Interrogation of Social Justice Contexts in Mathematical Modeling: The Use of Simulations of Practice in the Mathematical Preparation of Teachers

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Abstract

Research in prospective teachers' development of mathematical modeling knowledge for teaching is gaining momentum. The Mathematics of Doing, Understanding, Learning, and Educating for Secondary Students [MODULE(S2)]* project developed a curriculum in modeling for teacher education that includes *simulations of practice,* in which prospective teachers reflect on and plan a discussion around student thinking, their models, and the contextualization of their results. We present an analysis of prospective teachers' modeling work on the decreasing area of Indigenous reservation land in the U.S., and a *simulation of practice* which explores different methods for finding the area of land in connection to the injustice deeply rooted in the treatment of Indigenous people. This problem explores a critical social issue and calls for explicit attention to pedagogical knowledge in structuring discussions around the contextualization of the mathematical results.

Introduction

In an era of reconciliation, land acknowledgements that recognize First Peoples and traditional territory have become regular inclusions in academic presentations in Canada, Australia, the United States and elsewhere. These statements may raise awareness that Indigenous nations and communities have too often experienced dispossession and displacement through colonization. Yet, to address the social justice context of this issue in depth, this article focuses on a mathematical modeling task on this topic. The lesson featured here comes from the mathematical modeling curricular materials for secondary mathematics teacher preparation developed by the MODULE(S2)* project. MODULE(S2) aims to provide opportunities for faculty to facilitate development of prospective teachers' (PTs') Mathematical Knowledge for Teaching (MKT) in multiple curricular areas, and makes use of simulations of practice (SoP), versions of what Grossman et al. (2009) called approximations

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of practice, to reveal PTs' development of MKT. This study highlights one aspect of Mathematical Modeling Knowledge for Teaching (MMKT), namely, the interplay between social justice contexts and mathematical modeling approaches.

Theoretical Perspectives

MKT frameworks have influenced the development of mathematics teachers for the past thirty years (Ball et al., 2008; Rowland, 2014). MKT specialized for mathematical modeling is less developed and is in need of attention, as many teachers perceive themselves as being unprepared to teach mathematical modeling (Hayes 2019). However, there is evidence that engaging PTs in doing mathematical modeling enriches their conception of modeling and improves their competency in modeling (Anhalt and Cortez, 2016). Self-conception and modeling competency are critical elements of the knowledge quartet (KQ) framework conceived by Rowland (2014). This foundational knowledge may be transformed into effective mathematics teaching and connected and sequenced when planning or teaching lessons. The knowledge within the *foundational* (content knowledge), *transformation* (pedagogical representations) and *connection* (teaching cohesion) dimensions provides the readiness and confidence to deviate from planned activities in the classroom and pursue student-initiated solution approaches to mathematics problems. The latter is the *contingency* dimension (responding to student-initiated approaches) of the KQ and is particularly crucial in mathematical modeling since competency includes allowing students to propose and pursue appropriate choices of mathematics concepts that lead to adequate solutions.

Because the focus of the SoP is on student thinking, it serves as a bridge between *learning* about a teaching practice and actually *doing* it in the classroom. The SoP activities provide PTs opportunities to explore parts of teaching, such as anticipating student thinking in supportive environments where they are free to make mistakes and take risks and receive feedback from teacher educators and peers. They provide an opportunity to "rehearse and enact discrete components of complex practice in settings of reduced complexity" (p. 283). Therefore, they serve as a window to the profession so that PTs have a simplified, yet authentic experience to current professional practice (Grossman et al., 2009).

Currently, most teacher education programs do not engage PTs in sociopolitical contexts beyond what is required for accreditation and in doing so, preservice teachers miss opportunities to "gain self-knowledge, community knowledge, and skills for empathetic connection and sensitivity" (Ukpokodu, 2007, p. 9). They also miss opportunities to interrogate social norms of equity and mathematics in order to provide these opportunities to their future students (Gutierrez, 2017).

