

Rytov's EMT Applicability for Photonic Lattices

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Abstract: We address the properties of photonic lattices by Rytov's EMT method. The symmetric solution of Rytov's eigenvalue equations pertains to normal incidence angles with the asymmetric solution being relevant for off-normal illumination. © 2021 The Author(s)

1. Introduction

Effective medium theory (EMT) pertains to the study of subwavelength gratings modeled as thin films with an effective refractive index. There have been hundreds of papers that discuss EMT with some comparing its accuracy with other analytical methods [1-3]. One of these works is by Kikuta et al. [4] which represents the reflection and transmission of subwavelength lamellar gratings with rigorous numerical methods and EMT. The original EMT solution as related to 1D periodic lamellar gratings is by Rytov [5]. By employing Maxwell's equations and the periodicity and boundary conditions, effective refractive index as a function of wavevector is achieved. Most of the studies on the EMT apply to normal incidence. However, Botten et al. [6] propose zero-order EMT for lamellar gratings at non-normal incidence. Hemmati and Magnusson [7] applied EMT solutions by Rytov to describe applicable devices including wideband reflectors, polarizers, and bandpass filters by using the symmetric solution of Rytov. In this contribution, we consider application of Rytov's EMT method in symmetric and asymmetric cases as pertinent to the optical response of subwavelength gratings.

2. Elements of Rytov's EMT

To start with, we present Rytov's quadratic equation which is obtained by imposing continuity and periodicity of the electric and magnetic fields. Considering only the TE polarization, there results

$$(1 + \chi^2) \sin(\alpha_1 a) \sin(\alpha_2 b) + 2\chi(1 - \cos(\alpha_1 a) \cos(\alpha_2 b)) = 0 \quad (1)$$

Here, $\alpha_1 = k_0 \sqrt{n_H^2 - n_{TE}^2}$, $\alpha_2 = k_0 \sqrt{n_L^2 - n_{TE}^2}$, $k_0 = \frac{2\pi}{\lambda_0}$, $\chi = \frac{\alpha_1}{\alpha_2}$. For the layer system which Rytov proposes, n_H

and n_L are the refractive indices of the layers, n_{TE} is the effective refractive index, a and b are widths of grating ridges and throughs. After solving Eq. (1) and applying it to a grating structure with fill factor F , period Λ , and n_H and n_L as the refractive indices of high-index and low-index media, two equations are found as

$$(n_L^2 - n_{TE}^2)^{1/2} \tan\left[\frac{\pi\Lambda}{\lambda}(1-F)(n_L^2 - n_{TE}^2)^{1/2}\right] = -(n_H^2 - n_{TE}^2)^{1/2} \tan\left[\frac{\pi\Lambda}{\lambda}(F)(n_H^2 - n_{TE}^2)^{1/2}\right] \quad (2)$$

$$(n_L^2 - n_{TE}^2)^{1/2} \tan\left[\frac{\pi\Lambda}{\lambda}(1-F)(n_L^2 - n_{TE}^2)^{1/2}\right] = -(n_H^2 - n_{TE}^2)^{1/2} \tan\left[\frac{\pi\Lambda}{\lambda}(F)(n_H^2 - n_{TE}^2)^{1/2}\right] \quad (3)$$

Eqs. (2) and (3) relate to the symmetric and asymmetric solutions of Eq. (1) as discussed by Rytov [5]. In this paper, we study the response of subwavelength gratings with the asymmetric equation. In the work by Hemmati et al. [6], it is shown that the symmetric distribution of the field inside the grating forms in symmetric periodic structures at normal incidence. Non-symmetrical field distributions inside the grating will not be excited by normally-incident light.

