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Addressing complex state constraints in the integral barrier Lyapunov function-based adaptive tracking control

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ABSTRACT

For the state-constrained control problem with complex constrained regions, this paper presents an Integral Barrier Lyapunov Function-based adaptive backstepping scheme for the tracking control. The Barrier Lyapunov Function approach provides an effective tool to embed the barrier terms into the Lyapunov function, enabling the integration of barrier avoidance and closed-loop stabilisation. In the literature, previous works in the area of Barrier Lyapunov Function mostly considered a simple hyperrectangle shape of the constrained region. In this study, the complex barrier region is for the first time introduced to the Barrier Lyapunov Function framework. A novel recursive design procedure is constructed for a class of uncertain nonlinear parametric systems, ensuring the closed-loop signals are all bounded and the tracking errors are convergent. Finally, the proposed method is applied to a numerical example, illustrating the efficacy of this work.

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Barrier Lyapunov function; adaptive control; strict feedback; uncertain nonlinear system

1. Introduction

In many physical systems, constraints are ubiquitously encountered either as an intrinsic characteristic of system dynamics (Tang et al., 2016; Tee et al., 2009b), such as an obstacle in mechanical process and a saturation block in electrical components, or as a performance regulator to avoid undesired operating regimes (He et al., 2014; Tian & Song, 2019). In recent years, a surge of well-known studies are reported in the scope of constrained control, and many are successfully implemented in real-world applications. Among these approaches, two major routines are followed for the state-constrained control design. The first routine is to treat the state constraints as a separate condition or penalty term, in addition to the original control system. Herein, the model predictive control (MPC) handles the state constraints by solving an online finite-horizon optimal control problem using numerical optimizations in real-time (Mayne et al., 2000). The reference governor (RG) based approach modulates the reference signal that is fed into the feedback loop consistently using online nonlinear optimisation (Bemporad, 1998). Control Barrier Functions (CBF) proposed by Ames et al. (2017) unifies both control Lyapunov functions and control barrier function through combining two quadratic programmings to achieve the constrained control objective. The second routine is to integrate the state constraints into either the control design or the original system directly. The Control Lyapunov–Barrier Function (CLBF) in Romdlony and Jayawardhana (2016) and the set invariance notion in Blanchini (1999) design a level set that covers both constrained region and attractive region at the same time. The Barrier Avoidance Control converts a state-constrained problem into an unconstrained one using a diffeomorphic transformation and proves the equivalence of their stability properties (Tian et al., 2020). Motivated by the barrier method in the convex optimisation (Boyd & Vandenberghe, 2004), the Barrier Lyapunov Function (BLF) based method proposed in Tee et al. (2009a) and Tee and Ge (2012) deliberately assigns an infinity value to the Lyapunov candidate at the boundary of the barriers, which ensures the state constraints through designing the Lyapunov function.

In the past decade, the BLF based approaches have been intensively studied in the literature, including application to a Neural Network (NN) control (Ren et al., 2010), a switched system (Niu & Zhao, 2013), a pure-feedback system (Liu & Tong, 2016), and an unknown control direction system (Liu & Tong, 2017). Besides, the BLF has been implemented in a number of physical systems, such as attitude tracking control of multiple spacecrafts (Li et al., 2018), positioning control of a flexible crane system (He et al., 2014), boundary control for a flexible marine riser with vessel dynamics (He et al., 2011), and vibration mitigation of a downhole drilling system via active control (Tian & Song, 2021b). However, most of these work only considered a hyperrectangle (or orthotope) shape of the constrained region, where the state constraints can be generated separately in each dimension and are independent of other dimensions (for example, $|x_1| < k_1, \ldots, |x_n| < k_n$, k_i are positive constants). The study involving a complex state barrier shape rather than such a simple hyperrectangle shape has not been explored before. This extension is not trivial since a well-formulated description of the complex state constraints that can enable a systematic design has not been established, and integrating such complex barriers into the constrained control can largely increase the complexity of the control synthesis

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and thus remains a challenging task. In this work, a systematic way to address a class of complex state constraints in the scheme of Integral Barrier Lyapunov Function (IBLF) based backstepping design under a novel adaptive control law is investigated. To connect the complex barrier region to the BLF, we choose IBLF as our control framework. This is because the IBLF directly embeds the state constraints into the barrier term, and can avoid the need of formulating constraints on error dynamics to indirectly enforce the state constraints, which is adopted in the regular BLF logarithm (Tee et al., 2009a).

The contributions of this study are in twofold. Firstly, a class of complex constrained regions is for the first time integrated into the IBLF design. The barrier/boundary of the complex region is described in a cascade manner, i.e. the constraints in the ith dimension only depend on the 1st to the $i-1$th states. Note that a wide range of barriers can be described in this fashion, which include, for example, all the complex barriers that have convex shapes (Blanchini, 1999) and a class of the non-convex barriers (Dòria-Cerezo et al., 2014; Tian & Song, 2021a). For a strict-feedback nonlinear system, the description of the barriers in a cascade fashion enables the control design based on the backstepping scheme. This framework extends the original IBLF in Tee and Ge (2012), which only applies to the hyperrectangle shape of the barrier, to a broader range of the complex shapes of the constraints. Also, the equivalence of the original design and our design of IBLF can be established through a special choice of the barriers. Secondly, we introduce the parametric model uncertainty and the adaptive control into this IBLF design. Existing work on IBLF with adaptive control (Li et al., 2016) does not ensure the strictly negative definite property of the time-derivative of the Lyapunov candidate, which can induce great conservativeness. In this paper, a novel adaptive routine is proposed by modifying the algorithm in Krstic et al. (1995) and incorporating it into the IBLF framework, which can ensure the time-derivative of the Lyapunov function to be strictly negative definite. It is achieved by designing the adaptive law recursively along with the backstepping procedure, resulting in better control performance.

The rest of the article is organised as follows. In Section 2, we define the state-constrained tracking control problem for an uncertain nonlinear parametric system, and formulate the description for a class of complex state constraint regions in a cascade manner. Section 3 presents the main results of this study, where the complex state barriers are embedded into the IBLF candidate in a backstepping scheme. Adaptive law is included in the recursive steps of the control design to address the model uncertainty. Also, an analysis of the control system is provided to prove the convergence and boundedness of the closed-loop signals. Finally, a case study implements this method to a second-order nonlinear system under complex constrained regions in Section 4, demonstrating the effectiveness of the proposed algorithm.

2. Problem formulation

Throughout the paper, we define $\bar{x}_i = [x_1, x_2, \ldots, x_i]^T$, $\bar{z}_i = [z_1, z_2, \ldots, z_i]^T$, and $\bar{y}_{d_i} = [y_d, y_d^{(1)}, y_d^{(2)}, \ldots, y_d^{(n)}]^T$. We also denote $\|\cdot\|$ to be the Euclidean norm in $\mathbb{R}^i$, $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ to be the minimum and maximum eigenvalues of the matrix, and $\partial\mathcal{X}$ to be the boundary of the set $\mathcal{X}$.