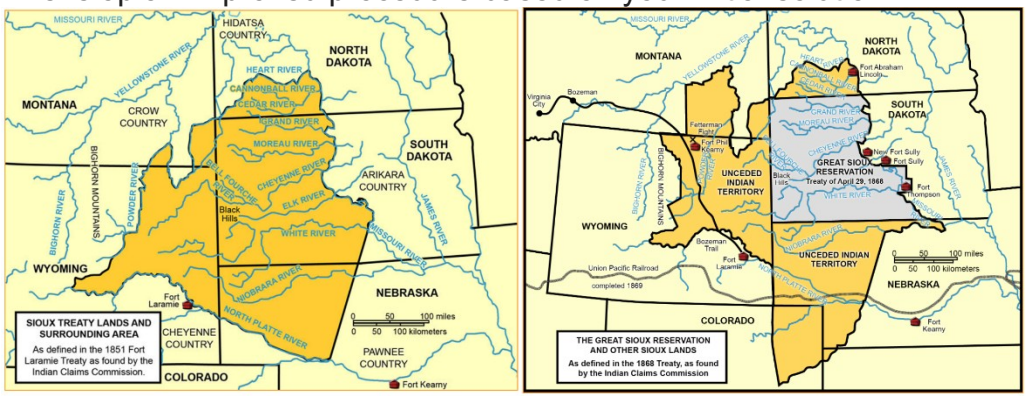
Methods

Our study took place in a secondary mathematics teacher preparation course with 16 prospective teachers (PTs) organized into five collaborative groups for a modeling project that took place over a period of four class sessions that

met twice per week. The PTs engaged with a modeling problem which required them to calculate the difference in areas between two historical maps from 1851 and 1876 of the Sioux Nation Reservation to determine the percentage decrease in land over 25 years (see Figure 1).

Based on the two maps of the Great Sioux Reservation in 1851 and 1876, develop a procedure that can be used to approximate the area of the Great Sioux Reservation and use it to calculate the percentage in area reduction between 1851 and 1876.

- Describe your method for estimating the area based on the map images.
- Estimate the accuracy of your solution and describe changes you would make to improve the accuracy.
- Develop an improved procedure based on your initial solution.



Credits: State Historical Society of North Dakota
https://www.ndstudies.gov/gr8/content/unit-iii-waves-development-1861-1920/lesson-4-alliances-and-conflicts/topic-2-sitting-bulls-people/section-3-treaties-fort-laramie-1851-1868

Fig. 1 The shrinking area of the Sioux Nation Reservation from 1851 to 1876 (in light gray) as a result of the Fort Laramie Treaties, 1851 and 1868.

This problem was the third of four modeling problems addressed during the semester. We introduced the problem by eliciting PTs' background knowledge about reservation land in the U.S. and from historical notes, such as a New York Times 1883 article, *New Lands for Settlers*, and discussions around the origin of the term *reservation*, land claims, and treaty agreements between native tribes and the U.S. government. Students metacognitively engaged in the modeling process as they participated in discussions and reflections about modeling while exploring critical social issues as contexts for the modeling problems. Our research aimed to capture the development of PTs' MMKT through the use of SoP, in which PTs reflected on and planned a discussion around student thinking in mathematical approaches and the contextualization of their results by structuring a discussion on the social justice issue of shrinking reservation land, its meaning in the context of social and political history and the current treatment of Indigenous populations (see Figure 2).

Our research questions include aspects of both the mathematical approaches and the contextualization of the results as manifested in the SoP: (1) How is the PT's engagement in the modeling problem reflected in the results of the SoP? and (2) How does integrating the SoP into the modeling activity

influence the PT's contextualization of the results in connection to social justice? Our data sources include the PTs' group modeling reports as virtual posters and individual SoP narrative essays. For this study, we examined the PTs' modeling reports, models, and SoP narratives and highlighted themes that emerged in their work. Using a grounded theory approach (Strauss & Corbin, 1990) allowed identifying and coding the emerging themes.

When designing a method for figuring out the area of the reservation land from the 1851 map of the Sioux Reservation, you make the following observations: a group of students choose to segment the area into familiar polygons to calculate the area of each polygon and then find the sum of the areas, while another group is overlaying a grid over the 1851 map and estimating the squares that cover the reservation land to calculate the area.

Describe a plan for how you would conduct a discussion the next day based on the two groups presenting their method for calculating the area of the reservation land and determining the percent reduction of the reservation land from 1851 to 1876.

