Exact and Efficient Protective Jamming in SINR-based Wireless Networks

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***Abstract*—A majority of research in communication in wireless networks is devoted to maximizing information flow, improving connectivity, or making the system robust against physical perturbations such as jamming. In this work we study how intentional jamming can be used for assuring privacy of wireless communication under the popular Signal-to-Interference-plus-Noise-Ratio (SINR) model. The considered problem, called Zone-restriction with Max-coverage, is as follows: how to place a number of jamming stations in order to generate interference that block the signal of given genuine stations in a specified restricted area, i.e., by making the SINR value of the genuine stations' signal below a pre-defined threshold in that area. In the construction of algorithms, we aim at optimizing both the accuracy – by minimizing the impact of the jamming stations to the area of genuine communication and by maximizing their influence to the area that should be jammed, as well as the energy consumption of the jamming stations. We present several solutions in various settings of the network, which often lead to challenging analysis even in relatively simple cases. Among others, we show that, surprisingly, it is possible to jam arbitrarily large areas by jammers using total energy arbitrarily close to zero.**

***Index Terms*—Wireless sensor networks, SINR, information hiding, jamming**

I. Introduction

In this work we pursue a non-traditional approach to wireless communication – how to (slightly) limit the genuine communication in order to protect it from eavesdropping? More specifically, we assume that there are some *restricted areas*, where we expect that the genuine wireless communication signal cannot be successfully received by any entity. At the same time, we would rather not affect the genuine communication that takes place outside the restricted areas. This problem can be motivated by many natural scenarios, spanning from military communication, preventing industrial espionage to protecting personal communication against eavesdropping, or providing wireless services in selected workspaces without being overheard in another ones.

One may be tempted to think that this problem can be solved by using standard cryptographic mechanisms. Note however that in many cases it is not possible to apply predeployment of any cryptographic material, especially in the case of real-life ad hoc systems. Moreover, in some environment the network devices are computationally restricted, and thus they cannot perform cryptographic operations required by even the simplest of cryptographic protocols. Finally, in many cases, e.g., on the battlefield, one needs to hide not only the content of the message, but also the fact that a communication takes place.

The outlined problem can be considered in various wireless communication models, however, in our paper, we consider popular Signal-to-Interference-plus-Noise-Ratio (SINR) model ([1]), in which the analysis of the problem is particularly challenging. On the other hand, the SINR model assumes that the power of the signal is fading with distance from the transmitting station and there is an interference from other network devices, which gives a model close to reality and acceptable from (most of) technology perspective. The SINR model has been proven significantly complex for analysis of even seemingly simple problems, such as answering the question if a chosen point can receive the transmission from any station (cf., [1], [3]). The complexity comes, among others, from the fact that transmission of a single station impacts the reception zones of even very distant stations.

In principle, the goal of limiting the communication in SINR model can be attained in two ways. The first is based on lowering the transmission power of stations, while the second is based on adding jamming stations to the original network that selectively reduce the reception zones by the introduction of an additional interference. In our paper we concentrate on the latter approach, as it has an inevitable advantage — we do not have to modify the initial network to apply it, we only add some jamming stations. This approach, sometimes called a *friendly jamming*, appeared in many previous papers that assumed other models of wireless networks (cf., [4], [5]), however the way how such policy can be applied is very different from our approach in SINR model.

Our contribution and paper organization: We consider several settings of SINR networks and construct algorithms for placing jammers to block the communication in a given restricted area while simultaneously minimizing unnecessary interferences in the rest of the network. Apart from such defined accuracy, we also consider the energy cost of jamming (total power of the jamming stations). It turns out, however,

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that the formal analysis of the effects of adding jamming stations leads to difficult analytical problems even in some simple settings of just a few stations.

In Section II we introduce the model of communication and formalize the Zone-restriction with Max-coverage problem, stated above. Section III presents our results for various settings in a one-dimensional model. In particular, as a warm-up, Section III-A studies the simplest case of a network with a single broadcasting station and a single jamming station. In Section III-B we present algorithm for finding an optimal solution for jamming a single station by two jamming stations. Despite very simple formulation of the problem, the analysis of the algorithm turned out to be surprisingly complex. The correctness of this algorithm is presented in Theorem 1.

In Section IV we present the idea of *noisy dust* – accurate positioning of multiple stations with small energy levels. Specifically, in Section IV-A we introduce an *adaptive noisy dust* scheme and prove somehow surprising result that the total energy of all jammers can get arbitrarily close to zero. We also utilize a similar scheme of *stripes*, more universal method of jamming arbitrary fragments of space in Section IV-B.

In Section V, we utilize the stripes scheme from 1D-model and hexagonal tessellation to provide a method of jamming rectangular shapes in a 2D-plane.

Due to space limitation, some technical proofs are to be presented in the full version of this paper. The related work is discussed in Section VI, while Section VII presents conclusions and most important future directions.

II. MODEL AND PROBLEM STATEMENT

A. Model of SINR network

The SINR **network** is a tuple[1] $\mathcal{A} = \langle D, S, N, \beta, P, \alpha \rangle$, where:

- $D \in \mathbb{N}^+$ is the dimension of the network, in realistic scenarios limited to $D \in \{1, 2, 3\}$,
- $S = \{s_1, \ldots, s_n\}$ is a set of positions of stations in $\mathbb{R}^D$,
- $N > 0$ is an ambient, background noise (fixed real number)[2],
- $\beta \geqslant 1$ is the reception threshold (fixed real number),
- $P : S \to \mathbb{R}$ is a stations' power function; by $P_i = P(s_i)$ we denote the power of station s_i[3],
- $\alpha \geqslant 2$ is a path-loss parameter (fixed real number).

For a network $\mathcal{A}$, we define the SINR function for station $s_i \in S$ and point $x \in \mathbb{R}^D \backslash (S \backslash \{s_i\})$ as:

$$\mathrm{SINR}_{\mathcal{A}}(s_i, x) = \frac{P_i \cdot d(s_i, x)^{-\alpha}}{N + \sum\limits_{s_j \in S \backslash \{s_i\}} P_j \cdot d(s_j, x)^{-\alpha}} \ ,$$

where d is a D-dimensional Euclidean metric. For $x \in S \backslash \{s_i\}$ we put $\mathrm{SINR}(s, x) = 0$. If the network $\mathcal{A}$ is known from context, we simplify the notation to $\mathrm{SINR}(s, x)$ for a station s.

