IJMEST

International Journal of Mathematical Education in Science and Technology

ISSN: (Print) (Online) Journal homepage: https://www.tandfonline.com/loi/tmes20

Development of prospective elementary teachers' mathematical modelling competencies and conceptions

W. Tidwell, C. O. Anhalt, R. Cortez & B. R. Kohler

To cite this article: W. Tidwell, C. O. Anhalt, R. Cortez & B. R. Kohler (2021): Development of prospective elementary teachers' mathematical modelling competencies and conceptions, International Journal of Mathematical Education in Science and Technology, DOI: 10.1080/0020739X.2021.2005170

To link to this article: https://doi.org/10.1080/0020739X.2021.2005170

Published online: 09 Dec 2021.

INTERNATIONAL JOURNAL OF MATHEMATICAL EDUCATION IN SCIENCE AND TECHNOLOGY
https://doi.org/10.1080/0020739X.2021.2005170

Development of prospective elementary teachers' mathematical modelling competencies and conceptions

W. Tidwell[a], C. O. Anhalt[b], R. Cortez[c] and B. R. Kohler[d]

[a]Morehead State University, Morehead, KY, USA; [b]The University of Arizona, Tucson, AZ, USA; [c]Tulane University, New Orleans, LA, USA; [d]Utah State University, Logan, UT, USA

ABSTRACT

This study examines prospective elementary teachers' growth and development of competencies, conceptions, and perceptions of mathematical modelling in a mathematics content course for elementary teachers. A series of lessons were implemented that engaged students in the modelling process through modelling tasks. The goal was to capture prospective teachers' thoughts and perceptions on the meaning of mathematical modelling and their conceptions of teaching and learning modelling at the elementary and middle school level. The research questions were: *(1) How do prospective teachers translate the mathematical modelling cycle into their practice of doing modelling?* and *(2) How do prospective teachers' conceptions of modelling and teaching and learning modelling evolve throughout the implementation of a series of mathematical modelling lessons?* Data sources included posters, modelling reports, responses to a pre- and post-intervention questionnaire, and semi-structured interview dialogue. Data analyses included mixed methods using provisional coding, open coding, and a categorical rubric. Findings from this study indicate that (1)Prospective elementary teachers translated the modelling cycle into their practice by developing their range of modelling competencies including multiple components of the modelling cycle, and (2) They developed a professionally appropriate conception of mathematical modelling along with productive perceptions of the benefits of teaching and learning modelling.

ARTICLE HISTORY
Received 19 April 2021

KEYWORDS
Elementary teacher education; prospective teachers; mathematical modelling

2020 MATHEMATICS SUBJECT CLASSIFICATION
97M10

1. Introduction

The aim of this work is to examine the preparation of teachers for mathematical modelling and interdisciplinary projects. Mathematical modelling continues to be incorporated into curriculum internationally with occurrences in Australia, the United States, Portugal, Germany, Ireland, Switzerland, Japan, Singapore, South Korea, Brazil, South Africa, and more (Lew, 2004; Stohlmann & Albarracín, 2016). Various authors have conjectured that mathematical modelling can develop creativity (Chamberlin & Moon, 2005), algebraic thinking (Lew, 2004), and problem solving (Noble, 1982). Additionally, Suh et al. posit that mathematical modelling is an ambitious teaching practice which 'aspires to teach all students

CONTACT W. Tidwell w.tidwell@moreheadstate.edu Morehead State University, Morehead, 40351 KY, USA

academic content and have them apply their knowledge to solve authentic academic problems' (Suh et al., 2018, p. 1122). When mathematical modelling is treated appropriately, it satisfies all five equitable teaching practices defined by Aguirre et al. (2013). However, despite these benefits to student learning, mathematical modelling continues to be less prominent in the K-12 curriculum than other areas of mathematics. The Guidelines for the Assessment and Instruction of Mathematical Modelling Education (GAIMME) Report (Consortium for Mathematics & Its Applications & Society for Industrial & Applied Mathematics, 2019) recommends that mathematical modelling be taught at all levels, K-16. In order for this to happen, interventions need to take place that create a space for prospective teachers to experience mathematical modelling in teacher preparation. We argue that an understanding of prospective elementary teachers' competencies in modelling will provide valuable insight into effective ways of preparing teachers to teach mathematical modelling.

This article reports findings after implementation of a mathematical modelling intervention in a content course for prospective elementary teachers. As an exploratory study with future elementary teachers, two main research questions guided this study:

(1) How do prospective teachers translate the mathematical modelling cycle into their practice of doing modelling?
(2) How do prospective teachers' conceptions of modelling and teaching and learning modelling evolve throughout the implementation of a series of mathematical modelling lessons?

2. Theoretical considerations

We focus the theoretical considerations on the development of mathematical modelling competencies, teacher education in modelling, assessment of modelling with a focus on the modelling process, and formation of conceptions throughout an intervention process.

2.1. Mathematical modelling competencies

Research literature suggests multiple descriptions and definitions of mathematical modelling, and we adapted D'Ambrosio (2015) description as the process of dealing with an isolated individualized reality to cope with and explain selected facts and phenomena of a given situation in terms of formal mathematics. This description articulates mathematical modelling as a process used to address situations originating from contexts within or outside mathematics. In some cases, mathematical modelling is defined as addressing a real-world phenomenon (see Consortium for Mathematics & Its Applications & Society for Industrial & Applied Mathematics, 2019; Stohlmann & Albarracín, 2016); however, modelers are typically not constrained by the context and approach any scenario of interest with mathematical modelling, including fictional contexts such as what the Earth would be like with two moons (Comins, 2010), video games (Cox, 2015), a zombie apocalypse (Lewis & Powell, 2016), and hypothetical cases of disease transmission (Rönn et al., 2020) or tobacco products (Vugrin et al., 2015).

We adopt the viewpoint of mathematical modelling as a process for which competency must be developed. Blomhøj and Jensen (2003) define mathematical modelling competency as 'being able to autonomously and insightfully carry through all parts of a mathematical modelling process in a certain context' (Blomhøj & Jensen, 2003, p. 126) and Maaß writes that 'Modelling [sub-]competencies include skills and abilities to perform modelling processes appropriately and goal-oriented as well as the willingness to put these into action' (Maaß, 2006, p. 117). These definitions imply that modelling competency is tied to the specific description of the modelling process, and that mathematical modelling competency is characterized by readiness for action (Jensen, 2007).

2.2. Mathematical modelling in teacher education

Research studies about mathematical modelling in teacher education have focused on secondary teachers. Evidence suggests that when preservice secondary teachers receive instruction by engaging with mathematical modelling tasks, a positive evolution in their conceptions of mathematical modelling, performance (Anhalt & Cortez, 2016), and willingness to implement mathematical modelling (Stohlmann et al., 2015) ensues. This conception of mathematical modelling has been positively tied to a preservice secondary teachers' conception of effective mathematical modelling instruction (Son et al., 2017). Furthermore, advancement in mathematical modelling competencies has been tied to the perception of mathematical modelling (Son et al., 2017). The inclusion of mathematical modelling in preservice secondary teacher instruction has shown positive impacts on various aspects of the belief structure surrounding mathematical modelling in this population. However, in the case of prospective elementary teachers, understanding of the relationships within this belief structure is in its early stages (Stohlmann & Albarracín, 2016). When interventions are effected to prospective elementary teachers, performance in mathematical modelling tasks increases (Karaci Yasa & Karatas, 2018). The present study investigates prospective elementary teachers' development of modelling competencies in parallel to the development of conceptions of mathematical modelling.

