborate in order to ensure that the resulting schedule has no tardiness.

The contributions of this work are as follows. (1) We present scheduling schemes for a team of chargers that periodically replenishes the battery in a team of robots deployed to work in aisle-like environments without interruption. (2) The schedule can handle arbitrary number of chargers and robots i.e, the technique scalable in terms of the number of robots deployed in the entire operation. (3) We explore the coordination needed between the two teams to achieve zero tardiness in both scheduling schemes.

The paper is organized as follows. In Section II, we present the problem formulation. In Section III, we analyze the scheduling problem for one charger serving an arbitrary number of robots. In Section IV, we generalize our previous analysis for an arbitrary number of chargers serving an arbitrary number of robots in a round robin schedule. In Section IV, we address the scheduling problem for an arbitrary number of chargers and robots based on a load-

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balancing scheme. Finally, we present the conclusions along with some future directions of research in Section V.

II. Problem Formulation

In this section, we describe the workspace, deployment strategy and parameters relevant to the scheduling problem. For the sake of clarity, we use the term *worker* for the robots deployed in the field, and *charger* for the ground/aerial robots used for battery replacement. Since our problem is motivated from agricultural applications, we consider workers that move in a row as shown in Figure 1. As the robot moves along a row, there is vegetation on both sides. Moreover, the space in which the robot moves is just wide enough to accommodate at most one robot at a time[1]. The worker can change rows only when it is at either end of the row. Such "aisle-like" environments [20] are encountered in warehouses and retail stores [21] in addition to farmlands.

We assume each worker is powered by a battery bank composed of m battery cells. The capacity of each battery cell is C_{cell}. All the workers have the same battery capacity $C = mC_{cell}$, and a discharging rate r_d. In the beginning, the workers enter the rows sequentially as shown in Figure 1. A charger replaces the discharged battery cells from the worker's battery bank using the swapping method described in the previous section. In order to prevent any interruption in the worker's task, the charger moves with the worker (in case of a ground vehicles) or lands on the worker (in case of an aerial vehicle) to perform the swapping operation. Since the charger can carry multiple battery packs, it can charge multiple workers before making the next visit to the charging station.

When a charger runs out of all its fully charged battery cells, it travels to the depot/charging station to recharge the swapped battery cells. We assume that all the chargers have the same charging rate $r_c = r_{swap}C_{cell}$, where r_{swap} is the swapping rate (number of cells per second) for all the chargers. We introduce the following notations to denote the pertinent time intervals during this operation:

1) T: Time for each charger to move between two adjacent workers.
2) $T_d(=\frac{C}{r_d})$: Time for a worker to get fully discharged.
3) $T_c(=\frac{C}{r_c - r_d})$: Time for a charger to fully charge a worker.

In the rest of the paper, we make the following assumptions. (i) The analysis is for the simple case of a rectangular space (ii) The chargers serve the workers to ensure that the robots are fully charged at the beginning of a new row. Given the above constraints associated with deployment, we want to analyze scheduling schemes for the chargers to replace battery on-board the workers that can ensure persistent operation without any stoppage. In the next section, we present our first scheduling scheme.

[1]Planters try to minimize the empty space between two rows of vegetation to maximize the yield

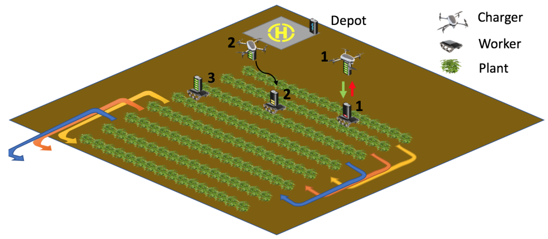

Fig. 1: Figure shows 3 workers and 2 chargers deployed in the field

III. Round Robin Scheduling Strategy

A. Round-robin scheme for a Single charger with Arbitrary Capacity

We consider the case in which a charger carries sufficient supply of cells to serve multiple workers before returning to the charging station. Initially, N labelled workers enter the work space as shown in Figure 2. They enter the work space sequentially with time gap ΔT. This prevents any two workers from getting fully discharged at the same time. We assume that all worker robots are identical. A single charger $(M = 1)$ serves them following the First-Come-First-Serve order i.e., priority is given to the worker that gets discharged first