[1]We follow the [6] definition of SINR model (different ones can be found in literature).

[2]The case $N = 0$ is also considered, but we may encounter anomalies like infinite reception zones.

[3]Slightly abusing the notation, in some cases we identify stations with their positions. In such case we assume that no two stations share the same position.

The SINR function calculates the relative power of the signal transmitted by station s_i in point x, taking into consideration the network parameters, distance between s_i and x and the interference of all other stations in this network. The threshold β is the network-wide minimal SINR value enabling communication. A *reception zone* of station s in network $\mathcal{A}$ is defined as $H_s^{\mathcal{A}} = \{x \in \mathbb{R}^D : \mathrm{SINR}_{\mathcal{A}}(s, x) \geqslant \beta\}$. In simple words, $H_s^{\mathcal{A}}$ is the space where the station s is heard. The reception zones for parameter $\beta \geqslant 1$ do not overlap, thus if some station s is heard at some point x, no other station is heard at that point for this network configuration. For convenience, instead of using the full name of a station s_i in lower indices, we will be using its unique order number i: $H_i^{\mathcal{A}}$ will be equivalent to $H_{s_i}^{\mathcal{A}}$. Finally, we define the *range* of station s for the network with positive noise value ($N > 0$) as $\mathrm{range}(s) = \left(\sqrt[\alpha]{\frac{N\beta}{P}} \right)^{-1}$, which is the radius of maximal reception zone of s (in a network consisting of a single station).

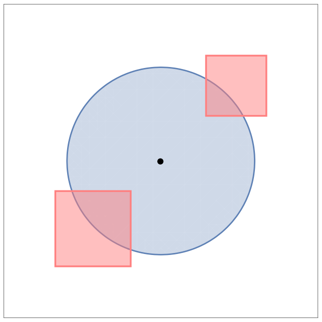
B. Formulation of the problem of restricting transmissions

For a network $\mathcal{A}$, there is given a *restricted area* $\mathcal{R}$: a subset of space, wherein no station can be heard. In other words, in all points in $\mathcal{R}$ the SINR function of all stations in a set S has to be lowered below the threshold β. This can be done using two techniques. First is to modify the network parameters – we can increase the threshold value β, decrease stations' powers, or increase the path-loss parameter α. Second, we can use special *jamming stations* added to the network in order to generate interference and change the shapes of the reception zones of the original set of stations in network $\mathcal{A}$.

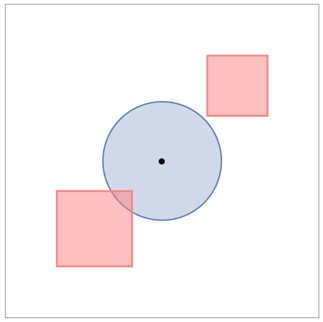
An illustration of such approaches for a single broadcasting station is presented in Figure 1. The red rectangles represent the restricted area and the blue field is the reception zone of the original station. In Figure 1a we see the initial status – the blue space overlaps with the red restricted areas, possibly compromising the transmission from the station. Using the first approach – changing some of the network parameters to reduce the reception zone – the reception zone stops overlapping with the upper restricted area, cf., Figure 1b, but it still does with the lower one. In Figure 1c we follow the second approach – adding jamming station (yellow field in the lower restricted area). It generates enough interference to prevent the original station from being heard at lower restricted area, but it is still heard at the upper one. Finally, in Figure 1d both approaches are combined – jamming station and network parameters modification solve the problem for both restricted areas.

In order to formalize our approaches, assume that there is a network $\mathcal{A} = \langle D, S, N, \beta, P, \alpha \rangle$ and some subspace $\mathcal{R} \subset \mathbb{R}^D$ representing the *restricted area* to be excluded from any communication involving stations from S.

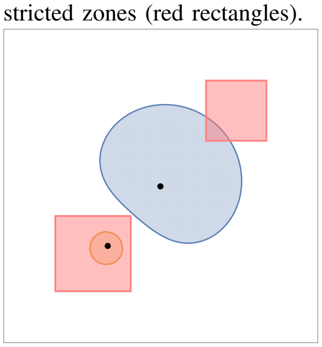
The problem of Zone-restriction with Max-coverage is defined as finding a set of *jamming stations* $\mathcal{J} = (S^{(J)}, P^{(J)})$ with positions in $S^{(J)}$ and powers defined by the function $P^{(J)}$ in such a way that in the resulting network $\mathcal{A}^{\mathcal{J}} = \langle D, S^{(J)} \cup S, N, \beta, P \cup P^{(J)}, \alpha \rangle$ the following condi-



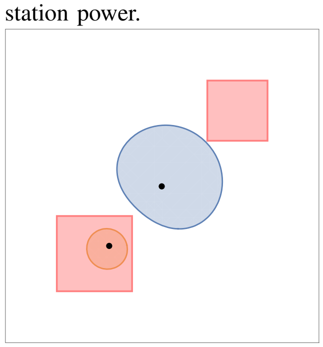
(a) Initial problem with one reception zone (blue) and two restricted zones (red rectangles).



(b) Single zone problem solved (upper rectangle) by reducing station power.



(c) Single zone problem solved (lower rectangle) by adding jamming station.



(d) Both zone problems solved by combining jamming station with power reduction.

Fig. 1: Sample problem for a single broadcasting station.

tion holds: $(\forall\ s \in S)(\forall\ x \in \mathcal{R})\quad \mathrm{SINR}(s,x) < \beta$. If this condition holds, we say that $S^{(J)}$ *correctly protects* $\mathcal{R}$.

This problem can be trivially solved by adding a single station in any restricted area and assigning to this station appropriately high transmission power. In typical scenarios this would however also suppress the desired communication in the reception zones of the genuine network.