2.3. Assessment in mathematical modelling

Research in the assessment of mathematical modelling in K-16 is less prominent than assessment in other areas of mathematics. This is possibly due to the nature of mathematical modelling, including its similarities to the scientific process, which requires the researcher to make observations about a problem situation, ask questions and do background research, form a hypothesis, conduct an experiment, analyze results and draw conclusions, and finally report the results and implications. And, modelling is similar to the writing process as described in the Common Core Writing standard (CCSS.ELA-Writing Standard Literacy.W.9–10.3), which calls to 'Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach, focusing on addressing what is most significant for a specific purpose and audience.' The modelling process involves understanding the problem situation, researching, reconsidering, formulating a model, interpreting the results, validating, revising, and refining, and consequently, recognizing that this iterative process is natural in modelling. It is not sufficient to assess solely the outcome or the model itself, and it is difficult to assess the process of modelling as is assessing engagement in the scientific method or the writing process.

Studies conducted with prospective teachers about their developed conceptions on assessment in mathematical modelling found that prospective teachers valued assessing the modelling process in addition to the models themselves, and yet recognized the difficulty in assessing the process (Anhalt & Cortez, 2016). Blomhøj and Jensen (2003) and Maaß (2006), independently define various competencies in mathematical modelling that come directly from the mathematical modelling process: understanding the problem situation, posing a simplified version of the situation, developing a model, computing a solution of the model, interpreting the solution and drawing conclusions, validating conclusions, and reporting out. In this research, we employ a holistic approach in assessing mathematical modelling competencies through a categorical rubric designed to provide feedback in the various competencies.

2.4. Conceptions about teaching

Regarding the formation of conceptions about teaching, we consider Thompson's (1992) notion of conception, which is 'a more general mental structure, encompassing beliefs, meanings, concepts, propositions, rules [and] mental images' (Thompson, 1992, p. 130) that is cognitive in nature (Da Ponte, 1994). This research aims to capture the developing and evolving conceptions of teaching mathematical modelling.

3. Methods

3.1. Description of the intervention

The intervention took place in a content course for future elementary education and special education teachers prior to entrance into a teacher education program. This course was chosen to implement this intervention for two reasons. First, the course already contains a module dedicated to problem solving, and second, it has a focus on algebraic thinking. Both of these elements have been linked to mathematical modelling (Lew, 2004; Noble, 1982). The intervention itself consisted of three multiday lessons described in Figures 1–3, and took place over nine class sessions. The first lesson prompted prospective teachers to investigate which conserved more water, a shower or a bath (for more information on this task see Felton et al., 2015). This first lesson lasted four days at the beginning of the semester and included student posters and reflections about this task. The second and third lesson took place at the end of the semester. The second lesson had a focus on the mathematical modelling process using the reflections from the first lesson. This lesson's primary purpose was to provide an opportunity to the prospective teachers to reflect on their experiences in mathematical modelling (for more information on this lesson see Anhalt et al., 2020). The third lesson prompted prospective teachers to investigate the Flint, Michigan water crisis (for more information on this task see Aguirre et al., 2019). As part of this task, prospective

> Some water conservationists say that showering uses less water than bathing. Others say this is not true! Keep in mind that older shower heads have a flow rate of up to 3.4 gallons/minute while energy-saving shower heads have a flow rate as low as 1.9 gallons/minute. Bathtubs also vary in size. **Provide a method to determine if a shower or a bath uses more water, and explain your approach.**

Figure 1. Lesson 1 was conducted over four class sessions at the beginning of the term. The modelling task is shown above.

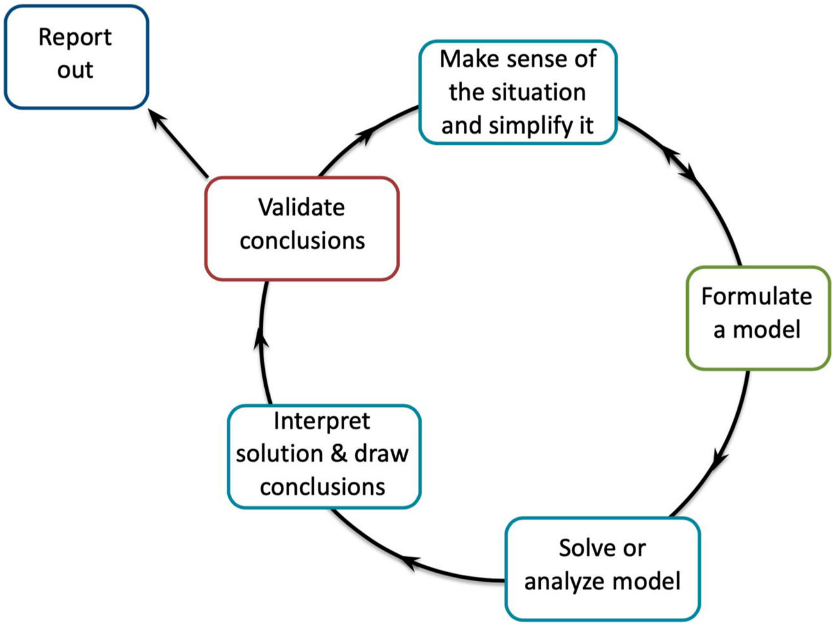

Figure 2. Lesson 2 was conducted over two class sessions at the end of the term, prior to Lesson 3. Students reflected on their process when they engaged in the modelling task of Lesson 1, and sorted components of their process in accordance with the modelling cycle shown.

Walmart, Coca Cola, Nestle, PepsiCo said that they will donate bottles of water for school children in Flint, Michigan, to help with the city's public health crisis over lead contaminated water. On January 26 the companies said that they are planning to collectively donate water to meet the daily needs of over 10,000 school children for the balance of the calendar year" (Bever, 2016, p. 1). To do so, the companies will send 176 truckloads of bottled water (up to 6.5 million bottles) to Flint.
How do we know how much water will be enough to meet the daily needs of Flint school children until December 31, 2016? Is the companies' plan a good one?

Figure 3. Lesson 3 was the final modelling task in this intervention and was conducted at the end of the term over three class sessions. The modelling task shown is based on a newspaper article from 2016.

teachers wrote individual reflections on their experience and collaborated to create group reports and presentations.

Originally, the planned intervention involved more modelling tasks. The first lesson had been slated to take two class periods, as it does when implemented with preservice or in-service secondary teachers. However, in the course for future elementary teachers, this lesson took twice as long. In addition, the COVID pandemic forced the course to move online. When the instructors conducting this intervention relayed this to the researchers, the rest of the intervention was modified to accommodate changes to the course outline and the shorter time available. These changes allowed prospective teachers more time to grapple with the mathematics and the context of the task within the presented learning environment.

3.2. Participants

This intervention took place at a predominantly white, large public university in the Intermountain West of the United States. In the beginning of the semester when this study took place, the course was taught completely in-person with no online component; however, exactly halfway through the intervention, the course went to an online format. The study consisted of 38 prospective elementary and special education teachers. The participants in the study came from two classes with different instructors, one with the 30 students that in the second half of the intervention went to asynchronous learning (we will call this group AG), and the other instructor with 8 students that went to synchronous online learning (we will call this group SG).

3.3. Data sources

Our main data sources were: (1) pre- and post-intervention questionnaire survey responses; (2) reports of two kinds, one of which was group posters and written reflections on the water conservation modelling task from the first lesson, and the other, structured group reports and reflections on the Flint Water Crisis task from the third lesson; and (3) individual semi-structured interviews with three of the prospective teachers.

