

# On Mitigating Acoustic Feedback in Hearing Aids with Frequency Warping by All-Pass Networks

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## Abstract

Acoustic feedback control continues to be a challenging problem due to the emerging form factors in advanced hearing aids (HAs) and hearables. In this paper, we present a novel use of well-known all-pass filters in a network to perform frequency warping that we call “freping.” Freping helps in breaking the Nyquist stability criterion and improves adaptive feedback cancellation (AFC). Based on informal subjective assessments, distortions due to freping are fairly benign. While common objective metrics like the perceptual evaluation of speech quality (PESQ) and the hearing-aid speech quality index (HASQI) may not adequately capture distortions due to freping and acoustic feedback artifacts from a perceptual perspective, they are still instructive in assessing the proposed method. We demonstrate quality improvements with freping for a basic AFC (PESQ: 2.56 to 3.52 and HASQI: 0.65 to 0.78) at a gain setting of 20; and an advanced AFC (PESQ: 2.75 to 3.17 and HASQI: 0.66 to 0.73) for a gain of 30. From our investigations, freping provides larger improvement for basic AFC, but still improves overall system performance for many AFC approaches.

**Index Terms:** hearing aids, hearables, frequency warping, all-pass, adaptive feedback cancellation, frequency shifting

## 1. Introduction

This research is part of the Open Speech Platform (OSP) [1, 2, 3, 4] funded by an NIH/NIDCD initiative to enable psychophysical research beyond what is currently capable in support of hearing healthcare. This paper is related to improving acoustic feedback reduction for form-factor accurate, audiology research in the field using behind the ear, receiver in the canal (BTE-RIC) transducers, hardware, embedded software, and application software we developed [3, 4]. In order to compensate for mild to moderate hearing loss, commercial hearing aids (HAs) and OSP provide an average gain of 35–38 dB. In the emerging form factors for advanced HAs and hearables, including conventional BTE-RICs, there is a significant acoustic coupling between the microphones and loudspeakers (called receivers in the telephony and HA communities). This acoustic coupling varies significantly based on surroundings (e.g. hats, scarves, hands, and walls that come in close proximity to the transducers) and can cause the system to become unstable, when the audio content includes characteristic frequencies of the system. This instability results in brief “howling” artifacts and they are of immense annoyance to the HA users.

Howling artifacts manifest when multiple factors collude to fulfill the magnitude and phase conditions of the Nyquist stability criterion (NSC) [5]. Adaptive feedback cancellation (AFC) has been the work horse for breaking NSC to avoid instabilities in many audio applications [6], including HAs

[7, 8, 9, 10]. Typically, the AFC deploys the least mean square (LMS) based approaches to mitigate the magnitude condition in NSC [11, 12, 13, 14]. On the other hand, frequency shifting (FS) [15, 16, 17] and other ad hoc methods [18, 19, 20] mainly deal with the phase condition. In this paper, we focus on spectral manipulations following LMS based approaches to break NSC in both magnitude and phase conditions.

In a 1972 paper, Oppenheim and Johnson [21] described discrete representation of continuous signals and systems and included detailed recipes to “transform the frequency axis in a nonlinear manner.” This frequency warping is accomplished using an all-pass network. The authors presented three applications (efficient spectral analysis with unequal resolution, vernier spectral analysis, and correcting for helium speech artifacts in underwater diving) and predicted additional applications in future. Surprisingly, this clever trick of using all-pass structures for manipulating signals seems to have attracted much less attention than warranted in the speech community.

We adopt teachings in [21] for HAs and call it “freping,” a portmanteau for frequency warping. A common type of hearing loss is the sloping hearing loss, where the impaired user has limited ability to perceive high-frequency content. Typically, the intervention is to boost the high-frequency components or move the content to lower frequencies [22]. The former introduces challenges for acoustic feedback control, while the latter facilitates better feedback reduction. Another less common type of hearing loss, but more challenging for providing meaningful interventions is the “cookie bite” hearing loss, wherein it is difficult for the impaired person to perceive mid-frequency content, compared with low- and high-frequency components. We posit that freping will provide an additional tool to the audiologist for managing individual hearing loss profiles. In this paper, we focus on the benefits of freping for mitigating NSC in conjunction with LMS based AFC approaches.

