Optimal photonic gates for quantum-enhanced telescopes

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(Dated: August 4, 2021)

We propose two optimal phase-estimation schemes that can be used for quantum-enhanced long-baseline interferometry. By using distributed entanglement, it is possible to eliminate the loss of stellar photons during transmission over the baselines. The first protocol is a sequence of gates using nonlinear optical elements, optimized over all possible measurement schemes to saturate the Cramér-Rao bound. The second approach builds on an existing protocol, which encodes the time of arrival of the stellar photon into a quantum memory. Our modified version reduces both the number of ancilla qubits and the number of gate operations by a factor of two.

Classical long-baseline interferometry has become a widely accepted method of determining stellar distances or imaging light sources [1, 2]. The central idea is to measure the coherence of the starlight incident at two or more telescopes as a function of their separation, then use the Van Cittert-Zernike theorem [3, 4] to extract information about the source. This has led to many significant advances, including the first observation of a black hole using radio telescopes [5, 6], exoplanet angular diameter estimation [7], and pulsar proper motion measurements [8]. However, there are fundamental limits to such classical interferometric techniques in the optical frequencies, such as quantum shot noise [9] and stellar photon loss during transmission through the long baselines.

Quantum-enhanced telescopy aims to overcome these difficulties by employing concepts from quantum information theory [10], some of which have been implemented in experiment, including long-distance entanglement distribution [11, 12], quantum logic gates [13, 14] and quantum memories [15, 16]. Therefore, it became attractive to design interferometric setups using these quantum resources. The development of quantum repeaters [17, 18] motivated the exploration of non-local setups to enable reliable, long-distance distribution of entangled quantum states. A *spatially local scheme* for a pair of telescopes does not allow bringing the light collected by the telescopes physically together nor distributing entangled quantum states between the telescope locations. For weak thermal light sources like starlight, spatially local schemes like heterodyne detection will always provide less information about the source when compared to the non-local proposals [19].

Gottesman et al. [20] suggested the pioneering proposal of overcoming the problem of transmission losses in the long baselines by establishing a quantum repeater link [17] between the telescopes, but this scheme requires a high rate of entanglement distribution, making it experimentally challenging. Essentially, one needs a distributed photon ready to interfere with every possible spectral-temporal mode of the starlight, which is extremely inefficient since nearly all these modes are unoccupied. Khabiboulline et al. [21, 22] showed that one can significantly reduce the needed rate of entanglement generation by implementing local quantum processing with appropriate quantum memories [23]. In the conceptually simplest scheme, they effectively proposed a quantum nondemolition measurement that identifies which spectral-temporal mode contains a stellar photon, without determining which telescope received the photon.

In this Letter, we introduce two optimal phase estimation schemes that can be applied to long-baseline interferometry. We describe the general two-telescope setup and define what makes a measurement scheme optimal. We then show how the idea of Gottesman et al. can be altered to improve the precision of the phase estimate by a factor of two by using nonlinear gate operations. We also consider a modification to the Khabiboulline et al. scheme that reduces the number of required resources and quantum operations by half. Both proposed protocols can be used to determine the time of arrival of the star photon while keeping the which-path information ambiguous.

Setup.—To explain the basic principle of our procedures, we will consider the case where there are two telescopes that can receive the stellar photons. For weak sources, the average photon number per mode ϵ is much less than one, so we model the source as a weak thermal state [19],

$$\begin{aligned}\rho_{\text{star}} =&(1-\epsilon)|0_L 0_R\rangle\langle 0_L 0_R| \\ &+\frac{\epsilon}{2}(|1_L 0_R\rangle\langle 1_L 0_R| + |0_L 1_R\rangle\langle 0_L 1_R| \\ &+\nu^*|1_L 0_R\rangle\langle 0_L 1_R| + \nu|0_L 1_R\rangle\langle 1_L 0_R|) + \mathcal{O}\left(\epsilon^2\right),\end{aligned} \tag{1}$$

where $|1_L 0_R\rangle$ corresponds to one photon coming to the left (L) telescope and zero photons coming to the right (R) telescope, and similar for the other terms.

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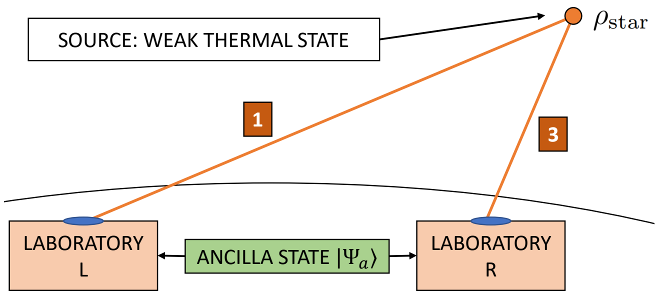

FIG. 1. Generalized setup for quantum-assisted telescopy. Modes 1 and 3 couple the starlight to the left and right laboratories, respectively.

Note that each mode is a single-rail optical qubit, where the computational basis states are the absence (0) and presence (1) of a photon. The protocol's goal is to determine the *complex visibility* ν, which depends on the source intensity distribution and is a function of the baseline connecting two telescopes. Given the visibility as a function of baseline, one can use the van Cittert-Zernike theorem to determine the intensity profile of the source [3, 4]. For all protocols, we will consider both a point source, for which $\nu = e^{-i\Phi}$, as well as an extended source, for which $\nu = ge^{-i\Phi}$, where g is a real and positive amplitude.

The star photons can arrive at two distant telescopes with separate laboratories at their locations, as shown in Fig. 1. The laboratories are provided a known shared ancilla quantum state $|\Psi_{\text{a}}\rangle$ as a resource. We allow local operations and measurements within each laboratory, as well as classical communication between the laboratories, but we do not allow the distribution of stellar photons between the two laboratories. By definition, this prevents the loss of stellar photons that occurs during transmission over long baselines.