Questionnaire. The pre- and post-questionnaire with five open-ended questions was administered at the beginning and end of the semester. The questionnaire was adapted from a survey instrument implemented by Anhalt and Cortez (2016). All 38 participants responded to the pre-questionnaire, and 11 of these 38 students – 8 from the synchronous group (SG) and 3 from the asynchronous group (AG) – responded to the post questionnaire. The questionnaire for this study can be found in Appendix 1.

Reports. Nine mathematical modelling collaborative group reports and 33 individual reflections were collected from the first lesson on the water conservation task. Additionally, group presentations were conducted to showcase the models created on posters. At the end of the third lesson, nine written collaborative group reports and 37 individual reflections were collected from the Flint water crisis task.

Data also included the observation notes from the researcher during the intervention. These field notes helped triangulate the analysis of the reports by providing context and insight into student thinking.

Interviews. The semi-structured interviews were conducted via online conference calls with a subset of participants approximately four months after the end of the intervention. The script for the semi-structured interviews can be found in Appendix 2. The participants of these interviews (2 SG and 1 AG) were selected from the group of 11 participants who completed both the pre- and post-questionnaires. The interviews followed a similar prompt structure to the questionnaires with additional prompts to discuss potential advantages, limitations, and disadvantages of mathematical modelling curriculum.

3.4. Data analysis

Questionnaire Responses and Interviews. The pre- and post-questionnaire response data are associated with research question 2 regarding how these elementary prospective teachers translate the mathematical modelling process into their modelling work.

Prompts 1, 3, and 4 on the questionnaire and interviews were open coded with emergent themes. The reflection, and prompt 2 and 5 on the questionnaire were provisionally coded. Prompt 2 on the questionnaire was coded on a 4-tiered Likert scale with an additional code, 'not applicable.' Prompt 5 initially used codes for *proficiency*, which are from the five strands described in *Adding It Up: Helping Children Learn Mathematics* (National Research Council, 2001) and *process* as the five processes described in the *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics, 2000). The data from prompt 5 were recoded using the eight Standards for Mathematical Practice (SMP) which come from the Common Core State Standards for Mathematics (CCSSM) (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). The three interviews conducted online via individual conference calls served as triangulation with the data from the pre- and post-questionnaires for particular associated content since the three participants were a subset of the participants who responded to both the pre- and the post-questionnaire prompts. Interview data are associated with research question 2.

Reports. The reports were structured following guidelines provided as part of the third lesson. These guidelines outlined focus areas of the report, including, the statement of the problem, research conducted on the topic, assumptions with justification, the model, related computations and solution, interpretations of the results, conclusions, improvements made to the model and recommendations based on the outcome. Student posters and reports were scored using a rubric with categories connected to the elements of the mathematical modelling process, and for assessing mathematical modelling reports discussed in Tidwell et al. (under review). These reports and posters were scored independently by two researchers according to that rubric (see Appendix 3), then the scores were discussed and the few score inconsistencies were reconciled. This rubric allows for the assessment of all seven competencies discussed in Section 2.1, Mathematical Modelling Competencies, and therefore, the data are associated with research question 1.

4. Results

This section provides a summary of the data analyses addressing the research questions stated in the introduction.

(1) How do prospective teachers translate the mathematical modelling cycle into their practice of doing modelling?
(2) How do prospective teachers' conceptions of modelling and teaching and learning modelling evolve throughout the implementation of a series of mathematical modelling lessons?

Section 4.1 addresses research question 1, and the results pertaining to research question 2 are summarized by theme in Sections 4.2 to 4.6.

4.1. Prospective teachers' mathematical modelling competencies

Prospective teachers' reports created after engaging in the two lessons with modelling tasks revealed their mathematical modelling competencies. The first modelling task, Shower versus Bath, was addressed in lesson 1. The second task was the Flint Water Task, addressed in lesson 3 of the intervention. Students created posters for the first task and written and oral reports for the second. These reports were scored using the rubric Appendix 3 that has descriptors of each of seven mathematical modelling components aligned with the modelling cycle addressed in Lesson 2 of the intervention: *Understand the problem*, *Pose a simplified version, Develop a model, Compute a solution, Interpret the solution, Validate the conclusions*, and *Report*. The rubric describes four proficiency levels with a maximum score of four for each component.

The left side of Figure 4 shows the box and whisker plot of the total scores for the reports out of a maximum of 28 points. The right side of Figure 4 shows the average score of each competency for the first and second reports. This level of detail provides a picture of the relative gains by students in each competency as a result of this intervention. The average scores show gains in every category except the *Develop a model* category, where there is a slight loss in the average proficiency score.

The competencies that show the most significant gains between the first and second reports include understanding, validate, and report. The posters created by the prospective teachers as the report for lesson one show that the communication of their choices, assumptions, and conclusions tended to be too brief to include complete explanations of the work. Four out of nine posters used assumptions implicitly in the models without explicitly listing the assumptions. In cases when the assumptions were listed, the justification was not provided. In six of the nine posters, the prospective teachers determined the water usage based on more than one shower flow rate and more than one tub size. Those reports contained several results based on different scenarios that can be the basis of a rich discussion. At this early stage in the prospective teachers' exposure to mathematical modelling, the posters did not provide enough evidence of certain competencies to receive a high rubric score.

The nine written reports that followed the Flint Water Crisis task included more details and justifications. The reports identified the context, objective of the solution, and included researched information to explain assumptions and choices made. For example, in one report the authors provided additional context. 'We researched the timeline of the Flint Water crisis, both from the perspective of Flint citizens and those whose authority led to the

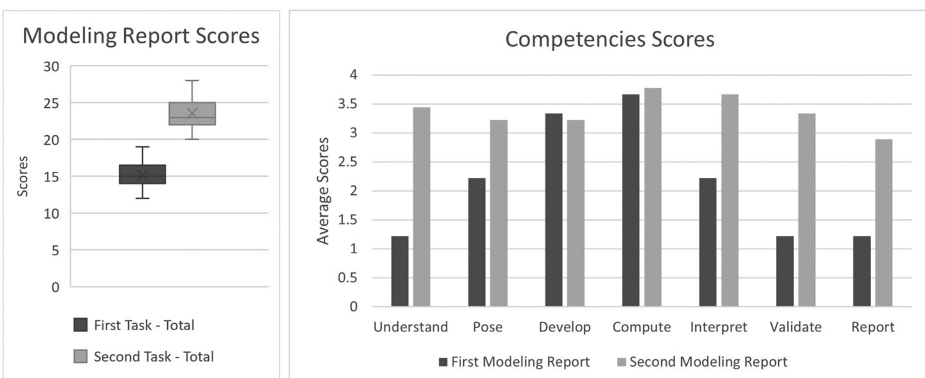

Figure 4. On the left, a box and whisker plot detailing the total scores on the first and second modelling reports and on the right, the averages of the modelling competencies for the first and second reports.

problem.' The reports were focused on evaluating the companies' plan for action. Another report stated, 'We chose to disregard other aspects that may make their plan "good" such as saving time and money and simply focus on whether the students would be receiving an adequate amount of water.' All nine reports included a justification for the daily water needed per child.

The reports included clear interpretation (contextualizing) and validation of the results often addressed in parallel. Four of the nine reports explicitly discussed strengths and limitations of their models. According to one report, 'In this model, we only represent one variable which is usually the most we want to consider at one time… [it should be] part of a larger group of models that represent multiple variables.'

This concludes our results referring to the first research question. The questionnaires and interviews were used as our data sources for the second research question. Responses to all five questionnaire prompts are summarized in Table 1. The coded responses of the 11 students who completed both questionnaires are listed in the middle and right columns, while the coded responses of all 38 students who completed the pre-intervention questionnaire are in the middle column. The next few sections describe how the questionnaires were coded and summarize the relevant interview responses pertaining to each of the themes below.