## 2. Revisiting all-pass networks

The all-pass networks described in [21] realize a nonlinear mapping of the frequency axis as controlled by a single warping parameter  $\alpha$ . Let  $\omega = 2\pi(f/f_s)$  be the normalized angular frequency where  $f$  is the original frequency and  $f_s$  is the sampling rate. The mapping  $\theta(\cdot)$  is according to [21]:

$$\hat{\omega} = \theta(\omega) = \omega + 2 \arctan \left( \frac{\alpha \sin \omega}{1 - \alpha \cos \omega} \right), \quad -1 < \alpha < 1, \quad (1)$$

where  $\hat{\omega} = 2\pi(\hat{f}/f_s)$  and  $\hat{f}$  is the warped frequency.

It can be shown that the nonlinear frequency mapping (1) between the original signal  $v(n)$  and the frequency-warped signal  $q(k)$  can be achieved by passing the time-reversed signal

$v(-n)$  through a linear time-invariant system  $H_k(z)$  given as:

$$H_k(z) = \begin{cases} \frac{(1-\alpha^2)z^{-1}}{(1-\alpha z^{-1})^2} \left( \frac{z^{-1}-\alpha}{1-\alpha z^{-1}} \right)^{k-1}, & k > 0 \\ \frac{1}{1-\alpha z^{-1}}, & k = 0 \end{cases}, \quad (2)$$

and taking the output of  $H_k(z)$  at  $n = 0$  as  $q(k)$ . It can thus be implemented as the network shown in Figure 1. The first two stages act as (i) low-pass filters when  $\alpha$  is positive and the network warps frequencies higher and (ii) high-pass filters when  $\alpha$  is negative and the network warps frequencies lower. The remaining stages realize the actual frequency warping based on the bilinear transformation [23]. Note that when  $\alpha = 0$ , it simply passes through the input without any spectral modifications.

The frequency-warped output is given by sampling  $\tilde{q}_k(n)$ , the output signal at the  $k$ -th stage, along the cascade chain at  $n = 0$ , i.e.,  $q(k) = \tilde{q}_k(0)$ . In other words, the input sequence is first flipped and then passed through the network; the last sample of the output sequence at the  $k$ -th stage is taken as the  $k$ -th sample of the final frequency-warped sequence [24].

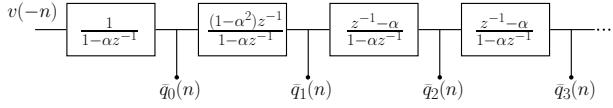


Figure 1: The all-pass network for frequency warping.

It is worth noting that in practice we need to truncate the signal for the all-pass network to be realizable. Therefore, the warping performance will depend on other factors such as the length and the type of the window function used.

### 3. Freping: real-time frequency warping

The all-pass networks described above are adopted for real-time frequency manipulations as illustrated in Figure 2. The input signal is first divided into overlapping frames and windowed using a proper window function. Each windowed segment then goes through the all-pass network to perform frequency warping with a specified warping parameter  $\alpha$ . Finally, the overlap-add method [25] is applied to produce the frequency-warped signal.

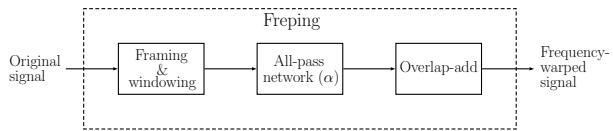


Figure 2: Short-time frequency warping using all-pass network.

To allow a more flexible way of manipulating spectral characteristics, we propose the multichannel freping as illustrated in Figure 3. The system utilizes a set of band-pass filters (BPFs) which divide the input signal into  $M$  frequency bands and a set of warping parameters  $\alpha = [\alpha_1, \dots, \alpha_M]^T$ . Each band goes through an independent all-pass network with the corresponding warping parameter. The output signals of all the frequency bands are summed up to produce the frequency-warped signal.