In this work, we discuss the behavior of a subwavelength grating at off-normal incidence. For this aim, we propose a grating design with fill factor $F=0.5$, period $\Lambda=1 \mu\text{m}$, $n_H=2.0$, $n_L=1$, $n_c=n_s=n_L=1$ as shown in Fig. 1(a). For this structure, two spectral reflectance maps for $\theta=0^\circ$ and $\theta=1^\circ$ are calculated as a function of grating thickness d_g . The computations are done with RCWA for two angles and are depicted in Fig. 1(b). As it is illustrated in Fig. 1(b) for $\theta=1^\circ$, the resonance peaks split into two resonances, one being the leaky edge and the other the edge that is non-leaky

at $\theta=0^\circ$. The separation of resonance peaks is visible when $\theta \neq 0^\circ$. There results a new resonance lines which is not seen for $\theta=0^\circ$.

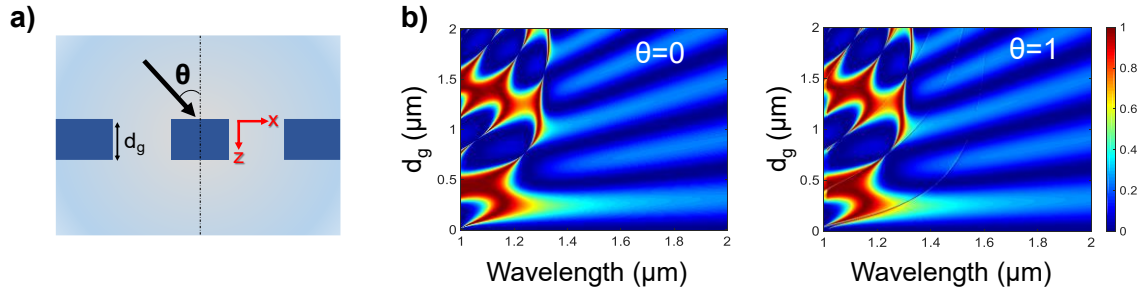


Fig. 1. (a) Schematic of the grating design under an arbitrary angle of incidence. (b) Reflection map pertinent to changing the grating thickness at $\theta=0$ for off-normal illumination at $\theta=1^\circ$.

In Fig. 2, we calculate the reflectance and electric field for the structure in Fig. 1(a) for $d_g=0.48 \mu\text{m}$. The reflectance spectra for $\theta=0^\circ$ and $\theta=1^\circ$ are shown in Fig. 2(a) for a particular spectral region. It is shown the structure possesses a resonance peak for $\theta=1^\circ$ at $\lambda=1.43 \mu\text{m}$. For this resonance wavelength, the electric field distribution shows an asymmetric shape as illustrated in Fig. 2(b).

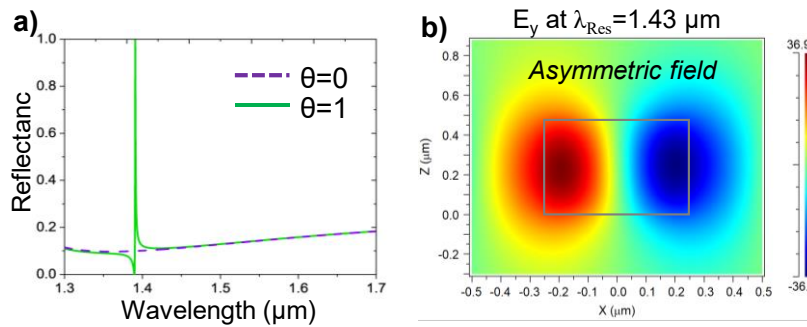


Fig. 2. Spectra and near fields for the grating design proposed in Fig. 1(a) with $d_g=0.48 \mu\text{m}$. (a) Reflectance spectra for $\theta=0^\circ$ and $\theta=1^\circ$. (b) The electric field distribution for $\theta=1^\circ$ calculated at resonance wavelength $\lambda=1.43 \mu\text{m}$.

3. Conclusion

We study Rytov's complete formalism which contains two sets of solutions. The symmetric solution is only valid for describing periodic photonic structures at normal incidence, whereas to explain the optical responses of these devices at off-normal angles, the asymmetric part of Rytov's EMT has to be considered.

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4. References

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