In order to take into account the above issue, we define a **coverage** – specifying how the new reception areas correspond to their original sizes, excluding the restricted area:

$$\mathrm{Cover}(\mathcal{J},\mathcal{A}) = \frac{\left|\bigcup_{s_i\in S}\left(H_i^{\mathcal{A}^{\mathcal{J}}} \cap (H_i^{\mathcal{A}}\setminus\mathcal{R})\right)\right|}{\left|\bigcup_{s_i\in S} H_i^{\mathcal{A}}\setminus\mathcal{R}\right|},$$

where $|A|$ denotes the volume of a set A. Note that $\mathrm{Cover}(\mathcal{J},\mathcal{A})$ is always properly defined, as long as $N > 0$, and $0 \leqslant \mathrm{Cover}(\mathcal{J},\mathcal{A}) \leqslant 1$. If the coverage equals to 1, then the jamming stations do not change the genuine reception zones apart from the restricted area. Our goal is to *maximize the coverage*. Finally, we define the *power cost* function $\mathrm{Cost}(\mathcal{J}) = \sum_{s\in S^{(J)}} P^{(J)}(s)$, which measures the total power used by the jamming stations.

The problem considered in this paper is specified as follows:

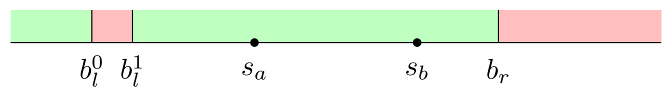
> *For a given network $\mathcal{A}$ and the restricted area $\mathcal{R}$, find a set of stations and their powers, $\mathcal{J} = (S^{(J)}, P^{(J)})$, correctly protecting $\mathcal{R}$ and maximizing* $\mathrm{Cover}(\mathcal{J},\mathcal{A})$.

Optionally, we also require minimizing $\mathrm{Cost}(\mathcal{J})$.

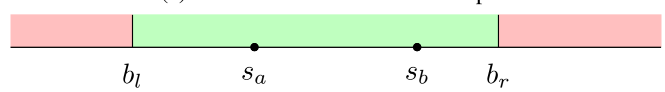
This work studies selected cases of $S^{(J)}$ and $P^{(J)}$, e.g., a limit on the number of jamming stations used for jamming.

III. Jamming in one-dimensional networks

In this section we consider one-dimensional SINR model ($D = 1$). The restricted area is a union of some (potentially infinite) disjoint intervals. For example, in Figure 2a there are two broadcasting stations located in s_a and s_b with the restricted area given as a union of two intervals (b_l^0, b_l^1) and (b_r, ∞). Such scenarios is often considered in VANET networks [7]. We say that the network is *uniform* if all the



(a) Non-uniform network example.



(b) Uniform network example.

Fig. 2: Example of 1D instance (restricted areas are red).

stations (both from S and $S^{(J)}$) have identical power level (that is, $P \cup P^{(J)} \equiv 1$).[4] In uniform setting all the reception zones are convex (cf., [3]), which substantially simplifies reasoning. For instance, in Figure 2a the interval (b_l^0, b_l^1) from the initial restricted zone can be replaced by $(-\infty, b_l^1)$. Indeed, if a given station located to the right of b_l^1 is not heard in b_l^1, then due to convexity, it is not heard in any point $x < b_l^0$ as well. Hence, we can model this constraint by a single point $b_l = b_l^1$. This configuration is presented in Figure 2b.

In the case of uniform networks the power level parameter is redundant, thus we identify the set of jamming stations $\mathcal{J} = (S^{(J)}, P^{(J)})$ with the set of their positions $S^{(J)}$, or even with a single position when we deal with a single jamming station.

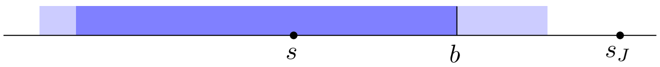
A. One side jamming

As an introduction, let us consider the simplest uniform network $\mathcal{A}_0$ with a single broadcasting station s and a single jamming station s_J. W.l.o.g. we can assume that $s = 0$ and a restricted area $\mathcal{R}_b = (b, \infty)$ for some $b > 0$. This problem can be solved by placing the jamming station at $s_J = b + r$ for some $r > 0$ in order to block communication in $\mathcal{R}_b$. Nevertheless, this jamming station also influences the initial part of the original communication of the station s, which should be taken into consideration in order to prevent a reduction of the coverage (see Figure 3b).

Lemma 1. *Let $\mathcal{A}_0$ be a network and s_J be a single jamming station placed at: $s_J = b + \sqrt[\alpha]{\frac{\beta}{b^{-\alpha} - \beta N}}$. Then*

[4] In principle, we can use any other fixed power level and re-scale other parameters (see e.g., [3]). Under uniformity, we skip the parameter P from formal descriptions in this section.

(a) An original reception zone.



(b) A reception zone with one jamming station s_J.

Fig. 3: Single side jamming example. The reception zone is indicated with darker color.

1) *s_J correctly protects $\mathcal{R}_b$,*
2) *guarantees coverage* $\mathrm{Cover}(\{s_J\}, \mathcal{A}_0)$ *in*

$$\left[\frac{b + (\beta(N + \mathrm{MaxI}))^{-\frac{1}{\alpha}}}{\mathrm{range}(s) + b}, \frac{b + (\beta(N + \mathrm{MinI}))^{-\frac{1}{\alpha}}}{\mathrm{range}(s) + b}\right] ,$$

where $\mathrm{MinI} := (s_J + \mathrm{range}(s))^{-\alpha}$ *and* $\mathrm{MaxI} := (s_J + b)^{-\alpha}$.

Proof: We skip a straightforward proof of point 1., based on monotonicity of the function $\mathrm{SINR}(s, x)$ w.r.t. argument x. The point 2. of the lemma follows from the limitation of the space, where some point $b_l < s$ can have $\mathrm{SINR}(s, b_l) = \beta$, to interval $[-\mathrm{range}(s), -b]$. The fact, that $b_l > -\mathrm{range}(s)$ is obvious and because of the monotonicity of the jamming station interference $I(s, x) = d(s_J, x)^{-\alpha}$, symmetry of the s energy function $E(s, x) = d(s, x)^{-\alpha}$ regarding point s and the fact that $d(s_J, b_l) > d(s_J, b)$, the inequality $b_l > -b$ follows.