In many practical situations, it is convenient to reuse the multichannel compression modules [26] in HA processing for freping. For specific types of hearing loss (e.g. sloping, cookie-bite, etc.), increasing the gain in higher frequency bands aids to fulfill the magnitude condition of NSC and freping hinders

the phase condition to occur. Thus, freping provides a way for simultaneously optimizing the parameters of multichannel compression and frequency lowering [27] in HAs for individual hearing loss. In this work, we limit ourselves to negative values of  $\alpha$  so that freping always shifts spectral content lower.

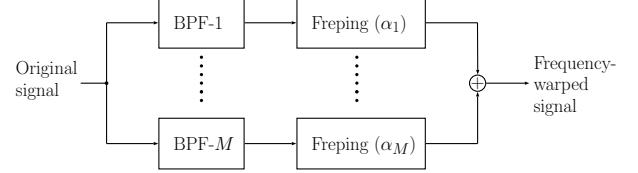


Figure 3: Multichannel freping.

### 4. Freping for acoustic feedback reduction

We investigate benefits of freping for mitigating acoustic feedback along with LMS based AFC, with the motivation of improving feedback control in the systems described in [1, 2, 3, 4].

#### 4.1. Adaptive feedback cancellation (AFC) system

We adopt the AFC framework used in [14] as depicted in Figure 4. The AFC filter  $W(z, n)$ , placed in parallel with the HA processing  $G(z, n)$ , is the transfer function of an  $L$ -tap adaptive filter  $\mathbf{w}(n) = [w_0(n), w_1(n), \dots, w_{L-1}(n)]^T$  that continuously adjusts its coefficients to capture the time-varying nature of the acoustic feedback path  $F(z, n)$ .  $d(n)$  is the microphone input which contains the clean signal  $x(n)$  and the feedback signal  $y(n)$  caused by the HA output  $o(n)$  passing through the feedback path.  $\hat{y}(n)$  is the feedback estimate.  $e(n) = d(n) - \hat{y}(n)$  is the feedback-compensated signal.  $A(z, n)$  is a time-varying pre-filter to decorrelate the input and output signals based on the prediction error method (PEM) [7].  $B(z)$  is a band-limited filter to concentrate on the frequency region where oscillation is more likely to occur [11].

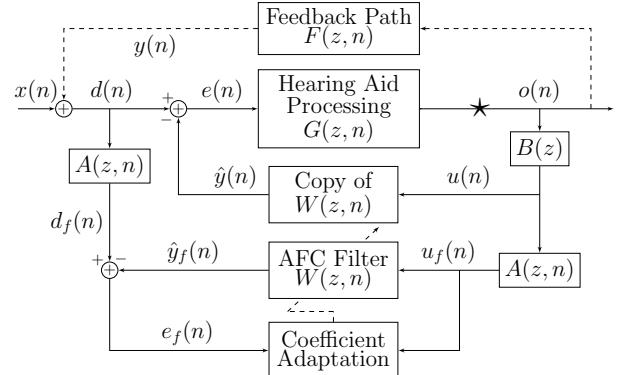


Figure 4: Block diagram of the AFC framework.

Typically, LMS-type algorithms are carried out for coefficient adaptation using the pre-filtered signals  $u_f(n)$  and  $e_f(n)$  to update the AFC filter  $\mathbf{w}(n)$  as:

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \frac{\mu}{L\hat{\sigma}^2(n) + \delta} \mathbf{u}_f(n) e_f(n), \quad (3)$$

where  $\mathbf{u}_f(n) = [u_f(n), u_f(n-1), \dots, u_f(n-L+1)]^T$ ,  $\mu > 0$  is the step size parameter,  $\delta > 0$  is a small constant to prevent division by zero, and  $\hat{\sigma}^2(n) = \rho\hat{\sigma}^2(n-1) + (1-\rho)(u_f^2(n) + e_f^2(n))$  is the power estimate with a forgetting factor  $0 < \rho \leq 1$ .

1. The update rule (3) is actually the “modified” LMS using the sum method [28] and has been widely used in AFC works [7, 11, 12, 14].