Fisher Information.—To quantify and compare the information obtained by specific measurement schemes, we use the Fisher information matrix [24]

$$f = \sum_k \frac{1}{p_k} \begin{pmatrix} \left(\frac{\partial p_k}{\partial \Phi}\right)^2 & \frac{\partial p_k}{\partial \Phi}\frac{\partial p_k}{\partial g} \\ \frac{\partial p_k}{\partial g}\frac{\partial p_k}{\partial \Phi} & \left(\frac{\partial p_k}{\partial g}\right)^2 \end{pmatrix}, \tag{2}$$

where p_k is the probability of obtaining measurement outcome k. According to the Cramér-Rao bound, the inverse of the Fisher information matrix sets a lower bound on the covariance matrix describing the phase and amplitude estimation problem [25, 26]. The upper bound on the Fisher information of a quantum measurement on the stellar photon state (1), optimized over all possible measurement schemes, is given by the quantum Fisher information (QFI), which has matrix elements [27–29]

$$h_{ij} = \text{Tr}\left[\rho_{\text{star}} \frac{L_i L_j + L_j L_i}{2}\right], \; i, j \in \{g, \Phi\}, \tag{3}$$

where L_i is the symmetric logarithmic derivative (SLD) corresponding to parameter i, defined by

$$\frac{L_i \rho_{\text{star}} + \rho_{\text{star}} L_i}{2} = \partial_i \rho_{\text{star}}. \tag{4}$$

This relation is satisfied by

$$\begin{aligned} L_\Phi &= ig \begin{pmatrix} 0 & -e^{-i\Phi} \\ e^{i\Phi} & 0 \end{pmatrix} \\ L_g &= \frac{1}{1-g^2} \begin{pmatrix} -g & e^{-i\Phi} \\ e^{i\Phi} & -g \end{pmatrix}, \end{aligned} \tag{5}$$

using the basis $|1_L 0_R\rangle$ and $|0_L 1_R\rangle$. For this calculation, we focus only on single-photon events, which occur with probability ϵ, so ρ_{star} can also be written in this basis:

$$\rho_{\text{star},1} = \frac{\epsilon}{2} \begin{pmatrix} 1 & \nu^* \\ \nu & 1 \end{pmatrix}. \tag{6}$$

We define any protocol whose Fisher information saturates the QFI, $f(\Phi, g) = h(\Phi, g)$, as an optimal measurement scheme. It is not always possible to saturate this bound in the multi-parameter case, as is the case for this particular system since the SLD matrices do not commute on the support of ρ_{star} [30]. Therefore, we will focus on estimating the phase Φ (by setting $g = 1$). We will present two protocols that are optimal for this single-parameter case.

The Gottesman et al. protocol presented in [20] has a Fisher information of $f = h/2$, so although it has certain advantages over classical interferometry, it is not an optimal scheme. This result reflects the fact that only half the star photons are used for the estimation in that particular scheme. We propose a protocol that uses all the star photons, and thus gives twice the precision in the estimate of Φ. The same improvement to the phase resolution is achieved in the two-parameter case where $\nu = ge^{-i\Phi}$.

CNOT-based protocol.—The protocol of Gottesman et al. [20] uses only linear optical elements to achieve half of the quantum Fisher information. We show in the Supplemental Material that this is the best it is possible to do with the ancilla from their proposal and linear optical elements. To achieve an optimal measurement scheme, we propose the use of nonlinear components. In this case, we make use of an optical CNOT gate in the Fock basis. That is, if there is a photon in the control mode, then the state of the target mode is flipped, i.e., a photon in the target mode is either created or destroyed; otherwise, nothing is done.

We consider a six-mode protocol and provide both laboratories with the ancilla state

$$|\Psi_{\text{a}}\rangle = \frac{1}{\sqrt{2}} |1_0\rangle \otimes \left(|1_2 0_4\rangle + e^{i\delta}|0_2 1_4\rangle\right) \otimes |1_5\rangle, \tag{7}$$

where the modes 2 and 4 are supplied by a single-photon entangled states, and δ is a tunable phase. In (7) we supply two extra photons to both laboratories, one in

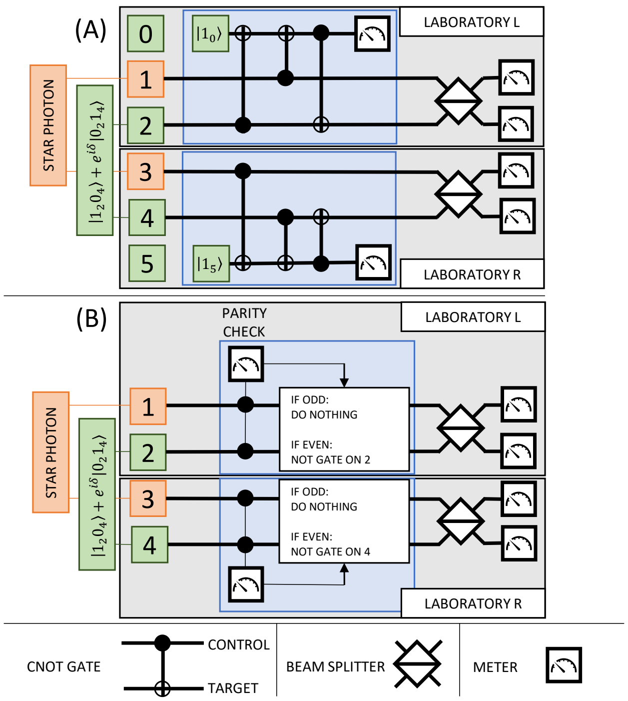

FIG. 2. Circuit representation of two versions of the non-local phase estimation protocol. Note that the blue boxes in (A) and (B) represent equivalent operations. We assume a phase shift of i upon reflection at the beam splitter. It is possible to trade the CNOT sequence in (A) for the local parity measurements in (B), followed by a conditioned NOT gate. One can observe that the blue boxes perform a quantum non-demolition measurement on the star photon and the ancilla, which allows us to herald events corresponding to the arrival of a star photon without localizing it to a particular telescope mode (1 or 3).

mode 0 and one in mode 5. The subscripts indicate the modes indicated in Figure 2A. The total state received by both laboratories is then $\rho_0 = \rho_{\text{star}} \otimes |\Psi_{\text{a}}\rangle\langle\Psi_{\text{a}}|$.