4.2. Prospective teachers' conceptions of mathematical modelling

Our analysis of prospective teachers' conceptions of mathematical modelling came primarily from responses to Prompt 1 of the questionnaire, *Explain what 'model with mathematics' means to you*, and Prompt 2 of the interview, *Could you describe the difference between modelling with mathematics and using manipulatives as models for mathematics?*

Responses to the first prompt of the questionnaire were classified using seven codes. The code *setting an example* was used when responses indicated that teachers or students modelaled some desirable behavior or practice as a role model for other students to follow. The *demonstrating or justifying* code referred to responses that claimed modelling was demonstrating a procedure or justifying work in a mathematical process. *Manipulatives or visuals* referred to responses claiming that model with mathematics had something to do with creating a visual or physical representation of a mathematical problem. One response was labeled *finding multiple solutions* since the response referenced open-middle problems where more than one solution path exists to obtain an answer. The difference between the *application* and *mathematical modelling* codes was taken from Stillman et al. (2007), who claim *application* is finding mathematics a home in reality, while *mathematical modelling* is finding the mathematics in reality. Responses that indicated a connection to problem solving, with a possible mention of one or more of the components of the modelling cycle (Figure 2) were coded as *mathematical modelling*. Notably, all post-test respondents showed this understanding of mathematical modelling.

In response to the second interview prompt, the students indicated that modelling with mathematics is different from using manipulatives as models. Student 1 said

> The process of modelling is: identifying the different pieces, gaining more understanding of these pieces, and then connecting them and maybe afterwards different things like other disciplines. While these three steps always show up in our process, we can go back and forth and do certain pieces over again or share this work with others.

Table 1. Summary of the responses of 38 students to the five prompts in the pre-intervention questionnaire (middle column) and the responses of 11 students who completed both the pre- and post-intervention questionnaire (middle and right columns). The codes for each prompt are listed in the left column. The asterisks indicate three codes that made up a disjoint partition of all responses.

Code	Pre-Intervention ($n = 38/n = 11$)	Post-Intervention ($n = 11$)
Prompt 1: Explain what 'model with mathematics' means to you.		
I don't know	15.8% / 45.5%	0%
Finding Multiple Solutions	2.6% / 0%	0%
Setting an Example	5.2% / 18.2%	0%
Demonstrating or Justifying	39.4% / 36.4%	0%
Manipulatives or Visuals	23.7% / 27.3%	0%
Application	31.6% / 27.3%	0%
Mathematical Modelling	10.5% / 18.2%	100%
Prompt 2: Are modelling with mathematics and solving word problems related?		
Same Thing	2.60% / 0%	0%
Related but different	84.2% / 81.8%	100%
Sometimes Related	5.2% / 9.1%	0%
Not Related	5.2% / 0%	0%
Not Applicable	2.6% / 9.1%	0%
Prompt 3: How can teachers prepare to teaching modelling?		
Practice	31.6% / 27.2%	45.4%*
Fully Understand	50.0% / 54.5 %	18.2%*
Creating/Finding Problems	15.8% / 27.3 %	36.4%*
Understand Social Setting	10.5% / 9.1 %	0%
Understand Learning Trajectories	10.5% / 9.1%	18.2%
Implementing	5.3% / 0%	0%
Understand Resources	0.0% / 0%	9.1%
Prompt 4: What role do 'real-life' contexts play in modelling problems?		
Demonstrate the Application	39.5% / 45.5%	36.3%
Tangible/Relatable	31.6% / 36.4%	54.5%
Provides purpose/exigency	39.5% / 36.4 %	45.5%
Increases Understanding	21.0% / 9.1%	0%
Prompt 5: What mathematical practices could children learn by engaging in mathematical modelling activities?		
Mathematical Process	**47.4% / 45.5%**	**54.5%**
Problem Solving	21.1% / 18.2%	36.4%
Connection	18.4% / 18.2%	18.2%
Representation	5.2% / 9.1%	0%
Communication	2.6% / 0%	0%
Reasoning & Proof	0% / 0%	18.2%
Mathematical Proficiency	**18.4% / 27.3 %**	**36.4%**
Strategic Competence	10.5% / 18.2 %	9.1%
Adaptive Reasoning	2.6% / 0%	9.1%
Productive Disposition	5.2% / 9.1%	18.2%
Mathematical Practice	**2.6% / 0%**	**0%**
MP1 Make sense of problems	2.6% / 0%	0%
MP2 Reason abstractly and quantitatively	2.6% / 0%	0%
MP3 Construct viable arguments	2.6% / 0%	0%
MP5 Use appropriate tools strategically	2.6% / 0%	0%
Mathematical Content	**44.7% / 45.5%**	**36.3%**

In this quote, the student discussed elements in the modelling process. This student described 'the pieces' in relation to pieces of information needed to solve the problem or gain a better understanding of the situation. In the second sentence of the quote, the student points out the cyclical nature of this process as well as the potential for reporting out the solution. Not quoted here, Student 1 claimed that manipulatives are used for the visualization of concepts, i.e. using pieces of pie to demonstrate fractions.

4.3. Prospective teachers' conception of preparation to teach mathematical modelling

In order to address prospective teachers' conception of preparation, data were analyzed from questionnaire prompt 3, *How can teachers prepare to teach modelling at the elementary school and middle school levels*. The majority of the student responses in the questionnaire were coded as *fully understood*. This code is for responses indicating that teachers can prepare to teach modelling by fully understanding key ideas involved in mathematical modelling without providing a method of attaining understanding. Responses indicating that practicing mathematical modelling was needed were coded as *practice*. The *implementing* code was used for responses stating that teachers need to prepare by implementing lessons in a classroom setting. The code *understand social setting* refers to when participants stated that they would need to know more about the students' interests and what tasks would be relevant to them. If a participant discussed requiring prior knowledge of the student relative to the curriculum, this was coded as *understand learning trajectories*. The *understand resources* code was only found once in a response that indicated the need to know all the technology and information available to the teacher in order to implement the modelling task.

4.4. Prospective teachers' perceptions of mathematical modelling in communication, language, and reasoning

Interview prompt 3 was, *Do you feel like your language, reasoning, or communication of mathematics has evolved by engaging in mathematical modelling activities over the semester?* All participants interviewed stated that in some capacity their communication, language and reasoning were improved. Student 2 said,

> Mathematical modelling helped connect the concepts and terms in applications. This elucidated how the terms and concepts affect each other and gave me a deeper understanding of these concepts. I feel that now I can better explain mathematics to other people.

Student 2 indicated that doing modelling tasks benefitted their communication and language just by going through the modelling process and went on to say that the act of modelling was beneficial for improving their understanding and relationships among mathematical concepts. Student 3 said,

> The mathematical modelling activities helped me view mathematical-based issues with a more open-minded approach since it's looking more at the approach to solving than the solution itself. Before, I was focused on trying to get one right answer instead of in a way that there may be multiple answers depending on the approach taken. This is beneficial because real-world answers do not have just one answer. It has … helped improve my communication since in the final project we had to present our results in a formal fashion.

Student 3 indicated that doing modelling tasks benefited their reasoning by helping them turn their focus away from the final answer to appropriate approaches. Student 3 also said that reporting out solutions was beneficial for improving communication and language.