An advanced AFC algorithm, based on the LMS (3), is the sparsity promoting LMS (SLMS) proposed in [14] which leverages the sparsity of the feedback path impulse response to achieve faster convergence for improvement. The SLMS update rule includes an additional sparsity promoting term  $\mathbf{S}(n)$  as:

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \frac{\mu}{L\hat{\sigma}^2(n) + \delta} \mathbf{S}(n) \mathbf{u}_f(n) e_f(n), \quad (4)$$

where  $\mathbf{S}(n) = \text{diag}\{s_0(n), s_1(n), \dots, s_{L-1}(n)\}$  is an  $L$ -by- $L$  diagonal matrix and the diagonal elements are updated according to  $s_i(n) = r_i(n)/(\frac{1}{L} \sum_{j=0}^{L-1} r_j(n))$  with

$$r_i(n) = (|w_i(n)| + c)^{2-p}, \quad (5)$$

where  $p \in (0, 2]$  is the sparsity control parameter and  $c > 0$  is a small positive constant to avoid stagnation of the algorithm.

#### 4.2. Mitigating Nyquist stability criterion (NSC)

Without any feedback control mechanism, the frequency responses of the HA processing  $G(e^{j\omega}, n)$  and the feedback path  $F(e^{j\omega}, n)$  form a closed-loop system which exhibits instability that leads to howling. The NSC [5] states that the closed-loop system becomes unstable whenever the following magnitude and phase conditions are both fulfilled [8]:

$$\begin{cases} |G(e^{j\omega}, n)F(e^{j\omega}, n)| \geq 1, & \text{(magnitude cond.)} \\ \angle G(e^{j\omega}, n)F(e^{j\omega}, n) = m2\pi, & m \in \mathbb{Z} \quad \text{(phase cond.)} \end{cases} \quad (6)$$

When AFC is employed, it becomes:

$$\begin{cases} |G(e^{j\omega}, n)(F(e^{j\omega}, n) - \hat{F}(e^{j\omega}, n))| \geq 1, \\ \angle G(e^{j\omega}, n)(F(e^{j\omega}, n) - \hat{F}(e^{j\omega}, n)) = m2\pi, & m \in \mathbb{Z} \end{cases}, \quad (7)$$

where  $\hat{F}(e^{j\omega}, n) = B(e^{j\omega})W(e^{j\omega}, n)$  is the estimated feedback path frequency response. The AFC aims at minimizing  $|F(e^{j\omega}, n) - \hat{F}(e^{j\omega}, n)|$  to mitigate the magnitude condition.

It is well-known that the LMS-type algorithms widely used in AFC suffer from biased estimation due to signal correlation [29]. Consequently, the feedback path estimate can be erroneous if decorrelation is not carefully considered. Although the PEM-based pre-filter [7] has provided certain amount of decorrelation, further improvement is achievable by inserting additional signal processing into the forward path of the HA [17], usually placed at  $\star$  as shown in Figure 4. Existing methods include frequency shifting (FS) [15, 16, 17], phase modulation [18], time-varying all-pass filters to introduce phase shifts [19], linear predictive coding vocoder [20], to name a few. In general, quality degradation might be introduced by these decorrelation methods and thus there is the trade-off between the sound quality and the decorrelation ability for AFC improvement.

Freping is an extreme version of FS [22] and it plays a similar role for decorrelation. It introduces nonlinear frequency shifts and the distortions appear to be perceptually benign based on informal subjective assessments. As instability is most likely to occur at the high-frequency region, it is reasonable to manipulate the high-frequency content while keeping the low-frequency region intact to avoid degradation in quality. By providing additional decorrelation, freping can reduce the

AFC bias and thus a better feedback path estimate can be obtained, thereby improving the magnitude condition in NSC. On the other hand, freping also helps avoid the microphone and receiver signals from remaining continuously in phase with each other. This prevents the phase condition in NSC to hold at the same frequency at two consecutive instants. Consequently, the input and output sounds could not build up in amplitude as effectively. Therefore, the likelihood of instability is reduced.

Note that the approach in [19] also utilizes all-pass filters to achieve decorrelation, in which time-varying poles are used for introducing phase shifts. This is different from freping which manipulates the spectral magnitude as well. Since freping is similar to the FS, we compare them in the following section.