Part 1. Write about the mathematical approaches:

- How would you facilitate a discussion so that students describe the benefits, drawbacks, and similarities of both approaches?
- Suppose that the groups calculate a wide range of percentages for the shrinkage of reservation land from 1851 to 1876. How would you facilitate a discussion to explain the variance in results?

Part 2. Attend to the contextualization of your results:

- How would you structure the discussion on the social justice issue of shrinking reservation land in addition to the mathematical objectives of this task? What are the implications of the results?
- Regardless of the methods for determining areas, what do the model results mean? What have you learned about the social and political history in the United States and current treatment of Indigenous populations?
- Address the purpose and utility of applying mathematics to address social issues.

Fig. 2 Simulation of practice (SoP) of the shrinking area of reservation land.

Findings

The PTs utilized various mathematical approaches to measure and compare the area of the reservation land on the given maps and all groups used the legend on the map for a scale factor. Groups 1 and 4 overlaid a trapezoid covering most of the area of reservation land, estimated the areas and their relative difference, and reported 61% and 66% reduction in land respectively over 25 years. Groups 2 and 3 used a similar approach by segmenting the land into large rectangles, calculated the sum of the areas for each of the two maps and calculated a 57% and 66% reduction in land respectively. Group 5 overlaid a square grid over the reservation land and counted the number of squares, found the difference between the two maps, and calculated a 64% reduction in land between the two maps.

Part 1 of the SoP activity about the benefits, drawbacks, and similarities of both approaches in finding the area of land on both maps yielded various themes in the responses, such as the value of multiple solution paths, a focus

on process and not on correct or incorrect responses, and valuing student thinking as a resource. The following sample data is from a PT's response that demonstrates the value in multiple solution paths: "*Problems like these can be solved using many different paths and approaches. A teacher needs to encourage the students to look at the ways to do it other than how they solved the problem. I would give the students the option to either answer the questions relating to benefits, drawbacks, and similarities for one approach first and then the other approach, or ... they could look at the benefits of both approaches, then look at the drawbacks for both approaches (PT1).*"

Part 2 of the SoP activity is about structuring a discussion around the contextualization of the mathematical results, which in this case is the social injustices and political history in the treatment of Indigenous populations. The PT responses about the results of the calculation of shrinking reservation land revealed their thinking about implications and possible reasons behind the social injustices taken by the U.S. government. The following is sample data: "*The model results are not only about how much land was taken from Indians. It shows the fear of white men to those who were different [from] them. So much that they felt the need to enclose the Indians and surround them to keep them under their control while taking away their resources from them (PT6).*"

Discussion

The model solutions and interpretations of the results led to authentic discussions of the social injustices deeply rooted in the mistreatment of Indigenous people. PTs' responses to the SoP activity revealed their willingness to go beyond the mathematical results and discuss the contextualization of the results. The first quote from PT1 shows that doing the problem in advance provided the readiness (or willingness) to embrace *contingency*, which encompasses listening to students' ideas and encouraging them to pursue their ideas. As a result of experiencing the SoP activity, the PTs considered future instructional planning to include student-initiated ideas.

PTs' responses in contextualizing the mathematical results went beyond the interpretation of the quantitative area of land to include an increased awareness of historical social injustices. The example quote from PT6 illustrates implications and possible reasons for the shrinking reservation land and revealed an emotional and deeper understanding of the mistreatment of Indigenous people. We posit that without this type of activity embedded in a social justice context, the PTs would not have had the opportunity to learn about this critical social history through mathematics.

Closing Remarks

In this study, PTs used various mathematical approaches to calculate the decrease in areas of reservation land over time, and the mathematical results enhanced the PTs' engagement in a SoP to critically examine the social injustices of the Indigenous People's lost land. The experience offered by curriculum of this type, in which PTs engage as learners of mathematics (to build foundation knowledge), consider student thinking (to build contingency knowledge) and then extend the experience further to a professional level, that

is, to examine the mathematics in the context of the critical social issues has the potential to have a lasting positive effect on the PTs' quality of future instruction. The results of this study serve to inform teacher education on the use of SoP activities and their role in developing MMKT, especially in focusing PTs' attention on student thinking, the contingency dimension of the KQ. The impact of preparing teachers to be cognitively and socially reflective practitioners is that they teach their students to question social injustices through mathematics.

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