The state ρ_0 undergoes the series of operations shown in Fig. 2, and is measured in the number basis. The probabilities of possible outcomes are

$$\begin{aligned}
p(1_0 1_1 0_2 1_3 0_4 1_5) = p(1_0 0_1 1_2 0_3 1_4 1_5) &= \frac{\epsilon}{8}\left[1 - \cos(\Phi + \delta)\right] \\
p(1_0 1_1 0_2 0_3 1_4 1_5) = p(1_0 0_1 1_2 1_3 0_4 1_5) &= \frac{\epsilon}{8}\left[1 + \cos(\Phi + \delta)\right] \\
p(0_1 1_1 0_2 1_3 0_4 0_5) = p(0_0 0_1 1_2 0_3 1_4 0_5) &= \frac{\epsilon}{8}\left[1 + \cos(\Phi - \delta)\right] \\
p(0_0 1_1 0_2 0_3 1_4 0_5) = p(0_0 0_1 1_2 1_3 0_4 0_5) &= \frac{\epsilon}{8}\left[1 - \cos(\Phi - \delta)\right].
\end{aligned} \tag{8}$$

Equations (8) allow one to estimate the relative phase shift Φ. Classical communication between the laboratories is required only to to determine which coincidence occurred after all the measurements are performed. For an extended source, one replaces $\cos(\Phi \pm \delta) \to \text{Re}\left\{\nu e^{\mp i\delta}\right\}$.

We can determine whether the protocol described in this section is optimal by evaluating the Fisher information f using (8) and then comparing it to the QFI. For the phase estimation problem, the resulting Fisher information is $f = \epsilon$, which saturates QFI. It shows that this protocol is an optimal measurement of the relative phase shift Φ, and gains twice as much information per stellar photon as the protocol using only linear elements. For the visibility estimation problem, we also achieve a factor of 2 improvement in Fisher information over the Gottesman et al. procedure.

In reality, the stellar photon is in a weak thermal state [19], so there is a large probability that no photon arrives at either telescope. Crucially, one can determine whether or not a photon arrived by comparing the measurement results of qubits 0 and 5. If they are the same, then a stellar photon arrived; if they are different, then no stellar photon arrived. More detailed calculations are given in the Supplemental Material.

One possible error is the loss of the entangled ancilla corresponding to $|0_2 0_4\rangle$ in the input. Such an error cannot be identified by a single detection event since it leads to a set of results similar to the one corresponding to the procedure without error, However, it can be identified by examining the frequency of the $(0_0 0_5)$ and $(1_0 1_5)$ events: if the error is introduced, the latter events occur more often. This detection scheme works only if the error appears on a recurrent basis. We discuss this in more detail in the Supplementary Material.

The CNOT-based protocol offers an improvement over the proposal of Gottesman et al., but it requires CNOT or NOT quantum gates for optimal performance. As shown in [31, 32], these gates can be achieved with single rail optics in a probabilistic way, while for our CNOT-based protocol, deterministic gates are necessary for optimal performance. Therefore, implementing the CNOT-based protocol optically would require deterministic non-linear optical gates, beyond what is currently available. Another approach, motivated by recent developments in quantum transduction [33, 34], is to use non-photonic ancilla qubits that are easier to manipulate, and to transduce then to photonic qubits before the beam splitters in figure 2.

Modified quantum memory protocol.—Even though our CNOT-based protocol is an optimal phase measurement scheme, it requires a copy of the ancilla state for each possible time-bin (more precisely, for each possible spectral-temporal mode within the duration of the measurement and over the bandwidth of the collected starlight); this requires a large amount of resources and is experimentally infeasible. Khabiboulline et al. [21] proposed an optimal phase measurement scheme that encodes the arrival time of the star photon in a quantum memory, for which the amount of required resources scales logarithmically with the number of time-bins. We propose a modification to their scheme that both simplifies it and reduces the required resources by half, which is critical for the practical implementation of these ideas.

Consider the modes provided by the star as single-rail qubits, where the logical 0 and 1 denote the absence

or presence of a single photon in a mo
can measure them in an arbitrary basis
star provided a photon, then the optima
ment is achieved when we directly mea
modes. One measurement is done in the
by $|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$, and the other m
in the rotated basis spanned by $|\pm_\delta\rangle =$
Given the setup in Fig. 1, performing
surement on mode 1 and rotated basis
mode 3 results in the probabilities co
stellar photon arrival

$$P(+,+_\delta) = P(-,-_\delta) = \frac{1}{4}\left[1 + \mathrm{co}\right.$$
$$P(+,-_\delta) = P(-,+_\delta) = \frac{1}{4}\left[1 - \mathrm{co}\right.$$

Such measurements can be done on sing
non-deterministic and heralded way [31]
will suffer from a lower Fisher informatio
tion could be transducing the single-rai
another type of qubit for which the me
be easier.

The Fisher information for this set of
urates the QFI, so this is also an opt
surement scheme. For extended sourc
$\cos(\Phi + \delta) \to \mathrm{Re}\left\{\nu e^{-i\delta}\right\}$.

The measurement on the stellar pho
issue: we cannot tell if the star provide
both $|0\rangle$ and $|1\rangle$ can return the same se
need to know whether or not the photon
if it has, then we must know when it happened. This is achieved by the procedure shown in Fig. 3.