## 5. Evaluation

We evaluate the proposed freping system using computer simulations in MATLAB at a sampling rate of 16 kHz. We implemented a 6-band system using a set of BPFs with non-uniform bandwidth whose center frequencies are 250, 500, 1000, 2000, 4000, and 6000 Hz, respectively. Frames of 128 samples with 50% overlap were utilized. The Hann function was applied for windowing. 25 male and 25 female speech signals from TIMIT database were used for simulations.

### 5.1. Speech quality considerations

In this experiment we directly performed freping on the speech signal and measured the frequency distortion at the output using the MATLAB implementation [30] of the (wide-band) perceptual evaluation of speech quality (PESQ) [31]. The PESQ score gives a good prediction of the mean opinion score and has been suggested for quantifying spectral distortion brought by FS [29, 8, 18]. Figure 5 shows the average PESQ score of the freping output over the 50 speech files as a function of the warping parameter  $\alpha$ , for the cases of operating on the full-band ( $\alpha = \alpha[1, 1, 1, 1, 1]^T$ ) and on the last two (high) frequency bands ( $\alpha = \alpha[0, 0, 0, 1, 1]^T$ ). We can see that quality degradation is minor in the latter case.

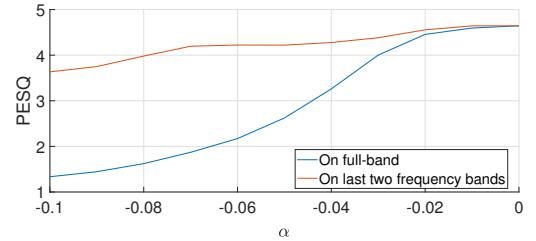


Figure 5: PESQ of freping output vs. warping parameter  $\alpha$ .

### 5.2. Acoustic feedback reduction with freping

Now we consider the practical scenario of HA as in Figure 4. We study freping with  $\alpha = \alpha[0, 0, 0, 1, 1]^T$  on top of the LMS (3) and the SLMS (4). The experimental setup was as follows. The HA processing  $G(z, n) = gz^{-\Delta}$  where  $g$  is the HA gain and  $\Delta$  is the sample delay chosen to have a total HA latency under 10 msec (from  $d(n)$  to  $o(n)$ ). The feedback path impulse response was measured using a BTE-RIC device with open fitting on a dummy head with a handset placed on the ear – the most challenging scenario for breaking NSC. For the AFC, we used  $L = 100$ ,  $\mu = 0.005$ ,  $\rho = 0.985$ , and  $\delta = 10^{-6}$  for both LMS and SLMS. For the SLMS we used  $p = 1.5$  and  $c = 10^{-6}$  as suggested in [14]. In all simulations, the AFC filter coefficients were initialized as all zeros.

### 5.2.1. HA output quality

Figure 6 shows the average PESQ score of the HA output over the 50 speech files for several values of the warping parameter  $\alpha$ . From the results we can see that when we increase  $\alpha$  in magnitude from 0, acoustic feedback gets better controlled, resulting in improved quality. However, further increasing  $\alpha$  in magnitude leads to higher spectral distortion and thus the quality drops. This indicates the trade-off between the reduction of feedback artifacts and frequency distortion, and is better seen in the case of a more aggressive gain setting.

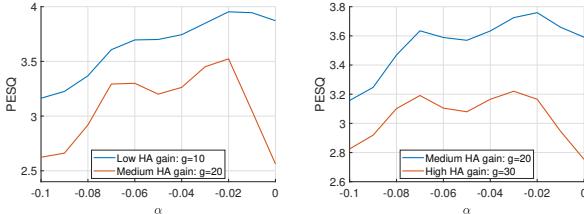


Figure 6: PESQ of HA output as a function of  $\alpha$  for AFC using LMS (left) and SLMS (right). With  $\alpha = -0.02$ , PESQ improvements of 2.56 to 3.52 and 2.75 to 3.17 can be seen for LMS with HA gain at 20 and SLMS with HA gain at 30, respectively.