B. Charging $> N$ workers

First, we analyze the case in which the charger carries a supply of cells sufficient enough to charge $n = N + k$ $(1 \leq k < N, k \in \mathbb{Z})$ workers before it returns to the depot. We define a *round* as the time during which $N + k$ workers get served once. A *cycle*[2] is defined as the number of rounds after which the charger starts a round from the same worker.

The workers need to collaborate (adjust their discharging rate i.e., moving speed) with the charger so that the worker at which the charger arrives after visiting the depot is just out of power. In any round, a worker can get charged at most twice. We assume that the discharging rate of worker i in round r for the t^{th} time is $r_{d,i}^{r,t}$, $(t \in 1,2)$. The discharging time for the worker i in round r to get fully discharged for the t^{th} time is $T_{d,i}^{r,t}$. The charging time of each worker is T_c which is a constant. During the time in which a worker runs out of power $T_{d,i}^{r,t}$, the charger serves all the other workers and may or may not visit the depot. For any round r, the first worker that will be run out of power is worker $w_r = [(r-1)(N+k)] \bmod N + 1$.

As shown in Figure 2, each round can be divided into two parts: the duration for which the first N workers get charged and the duration for which the last k workers get charged. In the first round, all the workers move at their maximum speed with discharging rate r_{dm} before they run out of power for the first time. Therefore, the discharging time for the first N workers in the first round is $\frac{C}{r_{dm}}$. For the last k workers in the first round, the charger needs to travel back from the last worker to the first and then visit all the other workers.

[2]Therefore, in each *cycle*, there are $\frac{l}{N+k}$ rounds, where l is the lowest common multiple of $N + k$ and N.

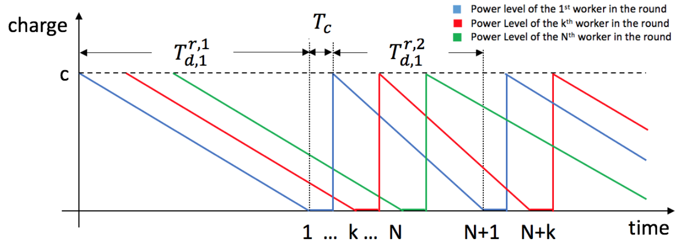

Fig. 2: Load variation of workers when $n = N + k$

Therefore, we obtain the following relation:

$$r_{d,i}^{1,2} = \frac{C}{T_{d,i}^{1,2}} = \frac{C}{(N-1)T_c + (N-1)T + T_{back}} \tag{1}$$

From round two, the workers will no longer move with their maximum speed. For any $r \geq 2$, the discharging time for the first N workers, which are workers 1 to N in this round is shown in Figure 2, $T_{d,i}^{r,t}$ contains $N-1$ charging time T_c, $N-2$ traveling time between adjacent workers, travel time from the last worker to the first, and a round-trip between the workers and the depot T_{add}. Assuming identical parameters for all workers, the workers will travel the same distance till they get fully discharged. Therefore, the travel time between adjacent workers as well as the travel time from the last worker to the first will always be a constant regardless of the variation in their speeds. Let T be the travel time between adjacent workers and T_{back} be the time required by the charger to travel from the last worker to the first.