4.5. Prospective teachers' perceived benefits of modelling for elementary and middle schoolers

Questionnaire prompt 5 asked, *What mathematical practices children could learn by engaging in mathematical modelling activities*. The responses were coded as *mathematical process* (National Council of Teachers of Mathematics, 2000), *mathematical proficiency* (National Research Council, 2001), *mathematical practice* (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) or *content*. The *PSSM* and *Adding it Up* documents were used to create provisional codes for the mathematical process and mathematical proficiency. Each of these documents detailed five sub-codes for both *mathematical process* and *mathematical proficiency*. The sub-codes for *mathematical process* were *problem solving, connection, representation, communication,* and *reasoning and proof*. The sub-codes for mathematical proficiency were *strategic competence, adaptive reasoning* and *productive disposition*.

Just one response coded *mathematical practice* on the pre-intervention questionnaire listed four of the standards for mathematical practice verbatim: *(MP1) make sense of problems and persevere in solving them*, *(MP2) reason abstractly and quantitatively*, *(MP3) construct viable arguments and critique the reasoning of others*, and *(MP5) use appropriate tools strategically*. Students in the course were likely not familiar with these standards. A subset of the responses were additionally coded with the Standards of Mathematical Practice (SMP), but this coding is not shown in Table 1. The responses in the post-intervention questionnaire that indicated some *mathematical process* or *mathematical proficiency* (n=9) were re-coded with provisional codes according to the standards of mathematical practice. After this re-coding, 5 responses were coded *(MP1) make sense of problems and persevere in solving them*, 2 responses were coded with *(MP3) construct viable arguments and critique the reasoning of others*, and 3 responses were coded *(MP4) model with mathematics*.

Interview prompt 4 asked, *Do you think mathematical modelling has a place in the curriculum of elementary and middle school?* All participants interviewed said that mathematical modelling has a place in the curriculum of elementary and middle school. Student 1 said,

> Upper grades tend to think a previous concept doesn't make sense but [this] doesn't matter … Mathematical modeling helps make sense of these topics so that concepts at lower levels make sense and show applicability prior to students getting to those upper grades … Mathematical modeling shouldn't just be used for one age group. It should be used in all age groups since it is something that will help students later in life.

Student 2 said that modelling allows elementary and middle school students to grapple with the real world and modelling provides a context for number sense. Student 2 said, '[Mathematical modeling] provides an opportunity to students to apply math to the real world in a tangible way. It's not just numbers on a page. This allows for students to try and deal with the real world.' At the end of the interview Student 3 went on to say,

> [Mathematical modeling] helps with the critical thinking in showing that problem solving is not always a super linear process … [It allows for one to] present or report your findings at the end, and I feel that it would be good for the students to communicate their mathematics.

Student 3 seems to have the perception that prospective teachers and their future students would benefit from doing modelling tasks for improving communication.

4.6. Prospective teachers' additional thoughts about modelling

Questionnaire prompt 2 was *Are modelling with mathematics and solving word problems related?* The codes are *same thing, related but different, sometimes related, not related*, and *not applicable*. The *not applicable* code was added as one participant responded to the wrong question due to a misprint. The other codes were seen as provisional codes similar to a Likert scale.

Questionnaire prompt 4 was *What role do 'real-life' contexts play in modelling problems?* The *demonstrate the application* code indicates the response saying that the real-life context provided an avenue to demonstrate the application of the mathematical concept used to solve the problem. The *tangible/relatable* code discussed providing a context to a mathematical concept to take away some of the abstractness. And lastly, the *provides purpose or exigency* code indicated that the real-life context provided a motivation to propel the learning of the concept.

Interview prompt 1 was *How do you feel your understanding of mathematical modelling changed over the semester, if it has changed?* All participants indicated that they felt that the intervention benefited their understanding of mathematical modelling. Student 1 said, "[I]t was beneficial to be able to have the hands-on experience where we got to actively use it and learn it." Student 2 said, 'We had the same information and different groups arrived at different yet similar answers to the problem. It was nice to be able to compare the responses.' Student 2 later commented that in the second task 'there was some difficulty … due to [the online learning format] in the comparison of response from groups.' Student 3 said, 'The projects helped show that [in mathematical modeling] there are multiple solutions that are dependent on your assumptions.'

5. Discussion

5.1. Growth of modelling competencies

The scores on the modelling reports, seen in Figure 4, show an improvement in the overall performance of mathematical modelling by these future elementary teachers. The data show that after this intervention the total modelling competencies improved, and specifically, *understand the problem, pose a simplified situation, interpret the solution and draw conclusions, validate the conclusions*, and *report the solution* improved over the course of this intervention. This finding is not surprising as the mathematical modelling tasks were chosen specifically because of their multiple mathematical entry points which allowed for students with varied mathematical backgrounds to participate. The intervention was beneficial in improving those competencies other than *computing a solution* and *developing a model*. The optimal mathematical level of tasks for future elementary teachers to maximize the growth of these modelling competencies requires further investigation, but this intervention yielded a positive growth. This finding supports the claim made by Karaci Yasa and Karatas (2018) that this type of intervention has positive effects on future elementary teachers mathematical modelling performance. The data from the reports and questionnaire prompt 1 suggest that there is a positive correlation between a future prospective teachers conception of mathematical modelling and mathematical modelling performance.

5.2. Evolution of mathematical modelling conception

The responses to prompt 1 on the pre-intervention questionnaire revealed initial conceptions of elementary teachers about modelling. This finding adds to the research about mathematical modelling views (Karali & Durmuş, 2015) and performance on modelling tasks (Bal & Doğanay, 2014; Karaci Yasa & Karatas, 2018). The literature and this data would suggest that if prospective elementary teachers do not have an exposure to mathematical modelling, their conceptions of mathematical modelling might be comparable to that of in-service elementary teachers.

The data from the pre- and post-intervention questionnaires on prompt 1 suggest that this intervention is successful in helping these prospective teachers have an appropriate conception of mathematical modelling. Appropriate conceptions of mathematical modelling are consistent with professional insight from various studies and resources. This finding supports that this kind of intervention is beneficial to future teachers of mathematics in progressing their conception of mathematical modelling (see for corroborating evidence Anhalt & Cortez, 2016). The interviews suggest that this specific intervention had long-lasting effects on the students' conception of mathematical modelling with interviewees showing a correct conception of mathematical modelling four months after the intervention. Not only did this intervention benefit the conception of mathematical modelling in this sample, but the evidence shows that the students were able to distinguish model with mathematics from using manipulatives as models, as the Standards for Preparing Teachers of Mathematics includes that '[Early childhood teachers of mathematics] understand that using manipulatives is not the same as mathematical modeling' (Association of Mathematics Teacher Educators, 2017, p. 49).

5.3. Prospective teachers' conception of preparation to teach mathematical modelling

All responses to prompt 3 of the post-intervention questionnaire were coded as mathematical modelling conception. This code means the primary ways prospective elementary teachers identify that elementary and middle school teachers should prepare to teach mathematical modelling were by (1) creating and finding mathematical modelling problems, (2) fully understanding the modelling process and associated competencies, and (3) practicing doing mathematical modelling tasks. These three ways were found in each of the responses. The data from the reports and prompt 1 on the questionnaires indicate that the intervention prepares students on key elements of mathematical modelling, so we can assert that a full understanding of the modelling process may be satisfied, at least partially, with practicing doing mathematical modelling tasks. Stohlmann et al. (2017) posits that in order to create mathematical modelling tasks, one must first have practice in doing mathematical modelling. In order to find appropriate mathematical modelling tasks, research suggests that teachers need to be proficient in mathematical modelling as typical resources provided to teachers do not address mathematical modelling, even when intended to (e.g. for textbooks see Son & Jung, 2018 and for online resources see Stohlmann & Yang, 2020). The literature and data discussed suggest that to prepare prospective elementary teachers to teach mathematical modelling, a positive intervention is to have this population practice doing mathematical modelling activities. This finding supports the belief that to prepare all teachers of mathematics, it is beneficial to use an intervention like the one described in this paper.

