Erratum to “The profile decomposition for the hyperbolic Schrödinger equation”

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In the appendix of this paper (Volume 62, Nos. 1–4, 293–320), it was erroneously stated that Gaussians are optimizers for the Strichartz inequality. This is incorrect, as pointed out in [1], and it remains a challenging open problem to determine their nature. This, however, does not affect the main content of the paper, which was independent of the appendix.

The following corrections are needed to the published version of the paper:

(1) Page 293, Equation (1.3) should have an unspecified constant C in place of $2^{-\frac{1}{4}}$.

(2) Appendix: After the first paragraph, the rest of the appendix should be replaced by the following paragraph:
“The original version of this paper stated erroneously that Gaussians were optimizers for the Strichartz norm above, and gave a corresponding numerical value for $\overline{C}$. In fact, it was later proved in [1] that Gaussians are not critical points of the Strichartz norm and thus cannot be optimizers. One can, however, use the profile decomposition to prove existence of an extremizer (see [3] for a similar proof).

PROPOSITION 1

The extremizers of (A.1) *exist, and there is a constant* C *such that*

$$\|e^{it\partial_x\partial_y} f\|_{L^4_{x,y,t}} \leq C\|f\|_{L^2_{x,y}} \tag{0.1}$$

for all $f \in L^2_{x,y}(\mathbb{R}^2)$.

The exact nature of the extremizer f above remains mysterious. They are not simple tensor products (i.e., there is no function g such that $f(x,y) = g(x+y)g(x-y)$), for otherwise, one could adapt arguments from [2] (see the proof in the previous version of this paper) to show that f

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would need to be Gaussian. Preliminary numerical investigations confirm that the optimizer should be a genuine function of x and y and suggest that it has nice decay and smoothness properties."

References

[1] E. Carneiro, L. Oliviera, and M. Sousa, *Gaussians never extremize Strichartz inequalities for hyperbolic paraboloids*, preprint, arXiv:1911.11796 [math.CA].

[2] D. Hundertmark and V. Zharnitsky, *On sharp Strichartz inequalities in low dimensions*, Int. Math. Res Not. **2006** (2006), art. ID 34080, 18. MR 2219206. DOI 10.1155/IMRN/2006/34080.

[3] S. Shao, *Maximizers for the Strichartz and the Sobolev–Strichartz inequalities for the Schrödinger equation*, Electron. J. Differential Equations **2009** (2009), no. 03, 13 pp. MR 2471112.

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