For the first N workers in a round r within a cycle, the following relation holds:

$$r_{d,i}^{r,1} = \frac{C}{T_{d,i}^{r,1}} = \frac{C}{(N-1)T_c + (N-2)T + T_{add} + T_{back}} \tag{2}$$

The last k workers, workers $N+1$ to $N+k$ in Figure 2, this is the second time they get served in the round. The working time for them contains $N-1$ times of charging time T_c, $N-1$ times of traveling between adjacent workers T and one time of traveling from the last worker to the first worker T_{back}. The charger doesn't visit the depot during this period. Therefore, for the last k workers in a round, we obtain the following relation:

$$r_{d,i}^{r,2} = \frac{C}{T_{d,i}^{r,2}} = \frac{C}{(N-1)T_c + (N-1)T + T_{back}} \tag{3}$$

At the end of any cycle $c \geq 1$, the charger replaces its discharged cells with fully charged cells at the depot after charging the last worker N. This implies that the travel time from the last worker to the first will be replaced by T_{add}. In this case, the initial discharging time of the first N workers in the first round in a cycle $c+1$ is given by the following expression:

$$r_{d,i}^{r,1} = \frac{C}{T_{d,i}^{r,1}} = \frac{C}{(N-1)T_c + (N-1)T + T_{add}} \tag{4}$$

This will repeat for the remaining cycles.

C. Charging $< N$ workers

Next, we consider the case when the charger charges $k < N$ workers before it visits the depot. In the first cycle, all the workers enter the field sequentially with time gap ΔT. However, only the first k workers move with maximum rate since the charger goes to the depot after the first round. The remaining workers need to slow down so that the charger arrives, just when the workers run out of power. For any round $r \geq 2$, the first worker that runs out of power is the worker $c_r = k(r-1) \bmod N + 1$. From round $\lceil \frac{N}{k} \rceil + 1$ (when all the workers get charged at least once), the workers can be divided into two groups in each round: the first $N \bmod k$ workers and the remaining $k - N \bmod k$ workers. Since there are N workers, the discharging time for the first $N \bmod k$ workers in a group contains an additional T_{add}. Figure 3 depicts the aforementioned scenario.

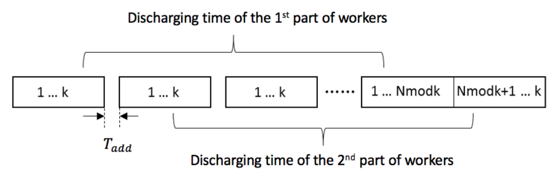

Fig. 3: Discharging time of the workers in a round

For any worker, during the time in which it moves, the charger needs to charge all the other workers once and charge itself multiple times.

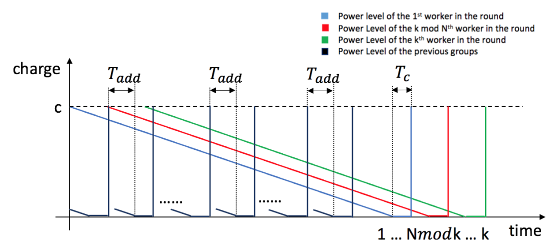

Fig. 4: Power level variation of workers when $n = k$

Figure 4 shows the power level variation of the workers for an arbitrary round. For the first $N \bmod k$ workers in a round, the charging time contains $\lceil \frac{N}{k} \rceil$ times of T_{add}, $N-1$ times of T_c, one time of traveling from the last worker to the first T_{back} and $N - 1 - \lceil \frac{N}{k} \rceil$ times of T. In this case, the discharging time is given as follows:

$$r_{d,i}^{r} = \frac{C}{T_{d,i}^{r}} = \frac{C}{(N-1)T_c + T_{back} + (N-1-\lceil \frac{N}{k} \rceil)T + \lceil \frac{N}{k} \rceil T_{add}}$$

For the remaining $k - N \bmod k$ workers, during their discharging time, the charger visits the depot one time less than the first $N \bmod k$ workers. Instead, it incurs an additional travel between adjacent workers. In this case, the discharging time can be expressed as follows:

$$r_{d,i}^{r} = \frac{C}{T_{d,i}^{r}} = \frac{C}{(N-1)T_c + T_{back} + (N-\lceil \frac{N}{k} \rceil)T + \lfloor \frac{N}{k} \rfloor T_{add}}$$

In any cycle, the discharging rate of the workers is given by the above expressions to satisfy the constraint that the worker is fully discharged when a charger reaches it.