Viewing results through the lens of Ball et al. (2008), all of the respondents of the post-intervention questionnaire indicated that they needed to have some amount of subject matter knowledge to prepare to teach mathematical modelling to elementary and middle school students. Six out of the 11 of these students indicated that some pedagogical content knowledge is needed as well. This provides a perspective on mathematical knowledge for teaching mathematical modelling.

5.4. Prospective teachers' perceptions of mathematical modelling in communication, language, and reasoning

The interview data showed that prospective teachers believe that their own communication, language, and reasoning improve through practicing mathematical modelling. This supports Yu & Chang's (2011) claim that when creating model-eliciting activities prospective teachers improve their communication, language, and reasoning, and Stohlmann et al.'s (2017) view that one must practice mathematical modelling activities in the process of learning how to create model-eliciting activities.

The data from the interviews support the claim that prospective teachers believe that mathematical modelling has the ability to help deepen understanding and connections of concepts. Student 2 said

> Mathematical modeling helped connect the concepts and terms in applications. This elucidated how the terms and concepts affect each other and gave me a deeper understanding of these concepts.

From this perception, students may come to realize that mathematical modelling is beneficial to better understand mathematical concepts. This is supported by the literature. Middleton et al. (2011) found that the implementation of discrete mathematics-based modelling tasks significantly improved teachers' content knowledge related to discrete mathematics. The data from the interviews suggest that this type of intervention may deepen understanding and connections of previously learned mathematical concepts and improve communication, language, and reasoning.

5.5. Prospective teachers' perceived benefits of modelling for elementary and middle schoolers

The data from prompt 5 on the post-intervention questionnaire show that these prospective teachers felt that elementary and middle school children can get better at problem solving and develop a productive disposition with mathematical modelling tasks. One prospective teacher said,

> They would have the opportunity to learn about failing and trying something new. Most of the time the first thing you try doesn't work out and it's important to understand that that's okay and you can still try different options until you find the right solution.

These data, viewed with the lens of the Standards of Mathematical Practice (SMP) (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), indicate that prospective teachers think that elementary and middle schoolers could learn mathematical practices (MP) 1, 3, and 4 by engaging in mathematical modelling activities. This finding is juxtaposed with research that claims preservice secondary teachers believe that modelling presents anywhere from four of the standards of mathematical practice (Anhalt & Cortez, 2016) to all eight (Stohlmann et al., 2015). The three SMPs perceived by prospective elementary teachers partially align with the SMPs found in Anhalt and Cortez (2016). In that study, MP 3 was not observed in the study of preservice secondary teachers, conversely SMPs 2, reason abstractly and quantitatively, and 6, attend to precision, were not perceived benefits of the students in this study. However, anecdotally, when the researchers piloted this intervention, prospective teachers perceived that all eight SMP standards would be potential benefits of engaging in mathematical modelling activities. The researchers hypothesize that this result was not seen in the data due to the prospective teachers lack of awareness of the SMPs. This hypothesis is grounded in the number of responses that claimed a mathematical concept was a mathematical practice (44.7% pre intervention and 36.3% post intervention).

In addition to elementary and middle schoolers' perceived benefits of mathematical practices, Student 1, in the interviews stated that '[M]athematical modeling helps make sense of these topics so that concepts at lower levels make sense and show applicability prior to students getting to those upper grades.' This emphasizes the finding that this intervention can effect student perceptions towards mathematical modelling being able to deepen the understanding of a concept or maybe even make the concept tangible so help students make sense of it. Student 2 discusses that modelling allows for elementary and middle school students to grapple with the real-world and that modelling provides a context to numbers to make them real. The grappling with the real-world allows prospective teachers to see mathematics as a tool 'to examine social and personal issues that arise throughout their lives' so that they may better 'communicate and model the power and utility of mathematics' to their students (Association of Mathematics Teacher Educators, 2017, p. 107).

5.6. Prospective teachers' additional thoughts on mathematical modelling

The responses to some of the questionnaire prompts revealed some additional thoughts prospective elementary teachers have about mathematical modelling and school mathematics. The data from prompt 2 of the questionnaires suggest the conception of mathematical modelling is correlated with what prospective elementary teachers think about the relationship of mathematical modelling and solving word problems. At the end of the intervention students felt that mathematical modelling and solving word problems are related but different. On the post-intervention questionnaire, one student said,

> Modeling with mathematics is more of us finding the word problem itself. It is us trying to discover what the problem is in the world and then finding a solution. Whereas solving word problems, ... [the word problems] are already written out for us, we don't have to do all the work to find the problem.

Data from prompt 4 of the questionnaire show evidence that students make progress toward an appropriate conception of mathematical modelling. They expressed that a real-world context provided an avenue to make mathematical concepts more tangible or relatable. One student said,

> I feel that real-life problems are what would make the most sense to most kids. When they see the problems in real-life can be solved with math it will make it not only make more sense, but help them find a purpose for math in their own lives.

This poses possible questions for further research. Are real-world contexts necessary for modelling prompts? Would more contextual diversity in mathematical modelling improve the conceptualization?

5.7. Online learning of mathematical modelling

This study shows a desired evolution of conceptions can be accomplished in an online environment. Of the 11 post-questionnaire respondents, all of whom completed the second half of the modelling activities online, 8 were in the synchronous group and 3 were in the asynchronous group. The data suggest that even in this online environment the students were able to get a correct conception of mathematical modelling and retain that conception months later. The data also suggest that this learning environment may not have hindered the progression of students mathematical modelling competencies; however, it is not possible to know how their competencies would have progressed if the entire intervention had been done in person. Despite the fact that the online learning of mathematical modelling was not in the study design, the results of this intervention are positive regardless of learning format and do not depend on face-to-face interactions. It is, however, acknowledged that the experience is different from face-to-face experiences with mathematical modelling and may provide difficulties, like noted by Student 2 in the interviews. This student noted difficulties in discussing the mathematical modelling reports and different solutions during and after the modelling task.

6. Conclusion

The sum of data collected reveals the evolution of conceptions of mathematical modelling and the improvement of mathematical modelling competencies. It extends the body of literature to suggest that a module of mathematical modelling activities positively affects teacher preparation (see Anhalt et al., 2018, for evidence for preservice secondary teachers). This intervention is beneficial for productively progressing prospective elementary teachers' conceptions and perceptions of mathematical modelling. When looked at with the lens of Ball et al. (2008), this intervention targets and effectively progresses subject matter knowledge surrounding mathematical modelling. It also can provide prospective elementary teachers with an experience of learning modelling in a productive way. Loucks-Horsley et al. (2010) suggest that one teaches the way one learned, so this intervention may influence the development relevant pedagogical content knowledge. The literature and this study suggest that this kind of intervention is necessary, but insufficient in the preparation of teachers of mathematics to teach mathematical modelling. In order for teachers of mathematics to gain the necessary mathematical knowledge for teaching modelling, some intervention targeting the pedagogical content knowledge is necessary. Interventions successful in progressing relevant pedagogical content knowledge are limited and focus on in-service teachers (see Doerr & English, 2006; Stohlmann et al., 2017; Suh et al., 2018). It seems beneficial to look into ways of progressing pedagogical content knowledge of mathematical modelling with prospective elementary teachers. If an effective intervention can be found and combined with this one, it stands to reason that prospective elementary teachers would have the mathematical knowledge for teaching and the willingness to implement mathematical modelling in their classrooms.