Consider a strict-feedback nonlinear system in the space of $\mathbb{R}^n$ as

$$\begin{aligned} \dot{x}_i &= f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1}, \quad i = 1, 2, \ldots, n-1 \\ \dot{x}_n &= f_n(\bar{x}_n) + g_n(\bar{x}_n)u \\ y &= x_1 \end{aligned} \tag{1}$$

where $f_1, f_2, \ldots, f_n, g_1, g_2, \ldots, g_n$ are smooth functions, $x_1, x_2, \ldots, x_n$ are the states, $u \in \mathbb{R}$ is the input, and $y \in \mathbb{R}$ is the output. The smooth function $f_i(\bar{x}_i)$ is linear-in-the-parameter and can be written as

$$f_i(\bar{x}_i) = \theta^T \phi_i(\bar{x}_i), \quad i = 1, 2 \ldots, n \tag{2}$$

where $\theta \in \mathbb{R}^m$ is an unknown constant vector of parameters and $\phi_i(\bar{x}_i) \in \mathbb{R}^m$ is a known nonlinear function vector satisfying $\|\theta\| \le \theta_M$ for $\theta_M > 0$.

The constrained region for each state with complex boundary is described as

$$\begin{aligned} \mathcal{D} = \Big\{ x \in \mathbb{R}^n \,\Big|\, & k_1^l < x_1 < k_1^u,\; k_2^l(x_1) < x_2 < k_2^u(x_1), \ldots, \\ & k_i^l(\bar{x}_{i-1}) < x_i < k_i^u(\bar{x}_{i-1}), \ldots, \\ & k_n^l(\bar{x}_{n-1}) < x_n < k_n^u(\bar{x}_{n-1}) \Big\} \end{aligned} \tag{3}$$

where $k_i^u(\bar{x}_{i-1})$ and $k_i^l(\bar{x}_{i-1})$ are smooth enough functions with respect to $\bar{x}_{i-1}$, indicating the upper and lower bounds of x_i. Note that the state constraints are constructed in a cascade manner, where the upper and lower bounds of x_i only depend on the states with index less than i. A wide class of complex barriers can be described in this cascade fashion (for example, all the convex barriers can be constructed in this cascade manner). A two-dimensional barrier is shown in Figure 1 for illustration, and this method of barrier description can also be extended to high-dimensional barriers. The region enclosed by the contour ($\widehat{ABDC}$) is commonly defined as a two-dimensional function as $\mathcal{D} = \{x \in \mathbb{R}^n \mid h(x_1, x_2) < 0\}$. Nevertheless, as an alternative, the region can also be defined in each dimension sequentially. First, in the x_1 dimension, the bounds of x_1 can be given by two scalars k_1^l and k_1^u (Figure 1). Next, with x_1 being constrained, the bounds of x_2 can be defined by functions of x_1 as $k_2^l(x_1)$ (as $\widehat{ACD}$ in Figure 1) and $k_2^u(x_1)$ (as $\widehat{ABD}$). Likewise, if there is one more dimension x_3, and barriers in x_3 can be defined as functions of x_1 and x_2 as $k_3^u(x_1, x_2)$ and $k_3^l(x_1, x_2)$. In this sequential manner, the description of the barrier can also be extended to higher orders of n where the bounds of x_n can be written as functions of $x_1, x_2, \ldots, x_{n-1}$ as $k_n^l(x_1, x_2, \ldots, x_{n-1}) < x_n < k_n^u(x_1, x_2, \ldots, x_{n-1})$. Note that, even if the barriers in the practical problem is sometimes non-smooth, we can instead choose an approximated barrier to ensure the smoothness of $k_i^u(\bar{x}_{i-1})$ and $k_i^l(\bar{x}_{i-1})$ with respect to $\bar{x}_{i-1}$ following the cascade rule (3). For example, as shown in Figure 2, the original constrained region is in the colour-shaded area that has four 'sharp' corners. To prevent the upper and lower bounds of x_2 dimension to become non-smooth due to these 'sharp' corners, we set $k_2^l(x_1)$ and $k_2^u(x_1)$ as third-order polynomials for approximation. As long as the state can avoid the approximated barrier

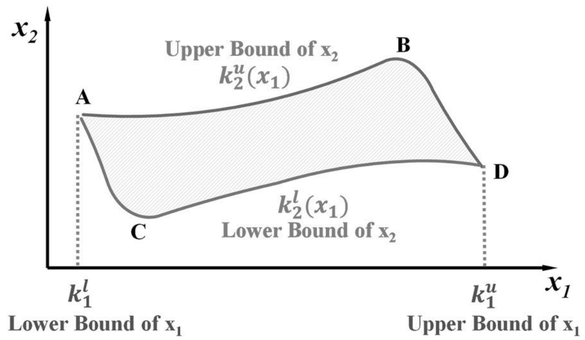

Figure 1. Illustration of a complex barrier.

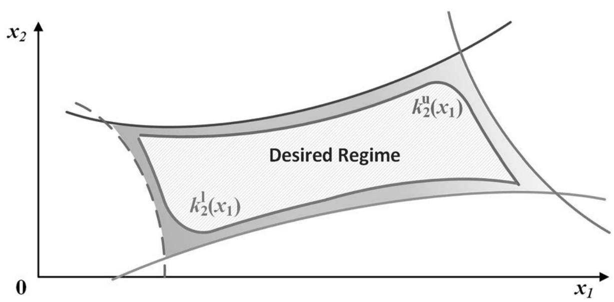

Figure 2. Approximating non-smooth barriers smooth barriers.

sufficiently close to the original barrier, the state-constrained control objective can still be achieved without being too conservative.

The control objective is given as:

The output of the system (1) *tracks the desired trajectory y_d and the states do not cross the boundary of the complex constrained region $\mathcal{D}$.*

Before the main result of this work, the following two assumptions are first provided:

Assumption 2.1: *There exists a positive constant g_0 such that $0 < g_0 \leq |g_i(\bar{x}_i)|$ for $k_j^l(\bar{x}_{j-1}) < x_j < k_j^u(\bar{x}_{j-1})$, $j = 1, 2, \ldots, i$. Without loss of generality, we can assume that $g_i(\bar{x}_i) > 0$ for $k_j^l(\bar{x}_{j-1}) < x_j < k_j^u(\bar{x}_{j-1})$, $j = 1, 2, \ldots, i$.*

Assumption 2.2: *The desired trajectory y_d and its derivatives satisfy*

$$k_1^l < y_d(t) < k_1^u \quad |y_d^{(i)}(t)| \leq B_i, \; i = 1, 2, \ldots, n \tag{4}$$

for all $t \geq 0$ and $B_i > 0$.