Suppose that within time T, we expect at most one photon to arrive from the star. We divide T into N short time-bins of length τ, corresponding to temporal modes, so that $T = N\tau$. To perform binary encoding of the time-bin, we need $2\log_2(N-1)$ ancilla qubits, each prepared in the state $|\Phi^+\rangle$, where $|\Phi^\pm\rangle = (|00\rangle \pm |11\rangle)/\sqrt{2}$ are maximally entangled Bell states. The first qubit from each pair is distributed to laboratory L, and the second qubit to R. The next step is to pass the ancilla through a set of controlled phase gates (CZ) that depends on the time-bin, where

$$\begin{aligned} &|0_c 0_t\rangle \xrightarrow{CZ} |0_c 0_t\rangle, \quad |0_c 1_t\rangle \xrightarrow{CZ} |0_c 1_t\rangle, \\ &|1_c 0_t\rangle \xrightarrow{CZ} |1_c 0_t\rangle, \quad |1_c 1_t\rangle \xrightarrow{CZ} -|1_c 1_t\rangle \end{aligned} \tag{10}$$

performs a standard phase shift gate Z on a target qubit when the state of the control qubit is 1. The index c denotes the control qubit, and t denotes the target qubit. A Z gate acting on one of the qubits in a Bell pair can be used to switch between Bell states, $\mathrm{Z}|\Phi^\pm\rangle = |\Phi^\mp\rangle$. In our case, the star supplies the control qubits for the gates, and the ancilla supplies the target qubits. If the star photon arrives during the nth time-bin, then the sequence of gates $\bigotimes_{i=1}^{2\log_2(N-1)} \mathrm{Z}^{n_i}$ is performed on the locally available ancilla qubits, where n_i is the ith digit of the integer n written in binary (see the Supplemental Material for an explicit example).

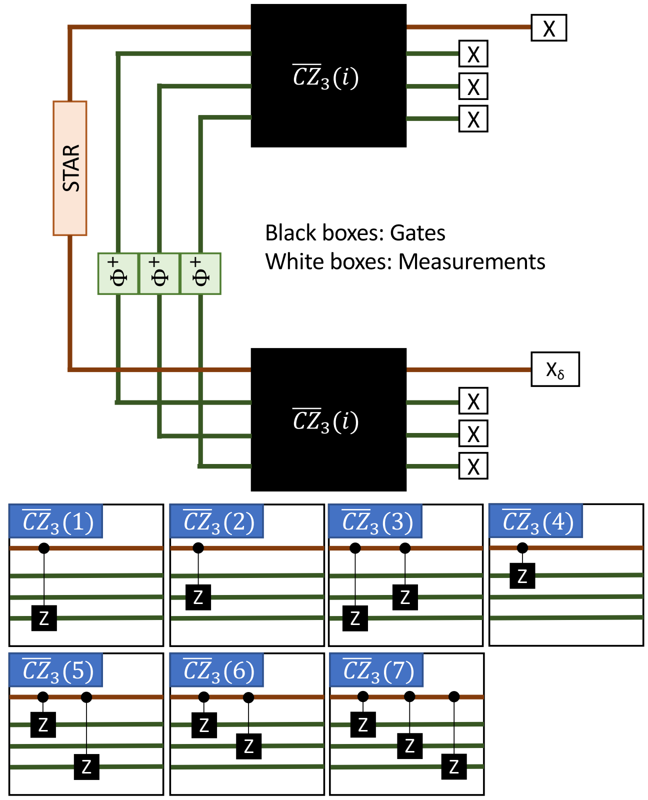

FIG. 3. Scheme of the modified version of Khabiboulline's protocol for $N = 7$ time-bins. A set of $2\log_2(N-1)$ Bell states are generated and distributed to the two laboratories to perform the time-bin encoding. $\overline{\mathrm{CZ}}_k(i)$ indicates a sequence of controlled phase gates with k target qubits (ancilla) corresponding to the i-th time-bin.

This encodes the time-bin information into the Bell states. A similar process was used in [21], but using an extra set of intermediary memory qubits which are the targets of a similarly modified CNOT gate before encoding the time-bin information into the Bell states.

As an example, if there are $N = 7$ total time bins as in Fig. 3, the protocol requires three Bell states to perform the binary encoding. If the star photon arrives within the third time-bin, then the ancilla is modified to

$$|\Phi^+\rangle|\Phi^+\rangle|\Phi^+\rangle \to |\Phi^+\rangle|\Phi^-\rangle|\Phi^-\rangle, \tag{11}$$

where the final two Bell states have been flipped in accordance with the binary representation of $n = 3$ (011). The states $|\Phi^+\rangle$ and $|\Phi^-\rangle$ can be distinguished by local measurements and classical communication by measuring both qubits in the X basis; if the results are the same, then the measured state is $|\Phi^-\rangle$, otherwise it is $|\Phi^+\rangle$. Applying the same procedure to all ancilla pairs returns the time-bin during which the star photon arrived. If no photon arrived, then the ancilla remains unchanged.

The final step is to measure the star pl
the X and rotated bases in the time-bi
the stellar photon has arrived; the possib
results are described by the probabilities
ified protocol reduces the number of an
gates by half when compared to the pr
[21], by eliminating the intermediary me

Conclusions.—We have proposed
enhanced long-baseline interferometry scl
improvements over two prior proposals.
et al. protocol [20] cannot be improve
ited by the ancilla, linear optics and meas
photon number basis, but the developme
photonic gates or quantum transducers
to improve it and achieve an optimal p
scheme. Such a protocol achieves the ma
value of Fisher information, but (similar
man et al. proposal) it consumes one cop
state for each time-bin. This linear scal
was improved to logarithmic by Khabibl
using binary encoding to store the time
stellar photon. We have modified their sc
the number of ancilla qubits and gate ope
This is done by encoding the time-bin
rectly into the Bell state ancilla qubits,
phase gates instead of using intermediary
with controlled NOT gates.