Disclosure statement

No potential conflict of interest was reported by the authors.

Funding

This work is a product of the Mathematics of Doing, Understanding, Learning, and Educating for Secondary Schools (MODULE(S2)) project, supported by the National Science Foundation (NFS) under Grant No. 1726707 (Association of Public and Land-grant Universities) 1726804 (Utah State University) and 1726098 (University of Arizona).

Data availability statement

The data that support the findings of this study are available on request from the corresponding author, WT. The data are not publicly available due to their containing information that could compromise the privacy of research participants.

References

Aguirre, J. M., Anhalt, C. O., Cortez, R., Turner, E. E., & Simic-Muller, K. (2019). Engaging teachers in the powerful combination of mathematical modeling and social justice: The flint water task. *Mathematics Teacher Educator*, *7*(2), 7–26. https://doi.org/10.5951/mathteaceduc.7.2.0007

Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013). *The impact of identity in K-8 mathematics: Rethinking equity-based practices*. National Council of Teachers of Mathematics.

Anhalt, C. O., & Cortez, R. (2016). Developing understanding of mathematical modeling in secondary teacher preparation. *Journal of Mathematics Teacher Education*, *19*(6), 523–545. https://doi.org/10.1007/s10857-015-9309-8

Anhalt, C. O., Cortez, R., & Bean Bennett, A. (2018). The emergence of mathematical modeling competencies: An investigation of prospective secondary mathematics teachers. *Mathematical Thinking and Learning*, *20*(3), 202–221. https://doi.org/10.1080/10986065.2018.1474532

Anhalt, C. O., Cortez, R., & Kohler, B. (2020). MODULE(S2): Mathematical modeling for secondary teaching. *MODULE(S2)*, *2019*. (NSF funded curriculum materials piloters version available by request).

Association of Mathematics Teacher Educators (2017). *Standards for preparing teachers of mathematics*. Association of Mathematics Teachers Educators. Available online at https://amte.net/standards.

Bal, A. P., & Doğanay, A. (2014). Improving primary school prospective teachers' understanding of the mathematics modeling process. *Educational Sciences: Theory and Practice*, *14*(4), 1375–1384. https://doi.org/10.12738/estp.2014.4.2042

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching what makes it special?. *Journal of Teacher Education*, *59*(5), 389–407. https://doi.org/10.1177/0022487108324554

Blomhøj, M., & Jensen, T. H. (2003). Developing mathematical modelling competence: Conceptual clarification and educational planning. *Teaching Mathematics and Its Applications*, *22*(3), 123–139. https://doi.org/10.1093/teamat/22.3.123

Chamberlin, S. A., & Moon, S. M. (2005). Model-eliciting activities as a tool to develop and identify creatively gifted mathematicians. *The Journal of Secondary Gifted Education*, *XVII*(1), 37–47. https://doi.org/10.4219/jsge-2005-393

Comins, N. F. (2010). *What if the earth had two moons?: And nine other thought-provoking speculations on the solar system*. St. Martin's Press.

Cox, R. T. (2015). *Mathematical modeling of Minecraft: Using mathematics to model the gameplay of video games* [M.S. thesis], The Ohio State University.

Consortium for Mathematics and Its Applications and Society for Industrial and Applied Mathematics (2019). *Guidelines for assessment and instruction in mathematical modeling education*. Consortium for Mathematics and Its Applications and Society for Industrial and Applied Mathematics.

Da Ponte, J. P. (1994). Mathematics teachers' professional knowledge. In J. P. Da Ponte and J. F. Matos (Eds.), *Proceedings of the Eighteenth International Conference for the Psychology of Mathematics Education* (pp. 194–210). University of Lisbon.

D'Ambrosio, U. (2015). Mathematical modelling as a strategy for building-up systems of knowledge in different cultural environments In G. A. Stillman, W. Blum, and M. S. Biembengut (Eds.), *Mathematical modelling in education research and practice* (pp. 385–395). Springer. https://doi.org/10.1007/978-3-319-18272-8_2.

Doerr, H. M., & English, L. D. (2006). Middle grade teachers' learning through students' engagement with modeling tasks. *Journal of Mathematics Teacher Education*, *9*(1), 5–32. http://doi.org/10.1007/s10857-006-9004-x

Felton, M. D., Anhalt, C. O., & Cortez, R. (2015). Going with the flow: Challenging students to make assumptions. *Mathematics Teaching in the Middle School*, *20*(6), 342–349. https://doi.org/10.5951/mathteacmiddscho.20.6.0342

Son, J. I. J., & Jung, H. (2018). Do social justice contexts matter in mathematical modeling: Modeling problem analysis. *New England Mathematics Journal*, *Li*(1), 0–10. https://lib.dr.iastate.edu/edu_pubs/96/

Jensen, T. H. (2007). Assessing mathematical modelling competency. In C. Haines, P. Galbraith, W. Blum, and S. Kahn (Eds.), *Mathematical modelling: Education, engineering and economics–ICTMA 12* (pp. 141–148). Woodhead Publishing. https://doi.org/10.1533/9780857099419.3.141.

Karaci Yasa, G., & Karatas, I. (2018). Effects of the instruction with mathematical modeling on pre-service mathematics teachers' mathematical modeling performance. *Australian Journal of Teacher Education*, *43*(8), 1–14. https://doi.org/10.14221/ajte.2018v43n8.1

Karali, D., & Durmuş, S. (2015). Primary school pre-service mathematics teachers' views on mathematical modeling. *Eurasia Journal of Mathematics, Science & Technology Education*, *11*(4), 803–815. https://doi.org/10.12973/Eurasia.2015.1440a

Lew, H. (2004). Developing algebraic thinking in early grades: Case study of Korean elementary school mathematics. *The Mathematics Educator*, *8*(1), 88–106. https://pdfs.semanticscholar.org/91b6/432abf8d00c3d6adee02573a1913c5b9173c.pdf

Lewis, M., & Powell, J. A. (2016). Modeling zombie outbreaks: A problem-based approach to improving mathematics one brain at a time. *PRIMUS*, *26*(7), 705–726. https://doi.org/10.1080/10611970.2016.1162236

Loucks-Horsley, S., Stiles, K. E., Mundry, S., Love, N., & Hewson, P. W. (2010). *Designing professional development for teachers of science and mathematics*. Corwin Press Inc. http://dx.doi.org/10.4135/9781452219103

Maaß, K. (2006). What are modelling competencies?. *ZDM*, *38*(2), 113–142. https://doi.org/10.1007/bf02655885

Middleton, J., Toncheff, M., & Haag, S. (2011). Growth in secondary teachers' content knowledge and practice in discrete mathematics. In L. R. Wiest, and T. Lamberg (Eds.), *Proceedings of the 33rd Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 425–433). University of Nevada.

National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*. National Council of Teachers of Mathematics.