3. Adaptive backstepping control design using Integral Barrier Lyapunov Function for complex state constraints

To address the complex state constraints $\mathcal{D}$ as described in (3), the boundary of the constraints $\partial\mathcal{D}$ is incorporated into the barrier terms in the IBLF candidate, and the control design is performed in an adaptive backstepping scheme. The Lyapunov function candidate for the system (1) is then written as

$$V(z, \alpha, x) = V_1(z_1, y_d) + \sum_{i=2}^{n} V_i(z_i, \alpha_{i-1}, \bar{x}_{i-1}) \tag{5}$$

where

$$V_1(z_1, y_d) = W_1(z_1, y_d) + \frac{1}{2}\tilde{\theta}_1^T \Gamma^{-1} \tilde{\theta}_1$$

$$V_i(z_i, \alpha_{i-1}, \bar{x}_{i-1}) = W_i(z_i, \alpha_{i-1}, \bar{x}_{i-1}) + \frac{1}{2}\tilde{\theta}_i^T \Gamma^{-1} \tilde{\theta}_i$$

$$W_1(z_1, y_d) = \int_0^{z_1} \frac{\tau \left(\frac{k_1^u - k_1^l}{2}\right)^2}{(k_1^u - \tau - y_d)(\tau + y_d - k_1^l)} \, d\tau$$

$$W_i(z_i, \alpha_{i-1}, \bar{x}_{i-1}) = \int_0^{z_i} \frac{\tau \left(\frac{k_i^u(\bar{x}_{i-1}) - k_i^l(\bar{x}_{i-1})}{2}\right)^2}{(k_i^u(\bar{x}_{i-1}) - \tau - \alpha_{i-1})(\tau + \alpha_{i-1} - k_i^l(\bar{x}_{i-1}))} \, d\tau \tag{6}$$

Here, $z_1 = x_1 - y_d$, $z_i = x_i - \alpha_{i-1}$, $i = 2, \ldots, n$, and $\Gamma = \Gamma^T > 0$ is the adaptation gain matrix. The stabilising functions $\alpha_1, \ldots, \alpha_{n-1}$ are continuously differentiable. We define $\hat{\theta}_i$ to be the ith estimation of θ and let $\tilde{\theta}_i = \hat{\theta}_i - \theta$. Note that, as can be verified by (6), if the states approach the boundary of the constrained region, i.e. $x_i \to k_i^u$ or $x_i \to k_i^l$, the integral will reach infinity at its upper limit, and the barrier Lyapunov candidate will result in an infinitely large value. In particular, when $k_i^u = -k_i^l = c_i$ holds (c_i is a positive constant), the IBLF candidate W_i becomes the standard type of IBLF (Tee & Ge, 2012) for the simple barrier with hyperrectangular shape. Thus, the expression of the IBLF w_i in this work is a generalised version of the standard IBLF.

Lemma 3.1: *The IBLF W_i defined in* (6) *satisfies the following condition if $k_i^l < \alpha_{i-1} < k_i^u$ holds*

$$\frac{z_i^2}{2} \leq W_i \leq \frac{(k_i^u - k_i^l)^2/4}{(k_i^u - x_i)(x_i - k_i^l)} z_i^2 \tag{7}$$

Proof: For the left part of the inequality, since $(k_i^u - (\tau + \alpha_{i-1}))((\tau + \alpha_{i-1}) - k_i^l) > 0$ we have

$$\frac{\left(\frac{k_i^u - k_i^l}{2}\right)^2}{(k_i^u - (\tau + \alpha_{i-1}))((\tau + \alpha_{i-1}) - k_i^l)} = \frac{\left(\frac{k_i^u - k_i^l}{2}\right)^2}{\left(\frac{k_i^u - k_i^l}{2}\right)^2 - \left(\frac{k_i^u + k_i^l}{2} - (\tau + \alpha_{i-1})\right)^2} \geq 1 \tag{8}$$

Thus, it can be derived that

$$W_i \geq \int_0^{z_i} \tau \, d\tau = \frac{z_i^2}{2} \tag{9}$$

Next, to prove the right part of the inequality, we first define

$$\sigma_i(\tau, \alpha_{i-1}, \bar{x}_{i-1}) = \frac{\tau \left(\frac{k_i^u - k_i^l}{2}\right)^2}{(k_i^u - (\tau + \alpha_{i-1}))((\tau + \alpha_{i-1}) - k_i^l)} \tag{10}$$

Then take derivative of σ_i with respect to τ

$$\frac{\partial \sigma_i}{\partial \tau} = \left(\frac{k_i^u - k_i^l}{2}\right)^2 \cdot \frac{(k_i^u - \alpha_{i-1})(\alpha_{i-1} - k_i^l)}{(k_i^u - (\tau + \alpha_{i-1}))^2((\tau + \alpha_{i-1}) - k_i^l)^2} \tag{11}$$

This partial derivative is positive when $k_i^l < \alpha_{i-1} < k_i^u$ holds. Therefore, σ_i is monotonically increasing with respect to τ, since $\sigma_i(0, \alpha_{i-1}) = 0$. Thus, we have

$$W_i = \int_0^{z_i} \sigma_i(\tau, \alpha_{i-1})\, \mathrm{d}\tau \leq z_i \sigma_i(z_i, \alpha_{i-1}) \tag{12}$$

This completes the proof of the right part of the inequality. ■

Following the backstepping scheme, a step-by-step control design is then proposed.

Step 1: In the first step, consider V_1 defined in (6). We write its time-derivative as

$$\begin{aligned}\dot{V}_1 &= \frac{\partial W_1}{\partial z_1}\dot{z}_1 + \frac{\partial W_1}{\partial y_d}\dot{y}_d + \tilde{\theta}_1^T \Gamma^{-1} \dot{\hat{\theta}}_1 \\ &= \frac{z_1 (k_1^u - k_1^l)^2/4}{(k_1^u - x_1)(x_1 - k_1^l)}(f_1 + g_1 z_2 + g_1 \alpha_1 - \dot{y}_d) \\ &\quad + \left(\frac{(k_1^u - k_1^l)^2/4}{(k_1^u - x_1)(x_1 - k_1^l)} - \frac{k_1^u - k_1^l}{4 z_1} \right. \\ &\quad \left. \times \ln \frac{(z_1 + y_d - k_1^l)(k_1^u - y_d)}{(k_1^u - z_1 - y_d)(y_d - k_1^l)}\right) z_1 \dot{y}_d + \tilde{\theta}_1^T \Gamma^{-1} \dot{\hat{\theta}}_1 \\ &= \frac{z_1 (k_1^u - k_1^l)^2/4}{(k_1^u - x_1)(x_1 - k_1^l)}(f_1 + g_1 z_2 + g_1 \alpha_1) \\ &\quad - \varrho_1(z_1, y_d) z_1 \dot{y}_d + \tilde{\theta}_1^T \Gamma^{-1} \dot{\hat{\theta}}_1 \end{aligned} \tag{13}$$

where

$$\varrho_1(z_1, y_d) = \frac{k_1^u - k_1^l}{4 z_1} \ln \frac{(z_1 + y_d - k_1^l)(k_1^u - y_d)}{(k_1^u - z_1 - y_d)(y_d - k_1^l)} \tag{14}$$