ACKNOWLEDGMENT

We thank Eric Chitambar, Virginia C
D. Monnier, Michael G. Raymer, and
for helpful discussions. This work was su
multi-university National Science Founda
193632 - QII-TAQS: Quantum-Enhanced

SUPPLEMENTAL MATER

Gottesman Protocol - Limitations of t Linear Optics

Consider the setup given in Fig. 4. Mo
supplied by the star, and modes 2 and 4
given by

$$|\Psi_{\mathrm{a}}\rangle = \frac{1}{\sqrt{2}}(|1_2 0_4\rangle + e^{i\delta}|0_2 1_4$$

where the indices indicate the relevant
quire that the measurements are perform
ton number basis, and that the manipul
are local, i.e., the black boxes evolve the input state according to a unitary operation $U = U_L \otimes U_R$. U_L acts only on modes 1 and 2, and U_R acts on 3 and 4. Assume that U_L and U_R represent sets of linear optical elements that do not change the local number of photons, but are otherwise arbitrary.

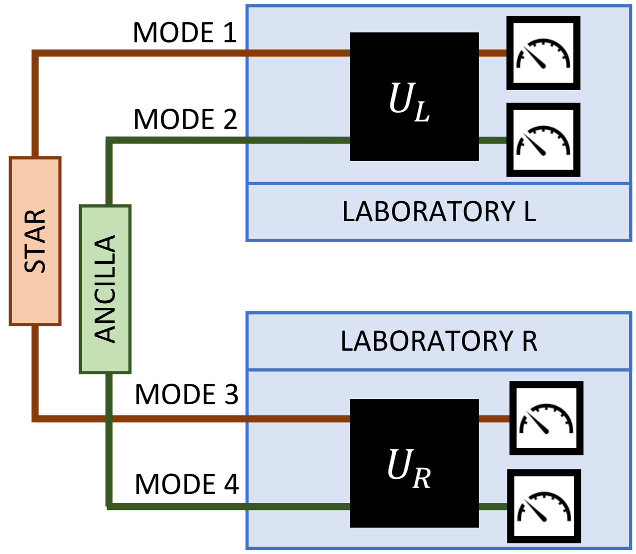

FIG. 4. Scheme of generalized Gottesman et al. protocol. The ancilla and measurements are kept the same as in the original proposal. We restrict this analysis to the local operations U_L and U_R.

For simplicity, we will take the star to be a point source that either supplies the vacuum or a single photon, in the state

$$\rho = (1-\epsilon)|0_L 0_R\rangle\langle 0_L 0_R| + \epsilon|\Psi_1\rangle\langle\Psi_1|, \tag{13}$$

where

$$|\Psi_1\rangle = \frac{1}{\sqrt{2}}\left(e^{i\Phi}|1_1 0_3\rangle + |0_1 1_3\rangle\right). \tag{14}$$

It is possible to filter out the vacuum events, since in those cases the two meters will detect exactly one total excitation, which comes from the ancilla, since the protocol preserves photon number. For the $|\Psi_1\rangle$ events, we can take the input state of the circuit to be

$$\begin{aligned} |\Psi_1\rangle \otimes |\Psi_{\mathrm{a}}\rangle = \frac{1}{2}\Big(& e^{i\Phi}|1_1 1_2 0_3 0_4\rangle + e^{i\delta}|0_1 0_2 1_3 1_4\rangle \\ & + e^{i(\Phi+\delta)}|1_1 0_2 0_3 1_4\rangle + |0_1 1_2 1_3 0_4\rangle\Big). \end{aligned} \tag{15}$$

Applying the U operator gives

$$\begin{aligned} |\Psi\rangle = \frac{1}{2}\Big(& e^{i\Phi} U_L|1_1 1_2\rangle \otimes |0_3 0_4\rangle + e^{i\delta}|0_1 0_2\rangle \otimes U_R|1_3 1_4\rangle \\ & + e^{i(\Phi+\delta)} U_L|1_1 0_2\rangle \otimes U_R|0_3 1_4\rangle \\ & + U_L|0_1 1_2\rangle \otimes U_R|1_3 0_4\rangle\Big), \end{aligned} \tag{16}$$

where we assume that U_L and U_R leave the vacuum unchanged. Regardless of the choice of U_L and U_R, the set of results due to the first and second terms will not overlap with the results due to the other terms because of the different numbers of photons. Since $e^{i\Phi}$ acts as a global phase shift for the first term, it will not affect the probabilities of the measurement results. Therefore, the first and second term will not provide any information about Φ, and we can post-select on the final two terms containing one photon. Note that it is possible to do so because the first two terms correspond to observing two excitations in one laboratory and no excitations in the other one.

The final two terms in (16) can have Φ-dependent interference fringes after performing U_L and U_R. Evaluating the QFI from these terms results in $h = 1$, which should be corrected for the fact that the probability of observing the corresponding events is $\epsilon/2$ (where ϵ is the probability that the star supplies a single photon to the receivers, and $1/2$ is the probability of observing an event due to the final two terms in (15) given that a photon has arrived). Therefore, the upper bound on the Fisher information, corresponding to the best choice of U_L and U_R, is $\epsilon/2$. This is exactly the value obtained for the Gottesman et al. protocol.

We conclude that the Gottesman et al. protocol cannot be improved if we restrict ourselves to setups as in Fig. 4, with an ancilla given by (12), using only linear optics and measurements in the number basis. Any improvement to the Gottesman protocol requires breaking one of these requirements in order to extract information about Φ from all the terms in (16). In our first protocol (Fig. 2, main article), we used nonlinear elements that enable us to perform the NOT or CNOT gates. Even though our scheme offers improvements over the Gottesman et al. protocol, it is a challenge to develop a physical operation that applies these gates.

