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Accurate Modeling and Rapid Synthesis Methods for Beamforming Metasurfaces

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***Abstract*—A general synthesis technique for beamforming metasurfaces is presented which utilizes accurate modeling techniques and rapid optimization methods. The metasurfaces considered consist of patterned metallic claddings supported by finite grounded dielectric substrates. The metasurfaces are modeled using integral equations which accurately account for all mutual coupling and finite dimensions. A beamforming metasurface is designed in three phases: an initial *Direct Solve* phase involving the solution of the integral equation via the method of moments to obtain a complex-valued initial design satisfying the desired far-field beam specifications, a subsequent *Optimization* phase to convert the complex-valued metasurface into a purely reactive metasurface, and a final *Patterning* phase to realize the metasurface as a patterned metallic cladding. The metasurface is optimized using an adjoint optimization method. The method calculates the gradient of the cost function in only two forward problem solutions. An example metasurface designed using this approach is presented.**

Keywords—metasurface; method of moments; adjoint optimization; surface waves

I. Introduction

Recently, several authors have reported synthesis methods for metasurfaces which utilize optimization techniques. These optimization approaches to metasurface design introduce surface waves [1], which can redistribute power transversally across the surface of the metasurface in order to achieve beamforming with lossless and passive structures [2-6]. Here, we present a general synthesis approach for beamforming metasurfaces. The geometry considered is depicted in Fig. 1. The metasurface consists of a patterned metallic cladding supported by a grounded dielectric substrate. The metasurface is finite in the x and y directions and invariant in the z direction. The metasurface is designed in three phases. In phase 1 *Direct Solve*, the patterned metallic cladding is homogenized, and an integral equation is directly solved to obtain the complex-valued sheet impedances required to obtain the desired field transformation. The reactive component of the complex-valued sheet is used as an initial guess in the optimization phase. In phase 2 *Optimization*, the reactive sheet is optimized such that the scattered field approaches the ideal solution obtained in phase 1. In phase 3 *Patterning*, the reactive sheet is realized by patterning a metallic cladding. In contrast to [5], our approach accounts for spatial dispersion by modeling the true thickness of the metasurface.

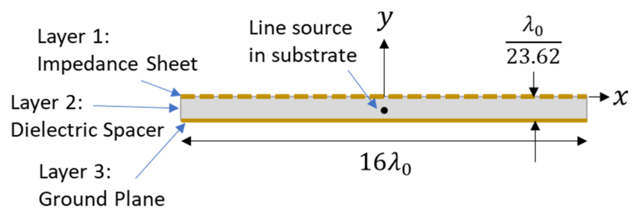

Fig. 1. Metasurface geometry. The finite metasurface sits atop a grounded dielectric substrate.

In contrast to [6], it does not require conducting baffles to separate the metasurface unit cells.

II. Phase 1: Initial Complex-Valued Sheet Design

An Electric Field Integral Equation (EFIE) is written to compute the metasurface's response to a line source illumination. The EFIE includes the mutual coupling between the homogenized unit cells of the inhomogeneous impedance sheet, the polarization currents in the dielectric substrate, and the surface currents on the ground plane [3,7]. The integral equation has the form

$$\sum_{i=1}^{3}\sum_{j=1}^{3}\left(\vec{E}_i^{inc}=\eta_i\vec{J}_i+\frac{\omega_0\mu}{4}\int H_0^{(2)}\left(k_0\left|\vec{\rho}_i-\vec{\rho}_j{}'\right|\right)\bullet\vec{J}_j\left(\vec{\rho}_j{}'\right)d\vec{\rho}_j{}'\right) \quad (1)$$

where ω is the angular frequency of operation, μ_0 is the magnetic permeability of free space, k_0 is the free space wavenumber, and $H_0^{(2)}(\,)$ is the Hankel function of the second kind of order 0. With reference to Fig. 1, when i or j is 1 or 3, $\vec{J}_{i\ or\ j}$ is an electric surface current density and when i or j is 2, $\vec{J}_{i\ or\ j}$ is the volumetric polarization current density. The impedance of layer i is denoted η_i (Note, $\eta_2=\left(j\omega\epsilon_0(1-\epsilon_r)\right)^{-1}$ and $\eta_3=0$) and $\vec{E}_i^{inc}$ is the incident field on layer i radiated by an infinite z-directed electric line source.

In the *Direct Solve* phase, the integral equation is solved to obtain the unknown sheet impedances that produce the desired aperture fields. The desired scattered aperture field amplitude distribution is defined based on the desired far field pattern. The amplitude level in V/m of the desired aperture field is determined by conserving global power on the metasurface

$$\int_{-w/2}^{w/2}\frac{1}{2}\left[\overrightarrow{E}_1^{sca}\times\overrightarrow{H}_1^{sca*}\right]dx=\int_{-w/2}^{w/2}\frac{1}{2}\left[\overrightarrow{E}_1^{inc}\times\overrightarrow{H}_1^{inc*}\right]dx=P^{inc} \quad (2)$$

where P^{inc} is the incident power on the metasurface in Watts and w is the width of the metasurface in the x direction. Specifically, the scattered electric field amplitude, $\vec{E}_1^{sca}$, is

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scaled to satisfy (2) so that the metasurface satisfies global power conservation.