Per L'Hôpital's rule, we can verify the limit of $\varrho_1(z_1, y_d)$ at $z_1 = 0$ as

$$\lim_{z_1 \to 0} \varrho_1(z_1, y_d) = \frac{(k_1^u - k_1^l)^2/4}{(k_1^u - y_d)(y_d - k_1^l)} \tag{15}$$

The stabilizing function α_1 is chosen as

$$\alpha_1 = \frac{1}{g_1}\left(-\hat{\theta}_1^T \phi_1 - p_1 z_1 + \frac{(k_1^u - x_1)(x_1 - k_1^l)}{(k_1^u - k_1^l)^2/4} \varrho_1 \dot{y}_d\right) \tag{16}$$

where the positive constant p_1 is the control gain, yielding

$$\begin{aligned}\dot{V}_1 &= \frac{(k_1^u - k_1^l)^2/4}{(k_1^u - x_1)(x_1 - k_1^l)}(-p_1 z_1^2 + g_1 z_1 z_2) \\ &\quad + \tilde{\theta}_1^T\left(\Gamma^{-1}\dot{\hat{\theta}}_1 - \frac{(k_1^u - k_1^l)^2/4}{(k_1^u - x_1)(x_1 - k_1^l)} z_1 \phi_1\right)\end{aligned} \tag{17}$$

Step i ($i = 2, \ldots, n$-1): In the ith step, we take time-derivative of V_i and write it as

$$\begin{aligned}\dot{V}_i &= \frac{\partial W_i}{\partial z_i}\dot{z}_i + \frac{\partial W_i}{\partial \alpha_{i-1}}\dot{\alpha}_{i-1} + \tilde{\theta}_i^T \Gamma^{-1}\dot{\hat{\theta}}_i \\ &= \frac{z_i (k_i^u - k_i^l)^2/4}{(k_i^u - x_i)(x_i - k_i^l)}(f_i + g_i z_{i+1} + g_i \alpha_i - \dot{\alpha}_{i-1}) \\ &\quad + \left(\frac{(k_i^u - k_i^l)^2/4}{(k_i^u - x_i)(x_i - k_i^l)} - \frac{k_i^u - k_i^l}{4 z_i}\right. \\ &\quad \left. \times \ln \frac{(z_i + \alpha_{i-1} - k_1^l)(k_i^u - \alpha_{i-1})}{(k_i^u - z_i - \alpha_{i-1})(\alpha_{i-1} - k_i^l)}\right) z_i \alpha_{i-1} \\ &\quad + \sum_{j=1}^{i-1}\left(\int_0^{z_i} \frac{\partial \sigma_i}{\partial x_j}\mathrm{d}\tau \cdot \dot{x}_j\right) + \tilde{\theta}_i^T \Gamma^{-1}\dot{\hat{\theta}}_i \\ &= \frac{z_i (k_i^u - k_i^l)^2/4}{(k_i^u - x_i)(x_i - k_i^l)}(f_i + g_i z_{i+1} + g_i \alpha_i) \\ &\quad - \varrho_i(z_i, \alpha_{i-1}, \bar{x}_{i-1}) z_i \dot{\alpha}_{i-1} \\ &\quad + z_i \sum_{j=1}^{i-1} \psi_{ij}(z_i, \alpha_{i-1}, \bar{x}_{i-1})\dot{x}_j + \tilde{\theta}_i^T \Gamma^{-1}\dot{\hat{\theta}}_i\end{aligned} \tag{18}$$

where

$$\varrho_i(z_i, \alpha_{i-1}, \bar{x}_{i-1}) = \frac{k_i^u - k_i^l}{4 z_i} \ln \frac{(z_i + \alpha_{i-1} - k_i^l)(k_i^u - \alpha_{i-1})}{(k_i^u - z_i - \alpha_{i-1})(\alpha_{i-1} - k_i^l)} \tag{19}$$

$$\psi_{ij}(z_i, \alpha_{i-1}, \bar{x}_{i-1}) = \frac{1}{z_i}\int_0^{z_i} \frac{\partial \sigma_i(\tau, \alpha_{i-1}, \bar{x}_{i-1})}{\partial x_j}\mathrm{d}\tau \tag{20}$$

$$\begin{aligned}\frac{\partial \sigma_i}{\partial x_j} &= \frac{\tau}{(k_i^u - \tau - \alpha_{i-1})^2(\tau + \alpha_{i-1} - k_i^l)^2} \\ &\quad \times \left[\frac{1}{2}(k_i^u - k_i^l)\left(\frac{\partial k_i^u}{\partial x_j} - \frac{\partial k_i^l}{\partial x_j}\right)\right. \\ &\quad \times (k_i^u - \tau - \alpha_{i-1})(\tau + \alpha_{i-1} - k_i^l) \\ &\quad - \left(\frac{k_i^u - k_i^l}{2}\right)^2\left(\frac{\partial k_i^u}{\partial x_j}(k_i^u - \tau - \alpha_{i-1})\right. \\ &\quad \left.\left. - \frac{\partial k_i^l}{\partial x_j} \times (\tau + \alpha_{i-1} - k_i^l)\right)\right]\end{aligned} \tag{21}$$

The limits of $\varrho_i(z_i, \alpha_{i-1}, \bar{x}_{i-1})$ and $\psi_{ij}(z_i, \alpha_{i-1}, \bar{x}_{i-1})$ at $z_i = 0$ are given as

$$\lim_{z_i \to 0} \varrho_i(z_i, \alpha_{i-1}, \bar{x}_{i-1}) = \frac{(k_i^u - k_i^l)^2/4}{(k_i^u - \alpha_{i-1})(\alpha_{i-1} - k_i^l)} \tag{22}$$

$$\lim_{z_i \to 0} \psi_{ij}(z_i, \alpha_{i-1}, \bar{x}_{i-1}) = \frac{\partial \sigma_i(z_i, \alpha_{i-1}, \bar{x}_{i-1})}{\partial x_j} \tag{23}$$

which are both well-defined in $x \in \mathcal{D}$ and $k_i^l < \alpha_{i-1} < k_i^u$. The existence of the high order derivatives of ϱ_i and ψ_{ij} can be validated through the following Lemma.