National Governors Association Center for Best Practices and Council of Chief State School Officers (2010). *Common core state standards mathematics*. National Governors Association Center for Best Practices; Council of Chief State School Officers. Available at http://www.corestandards.org/Math/

National Research Council (2001). *Helping children learn mathematics*. National Academy Press.
Noble, R. D. (1982). Mathematical modelling in the context of problem solving. *Mathematical Modelling*, *3*(3), 215–219. https://doi.org/10.1016/0270-0255(82)90026-4
Rönn, M. M., Dunville, R., Wang, L. Y., Bellerose, M., Malyuta, Y., Menzies, N. A., Aslam, M., Lewis, F., Walker-Baban, C., Asbel, L., & Parchem, S. (2020). Mathematical modeling study of school-based chlamydia screening: Potential impact on chlamydia prevalence in intervention schools and surrounding communities. *BMC Public Health*, *20*(1963), 1–9. https://doi.org/10.1186/s12889-020-09466-y
Son, J., Jung, H., & I, J. Y. (2017). How much do I know about mathematical modeling? In E. Galindo and J. Newton (Eds.), *Proceedings of the 39th Annual Meeting of the North America Chapter of the International Group for the Psychology of Mathematics Education* (pp. 913–916). Hoosier Association of Mathematics Teacher Educators.
Stillman, G. A., Galbraith, P., Brown, J. P., & Edwards, I. (2007). A framework for success in implementing mathematical modeling in the secondary classroom. In J. Watson and K. Beswick (Eds.), *Proceedings of the 30th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 688–697). University of Tasmania. https://eric.ed.gov/?id=ED5037468.
Stohlmann, M. S., & Albarracín, L. (2016). What is known about elementary grades mathematical modeling. *Education Research International*, *2016*, 1–9. http://dx.doi.org/10.1155/2016/5240683
Stohlmann, M. S., Maiorca, C., & Olson, T. A. (2015). Preservice secondary teachers' conceptions from a mathematical modeling activity and connections to the common core state standards. *The Mathematics Educator*, *24*(1), 21–43. https://eric.ed.gov/?id=EJ1075824
Stohlmann, M. S., Maiorca, C., & Allen, C. (2017). A case study of teachers' development of well-structured mathematical modelling activities. *Mathematics Teacher Education and Development*, *19*(2), 4–24. https://eric.ed.gov/?id=EJ1160834
Stohlmann, M. S., & Yang, Y. (2020). The alignment of teacher created online curricula to mathematical modeling. In R. Brandl and J. Herron (Eds.), *Proceedings of the 119th Annual Convention of the School Science and Mathematics Association* (pp. 40–47). SSMA. `https://www.ssma.org/assets/Proceedings/2020%20Proceedings%20 Volume%207.pdf`.
Suh, J. M., Matson, K., & Seshaiyer, P. (2018). Mathematical modeling competencies essential for elementary teachers: The case of teachers in transition. In T. E. Hodges, G. J. Roy, and A. M. Tyminski (Eds.), *Proceedings of the 40th Annual Meeting of the North America Chapter of the International Group for the Psychology of Mathematics Education* (pp. 1122–1125). University of South Carolina and Clemson University.
Thompson, A. G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 127–146). Macmillan.
Tidwell, W., Anhalt, C. O., Cortez, R., & Kohler, B. R. (under review). Modeling memorization: A data collection and mathematical modeling experience. *PRIMUS*.
Vugrin, E. D., Rostron, B. L., Verzi, S. J., Brodsky, N. S., Brown, T. J., Choiniere, C. J., Coleman, B. N., Paredes, A., & Apelberg, B. J. (2015). Modeling the potential effects of new tobacco products and policies: A dynamic population model for multiple product use and harm. *PloS One*, *10*(3), 1–26. https://doi.org/10.1371/journal.pone.0121008
Yu, S.-Y., & Chang, C.-K.2011). What did Taiwan mathematics teachers think of model-eliciting activities and modelling teaching? In G. Kaiser, W. Blum, R. Borromeo Ferri, and G. Stillman (Eds.), *Trends in Teaching and Learning of Mathematical Modelling* (pp. 147–156). Springer. https://doi.org/10.1007/978-94-007-0910-2_16.

Appendices

Appendix 1. Questionnaire prompts

(1) One of the standards for mathematical practice in the Common Core State Standards is "Model with mathematics.' Explain what this means to you

(2) Are modeling with mathematics and solving word problems related? Explain.
(3) How can teachers prepare to teach modeling at the elementary school and middle school levels?
(4) What role do you suppose that 'real-life' contexts play in modeling problems?
(5) What mathematical practices could children learn in elementary and middle school by engaging in mathematical modeling activities?

Appendix 2. Interview protocol

Note that the most essential questions are 1 and 3. Other questions and lettered questions may be omitted at the interviewer's discretion.

(1) How do you feel your understanding of mathematical modeling has changed over the semester, if it has changed?
(2) Could you describe the difference between modeling with mathematics and using manipulatives as models for mathematics?
(3) Do you feel like your language, reasoning, or communication of mathematics has evolved by engaging in mathematical modeling activities over the semester?
    (a) If yes, could you provide an example of growth?
(4) Do you think that mathematical modeling has a place in the curriculum of elementary and middle school?
    (a) If yes, why would you think that modeling is beneficial to the learning experience?
    (b) If no, what about modeling do you think makes it unsuitable for this curriculum?
    (c) If indifferent, do you have reservations for including mathematical modeling in elementary or middle school curriculum? If so, what are they?
(5) Any other comments?

Appendix 3. Mathematical modeling competencies rubric

Rubric for Assessment of Mathematical Modeling

Modeling Element	4 Proficient Evidence	3 Emergent Evidence	2 Limited Evidence	1/0 Little/no evidence
Understand the problem situation	Fully identifies the context, objective of the solution, and factors that affect the solution. Uses background knowledge of the context or researches the context.	Partially identifies the context, objective of the solution, or factors that affect the solution. Uses partial background knowledge of context or partially researches the context.	Limited in identifying the context, objective of the solution, and factors that affect the solution. Does not use available background knowledge and does little to no research of the context.	Demonstrates little to no evidence.
Pose a simplified version of the situation	Fully determines useful information given. Makes useful, appropriate assumptions and choices. Uses background knowledge and experience or research as additional information.	Partially determines useful given information Partially makes useful, appropriate assumptions and choices Partially uses background knowledge and experience or research as additional information	Limited in determining given information, making necessary assumptions, and making appropriate choices,	Demonstrates little to no evidence.
Develop a model	Translates the information into mathematical notation (decontextualize). The model uses all relevant assumptions made.	Partially translates the information into mathematical notation (decontextualize). The model uses some assumptions made.	Limited in translating the information into mathematical notation (decontextualize). The model uses assumptions different from those made (modified or implicit).	Does not translate information into mathematics.
Compute a solution of the model	Performs calculations correctly in the model (possibly one minor error). Checks for precision.	Performs calculations correctly in the model (with few errors). Is aware of checking for precision.	Limited with calculations in the model (with multiple errors) and is not aware of needing to check for precision.	Demonstrates little to no evidence.
Interpret the solution and draw conclusions	Interprets the mathematical solution in terms of the original situation (contextualizing). Draws conclusions that the solution implies about the original situation.	Partially interprets the mathematical solution in terms of the original situation (contextualizing). Is aware of drawing conclusions that the solution implies about the original situation.	Limited with interpreting the mathematical solution in terms of the original situation (contextualizing) and with drawing conclusions that the solution implies about the original situation.	Demonstrates little to no evidence.
Validate the conclusions	Determines if the mathematical answer makes sense in terms of the original situation and verifies the answer is within a valid range of values. Determines if conclusions are satisfactory in all respects; if not, shows evidence of iteration of the process to improve the model.	Partially determines if the mathematical answer makes sense in terms of the original situation. Demonstrates awareness that the answer is within a valid range of values. Partially determines if conclusions are satisfactory.	Limited in determining if the mathematical answer makes sense in terms of the original situation. Shows little awareness that verification of the solution should be made.	Demonstrates little to no evidence.
Report the solution	Communicates the model with full explanations and justifications of assumptions and choices made.	Communicates the model with partial explanations and justifications of assumptions and choices made.	Limited in communicating the model, explanations, and justifications demonstrating little understanding of the problem and solution.	Demonstrates little to no evidence.