CNOT-Based Protocol: Example Calculation

Let us take the star to be a point source that either sends a vacuum state or a single photon to the observer. The ancilla is given by (7). If the star sends the vacuum, then the input state is

$$\frac{1}{\sqrt{2}}\left(|1_0 0_1 1_2 0_3 0_4 1_5\rangle + e^{i\delta}|1_0 0_1 0_2 0_3 1_4 1_5\rangle\right), \tag{17}$$

where the indices denote the qubits/modes as in Fig. 2A of the main paper. Passing that state through the sequence of gates results in

$$\begin{aligned}\frac{1}{2\sqrt{2}}\Big(&|0_0 1_1 0_2 1_3 0_4 1_5\rangle + e^{i\delta}|1_0 1_1 0_2 1_3 0_4 0_5\rangle \\ &+ i|0_0 1_1 0_2 0_3 1_4 1_5\rangle + ie^{i\delta}|1_0 1_1 0_2 0_3 1_4 0_5\rangle \\ &+ i|0_0 0_1 1_2 1_3 0_4 1_5\rangle + ie^{i\delta}|1_0 0_1 1_2 1_3 0_4 0_5\rangle \\ &- |0_0 0_1 1_2 0_3 1_4 1_5\rangle - e^{i\delta}|1_0 0_1 1_2 0_3 1_4 0_5\rangle\Big),\end{aligned} \tag{18}$$

where we use a phase shift of i upon reflection at the beam splitter.

If the star sends a photon, its state is

$$|\Psi_s\rangle = \frac{1}{\sqrt{2}}\left(e^{i\Phi}|1_1 0_3\rangle + |0_1 1_3\rangle\right), \tag{19}$$

resulting in the input

$$\begin{aligned}\frac{1}{2}\Big(&e^{i\Phi}|1_0 1_1 1_2 0_3 0_4 1_5\rangle + e^{i\delta}|1_0 0_1 0_2 1_3 1_4 1_5\rangle \\ &+ e^{i(\Phi+\delta)}|1_0 1_1 0_2 0_3 1_4 1_5\rangle + |1_0 0_1 1_2 1_3 0_4 1_5\rangle\Big).\end{aligned} \tag{20}$$

After the CNOT gates, we get

$$\begin{aligned}\frac{1}{2}\Big(&e^{i\Phi}|1_0 1_1 0_2 0_3 1_4 1_5\rangle + e^{i\delta}|1_0 0_1 1_2 1_3 0_4 1_5\rangle \\ &e^{i(\Phi+\delta)}|0_0 1_1 0_2 0_3 1_4 0_5\rangle + |0_0 0_1 1_2 1_3 0_4 0_5\rangle\Big).\end{aligned} \tag{21}$$

Performing the measurement on qubits 0 and 5 returns the state

$$\frac{1}{\sqrt{2}}\left(e^{i(\Phi\pm\delta)}|1_1 0_2 0_3 1_4\rangle + |0_1 1_2 1_3 0_4\rangle\right), \tag{22}$$

where $(-)$ corresponds to the results $|1_0 1_5\rangle$, and $(+)$ to $|0_0 0_5\rangle$. After passing through the beam splitters, the state becomes

$$\begin{aligned}\frac{1}{\sqrt{2}}\Big[&\cos\left(\frac{\Phi+\delta}{2}\right)\left(|0_1 1_2 0_3 1_4\rangle + |1_1 0_2 1_3 0_4\rangle\right) \\ &+ \sin\left(\frac{\Phi+\delta}{2}\right)\left(|0_1 1_2 1_3 0_4\rangle - |1_1 0_2 0_3 1_4\rangle\right)\Big].\end{aligned} \tag{23}$$

The resulting probabilities would be conditioned on the stellar photon arrival. To recover the unconditioned probabilities, one should multiply them by ϵ: the probability of the stellar photon arrival, resulting in

$$\begin{aligned}p(1_0 1_1 0_2 1_3 0_4 1_5) &= p(1_0 0_1 1_2 0_3 1_4 1_5) = \frac{\epsilon}{8}\left[1 - \cos(\Phi+\delta)\right] \\ p(1_0 1_1 0_2 0_3 1_4 1_5) &= p(1_0 0_1 1_2 1_3 0_4 1_5) = \frac{\epsilon}{8}\left[1 + \cos(\Phi+\delta)\right] \\ p(0_1 1_1 0_2 1_3 0_4 0_5) &= p(0_0 0_1 1_2 0_3 1_4 0_5) = \frac{\epsilon}{8}\left[1 + \cos(\Phi-\delta)\right] \\ p(0_0 1_1 0_2 0_3 1_4 0_5) &= p(0_0 0_1 1_2 1_3 0_4 0_5) = \frac{\epsilon}{8}\left[1 - \cos(\Phi-\delta)\right].\end{aligned} \tag{24}$$

Note that one can determine if the star supplied the vacuum or a single photon based on the results of the measurements on qubits 0 and 5. If the star supplied a photon, these results agree as in (21). If the source supplied the vacuum, the results disagree as in (18).