Lemma 3.2: *The functions* $\varrho_i(z_i, \alpha_{i-1}, \bar{x}_{i-1})$ *and* $\psi_{ij}(z_i, \alpha_{i-1}, \bar{x}_{i-1})$ *are* $\mathcal{C}^{n-i}$ *in the set*

$$\Psi = \Big\{ \bar{x}_{i-1} \in \mathbb{R}^{i-1}, \alpha_{i-1} \in \mathbb{R} \,\Big|\, k_1^l < x_1 < k_1^u,\, k_2^l(x_1) < x_2 < k_2^u(x_1), \ldots, k_{i-1}^l(\bar{x}_{i-2}) < x_{i-1} < k_{i-1}^u(\bar{x}_{i-2}), k_i^l(\bar{x}_{i-1}) < \alpha_{i-1} < k_i^u(\bar{x}_{i-1}) \Big\} \quad (24)$$

Proof: We will first prove $\psi_{ij}(z_i, \alpha_{i-1}, \bar{x}_{i-1})$ is $\mathcal{C}^{n-i}$, and the same procedure can be applied to $\varrho_i(z_i, \alpha_{i-1}, \bar{x}_{i-1})$.

Define $\xi_i = z_i \psi_{ij}$. We can obtain the following equality by taking partial derivative recursively

$$\begin{aligned}\frac{\partial^k \xi_i}{\partial z_i^k} &= \frac{\partial^{k-1}}{\partial z_i^{k-1}} \left(z_i \frac{\partial \psi_{ij}}{\partial z_i} + \psi_{ij} \right) \\ &= \frac{\partial^{k-1} \psi_{ij}}{\partial z_i^{k-1}} + \frac{\partial^{k-2}}{\partial z_i^{k-2}} \left(z_i \frac{\partial^2 \psi_{ij}}{\partial z_i^2} + \frac{\partial \psi_{ij}}{\partial z_i} \right) \\ &= k \frac{\partial^{k-1} \psi_{ij}}{\partial z_i^{k-1}} + z_i \frac{\partial^k \psi_{ij}}{\partial z_i^k} \end{aligned} \quad (25)$$

From (25), we can write the kth order partial derivative of ψ_{ij} with respect to z_i as

$$\frac{\partial^k \psi_{ij}}{\partial z_i^k} = \frac{1}{z_i} \left(\frac{\partial^k \xi_i}{\partial z_i^k} - k \frac{\partial^{k-1} \psi_{ij}}{\partial z_i^{k-1}} \right) = \frac{\eta}{z_i^{k+1}} \quad (26)$$

$$\text{where} \quad \eta = z_i^k \left(\frac{\partial^k \xi_i}{\partial z_i^k} - k \frac{\partial^{k-1} \psi_{ij}}{\partial z_i^{k-1}} \right) \quad (27)$$

The following limit can be obtained by L'Hôpital's rule and (25)

$$\begin{aligned}\lim_{z_i \to 0} \frac{\partial^k \psi_{ij}}{\partial z_i^k} &= \lim_{z_i \to 0} \frac{1}{(k+1) z_i^k} \frac{\partial \eta}{\partial z_i} \\ &= \lim_{z_i \to 0} \frac{1}{(k+1) z_i^k} \Bigg(k z_i^{k-1} \frac{\partial^k \xi_i}{\partial z_i^k} + z_i^k \frac{\partial^{k+1} \xi_i}{\partial z_i^{k+1}} \\ &\quad - k^2 z_i^{k-1} \frac{\partial^{k-1} \psi_{ij}}{\partial z_i^{k-1}} - k z_i^k \frac{\partial^k \psi_{ij}}{\partial z_i^k} \Bigg) \\ &= \lim_{z_i \to 0} \frac{1}{k+1} \frac{\partial^{k+1} \xi_i}{\partial z_i^{k+1}} \end{aligned} \quad (28)$$

By (21), we have $\xi_i = \int_0^{z_i} (\partial \sigma_i / \partial x_j)\, \mathrm{d}\tau$ to be $\mathcal{C}^\infty$ in the set Ψ. Thus, the pure partial derivative of ψ_{ij} with respect to z_i is at least $n-i$ times continuously differentiable.

Then, we consider the pure partial derivatives with respect to α_{i-1} or x_j and mixed partial derivatives of ψ_{ij}. Per Clairaut's Theorem, we can obtain any mixed partial derivative of ψ_{ij} regardless of the differentiation order, given the following

$$\begin{aligned}&\lim_{z_i \to 0} \frac{\partial^{k+l+m_1+\cdots+m_{i-1}} \psi_{ij}}{\partial z_i^k \partial \alpha_{i-1}^l \partial x_1^{m_1} \cdots \partial x_{i-1}^{m_{i-1}}} \\ &= \frac{\partial^{l+m_1+\cdots+m_{i-1}}}{\partial \alpha_{i-1}^l \partial x_1^{m_1} \cdots \partial x_{i-1}^{m_{i-1}}} \left(\lim_{z_i \to 0} \frac{\partial^k \psi_{ij}}{\partial z_i^k} \right) \end{aligned} \quad (29)$$

where $(k + l + m_1 + \cdots + m_{i-1}) \in \{1, \ldots, n-i\}$ and $k, l, m_1, \ldots, m_{i-1}$ are positive integers. Due to the smoothness of k_i^u and k_i^l, the limits (29) exist. Thus, the pure partial derivatives with respect to α_{i-1} or x_j and mixed partial derivatives of ψ_{ij} up to $n-i$th order exist and are continuous in the set Ψ. ■

Next, the time-derivative of α_{i-1} can be expanded as

$$\dot{\alpha}_i = \sum_{j=1}^{i} \frac{\partial \alpha_i}{\partial \hat{\theta}_j} \dot{\hat{\theta}}_j + \sum_{j=1}^{i} \frac{\partial \alpha_i}{\partial x_j} \dot{x}_j + \sum_{j=1}^{i} \frac{\partial \alpha_i}{\partial y_d^{(j)}} y_d^{(j+1)} \quad (30)$$