With both $\vec{E}_1^{inc}$ and $\vec{E}_1^{sca}$ defined, (1) is used to obtain the induced polarization and surface currents. With the induced surface currents on layer 1, the sheet impedances η_1 can then be obtained as $\eta_1 = E_{z1}^{tot}/J_{z1}$.

III. Phase 2: Rapid Optimization Of Metasurfaces

The obtained sheet impedances will in general be complex-valued. Surface waves which carry power transversely can be added, through optimization, in order to satisfy a reactive boundary condition, and hence obtain a passive and lossless metasurface. The adjoint method is used to optimize the metasurface impedances and introduce surface waves [8,9].

The optimizer is seeded with the reactances of the complex-valued sheet obtained in the *Direct Solve* phase (the resistances are discarded). To avoid surface waves containing high tangential wavenumbers, the impedances are limited to lie within the range of $-4700 < imag(\eta_s) < -40$ during the optimization. The fitness function is defined as

$$f = \frac{1}{2}\Delta E^T \Delta E, \quad (3)$$

where T denotes the matrix transpose and ΔE is

$$\Delta E = \frac{1}{M}\left[20\log\left(\left|E_{ff}^{sca,calc}(\phi)\right|\right) - 20\log\left(\left|E_{ff}^{sca,tar}(\phi)\right|\right)\right]. \quad (4)$$

In (4), M is the number of far field observation angles and $E_{ff}^{sca,calc}$ and $E_{ff}^{sca,tar}$ are the calculated and target scattered far field amplitude patterns, respectively. The far fields are calculated from the vector potential

$$E_{ff}^{sca}(\phi) = -\frac{k_0\eta_0}{4}\sqrt{\frac{2j}{\pi k_0 \rho}}e^{-jk_0\rho}\iint_{S'} J_i(\eta_s)e^{jk(x'\cos\phi + y'\sin\phi)}dS' \quad (5)$$

where the currents $J_i(\eta_s)$ result from solving the linear system associated with (1) for a given vector of sheet reactances η_s.

The adjoint field method calculates an analytic gradient of (3) by forming its components as

$$\nabla f = real\left(j\lambda^\dagger diag(J)\right)^T \quad (6)$$

where J is the solution to the original problem,†denotes the Hermitian transpose, and λ is the solution to the adjoint problem

$$Z^\dagger \lambda = R^\dagger \Delta E^{adj}. \quad (7)$$

In (7), R is an M-by-N matrix formed by evaluating (5) at the M observation angles in the far field with the currents J on all N elements replaced by $1\angle 0°$ (when pulse basis functions are used). Also, Z is the method of moments impedance matrix used to solve (1) for the forward problem and ΔE^{adj} is the adjoint field. The gradient in (6) is used in the classical gradient descent optimization method to optimize the impedance sheets.

IV. Design Metasuface Examples

An example design of a directly-fed metasurface from a line source placed within the substrate is provided (see Fig. 1). The metasurface is designed to convert the cylindrical wave of the line source into an aperture field with uniform phase and amplitude at 10GHz. The results of phases 1 and 2 of the three phase design approach are provided in Fig. 2. The optimization took approximately 200 iterations over 5 hours to complete.

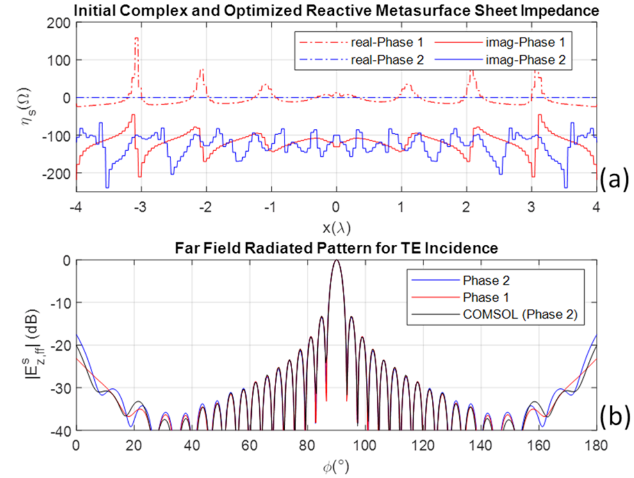

Fig. 2. *(a)* Metasurface sheet impedances for both phase 1 *Direct Solve* and phase 2 *Optimization*. *(b)* Scattered far-field patterns for phase 1 *Direct Solve*, phase 2 *Optimization* and a COMSOL full-wave verification of the optimized reactive sheet obtained in phase 2.

V. Conclusion

In conclusion, a general synthesis technique for beamforming metasurfaces was presented. The technique is based on integral equation modeling and adjoint variable optimization. An example of a metasurface designed using this approach was presented. Full-wave simulations validate the approach.

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