### 5.2.2. Feedback reduction improvement

We now focus on quantifying the improvement brought by freping in reducing feedback artifacts. In the remaining experiments,  $\alpha = -0.02$  was used as suggested by the results in Figure 6. Also from Figure 5, this choice of  $\alpha$  corresponds to an average PESQ of 4.55 which indicates good quality. Furthermore, based on informal subjective assessments, distortions introduced with  $\alpha$  in the vicinity of this choice are fairly benign. According to (1), for this choice of  $\alpha$ , the center frequencies of the fifth and sixth frequency bands would move from 4000 and 6000 Hz to 3898 and 5927 Hz, respectively.

We compare performance with an existing FS method based on the analytical representation of signal using the Hilbert transform [6, 15]. The amount of shift was set to 12 Hz, only applied to frequency region above 1.5 kHz as suggested by [16, 17]. When directly performed, this setup gives an average PESQ score of 4.47 of the FS output over the 50 speech files, which is comparable but slightly lower than that of the freping result.

For evaluation, we compare the feedback-compensated signal  $e(n)$  with the clean signal  $x(n)$ , using the hearing-aid speech quality index (HASQI) [32] which has been adopted in prior AFC works [14, 10, 33]. The HASQI score ranges from 0 to 1, where a higher value indicates better quality.

Figure 7 presents example spectrograms of the feedback-compensated signal for several cases. We can see that freping effectively reduces the howling components present in the red boxes, resulting in improved quality.

Figure 8 demonstrates advantage of using freping by showing the average HASQI score over the 50 speech files for various gain settings. From the results we see that both the basic (LMS) and advanced (SLMS) AFC algorithms can benefit from freping. This indicates the ability of the proposed frequency warping method to further improve feedback reduction on top of many AFC approaches. Moreover, compared to the FS, freping demonstrates better performance under all the gain settings.

Finally, we compare the added stable gain (ASG), which is the additional gain due to feedback control mechanism that the HA can still operate in the stable state, for the cases of AFC,

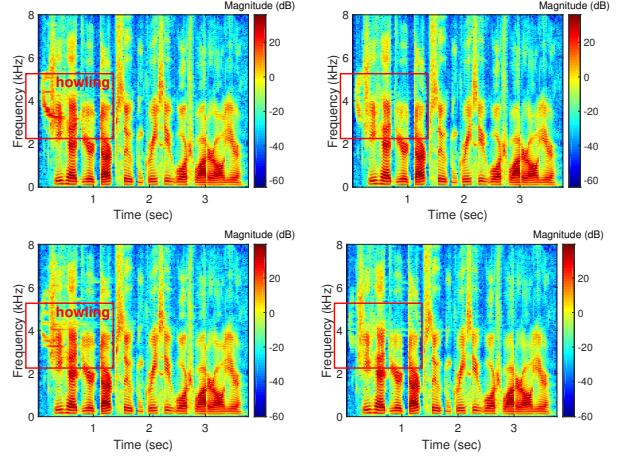


Figure 7: Spectrograms of feedback-compensated signal. The top row is for LMS with HA gain at 20 and the bottom row is for SLMS with HA gain at 30. Frepding is disabled in the left column and enabled with  $\alpha = -0.02$  in the right column. The HASQI scores are 0.81 (top left), 0.84 (top right), 0.79 (bottom left), and 0.82 (bottom right).

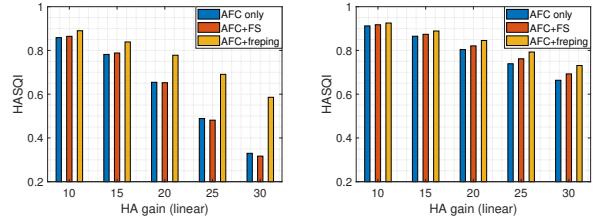


Figure 8: HASQI of feedback-compensated signal for AFC using LMS (left) and SLMS (right). With freping, HASQI improvements of 0.65 to 0.78 and 0.66 to 0.73 can be seen for LMS with HA gain at 20 and SLMS with HA gain at 30, respectively.

AFC with FS, and AFC with freping. We used the ASG estimation approach proposed in [10], where a HASQI below 0.8 was considered of unacceptable quality. The results are shown in Table 1, obtained from the average of 5 male and 5 female speech files. We can see that freping can improve the ASG on top of both the basic and advanced AFC algorithms. Compared to the FS, a higher ASG can be achieved by using freping.