We define maximal interference of s_J, achieved at point $-b$ and denoted as MaxI and minimal interference achieved at point $-\mathrm{range}(s)$, denoted as MinI. Let us define the following, simplified, version of SINR function:

$$\mathrm{SNR}(s, x, I) = \frac{d(s, x)^{-\alpha}}{N + I} ,$$

where we essentially replace the jamming interference with some constant value I. By using it for MinI and MaxI, we solve the equations:

$$\mathrm{SNR}(s, x_l, MinI) = \beta \qquad \mathrm{SNR}(s, x_r, MaxI) = \beta ,$$

getting the following results:

$$x_l = (\beta(N + \mathrm{MinI}))^{-\frac{1}{\alpha}} \qquad x_r = (\beta(N + \mathrm{MaxI}))^{-\frac{1}{\alpha}} .$$

Based on the monotonicity of $I(s, x)$, we know that $-x_l \leqslant b_l \leqslant -x_r$ (Figure 4), what means that the final reception zone of s can be maximally of size of the segment $[-x_l, b]$ and at least of the size of the segment $[-x_r, b]$. Additionally, the maximal reception zone of s, excluding restricted area has the following size:

$$\left|H_s^{\mathcal{A}} \setminus \mathcal{R}\right| = \mathrm{range}(s) + b ,$$

which finalizes the proof for the Cover value bounds. ■

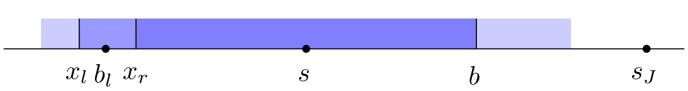


Fig. 4: Bounding the possible values of b_l by segment $[x_l, x_r]$.

Let us note that a careful inspection of possible choices of the place for the jamming stations s_J show that the chosen place is optimal in terms of coverage.

B. Two precise jammers in uniform network

Let us again consider the network $\mathcal{A}_0$ and restricted area of the form $\mathcal{R}_{b_l,b_r} := (-\infty, -b_l) \cup (b_r, \infty)$ for some $0 < b_l, b_r$ and **two** jamming stations. Assume that $b_l, b_r \leqslant \mathrm{range}(s)$ (otherwise the problem either simplifies to a single side jamming or becomes trivial). Clearly one can use directly the positions described in the previous section to jam left and right side independently. However, this way network is burdened with a significant and unnecessary reduction of the coverage.

If the jamming stations $\mathcal{J}^* = \{-x^*, y^*\}$ guarantees $H_s^{\mathcal{A}_0^{\mathcal{J}^*}} = [-b_l, b_r]$, then we call $\mathcal{J}^*$ an *optimal arrangement*.

Below, we briefly present an iterative algorithm that returns positions $-x, y$ of jamming stations that guarantee correct protection of the restricted zone and are δ-close to optimal arrangement $-x^*, y^*$ (see Theorem 1 for precise formulation).

1) Short description of the algorithm: Apart from parameters which describe the SINR model and the restricted area $\mathcal{R}_{b_l,b_r}$, the algorithm takes a precision parameter δ as an input. We use the following notation:

- $C_i = \frac{1}{\beta} - \frac{N}{P} b_i^{\alpha}$, for $i \in \{l, r\}$,
- for $b > 0$ and $x > b^{-\frac{1}{\alpha}}$, let us define $f(a, b; x) = 1 + a\left(1 + (b - x^{-\alpha})^{-\frac{1}{\alpha}}\right)$,
- $h(x) = f\left(\frac{b_l}{b_r}, C_l; f\left(\frac{b_r}{b_l}, C_r; x\right)\right)$.

Algorithm 1: AssignJammingStations(δ)

Algorithm `AssignJammingStations`(δ)
  $t_0 = 1 + \frac{b_l}{b_r}\left(1 + C_l^{-\frac{1}{\alpha}}\right)$,
  $t =$ `AlignPosition`(t, δ)
  $D_f = \dfrac{\left|\frac{\partial}{\partial t} f\left(\frac{b_r}{b_l}, C_r; t\right)\right| b_l}{b_r}$
  if $D_f \geqslant 1$ **then**
    $\delta = \frac{\delta}{D_f}$, $t =$ `AlignPosition`(t, δ)
  $y = \left(f\left(\frac{b_r}{b_l}, C_r; t\right) - 1\right) b_l$, $x = (t - 1) b_r$
  return $(-x, y)$
Procedure `AlignPosition`(t, δ)
  $\zeta = h'(t)$
  $k = \left\lceil \dfrac{\ln\left(\frac{\frac{\delta}{b_r}(1-\zeta)}{h(t)-t}\right)}{\ln \zeta} \right\rceil$
  for $i \in \{1, \ldots, k\}$ **do**
    $t = h(t)$
  return t

Without going into low-level details, the function f entangles positions of optimally set jamming stations. Thence we assume the same in our approach. The function h adapts position of the left jamming station to be closer to optimal. Initially we carefully set admissible position t_0 and then the lion's share of the execution, Algorithm 1 rectifies t, which is responsible for the position of the left jamming station and at the end, it returns positions $-x$ and y, which are δ-close to the optimal ones. In fact, there are two adaptive phases, the first one is performed in order to guarantee that $|x - x^*| < \delta$, and the second, to affirm dual condition $|y - y^*| < \delta$.

2) Result and ideas:

Theorem 1. *Consider a uniform* SINR *network* $\mathcal{A}_0$ *with a single station* $s = 0$ *and parameters* $N > 0, \alpha \geqslant 1$ *and a restricted area* $\mathcal{R}_{b_l,b_r}$ *such that*

1) $0 < b_l \leqslant b_r \leqslant \text{range}(s)$,
2) $C_r^{-\frac{1}{\alpha}} < t_0 = 1 + \frac{b_l}{b_r}\left(1 + C_l^{-\frac{1}{\alpha}}\right)$,
3) $C_l^{-\frac{1}{\alpha}} < 1 + \frac{b_r}{b_l}\left(1 + C_r^{-\frac{1}{\alpha}}\right)$.