After completing a cycle, the charger moves from the depot to the first worker to start a new cycle. However, the discharging rate for the first N workers will change since there is no travel back from the last worker to the first in this case. T_{back} is replaced by T_{add}. Therefore, from the second cycle, the discharging rate of the first N workers being served can be obtained as follows: For the first N mod k workers in a round:

$$r_{d,i}^{r} = \frac{C}{T_{d,i}^{r}} = \frac{C}{(N-1)T_c + (N - \lceil \frac{N}{k} \rceil)T + \lceil \frac{N}{k} \rceil T_{add}} \quad (5)$$

For the remaining $k - N$ mod k workers in a round:

$$r_{d,i}^{r} = \frac{C}{T_{d,i}^{r}} = \frac{C}{(N-1)T_c + (N - \lfloor \frac{N}{k} \rfloor)T + \lfloor \frac{N}{k} \rfloor T_{add}} \quad (6)$$

From the second cycle, the same procedure will be repeated.

IV. Round-Robin Scheme for Multiple Chargers with Arbitrary Capacity

In this section, we present the routing plan for arbitrary number of workers and chargers for the round-robin scheme. There are N workers and M chargers. Each charger recharges itself after charging n workers. We define a *round* as the period during which N workers get charged once. In each round, the charging order is from worker 1 to N. For any worker i in round k (worker (i,k)), we need to know the discharging time $T_d(i,k)$. In round-robin scheme, a worker will be charged by a different charger each time. For worker (i,k), let m_{i_k} denote the charger that charges it.

$$m_{i_k} = ((k-1)N + i - 1) \bmod M + 1 \quad (7)$$

Since the charger goes to the depot regularly, whether the charger m_{i_k} was at the depot or another worker just before visiting worker (i,k) can be determined by the following rule:

$$\begin{cases} \frac{(k-1)N+i-m_{i_k}}{M} \bmod n = 0 \Rightarrow m_{i_k} \ \textit{at depot} \\ \frac{(k-1)N+i-m_{i_k}}{M} \bmod n \neq 0 \Rightarrow m_{i_k} \ \textit{at another worker} \end{cases} \quad (8)$$

We let the previous worker served by charger m_{i_k} be worker (i',k'). (i',k') is M workers prior to worker (i,k). Since $M < N$, (i,k) may or may not be in the same round.

$$i' = ((k-1)N + i - M) \bmod N, \quad k' = \lceil \frac{(k-1)N + i - M}{N} \rceil$$

$$i \le M \Rightarrow \begin{cases} i' = N + i - M \\ k' = k - 1 \end{cases} \quad , i > M \Rightarrow \begin{cases} i' = i - M \\ k' = k \end{cases}$$

Figure 5 shows the power level variation of worker (i,k) and (i',k'). From the figure, the discharging time $T_d(i,k)$ can be given by the following expression:

$$T_d(i,k) = \begin{cases} T_r + \Delta T(i',k') & \frac{(k-1)N+i-m_{i_k}}{M} \bmod n = 0 \\ T_{i'i} + \Delta T(i',k') & \frac{(k-1)N+i-m_{i_k}}{M} \bmod n \neq 0 \end{cases}$$

where $T_{i'i}$ is the traveling time for the charger from worker (i',k') to (i,k), T_r is the recharging time of the charger at the depot and $\Delta T(i',k')$ is the time difference between worker

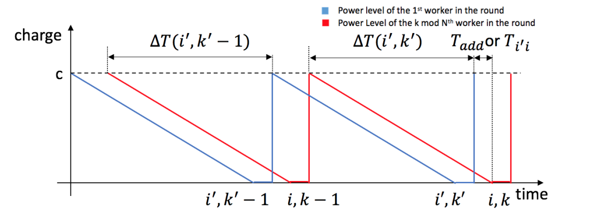

Fig. 5: Power level variation workers with general round-robin scheme

(i',k') and worker $(i,k-1)$ getting fully discharged. The workers enter the workspace sequentially with time gap ΔT. As shown in Figure 5, before coming to a worker, the charger may be at the depot or another worker. The non-uniform travel times between the workers needs cooperation between the workers and the chargers in order to eliminate any tardiness. Therefore, for a different worker (i,k), $\Delta T(i',k')$ may be different. The expression for the time difference is given by the following expression:

$$\Delta T(i',k') = T_d(i',k') + \Delta T(i',k'-1) - T_d(i,k-1) \quad (9)$$

For every cycle, each charger starts with the same serving order, the last phase in the cycle in which all the workers visit the depot is at the end of a round. For all the chargers to run out of cells, nM workers are charged and we assume each charger visits the depot $\alpha, \alpha \in \mathbb{Z}^+$ times in a cycle. Since the end of a cycle is also the end of a round, we assume that there are $\beta, \beta \in \mathbb{Z}^+$ rounds in a cycle. Since the number of workers that get charged in a cycle is constant, we obtain the following expression:

$$\alpha n M = \beta N \quad (10)$$

To have the smallest number of rounds in a cycle,

$$\alpha n M = \beta N = LCM(nM, N) \Rightarrow \beta = \frac{LCM(nM,N)}{N}, \quad (11)$$

where LCM stands for least common multiple. Therefore, in each cycle, there are $p = \frac{LCM(nM,N)}{N}$ rounds.

Lemma 1. *For any worker (i,k), the discharging time $T_d(i,k) = T_d(i,k+p)$ where $p = \frac{LCM(nM,N)}{N}$.*

V. Scheduling with Partitioning Schemes

The battery capacity of the workers is dictated by the frequency at which it can be served by the chargers. As the distance traveled by the charger increases, the frequency at which the workers get served decreases. In order to alleviate this problem, we propose partitioning schemes that allocate a smaller group of workers to individual chargers so that the distance traveled by the chargers is minimized compared to the round-robin scheme proposed in the previous subsection. A distinct characteristic of the partitioning strategy is that the minimum battery capacity of the workers depends on the ratio between N and M instead of their actual values which is the case with the round-robin scheme.

As in the round-robin scheme, N workers start moving with a time gap ΔT. However, instead of serving all the workers together, each charger is allocated a specific group of workers to serve. Let $n = \frac{N}{M}$. If n is an integer, then the workers are divided in M groups, $G_1, \ldots, G_M$. G_i contains all the workers from worker $(i-1)n+1$ till worker ni. In each group, the charger will follow the round-robin strategy proposed in Section III-A. After serving every vehicle in the group, the charger goes to the depot to recharge itself before starting a new round. Based on the values of N and M, the following scenarios arise:

If $n(=\frac{N}{M})$ is an integer, no workers need to be shared between two groups. All chargers serve the same number of workers, and the scheduling strategy for each charger and the workers in its corresponding group can be given by the round-robin strategy proposed in Section III-A.

If n is not an integer, we call it the *Load Balancing without Integer Constraints*. N workers are divided into M groups. A shared worker is served by two chargers, i.e., it is assumed to belong to two groups. Based on this assumption, each group contains $\lceil \frac{N}{M} \rceil$ or $\lceil \frac{N}{M} \rceil + 1$ workers and $\frac{N}{M}C$ amount of power need to be charged. If a group contains $\lceil \frac{N}{M} \rceil + 1$ workers, we call it a large group. With LB/IC, each charger charges its own group. The chargers start charging a shared worker only after both have arrived at the worker. We define a *round* to be the time in which all the workers get served once. During the time in which a worker is working, the charger charges other workers in its group and may go to the depot and recharge itself. We assume M and N are co-prime, otherwise the workers and chargers can be divided into several groups with exactly the same number of vehicles. In this case, with LB/IC, the first and last worker in a group (except the first and the last group) need to be shared between two groups. Since the chargers in the two groups may not reach the shared worker at the same time, the one which arrives first needs to wait for the other charger to arrive, and subsequently, wait till the worker gets fully discharged. In LB/IC, there are M groups and $M-1$ shared workers. Since each charger charges $\frac{N}{M}C$ amount of power, from worker 1 to the last worker in any group g, $\frac{gN}{M}C$ amount of power is charged. The last worker in group g needs to be shared with the next group. Therefore, we can conclude that worker i is a shared worker when $i = \lceil \frac{gN}{M} \rceil$, where $1 \leq g < M$, $g \in \mathbb{Z}$. Since a shared worker belongs to both groups, worker i is the last worker in group g and the first worker in group $g+1$.