Choose the stabilizing function as

$$\begin{aligned}\alpha_i = \frac{1}{g_i} \Bigg[&-p_i z_i - \frac{(k_{i-1}^u - k_{i-1}^l)^2 (k_i^u - x_i)(x_i - k_i^l) g_{i-1} z_{i-1}}{(k_i^u - k_i^l)^2 (k_{i-1}^u - x_{i-1})(x_{i-1} - k_{i-1}^l)} \\ &- \hat{\theta}_i^T \Bigg(\phi_i - \frac{(k_i^u - x_i)(x_i - k_i^l)}{(k_i^u - k_i^l)^2/4} \sum_{j=1}^{i-1} \left(\frac{\partial \alpha_{i-1}}{\partial x_j} \varrho_i - \psi_{ij} \right) \\ &\times \phi_j \Bigg) + \frac{(k_i^u - x_i)(x_i - k_i^l)}{(k_i^u - k_i^l)^2/4} \sum_{j=1}^{i-1} \Bigg(\left(\frac{\partial \alpha_{i-1}}{\partial x_j} \varrho_i - \psi_{ij} \right) \\ &\times g_j x_{j+1} + \varrho_i \frac{\partial \alpha_{i-1}}{\partial y_d^{(j)}} y_d^{(j+1)} + \varrho_i \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}_j} \dot{\hat{\theta}}_j \Bigg) \Bigg] \end{aligned} \quad (31)$$

Substituting (31) into (18) gives

$$\begin{aligned}\dot{V}_i = &\frac{(k_i^u - k_i^l)^2/4}{(k_i^u - x_i)(x_i - k_i^l)} (-p_i z_i^2 + g_i z_i z_{i+1}) \\ &- \frac{(k_{i-1}^u - k_{i-1}^l)^2/4}{(k_{i-1}^u - x_{i-1})(x_{i-1} - k_{i-1}^l)} g_{i-1} z_{i-1} z_i \\ &+ \tilde{\theta}_i^T \Bigg[\Gamma^{-1} \dot{\hat{\theta}}_i - z_i \Bigg(\frac{(k_i^u - k_i^l)^2/4}{(k_i^u - x_i)(x_i - k_i^l)} \phi_i \\ &- \sum_{j=1}^{i-1} \left(\frac{\partial \alpha_{i-1}}{\partial x_j} \varrho_i - \psi_{ij} \right) \phi_j \Bigg) \Bigg] \end{aligned} \quad (32)$$

Step n: The time-derivative of z_n is

$$\dot{z}_n = \dot{x}_n - \dot{\alpha}_{n-1} = f_n(\bar{x}_n) + g_n(\bar{x}_n) u - \dot{\alpha}_{n-1} \quad (33)$$

Use the convenient notation

$$\begin{aligned} h_1 &= \frac{(k_1^u - k_1^l)^2/4}{(k_1^u - x_1)(x_1 - k_1^l)} z_1 \phi_1 \\ h_i &= z_i \left(\frac{(k_i^u - k_i^l)^2/4}{(k_i^u - x_i)(x_i - k_i^l)} \phi_i - \sum_{j=1}^{i-1} \left(\frac{\partial \alpha_{i-1}}{\partial x_j} \varrho_i - \psi_{ij} \right) \phi_j \right) \\ &\quad i = 2, \ldots, n \end{aligned} \quad (34)$$

Choose the input and the adaptive law as

$$u = \alpha_n, \quad \dot{\hat{\theta}}_i = \Gamma h_i, \quad i = 1, \ldots, n \quad (35)$$

We can generate the time-derivative of $V = \sum_{i=1}^{n} V_i$ as

$$
\begin{aligned}
\dot{V} &= \sum_{i=1}^{n} \dot{V}_i \\
&= -\sum_{i=1}^{n} \frac{(k_i^u - k_i^l)^2/4}{(k_i^u - x_i)(x_i - k_i^l)} p_i z_i^2 + \sum_{i=1}^{n} \tilde{\theta}_i^T (\Gamma^{-1}\dot{\hat{\theta}}_i - h_i) \\
&= -\sum_{i=1}^{n} \frac{(k_i^u - k_i^l)^2/4}{(k_i^u - x_i)(x_i - k_i^l)} p_i z_i^2 \\
&\le -\rho \sum_{i=1}^{n} W_i
\end{aligned} \tag{36}
$$

where $\rho = \min\{p_i, i = 1, .., n\}$.

The closed-loop system is written as

$$
\begin{aligned}
\dot{z}_1 &= -\tilde{\theta}_1^T \phi_1 - p_1 z_1 + \left(\frac{(k_1^u - x_1)(x_1 - k_1^l)}{(k_1^u - k_1^l)^2/4} \varrho_1 - 1 \right) \dot{y}_d \\
&\quad + g_1 z_2 \\
\dot{z}_i &= -\tilde{\theta}_i^T \phi_i - \frac{(k_{i-1}^u - k_{i-1}^l)^2 (k_i^u - x_i)(x_i - k_i^l) g_{i-1} z_{i-1}}{(k_i^u - k_i^l)^2 (k_{i-1}^u - x_{i-1})(x_{i-1} - k_{i-1}^l)} \\
&\quad - p_i z_i + g_i z_{i+1} + \frac{(k_i^u - x_i)(x_i - k_i^l)}{(k_i^u - k_i^l)^2/4} \sum_{j=1}^{i-1} \Bigg(\Bigg(\frac{\partial \alpha_{i-1}}{\partial x_j} \varrho_i \\
&\quad - \psi_{ij} \Bigg) (\hat{\theta}_i^T \phi_j + g_j x_{j+1}) + \varrho_i \frac{\partial \alpha_{i-1}}{\partial y_d^{(j)}} y_d^{(j+1)} \\
&\quad + \varrho_i \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}_j} \dot{\hat{\theta}}_j \Bigg) - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}_j} \dot{\hat{\theta}}_j + \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_d^{(j)}} y_d^{(j+1)} \\
&\quad + \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} (\hat{\theta}_j \phi_j + g_j x_{j+1}) \quad i = 2, \ldots, n-1 \\
\dot{z}_n &= -\tilde{\theta}_n^T \phi_n - \frac{(k_{n-1}^u - k_{n-1}^l)^2 (k_n^u - x_n)(x_n - k_n^l) g_{n-1} z_{n-1}}{(k_n^u - k_n^l)^2 (k_{n-1}^u - x_{n-1})(x_{n-1} - k_{n-1}^l)} \\
&\quad - p_n z_n + \frac{(k_n^u - x_n)(x_n - k_n^l)}{(k_n^u - k_n^l)^2/4} \sum_{j=1}^{n-1} \Bigg(\Bigg(\frac{\partial \alpha_{n-1}}{\partial x_j} \varrho_n \\
&\quad - \psi_{nj} \Bigg) (\hat{\theta}_n^T \phi_j + g_j x_{j+1}) + \varrho_n \frac{\partial \alpha_{n-1}}{\partial y_d^{(j)}} y_d^{(j+1)} \\
&\quad + \varrho_n \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}_j} \dot{\hat{\theta}}_j \Bigg) - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}_j} \dot{\hat{\theta}}_j + \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial y_d^{(j)}} y_d^{(j+1)} \\
&\quad + \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} (\hat{\theta}_j \phi_j + g_j x_{j+1})
\end{aligned} \tag{37}
$$

With the closed-loop system well formulated, the main result of this study is provided as follows.