The protocol will not work properly if the entangled ancilla photon is lost (corresponding to $0_2 0_4$ in the input state). If both stellar and entangled ancilla photons have not arrived (corresponding to the input state $|1_0 0_1 0_2 0_3 0_4 1_5\rangle$), then the following results can occur with equal probabilities

$$\begin{aligned}&p(1_0 1_1 0_2 1_3 0_4 1_5|\mathbf{0}_{\text{star}}, \mathbf{0}_{\text{ancilla}}) \\ &= p(1_0 0_1 1_2 0_3 1_4 1_5|\mathbf{0}_{\text{star}}, \mathbf{0}_{\text{ancilla}}) \\ &= p(1_0 1_1 0_2 0_3 1_4 1_5|\mathbf{0}_{\text{star}}, \mathbf{0}_{\text{ancilla}}) \\ &= p(1_0 0_1 1_2 1_3 0_4 1_5|\mathbf{0}_{\text{star}}, \mathbf{0}_{\text{ancilla}}) = \frac{1-\epsilon}{4}.\end{aligned} \tag{25}$$

where by $(\mathbf{0}_{\text{star}}, \mathbf{0}_{\text{ancilla}})$ we have explicitly indicat these probabilities are conditioned on the stellar a cilla photon not arriving. We observe that the possible measurement results for the properly per procedure overlaps the set of possible results corre ing to the error of the entangled ancilla photon riving. We conclude that it is impossible to ident error based on a single detection event.

However, it should be possible to identify it if th occurs frequently within the measurement series pose that the procedure is performed correctly wit ability η and the error of not supplying the entang cilla photon happens with probability $(1-\eta)$. C the results of the measurements of the 0 and 5 Based on (24) and (25) we have

$$
\begin{aligned}
p(0_0 0_5) &= \frac{\eta\epsilon}{2} \\
p(1_0 1_5) &= \frac{\eta\epsilon}{2} + (1-\eta)(1-\epsilon).
\end{aligned}
$$

The $\eta\epsilon/2$ terms result from the proper operation procedure. The $(1-\eta)(1-\epsilon)$ term results fron ducing the error. We conclude that one can ident error of the loss of the entangled ancilla photon b paring the frequency of $(0_0 0_5)$ and $(1_0 1_5)$: if th is introduced, then the frequency of the latter ev higher.

Finally, we should consider the events for wh stellar photon arrives and the entangled ancilla is lost. In this case, the results of the measuren the 0 and 5 modes will be different from each other (one gets either $0_0 1_5$ or $1_0 0_5$), and such events are not taken into account when estimating the visibility. The protocol still works, but it does not extract information about the visibility from all the stellar photons. The Fisher information is therefore reduced by a factor of η, which is the probability that the arriving stellar photon will be used for parameter estimation.

Unmodified Quantum Memory Protocol: Example Calculation

We will summarize the protocol proposed by Khabiboulline et al. The ancilla state is

$$
\begin{aligned}
|\Psi_{\text{a}}\rangle =& |0...000\rangle_{\text{M,L}} |0...000\rangle_{\text{M,R}} \otimes \\
& \otimes |\Phi^+\rangle ... |\Phi^+\rangle |\Phi^+\rangle |\Phi^+\rangle
\end{aligned} \tag{27}
$$

Given that the total measurement time is $T = N\tau$, we have $4\log_2(N+1)$ qubits. A quarter of them are prepared in the state $|0...000\rangle_{\text{M,L}}$ and located in one of the local laboratories denoted by L, and another quarter $|0...000\rangle_{\text{M,R}}$ is located in laboratory R. We will call them *memory qubits*. The Bell pairs $|\Phi^+\rangle$ (consisting of $2\log_2(N+1)$ qubits) are distributed to the laboratories, each laboratory receiving one qubit from each pair. The procedure for $N = 3$ is summarized in Fig. 5.

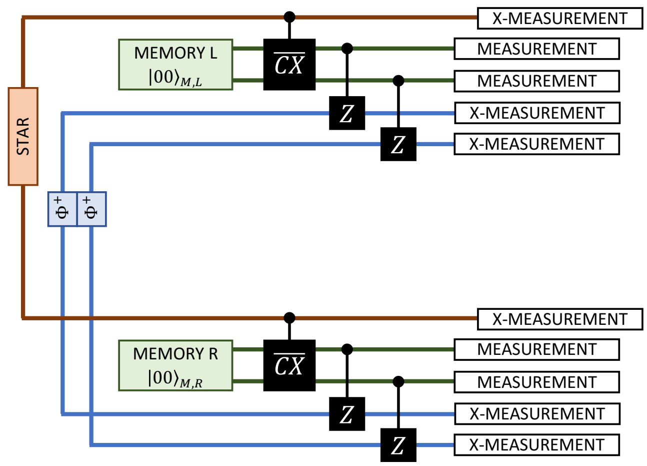

FIG. 5. Schematic representation of Khabiboulline's protocol for $N = 3$ time-bins.

We then use a modified controlled NOT $\overline{\text{CX}}$ gate, whose action is dependent on the time-bin during which the star photon arrived. The modes supplied by the star act as control qubits, and the memory provides the target qubits. The $\overline{\text{CX}}$ gate follows the pattern

$$
\begin{aligned}
\text{No photon arrival :} &|0\rangle|0...000\rangle_{\text{M}} \to |0\rangle|0...000\rangle_{\text{M}} \\
\text{Time-bin 1 :} &|1\rangle|0...000\rangle_{\text{M}} \to |1\rangle|0...001\rangle_{\text{M}} \\
\text{Time-bin 2 :} &|1\rangle|0...000\rangle_{\text{M}} \to |1\rangle|0...010\rangle_{\text{M}} \\
\text{Time-bin 3 :} &|1\rangle|0...000\rangle_{\text{M}} \to |1\rangle|0...011\rangle_{\text{M}} \\
&... \\
\text{Time-bin N :} &|1\rangle|0...000\rangle_{\text{M}} \to |1\rangle|1...111\rangle_{\text{M}}.
\end{aligned} \tag{28}
$$

This gate performs the *encoding* step: the arrival time-bin is encoded in binary in the memory qubits.