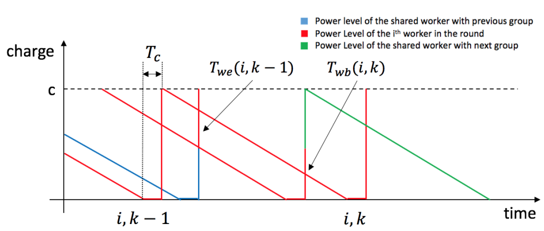

Fig. 6: Power level variation of a worker with general LB/IC scheme

Figure 6 shows the load variation of an arbitrary worker (i,k) which is not shared. For an unshared worker i, we let it belong to group g. Since i is in between the last worker of group $g-1$ and g, we can obtain g from the following inequality:

$$\lceil \frac{(g-1)N}{M} \rceil < i < \lceil \frac{gN}{M} \rceil, \ g \in \mathbb{Z} \tag{12}$$

After charging it in round $k-1$, the charger goes to charge all the other workers in the group. It may need to wait at the shared workers or visit the depot. The discharging time can be expressed as follows:

$$T_d(i,k) = \frac{N}{M}T_c + T^i_{travel}(k-1,k) + T_{wb}(g,k) + T_{we}(g,k-1),$$

where $\frac{N}{M}T_c$ is the total charging time in a round in a group. $T^i_{travel}(k-1,k)$ is the total travel time of charger between serving worker i in round $k-1$ and round k. $T_{wb}(g,k)$ and $T_{we}(g,k-1)$ are the waiting times the charger of group g takes at the first and the last worker in the round.

In LB/IC, a group contains $\lceil \frac{N}{M} \rceil$ or $\lceil \frac{N}{M} \rceil + 1$ workers if $N-1$ is a multiple of M. We let group g contain $\lceil \frac{N}{M} \rceil + j$ workers where $j = 0$ *or* 1 is a group size variable. From worker 1 to the last worker in group g, $\frac{gN}{M}$ workers are charged. Therefore, the number of workers in group g N_g is given by the following expression:

$$\begin{aligned} N_g &= \lceil \frac{gN}{M} \rceil - \lceil \frac{(g-1)N}{M} \rceil + 1 \\ if\ N_g &= \lceil \frac{N}{M} \rceil \Rightarrow j = 0,\ if\ N_g = \lceil \frac{N}{M} \rceil + 1 \Rightarrow j = 1 \end{aligned} \tag{13}$$

If the charger doesn't go to the depot, $T^i_{travel}(k-1,k) = (\lceil \frac{N}{M} \rceil - 1 + j)T + T_{back}$ where T is the traveling time between any two adjacent workers and T_{back} is the traveling time from the last worker to the first in the group. When the charger needs to go to the depot, some of the traveling time between the workers will be replaced by travel time to the depot. From the beginning of the task, assume q workers have been charged before worker(i,k) in its group. $q = (k-1)(\lceil \frac{N}{M} \rceil + j) + i - \lceil \frac{(g-1)N}{M} \rceil + 1$. T_{travel} can be determined with the following criterion:

$$\begin{aligned} &if\ \lfloor \frac{q}{n} \rfloor = \lfloor \frac{q - \lceil \frac{N}{M} \rceil - j}{n} \rfloor \Rightarrow \\ &T^i_{travel}(k-1,k) = (\lceil \frac{N}{M} \rceil + j - 1 - \lceil \frac{\lceil \frac{N}{M} \rceil + j}{n} \rceil)T \\ &\quad + \lceil \frac{\lceil \frac{N}{M} \rceil + j}{n} \rceil T_d + T_{back} \end{aligned} \tag{14}$$