Theorem 3.1: *Consider the closed-loop system* (1) *under the control input and the adaptive law* (35)*. If the initial condition satisfies $x(0) \in \mathcal{D}$ and the condition is provided as*

$$
k_i^l(\bar{x}_{i-1}) < \alpha_{i-1} < k_i^u(\bar{x}_{i-1}), \quad i = 2, \ldots, n, \quad \forall (\bar{z}_n, \bar{y}_{d_n}) \in \Omega \tag{38}
$$

where

$$
\begin{aligned}
\Omega = \Big\{ &\bar{z}_n \in \mathbb{R}^n, \bar{y}_{d_n} \in \mathbb{R}^{n+1} \,\Big|\, |z_i| \le \sqrt{2V_M}, \\
&k_1^l < y_d < k_1^u, |y_d^{(i)}| \le B_i, i = 1, \ldots, n \Big\}
\end{aligned} \tag{39}
$$

$$
V_M = \sum_{i=1}^{n} W_i(0) + \frac{1}{2} \lambda_{\max}(\Gamma^{-1}) \sum_{i=1}^{n} (\|\hat{\theta}_i(0)\| + \theta_M)^2 \tag{40}
$$

Then the following properties hold.

(i) *The tracking error $\bar{z}_n$ and the estimator $\hat{\theta}_i$ remain in the compact set given as*

$$
\Omega_z = \left\{ \bar{z}_n \in \mathbb{R}^n \,\middle|\, \|\bar{z}_n\| \le \sqrt{2V_M} \right\} \tag{41}
$$

$$
\Omega_{\hat{\theta}_i} = \left\{ \hat{\theta}_i \in \mathbb{R}^m \,\middle|\, \|\hat{\theta}_i\| \le \theta_M + \sqrt{\frac{2V_M}{\lambda_{\min}(\Gamma^{-1})}} \right\} \tag{42}
$$

Also, z_i and $\dot{\hat{\theta}}_i$, $i = 1, \ldots, n$, converge to zero.

(ii) *The state x stays in $\mathcal{D}$ for all $t > 0$.*

(iii) *The stabilizing functions α_i, $i = 1, \ldots, n-1$, and control input u are bounded for all $t > 0$.*

Proof: (i) From $\dot{V}(t) \le 0$, we have $V(t) \le V(0)$. Also, since $\|\theta\| \le \theta_M$, it follows that $V(0) \le V_M$. Given Lemma 3.1, we obtain $(1/2)\Sigma_{i=1}^n z_i^2(t) \le V(t) \le V(0) \le V_M$. Thus, we can show that $\|\bar{z}_n\| \le \sqrt{2V_M}$ and $\bar{z}_n \in \Omega_z$. Furthermore, as $\lambda_{\min}(\Gamma^{-1})\|\hat{\theta}_i - \theta\|^2 \le 2V_M$, it follows that $\hat{\theta}_i \in \Omega_{\hat{\theta}_i}$.

Let $\int_0^t (-\dot{V}(t))\,\mathrm{d}t = V(0) - V(t)$. Since $V(0)$ is bounded and $\dot{V}(t) \le 0$, we have $V(t)$ non-increasing and thus bounded. It can be shown that $\int_0^\infty (-\dot{V}(t))\,\mathrm{d}t$ is bounded. Also, as $-\dot{V}(t)$ is uniformly continuous, by Barbalat's Lemma, we have $-\dot{V}(t)$ converge to zero and then $z_i \to 0$ as $t \to \infty$. By the definition of $\dot{\hat{\theta}}_i = \Gamma h_i$, it follows that $\dot{\hat{\theta}}_i \to 0$ as $t \to \infty$. (ii) By proof of contradiction, we assume that there exists a time instance $t = \bar{t}$ such that $x \in \partial\mathcal{D}$, i.e. $x_i = k_i^u$ or $x_i = k_i^l$, given the initial condition $x(0) \in \mathcal{D}$. From $\dot{V}(t) \le 0$, we have $V(\bar{t}) = \Sigma_{i=1}^n V_i(\bar{t}) \le V(0)$ and thus $V_i(\bar{t})$ is bounded. Meanwhile, integrating $V_i(\bar{t})$ gives

$$
\begin{aligned}
V_i(\bar{t}) &= \frac{k_i^u - k_i^l}{4} \int_0^{z_i} \tau \left(\frac{1}{k_i^u - (\tau + \alpha_{i-1})} \right. \\
&\qquad \left. + \frac{1}{(\tau + \alpha_{i-1}) - k_i^l} \right) \mathrm{d}\tau + \frac{1}{2} \tilde{\theta}_i^T \Gamma^{-1} \tilde{\theta}_i \\
&= \frac{k_i^u - k_i^l}{4} \left(\int_0^{z_i} \left(\frac{k_i^u - \alpha_{i-1}}{k_i^u - (\tau + \alpha_{i-1})} - 1 \right) \mathrm{d}\tau \right. \\
&\qquad \left. + \int_0^{z_i} \left(\frac{-\alpha_{i-1} + k_i^1}{(\tau + \alpha_{i-1}) - k_i^l} + 1 \right) \mathrm{d}\tau \right) + \frac{1}{2} \tilde{\theta}_i^T \Gamma^{-1} \tilde{\theta}_i
\end{aligned}
$$

$$= \frac{k_i^u - k_i^l}{4}\left((\alpha_{i-1}(\bar{t}) - k_i^l)\ln\frac{\alpha_{i-1}(\bar{t}) - k_i^l}{x_i(\bar{t}) - k_i^l}\right.$$
$$\left. + (k_i^u - \alpha_{i-1}(\bar{t}))\ln\frac{k_i^u - \alpha_{i-1}(\bar{t})}{k_i^u - x_i(\bar{t})}\right) + \frac{1}{2}\tilde{\theta}_i^T\Gamma^{-1}\tilde{\theta}_i \quad (43)$$

As the condition (38) holds, if $x_i = k_i^u$ or $x_i = k_i^l$, the value of $V_i(\bar{t})$ becomes unbounded, which contradicts the above discussion. Therefore, the state x will not reach $\partial\mathcal{D}$.