Table 1: ASG (in dB) comparison.

| AFC algorithms | AFC only | AFC+FS | AFC+freping  |
|----------------|----------|--------|--------------|
| LMS            | 14.41    | 15.05  | <b>16.90</b> |
| SLMS           | 17.87    | 18.47  | <b>19.31</b> |

## 6. Conclusions

In this paper, we proposed a novel use of all-pass networks for frequency warping that we call “freping.” We described real-time realization of multichannel freping for use in HAs and its use for breaking the NSC in acoustic feedback control. Experimental results demonstrate quality improvements with freping for basic and advanced AFC approaches. For a desired quality lower bound (e.g. HASQI = 0.8), we found ASG improvements of 2.5 and 1.4 dB for LMS and SLMS with freping, respectively.

## 7. Acknowledgements

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## 8. References

- [1] Open Speech Platform, <http://openspeechplatform.ucsd.edu/>. Univ. California, San Diego, 2019.
- [2] H. Garudadri, A. Boothroyd, C.-H. Lee, S. Gadiyaram, J. Bell, D. Sengupta, S. Hamilton, K. C. Vastare, R. Gupta, and B. D. Rao, “A realtime, open-source speech-processing platform for research in hearing loss compensation,” in *Proc. Asilomar Conf. Signals, Syst., Comput. (ACSSC)*, 2017, pp. 1900–1904.
- [3] L. Pisha, S. Hamilton, D. Sengupta, C.-H. Lee, K. C. Vastare, T. Zubatiy, S. Luna, C. Yalcin, A. Grant, R. Gupta, G. Chockalingam, B. D. Rao, and H. Garudadri, “A wearable platform for research in augmented hearing,” in *Proc. Asilomar Conf. Signals, Syst., Comput. (ACSSC)*, 2018, pp. 223–227.
- [4] Release 2019a, <https://github.com/nihospr01/OpenSpeechPlatform-UCSD>. Open Speech Platform, 2019.
- [5] H. Nyquist, “Regeneration theory,” *Bell Syst. Tech. J.*, vol. 11, no. 1, pp. 126–147, 1932.
- [6] T. van Waterschoot and M. Moonen, “Fifty years of acoustic feedback control: State of the art and future challenges,” *Proc. IEEE*, vol. 99, no. 2, pp. 288–327, 2011.
- [7] A. Sprriet, S. Doclo, M. Moonen, and J. Wouters, “Feedback control in hearing aids,” *Springer Handbook of Speech Process.*, pp. 979–1000, 2008.
- [8] M. Guo, *Analysis, design, and evaluation of acoustic feedback cancellation systems for hearing aids*. Ph.D. dissertation, Aalborg Univ., 2012.
- [9] C. R. C. Nakagawa, S. Nordholm, and W.-Y. Yan, “New insights into optimal acoustic feedback cancellation,” *IEEE Signal Process. Lett.*, vol. 20, no. 9, pp. 869–872, 2013.
- [10] C.-H. Lee, J. M. Kates, B. D. Rao, and H. Garudadri, “Speech quality and stable gain trade-offs in adaptive feedback cancellation for hearing aids,” *J. Acoust. Soc. Am.*, vol. 142, no. 4, pp. EL388–EL394, 2017.
- [11] H.-F. Chi, S. X. Gao, S. D. Soli, and A. Alwan, “Band-limited feedback cancellation with a modified filtered-X LMS algorithm for hearing aids,” *Speech Commun.*, vol. 39, no. 1-2, pp. 147–161, 2003.
- [12] H. Schepker, L. T. T. Tran, S. Nordholm, and S. Doclo, “Improving adaptive feedback cancellation in hearing aids using an affine combination of filters,” in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Process. (ICASSP)*, 2016, pp. 231–235.
- [13] L. T. T. Tran, H. Schepker, S. Doclo, H. H. Dam, and S. Nordholm, “Proportionate NLMS for adaptive feedback control in hearing aids,” in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Process. (ICASSP)*, 2017, pp. 211–215.
- [14] C.-H. Lee, B. D. Rao, and H. Garudadri, “Sparsity promoting LMS for adaptive feedback cancellation,” in *Proc. Europ. Signal Process. Conf. (EUSIPCO)*, 2017, pp. 226–230.