Then

1) *there exists a unique optimal arrangement* $\mathcal{J}^* = \{-x^*, y^*\}$
2) `AssignJammingStations(`δ`)` *returns* $\mathcal{J} = \{-x, y\}$ *such that:*
   - $\mathcal{J}$ *correctly protect* $\mathcal{R}_{b_l,b_r}$,
   - $|x - x^*| \leqslant \delta$ *and* $|y - y^*| \leqslant \delta$ *(we then say that the arrangement* $\mathcal{J}$ *is* δ*-close).*

The proof of Theorem 1 is technical and it is postponed to be presented in a full version of this paper. However, we roughly sketch its idea in here as well. First, we show that with the assumptions of Theorem 1, h function is rising and concave. Then, by Banach fixed point theorem, we prove that a sequence given by the relations $x_0 = t_0$ and $x_{n+1} = h(x_n)$ converges monotonically to $1 + \frac{x^*}{b_r}$ and can be used in order to provide the positions of the left jamming station. Further we obtain that then the position of the second jamming station can be computed in terms of f function. Next we provide how many steps in each adaptation phase are needed before termination of `AlignPosition(`δ`)` in order to satisfy δ-closeness condition, which finalizes the proof. Let us mention that it can be proved that each of the adapting phases are executed faster than a Newton-Raphson method for a function h (which has very fast, quadratic rate of convergence).

IV. Jamming by noisy dust

In this section we introduce the idea of *noisy dust* — universal strategy for utilizing numerous jamming stations for jamming arbitrary fragments of space. Let us consider a network $\mathcal{A}$, where $S = \{s_1, s_2, \ldots, s_k\}$ and $s_i < s_j$ for any $i < j$, with $P \equiv 1$. Moreover, for any $s \in S$, let us introduce border points $b_\iota(s)$, for $\iota \in \{l, r\}$ such that $s - \text{range}(s) < b_l(s) < s < b_r(s) < s + \text{range}(s)$ and $b_r(s_i) < b_l(s_j)$ whenever $i < j$. Let $\mathcal{R} = \mathbb{R} \setminus \bigcup_{i=1}^{k} [b_l(s_i), b_r(s_i)]$ be a restricted area. By a *noisy dust* we understand a set of stations $\mathcal{J} = (S^{(J)}, P^{(J)})$ placed onto the restricted area, where $P^{(J)} \equiv p$ and $S^{(J)}$ is a disjoint union $\bigcup_{s \in S} \bigcup_{\iota \in \{l,r\}} S^{(J)}(b_\iota(s))$, where $S^{(J)}(b_\iota(s))$ is a set of positions of jamming stations, which intuitively are close to $b_{\iota(s)}$ and correctly protects the nearby restricted area (e.g. $S^{(J)}(b_r(s_{k-1}))$ correctly protects the segment $(s_{k-1}, s_{k-1} + \text{range}(s_{k-1})) \cap (b_r(s_{k-1}), b_l(s_k))$. Note that p intuitively should be small and let $F(p) = (p\beta)^{\frac{1}{\alpha}}$.

Below we present a theorem which describes a space which a single station with small power p can correctly protect:

Theorem 2. *Assume a network* $\mathcal{A}$ *with a station* s_0*, a power* $P_0 = 1$ *and some point* b*, such that* $s_0 < b < \text{range}(s_0)$*. Let* P_j *be the power of a jamming station placed at* $s_j = b + r$*. Let us assume that* $d(s_j, b) = d(s_0, b)F(P_j)$ *. Then* s_j *correctly protects the segment* (b, s_j)*.*

Proof: Realize that energy functions of s_0 and s_j are monotonic for the segment (b, s_j): decreasing for s_0 and increasing for s_j. Hence if we will show that $\text{SINR}(s_0, b) = \beta$, it will be enough to prove the jamming property for that segment. We can do it by rearranging the noiseless SINR equation to obtain the relations below:

$$\text{SINR}(s_0, b) = \frac{d(s_0, b)^{-\alpha}}{P_j \ d(s_j, b)^{-\alpha}} = \beta \ ,$$

$$F(P_j) = (P_j\beta)^{\frac{1}{\alpha}} = \frac{d(s_j, b)}{d(s_0, b)} \ , \qquad d(s_j, b) = d(s_0, b)F(P_j) \ .$$

With the addition of the noise, the desired property of correct protection will not be affected. ■

This simple equation can be used in positioning schemes for multiple jamming stations and let us freely calculate jamming station configuration by modifying the border point position, jamming station power and distance of the station from the border point b. We utilize it for two jamming schemes: an adaptive noisy dust in subsection IV-A, where we optimize the number of required stations by taking into account how the jamming zone of station changes with distance from the jammed station, and special noisy dust stripes in subsection IV-B, which let us create universal jamming networks. Finally, in subsection IV-C we present general lower coverage bound for the noisy dust scheme.

A. Adaptive noisy dust

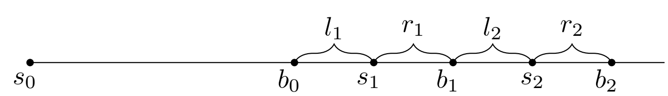


Fig. 5: Positioning of multiple small stations.

Let us assume that we have single station $s_0 = 0$ with selected area $\mathcal{R}_b = (b, \infty)$ for some proper border point b, which we want protect for some network $\mathcal{A}_0$ and power of station s is equal to $P \equiv 1$. In adaptive noisy dust scheme we will iteratively position consecutive jamming stations with uniform powers equal to $p = P_j$ (arbitrarily chosen), for which $F = F(p)$. Starting from point $b_0 = b$ (Figure 5) and using it as initial border, we assign a position of the first station, then we calculate the borders of segment (b_0, b_1) protected by this station and use this information to set the next jamming station in b_1, which will serve as a new border point. We will continue this process until we fill the whole $\mathcal{R}_b$ area. The details of the process are described in Theorem 3 along with a surprising result concerning cost of such jamming network, which converges to zero as $p \to 0^+$ (and increase in the number of jamming stations). Slightly abusing notation, we will denote the distance between station s and point b by using $d(s,b)=b$.