(iii) Due to the condition (38), Lemma 3.2 holds and $\varrho_i(z_i, \alpha_{i-1}, \bar{x}_{i-1})$, $\psi_{ij}(z_i, \alpha_{i-1}, \bar{x}_{i-1})$ are $\mathcal{C}^{n-i}$. Given the definition of α_i in (16), (31) and the choice of control input $u = \alpha_n$, it is clear that the closed-loop signals of α_i and u are bounded. ■

4. Numerical example

The simulation is performed on a second-order nonlinear system given as

$$\dot{x}_1 = 0.1x_1^2 + x_2$$
$$\dot{x}_2 = 0.1x_1x_2 - 0.2x_1 + (1 + x_1^2)u \quad (44)$$

where we choose the nonlinear function vectors as $\phi_1 = [0.01x_1^2, 0, 0]^T$, $\phi_2 = [0, 0.01x_1x_2, -0.02x_1]^T$, and the unknown parameter vector as $\theta = [10, 10, 10]^T$. The desired trajectory is given as

$$y_d(t) = 0.5\sin(t) \quad (45)$$

The state-constrained region is defined in the coordinates of $x_1 - x_2$, as shown in Figure 3. We provide the expressions of the boundaries of this constrained region as

$$k_1^u = 1$$
$$k_1^l = -1$$
$$k_2^u(x_1) = -1.102x_1^2 - 0.987x_1 + 1.270$$
$$k_2^l(x_1) = 0.897x_1^2 - 0.989x_1 - 1.270 \quad (46)$$

To design the controller under IBLF based adaptive backstepping scheme as proposed in this study, we choose the control gains as $p_1 = p_2 = 1$ through a feasibility check of the condition (38). Also, the parameters for adaptive laws are designed as

$$\hat{\theta}_1 = \begin{bmatrix}\vartheta_1\\0\\0\end{bmatrix},\quad \hat{\theta}_2 = \begin{bmatrix}\vartheta_2\\\vartheta_3\\\vartheta_4\end{bmatrix},\quad \Gamma = \begin{bmatrix}5000 & 0 & 0\\0 & 5000 & 0\\0 & 0 & 5000\end{bmatrix} \quad (47)$$

The initial conditions of the numerical test are $x_1(0) = -0.8$, $x_1(0) = 0.1$, and $\vartheta_1(0) = \vartheta_2(0) = \vartheta_3(0) = \vartheta_4(0) = 15$. In the simulation, the phase portrait of $x_1 - x_2$ is drawn in Figure 3, where the trajectory of x approaches a periodic cycle that matches the desired sinusoidal trajectory without any violation of the state constraints. Also, Figure 4 shows that the phase portrait of $z_1 - z_2$ approaches to the origin, indicating the convergence of z_1 and z_2 to zero. The time history of the adaptive gain $\hat{\theta}$ is depicted in Figure 5, which verifies the fact that $\dot{\hat{\theta}}$ converges to zero. Finally, the boundedness of the control input is shown in Figure 6.

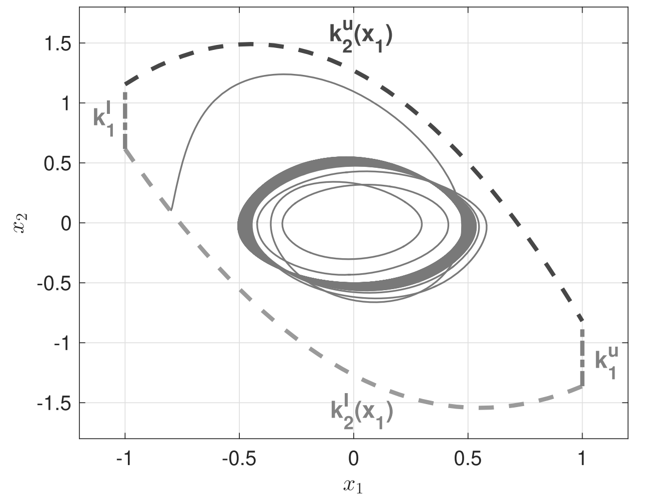

Figure 3. Complex state-constrained region and phase portrait of $x_1 - x_2$ (boundaries denoted by dashed lines).

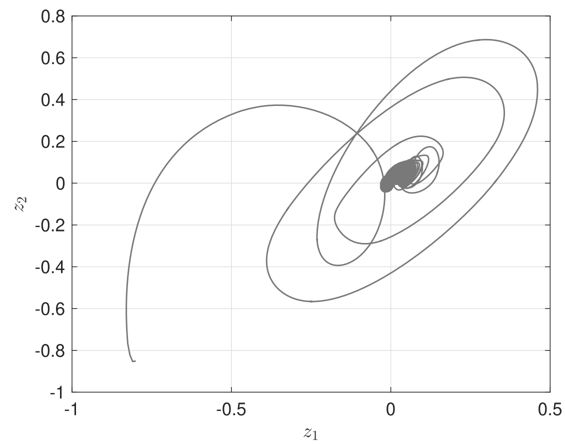

Figure 4. Phase portrait of $z_1 - z_2$.

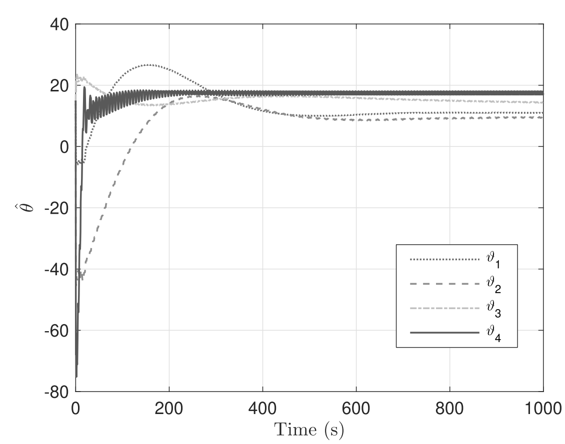

Figure 5. Time history of the adaptive gain $\hat{\theta}$.

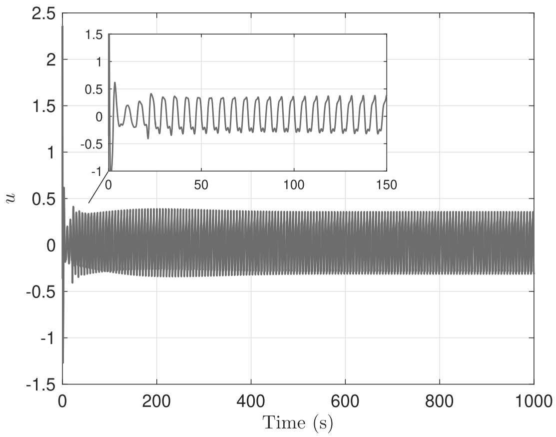

Figure 6. Time history of the control input *u*.

5. Conclusion

This paper investigates a novel control technique to address the complex state constraints in an IBLF based backstepping scheme. Unlike the previous studies that only consider a hyperrectangle shape of the constraint, a complex state barrier described in a cascade manner is introduced in a step-by-step control design. An adaptive law is also employed in the systematic construction of the controller to estimate the unknown system parameters. Proof is provided to show the assurance of the state constraints, the boundedness of the closed-loop signals, and the convergence of the tracking errors, which is verified by a numerical study at the end.

In the future study, guideline for control design under state constraints with a generic shape will be further explored, and the method of evaluating the feasibility of control design will be established.

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