- [15] T. van Waterschoot and M. Moonen, “Assessing the acoustic feedback control performance of adaptive feedback cancellation in sound reinforcement systems,” in *Proc. Europ. Signal Process. Conf. (EUSIPCO)*, 2009, pp. 1997–2001.
- [16] F. Strasser and H. Puder, “Sub-band feedback cancellation with variable step sizes for music signals in hearing aids,” in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Process. (ICASSP)*, 2014, pp. 8207–8211.
- [17] ———, “Adaptive feedback cancellation for realistic hearing aid applications,” *IEEE/ACM Trans. Audio, Speech, Lang. Process.*, vol. 23, no. 12, pp. 2322–2333, 2015.
- [18] M. Guo, S. H. Jensen, J. Jensen, and S. L. Grant, “On the use of a phase modulation method for decorrelation in acoustic feedback cancellation,” in *Proc. Europ. Signal Process. Conf. (EUSIPCO)*, 2012, pp. 2000–2004.
- [19] C. Boukis, D. P. Mandic, and A. G. Constantinides, “Toward bias minimization in acoustic feedback cancellation systems,” *J. Acoust. Soc. Am.*, vol. 121, no. 3, pp. 1529–1537, 2007.
- [20] G. Ma, F. Gran, F. Jacobsen, and F. T. Agerkvist, “Adaptive feedback cancellation with band-limited LPC vocoder in digital hearing aids,” *IEEE Trans. Audio, Speech, Lang. Process.*, vol. 19, no. 4, pp. 677–687, 2011.
- [21] A. V. Oppenheim and D. H. Johnson, “Discrete representation of signals,” *Proc. IEEE*, vol. 60, no. 6, pp. 681–691, 1972.
- [22] H. Dillon, *Hearing aids*. 2nd edition, Boomerang Press, 2008.
- [23] C. Braccini and A. Oppenheim, “Unequal bandwidth spectral analysis using digital frequency warping,” *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 22, no. 4, pp. 236–244, 1974.
- [24] W. P. M. Allen, D. G. Bailey, S. N. Demidenko, and V. Piuri, “Analysis and application of digital spectral warping in analog and mixed-signal testing,” *IEEE Trans. Reliab.*, vol. 52, no. 4, pp. 444–457, 2003.
- [25] J. B. Allen, “Short term spectral analysis, synthesis, and modification by discrete fourier transform,” *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 25, no. 3, pp. 235–238, 1977.
- [26] J. M. Kates, “Principles of digital dynamic-range compression,” *Trends Amplif.*, vol. 9, no. 2, pp. 45–76, 2005.
- [27] A. Simpson, “Frequency-lowering devices for managing high-frequency hearing loss: A review,” *Trends Amplif.*, vol. 13, no. 2, pp. 87–106, 2009.
- [28] J. E. Greenberg, “Modified LMS algorithms for speech processing with an adaptive noise canceller,” *IEEE Trans. Speech Audio Process.*, vol. 6, no. 4, pp. 338–351, 1998.
- [29] M. Guo, S. H. Jensen, and J. Jensen, “Evaluation of state-of-the-art acoustic feedback cancellation systems for hearing aids,” *J. Audio Eng. Soc.*, vol. 61, no. 3, pp. 125–137, 2013.
- [30] P. C. Loizou, *Speech enhancement: theory and practice*. 2nd edition, CRC press, 2013.
- [31] ITU-T Recommendation P.862.2, “Wideband extension to recommendation p. 862 for the assessment of wideband telephone networks and speech codecs,” *Int. Telecommun. Union*, 2005.
- [32] J. M. Kates and K. H. Arehart, “The hearing-aid speech quality index (HASQI) version 2,” *J. Audio Eng. Soc.*, vol. 62, no. 3, pp. 99–117, 2014.
- [33] G. Bernardi, T. van Waterschoot, J. Wouters, and M. Moonen, “Subjective and objective sound-quality evaluation of adaptive feedback cancellation algorithms,” *IEEE/ACM Trans. Audio, Speech, Lang. Process.*, vol. 26, no. 5, pp. 1010–1024, 2018.