Theorem 3. *Let us consider a single station network $\mathcal{A}_0$ with a restricted area $\mathcal{R}_b=(b,\infty)$. Let*

$$n=\left\lceil\frac{\ln\left(\frac{\text{range}(s)}{b}\right)}{\ln\left(\frac{1+F}{1-F}\right)}\right\rceil\ ,\qquad s_i=\frac{b(1+F)^i}{(1-F)^{i-1}}\ .$$

Then, for the set $\mathcal{J}_p=(\{s_i|\ i\in[n]\},\{s_i\to p|\ i\in[n]\})$ of jamming stations:

1) *$\mathcal{J}_p$ correctly protects $\mathcal{R}_b$,*
2) $\lim_{p\to 0^+}\text{Cost}(\mathcal{J}_p)=0$.

Proof: Using the Theorem 2, we can easily calculate the position of a single station based on the border point, ex. for the first border point $b_0=b$ we will get $l_1=b_0\ F$. Now for station $s_1=b_0+l_1$, we have to know how far it can protect the space on the side opposite to the jammed station — indeed, we are searching for r_1 from Figure 5. Consider point $b_1=s_1+r_1$ and the equation $\text{SINR}(s,b_1)=\beta$. Note that $\text{SINR}(s,x)$ is continuous for $x>0$ and $\text{SINR}(s,x)\overset{x\to s_1}{\longrightarrow}0$, so $\text{SINR}(s,x)<\beta$ for $x\in(s_1-l_1,s_1+r_1)$. Since $\text{SINR}(s,s_1+r_1)=\beta$ is a symmetrical to the case of $\text{SINR}(s,s_1-l_1)$, we conduct very similar argument and obtain $r_1=b_1F=(s_1+r_1)F$. Therefore $r_1(1-F)=s_1F=(b_0+l_1)F=b_0(1+F)F$, so we can obtain

$$s_1=b_0(1+F)\ ,\qquad r_1=\frac{F(1+F)b_0}{1-F}=\frac{Fs_1}{1-F}\ ,$$
$$b_1=b_0\left(1+F+\frac{F(1+F)}{1-F}\right)=b_0\frac{1+F}{1-F}\ . \tag{1}$$

This concludes, that by setting $s_1=b_0(1+F)$, we will correctly protect the interval $(s_1-l_1,s_1+r_1)=(b_0,b_1)$. Realize, that this is true also for network with noise ($N>0$).

Now we want to extend this result to multiple stations. We use similar approach, where next station s_2 is positioned with the assumption that point b_1 is its border point, and it will drowns out some interval (b_1,b_2). We will put recursively subsequent stations, until we reach $\text{range}(s)$ and cover interval $(b_0,\text{range}(s))$ with small jamming fields. Note that we do not need to care about points b_i, $i>0$, since the interference of multiple stations is bigger than for a single one and in result $\text{SINR}(s,b_i)<\beta$ for $i>0$. By using equations (1) as a base, we can extend our notation for other stations as follows:

$$b_i=b_{i-1}\frac{1+F}{1-F}\ ,\quad s_i=(1+F)b_{i-1}\ ,\quad l_i=Fb_{i-1}\ ,$$
$$r_i=\frac{F(1+F)b_{i-1}}{1-F}=\frac{Fs_i}{1-F}\quad\text{for } i\in\{1,2,\dots,n\}\ . \tag{2}$$

Thus, we instantly get the positions of the jamming stations:

$$s_i=(1+F)b_{i-1}=(1+F)b_0\left(\frac{1+F}{1-F}\right)^{i-1}=s_1\left(\frac{1+F}{1-F}\right)^{i-1},$$

with the first one given by $s_1=(1+F)b_0$.

We are also interested in the number n of jamming stations that we need to correctly drown out the region $(b_0,\text{range}(s)]$. More precisely, we search for the minimal n such that $b_n>\text{range}(s)$. As $b_n=b_0\left(\frac{1+F}{1-F}\right)^n$, we need $\left(\frac{1+F}{1-F}\right)^n>\frac{\text{range}(s)}{b_0}$ to be fulfilled, hence we attain:

$$n=\left\lceil\frac{\ln\left(\frac{\text{range}(s)}{b_0}\right)}{\ln\left(\frac{1+F}{1-F}\right)}\right\rceil\ . \tag{3}$$

For our convenience we denote the internal value of the ceiling function in the right hand side of Equation 3 as $\tilde{n}$. It finishes the proof for statement 1. of Theorem 3.

To evaluate the energy efficiency of the algorithm, depending on the power p of the jamming stations, we have to analyze the value $\text{Cost}(\mathcal{J}_p)=np$ or easier to consider value of $\text{Cost}'(\mathcal{J}_p)=\tilde{n}p$. Let $C(p)=\frac{1}{p\beta}$ and $F(p)=(C(p))^{-\frac{1}{\alpha}}=(p\beta)^{\frac{1}{\alpha}}$. Notice that $\frac{\text{range}(s)}{b_0}=\frac{(N\beta)^{-\frac{1}{\alpha}}}{b_0}$ is not dependent on p. We see that as $p\to 0^+$, the limits of numerator $\text{num}(p)$ and denominator $\text{den}(p)$ of the full form of $\text{Cost}'(\mathcal{J}_p)$ tend to 0:

$$\lim_{p\to 0^+}p\ln\left(\frac{\text{range}(s)}{b_0}\right)=0\ ,\quad \lim_{p\to 0^+}\ln\left(\frac{1+F(p)}{1-F(p)}\right)=0\ .$$