$$\begin{aligned} &if\ \lfloor \frac{q}{n} \rfloor = \alpha LCM(n, \lceil \frac{N}{M} \rceil + j), where\ \alpha \in \mathbf{N}^+ \Rightarrow \\ &T^i_{travel}(k-1,k) = (\lceil \frac{N}{M} \rceil + j - \lceil \frac{\lceil \frac{N}{M} \rceil + j}{n} \rceil)T + \lceil \frac{\lceil \frac{N}{M} \rceil + j}{n} \rceil T_d \end{aligned}$$

The waiting time T_{wb} and T_{we} are the time difference of the two chargers in the two group reaching the shared workers. For worker 1, $T_{wb} = 0$ and for worker N, $T_{we} = 0$. For a shared worker (i,k) where $i \neq 1$ *or* N, its discharging time can be calculated from the perspective of the chargers in the previous and the next group:

$$T_d(i,k)_{pre} = \frac{N}{M} r_c + T_{travel} + T_{wb}(g-1,k-1) \quad (15)$$
$$T_d(i,k)_{next} = \frac{N}{M} r_c + T_{travel} + T_{we}(g+1,k-1)$$
$$T_d(i,k) = max(T_d(i,k)_{pre}, T_d(i,k)_{next}),$$

where $T_d(i,k)_{pre}$ is the discharging time associated with the previous group, and $T_d(i,k)_{next}$ is the discharging time associated with the next group. The waiting times can be obtained as the difference between the two discharging times:

$$if\ T_d(i,k)_{pre} > T_d(i,k)_{next} \Rightarrow \begin{cases} T_{we}(g,k) = 0 \\ T_{wb}(g,k) = T_d(i,k)_{pre} - T_d(i,k)_{next} \end{cases} \quad (16)$$

$$if\ T_d(i,k)_{pre} \leq T_d(i,k)_{next} \Rightarrow \begin{cases} T_{we}(g,k) = T_d(i,k)_{next} - T_d(i,k)_{pre} \\ T_{wb}(g,k) = 0 \end{cases} \quad (17)$$

From (13) and (15), the discharging time of any worker in any round can be calculated. Simulation result of the scheduling scheme is shown as follows. We assume that $T_{add} = 6\ time\ units$, $T_c = 1\ time\ unit$, $\Delta T = 1\ time\ unit$ and $T = 1.5\ time\ units$. In Figure 7, nine workers are charged by four chargers where each charger goes to depot after fully charging four workers. The discharging time of each worker is almost the same in every round, so the curves are stacked together.

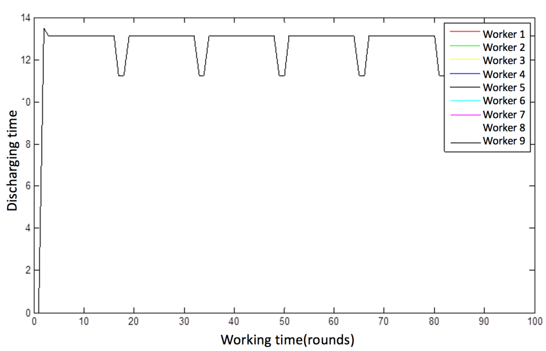

Fig. 7: Variation of discharging time for nine workers and four chargers

VI. Conclusion

In this paper, we considered the scheduling planning problem for multiple autonomous (aerial) chargers serving multiple ground robots. Two different scheduling schemes were investigated based on the allocation of the ground robots to the chargers. As a part of our future work, we plan to address the problem of uncertainty and explore the possibility of non-cooperation between the ground robots and aerial chargers for the task. The theoretical foundations of the framework will be based on ideas from queuing theory and game theory. Based on the theoretical results, a planner will be developed for a heterogenous multi-robot systems with self charging capabilities for persistent operation.

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