For simplicity, suppose that the star emits a photon in the third time-bin and is a point source, then $\nu = e^{-i\Phi}$, and the phase Φ is the parameter to be estimated. The state of the emitted photon is given by

$$
|\Psi_{\text{star}}\rangle = \frac{1}{\sqrt{2}}(e^{i\Phi}|1_L 0_R\rangle + |0_L 1_R\rangle). \tag{29}
$$

The combined state of the stellar photon and the ancilla is $|\Psi_{\text{star}}\rangle \otimes |\Psi_{\text{a}}\rangle$. Performing the $\overline{\text{CX}}$ gate results in the state

$$
\begin{aligned}
|\Psi'\rangle = & \frac{e^{i\Phi}}{\sqrt{2}} |1_L 0_R\rangle |0...011\rangle_{\text{M,L}} |0...000\rangle_{\text{M,R}} \\
& \otimes |\Phi^+\rangle ... |\Phi^+\rangle |\Phi^+\rangle |\Phi^+\rangle \\
& + \frac{1}{\sqrt{2}} |0_L 1_R\rangle |0...000\rangle_{\text{M,L}} |0...011\rangle_{\text{M,R}} \\
& \otimes |\Phi^+\rangle ... |\Phi^+\rangle |\Phi^+\rangle |\Phi^+\rangle.
\end{aligned} \tag{30}
$$

The next step is to perform a set of standard CZ gates. Each memory qubit acts as a control and is assigned a corresponding Bell state as the target. Performing the CZ gates results in the state

$$
\begin{aligned}
|\Psi''\rangle = &\frac{e^{i\Phi}}{\sqrt{2}}|1_L 0_R\rangle|0...011\rangle_{\mathrm{M,L}}|0...000\rangle_{\mathrm{M,R}}\\
&\quad\otimes|\Phi^+\rangle...|\Phi^+\rangle|\Phi^-\rangle|\Phi^-\rangle\\
&+\frac{1}{\sqrt{2}}|0_L 1_R\rangle|0...000\rangle_{\mathrm{M,L}}|0...011\rangle_{\mathrm{M,R}}\\
&\quad\otimes|\Phi^+\rangle...|\Phi^+\rangle|\Phi^-\rangle|\Phi^-\rangle,
\end{aligned}
\tag{31}
$$

which transfers the time-bin information from the memory qubits to the Bell pairs. Note that the Bell pairs are separable from the other qubits. One can distinguish between $|\Phi^+\rangle$ and $|\Phi^-\rangle$ by using local measurements and classical communication, since they can be rewritten in the X basis as

$$
\begin{aligned}
|\Phi^+\rangle &= (|+-\rangle + |-+\rangle)/\sqrt{2}\\
|\Phi^-\rangle &= (|++\rangle + |--\rangle)/\sqrt{2}.
\end{aligned}
\tag{32}
$$

If the result of an X-measurement gives the same result in both laboratories, then we have the state $|\Phi^-\rangle$, otherwise we have $|\Phi^+\rangle$. This allows the parties to determine the time-bin during which the photon arrived. It also allows us to determine which memory qubits were affected by the $\overline{\mathrm{CX}}$ gate. The other memory qubits can be traced out. After measuring the Bell pairs and tracing out the irrelevant memory qubits, the analyzed state is

$$
\begin{aligned}
|\Psi'''\rangle = \frac{1}{\sqrt{2}}\Big(&e^{i\Phi}|1_L 0_R\rangle|11\rangle_{\mathrm{M,L}}|00\rangle_{\mathrm{M,R}}\\
&+|0_L 1_R\rangle|00\rangle_{\mathrm{M,L}}|11\rangle_{\mathrm{M,R}}\Big).
\end{aligned}
\tag{33}
$$

The star photon mode is decoupled from the memories by the measurement in the X basis. In any order, all but one of the memory qubits are measured in the X basis, and the final memory qubit is measured in the rotated basis spanned by $|\pm_\delta\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm e^{i\delta}|1\rangle)$. If n_- denotes the number of times the X measurements return the $|-\rangle$ result, then the probabilities of the measurement results in the rotated basis are

$$
P(\pm_\delta) = \frac{1}{2}\left[1 \pm (-1)^{n_-}\cos(\Phi+\delta)\right], \tag{34}
$$

and for an extended source,

$$
P(\pm_\delta) = \frac{1}{2}\left[1 \pm (-1)^{n_-}\mathrm{Re}\left\{\nu e^{-i\delta}\right\}\right]. \tag{35}
$$

Modified Quantum Memory Protocol: Example Calculation

Suppose the star provides a photon in the state (29), which arrives at the telescopes in the third time-bin. The combined state of the star photon and the ancilla qubits is

$$
\frac{1}{\sqrt{2}}(e^{i\Phi}|1_L 0_R\rangle + |0_L 1_R\rangle)\otimes|\Phi^+\rangle...|\Phi^+\rangle|\Phi^+\rangle|\Phi^+\rangle. \tag{36}
$$

Performing the modified controlled phase gate results in

$$
\frac{1}{\sqrt{2}}(e^{i\Phi}|1_L 0_R\rangle + |0_L 1_R\rangle)\otimes|\Phi^+\rangle...|\Phi^+\rangle|\Phi^-\rangle|\Phi^-\rangle. \tag{37}
$$

Both laboratories measure the ancilla qubits in the X basis and establish the time-bin during which the star photon arrived. After these measurements, the star photon is left in the state (29). The stellar photon provides us with two single rail qubits which we can rewrite in different bases. Rewriting the state of the qubit in laboratory L in the X basis and the qubit in R in the rotated basis gives

$$
\begin{aligned}
|\Psi_{\mathrm{star}}\rangle = \frac{1}{\sqrt{2}}\Big[&\cos\left(\frac{\Phi+\delta}{2}\right)(|+,+_\delta\rangle - |-,-_\delta\rangle)\\
&+i\sin\left(\frac{\Phi+\delta}{2}\right)(|+,-_\delta\rangle - |-,+_\delta\rangle)\Big],
\end{aligned}
\tag{38}
$$

which results in the probabilities (9).

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