We are going to use L'Hôpital's rule to find the limit of $\text{Cost}'(\mathcal{J}_p)=\tilde{n}p$ as p tends to 0. In order to proceed, we need to calculate the derivatives of $F(p)$ and both numerator and denominator of the full form of $\text{Cost}'(\mathcal{J}_p)$:

$$\frac{\partial F(p)}{\partial p}=\frac{\beta\,(C(p))^{\frac{1}{\alpha}-1}}{\alpha}=\frac{F}{\alpha\frac{1}{\beta}C(p)}=\frac{F}{\alpha p}$$
$$\frac{\partial\,\text{num}(p)}{\partial p}=\ln\left(\frac{\text{range}(s)}{b_0}\right)$$
$$\frac{\partial\,\text{den}(p)}{\partial p}=\frac{1-F}{1+F}\,\frac{\frac{\partial F(p)}{\partial p}(1-F)+\frac{\partial F(p)}{\partial p}(1+F)}{(1-F)^2}=\frac{\partial F(p)}{\partial p}\,\frac{2}{(1-F)(1+F)}=\frac{2F}{\alpha p(1-F^2)}$$

From L'Hôpital's rule:

$$\lim_{p\to 0^+}\text{Cost}'(\mathcal{J}_p)=\lim_{p\to 0^+}\frac{\text{num}(p)}{\text{den}(p)}=\lim_{p\to 0^+}\frac{\frac{\partial\,\text{num}(p)}{\partial p}}{\frac{\partial\,\text{den}(p)}{\partial p}}$$
$$=\lim_{p\to 0^+}\ln\left(\frac{\text{range}(s)}{b_0}\right)\frac{\alpha p(1-F^2)}{2F}$$
$$=\alpha\ln\left(\frac{\text{range}(s)}{b_0}\right)\lim_{p\to 0^+}\frac{p\left(1-(\beta p)^{\frac{2}{\alpha}}\right)}{2\,(\beta p)^{\frac{1}{\alpha}}}$$
$$=\alpha\ln\left(\frac{\text{range}(s)}{b_0}\right)\lim_{p\to 0^+}p^{1-\frac{1}{\alpha}}\frac{\left(1-(\beta p)^{\frac{2}{\alpha}}\right)}{2\,(\beta)^{\frac{1}{\alpha}}}=0\ ,$$

therefore also $\text{Cost}(\mathcal{J}_p)$ tends to 0 as $p\to 0^+$, because $|\text{Cost}(\mathcal{J}_p)-\text{Cost}'(\mathcal{J}_p)|<p$, what concludes the proof. ■

The idea presented here can be easily used also for multiple stations jamming scenario, still keeping its property of low energy usage.

B. Noisy dust stripes

In this subsection we present a scheme of positioning jamming stations with some fixed power in a form of *jamming stripes* – uniformly spaced stations, which provide protection for some chosen space interval. The next theorem shows how to form a single *stripe*:

Theorem 4. *Let $\mathcal{A}_0$ be a single station network with a restricted area $\mathcal{R}_b = (b_0, b_1)$, where $s < b_0 < b_1 <$ range(s). Let*

$$n = \left\lceil \frac{b_1 - b_0}{2F(p)b_0} \right\rceil \ , \qquad s_i = b_0(1 + F(p) + 2(i-1)F(p)) \ .$$

Then the set $\mathcal{J}_p = (\{s_i | \ i \in [n]\}, \{s_i \to p | \ i \in [n]\})$ of jamming stations correctly protects $\mathcal{R}_b$.

Proof: Let us look at the jamming station closest to the b_0:

$$s_1 = b_0 + F(p)b_0 \ .$$

Basing on Theorem 2, it correctly protects interval (b_0, s_1). On the other hand, due to energy function of s being monotonically decreasing for $x > s$, we know that any point $x > s_1$ requires less interference than point b_0 to be jammed. However, we have already generated enough interference at range of $F(p)b_0$ from s_1 to jam such points, thus the effective jamming interval of s_1 is $(b_0, b_0 + 2F(p)b_0)$. Then each of the stations s_i, for $i > 1$, is positioned with identical spacing and power, so they will preserve that property, filling whole $\mathcal{R}_b$ with enough interference to protect it. ■

The idea of Theorem 4 with single *stripe* set, can be easily extended for jamming arbitrary networks — it only requires to find the closest points to protect (border points for each broadcasting station) and then configuring *stripes* accordingly.

C. Noisy dust coverage

The *noisy dust* coverage value can be limited from below in generic case as presented in Theorem 5:

Theorem 5. *Assume a network $\mathcal{A}$ and a restricted area $\mathcal{R}$ as in section IV. Let $\mathcal{J}$ be a noisy dust for $\mathcal{A}$ with $\mathcal{R}$. Then*

$$\mathrm{Cover}(\mathcal{J}, \mathcal{A}) \geqslant \frac{\beta^{-\frac{1}{\alpha}} \sum_{s \in S} (N + \mathrm{MaxI}_l(s, \mathcal{J}))^{-\frac{1}{\alpha}}}{\sum_{s \in S} b_r(s) - b_l(s)} + \frac{\beta^{-\frac{1}{\alpha}} \sum_{s \in S} (N + \mathrm{MaxI}_r(s, \mathcal{J}))^{-\frac{1}{\alpha}}}{\sum_{s \in S} b_r(s) - b_l(s)} \ , \qquad (4)$$

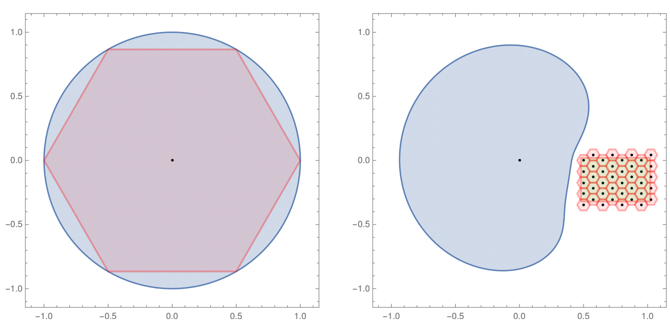
where, for $\iota \in \{l, r\}$,

$$\mathrm{MaxI}_\iota(s, \mathcal{J}) = p \sum_{s' \in S_\iota^{(J)}(s)} d(s', b_\iota(s))^{-\alpha} + p \sum_{s' \in S_{\iota'}^{(J)}(s)} d(s', s)^{-\alpha} \ .$$

Proof: We utilize the similar argument to this from the proof of Lemma 1. Remark that for s, there always exists $\varepsilon(s)$ that s is heard in its vicinity $(s - \varepsilon, s + \varepsilon)$, irrespective of $\mathcal{J}$. From the argument in the proof of Lemma 1, each reception zone of $s \in S$ is convex, so let $x_l(s) < s$ and $x_r(s) > s$ be such the points that $\mathrm{SINR}(s, x_l(s)) = \mathrm{SINR}(s, x_r(s)) = \beta$. Therefore $d(s', s) < d(s', x_{\iota'})$ and $d(s', b_\iota(s)) < d(s', x_\iota)$, for any $s' \in S_\iota^{(J)}(s)$ and $\iota \in \{l, r\}$. Therefore, for every $s \in S$, there exist $y_l(s)$ and $y_r(s)$ such that $x_l(s) < y_l(s) < s < y_r(s) < x_r(s)$ and $\mathrm{SNR}(s, y_l(s), \mathrm{MaxI}_l(s, \mathcal{J})) = \mathrm{SNR}(s, y_r(s), \mathrm{MaxI}_r(s, \mathcal{J})) = \beta$. Therefore $y_\iota(s) = (\beta(N + \mathrm{MaxI}_\iota(s, \mathcal{J})))^{-\frac{1}{\alpha}}$ for any $s \in S$ and $\iota \in \{l, r\}$, just like in Lemma 1. By summing all $d(y_l(s), y_r(s))$ we attain the numerator of inequality (4). The denominator is just the (finite) volume of $\mathbb{R} \setminus \mathcal{R}$. ■

V. Jamming in 2D

An extension of our problem to a two-dimensional case bears some new interesting challenges. For instance, reception zones are no longer reduced to intervals and can form complex shapes on a plane, what results in even more complicated analysis. In this section we propose a simple and natural generalization our ideas from section IV. Let us consider a simple network $\mathcal{A}$ with a single broadcasting station and $D = 2$. On the plane, there is a broad class of reasonable restricted areas. For a sneak peak, let us examine a simple case of rectangular restricted zone with sides parallel to the axes: $\mathcal{R} = \{(x, y) \in \mathbb{R}^2 | \ x_l < x < x_r, \ y_b < y < y_t\}$, where parameters $x_l < x_r$ and $y_b < y_t$ define the rectangle.



(a) A blue area is a full effective jamming zone of a single station and a red one is a simplified jamming area reduced to a hexagon.

(b) An example od 2D jamming — a red rectangle is $\mathcal{R}$ and small hexagons are the simplified jamming zones of added stations.

Fig. 6

In order to provide 2D *noisy dust*, let us reuse the *jamming stripes* idea from subsection IV-B. First, we select a point $b = (b_x, b_y) \in \mathcal{R}$, which is the closest from the broadcasting station $s = (0, 0)$. Note that two-dimensional undisturbed reception zone is a circle. Instead of covering the rectangle with such the figures, we utilize a hexagonal tessellation, which covers $\mathcal{R}$. For simplification we approximate a circle drown out by a jamming station via a regular hexagon (see Figure 6a). This approach has a slight impact on the jammed space, however it still covers the area properly. If we assume that each jamming station has the same arbitrary chosen small power p, then we can calculate a range of each jamming station as $r = d(b, s)F(p)$ (with $F(p) = (p\beta)^{\frac{1}{\alpha}}$). Finally, we cover $\mathcal{R}$ with regular hexagons of side r, with a first hexagon centered in b. An argumentation from subsection IV-B together with triangle inequality shows that such the approach correctly covers $\mathcal{R}$. An exemplary result is presented in Figure 6b.

The aforementioned technique can be easily extended to cases with restricted zones defined as arbitrary unions of the rectangles (also rotated). Nevertheless, methods of restriction of areas of more complicated shape require extensive research.

VI. Related work

The SINR model is well established in wireless networks, including older and newer technologies such as 5G mobile networks [8]. It was used as a measurement of connection quality [9], [10]. SINR is also widely used in theoretical models of wireless communication. Its geometrical properties were studied by Avin et al. [3], who analyzed the properties of reception zones under uniform SINR model, showing, among others, their convexity (the result heavily utilized in this paper). Non-uniform network properties were analyzed in [11], along with new point location algorithm, and in [12], where non-uniform SINR network model, combined with Voronoi Diagrams, proved to retain some of the useful properties of the uniform setting. There is also a large amount of work considering the basic problems under the SINR model, such as *broadcasting* [13], *link scheduling* [14] or *power control* [15]. Quickly evolving and growing wireless communication technology is prone to many security threats (ex. [16], [17]) and more then ever require effective and efficient solutions to protect users privacy. Most of such protective measures are based on cryptographic solutions ([18], [19]). The approach taken in this paper, using jamming stations as a part of the security mechanism, has been considered in [20]–[22] in the context of simpler models and the idea was proved to be practically feasible ([23]). Regarding the SINR model, in [24], the authors considered a model similar to ours, but focusing on a practical 2D scenario, where the space is divided into a *storage*, in which the legitimate communication is supposed to take place, a *jamming space*, where jammers can be placed, delimited by a *fence*, and the rest of the space, where the adversary can eavesdrop. In such settings, the optimization problems of jammers' positioning and power assignment were presented with approximation algorithms working for continuous space. This work has been further extended in [4], where SIR model is used as a connection quality measurement, the solution is based on performing *temporal jamming*, and the channel quality is modeled by the *bit-error probability*. In our paper we extensively use some of previous (e.g. convexity of some SINR diagrams proved in [3]). Nevertheless, to the best of our knowledge, the general results of the current paper cannot be reduced to techniques used in previous literature. This is because the small changes in the problem formulation or the assumed model lead to significantly changed analysis in SINR-based networks.

VII. Conclusions and open problems

We introduced the problem of Zone-restriction with Max-coverage in SINR wireless networks and considered three scenarios. Finding a general solution for this problem seems to be very challenging due to the complexity of the model. The examples of important open directions are: to consider two thresholds $\beta' < \beta$, which models the case where the eavesdropping adversary may have a more sensitive receivers than regular users, cf. [4], and jamming only selected stations; add mobility and directional antennas to the model.

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