Fair and Diverse Allocation of Scarce Resources

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Abstract

We aim to design a *fairness-aware* allocation approach to maximize the geographical diversity and avoid *unfairness* in the sense of demographic disparity. During the development of this work, the COVID-19 pandemic is still spreading in the U.S. and other parts of the world on large scale. Many poor communities and minority groups are much more vulnerable than the rest. To provide sufficient vaccine and medical resources to all residents and effectively stop the further spreading of the pandemic, the average medical resources per capita of a community should be independent of the community's demographic features but only conditional on the exposure rate to the disease. In this article, we integrate different aspects of resource allocation and create a synergistic intervention strategy that gives vulnerable populations higher priority in medical resource distribution. This prevention-centered strategy seeks a balance between geographical coverage and social group fairness. The proposed principle can be applied to other scarce resources and social benefits allocation.

1 Introduction

The COVID-19 pandemic has been widely spreading around the globe and caused hundreds of thousands of deaths, crashed the healthcare systems of many countries, and stalled almost all social and economic activities, which leads to an astronomical amount of financial loss. To battle the spreading COVID-19 pandemic, several leading countries and organizations have devoted a significant amount of resources to develop vaccines, new diagnostics, and anti-infective treatments for the novel coronavirus [17]. More than 60 candidate vaccines are now in development worldwide, and several have entered early clinical trials in human volunteers, according to the World Health Organization [39]. Once the development of the treatments is approved for public use, it is of paramount importance that these resources are quickly dispatched to all the communities in a country because one weak link in the defense against the virus would leave the neighboring communities, even the whole state vulnerable and exposed to the spreading of the disease. However, the existing healthcare infrastructures in many states and cities are insufficient to provide universal supplies of vaccinations and treatments. Hence, targeting the high risk population and setting fair principles for prioritizing the allocation accordingly will save lives, curb the further spreading of the virus, and prevent sequential global pandemic outbreaks in the following years.

In the United States, due to the disparities [43] between different population subgroups in terms of health, financial stability, and accessibility to health care services, certain low-income, and minority populated communities are particularly vulnerable to COVID-19. For example, among Chicago's minority groups, the Latinx and Black groups have been hit much harder by the COVID-19 pandemic than any other ethnic group. By the latest report (September 15, 2020) from the Chicago Department

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of Public Health (CDPH), among the death tolls caused by COVID-19, 33.0% are from the Latinx group, and 42.8% are from Black & non-Latinx group [49]. These two ethnic groups have also led to a substantial increase in the number of COVID-19 confirmed cases by a large margin compared to the rest ethnic groups. These ratios are disproportional to the percentages of these ethnic groups of the total population in Chicago, which mainly consists of 32.3% white, 28.7% Hispanic, and 30.9% African American, according to US Census Bureau [3].

The development of vaccines and other effective treatment medicines against any novel infectious disease generally takes a long time due to the high uncertainties in all the stages in the process, including clinical trials. Even if the vaccines and treatments are developed successfully, they will only be available in limited quantities initially due to manufacturing, logistic and financial constraints. Due to the huge gap between available medical resources and the entire population in need of them, especially during the early production stages, the allocation of such resources becomes a difficult yet pressing issue. Moreover, due to demographic distribution and occupational factors of the residents in different geographical regions, resource allocation based solely on maximum geographical uniformity can lead to an *unfair* distribution system and propagate biases across population subgroups of the population.

We design a *fairness-aware* allocation framework for vaccine and scarce treatment resources considering both *Geographical Diversity* and *Social Group Fairness* as the guiding principles for prevention-centered strategies. The social group fairness is based on the general fairness notion of *Equality of Opportunity* [25,31]. An allocation strategy is fair if the average amount of resources an individual receives only depends on the individual's exposure rate to the disease and is independent of the individual's demographic or social-economic background. We also consider an allocation strategy diverse if the geographical location does not affect the averaged resources an individual can receive. Based on such notions of fairness and diversity, we formally define and formulate them into inequality constraints. Our proposed research provides a solution to distribute the scarce medical resources in a fair manner to all communities and protect certain minorities and low-income groups that are more vulnerable to pandemic's impact. Not only can such a solution help stop the spreading pandemic more effectively, but also push for justice and fairness in healthcare decision-making.

1.1 Related Work

Resource allocation has been a classic and important problem in many domains such as economy (e.g, [30]), management science (e.g, [14]), emergency response (e.g, [28]), etc. More recent works have addressed the need for an integrated approach with different purposes such as material allocation in a production plan [8], supplier selection, and order allocation application [51]. Healthcare resource allocation is one of the most challenging allocation problems. A large number works have been developed to identify effective strategies for it [7,12,26,27,33,41,55,61].

Identifying and assessing the type of populations at risk in emergency resource allocation is yet another challenge that appears in various crises. The risk equity in the field of emergency resources location-allocation has been studied in the existing literature. [65] developed a bi-level optimization based on the game theory for emergency logistics. The risk is assessed based on the multiplication of incident probability by the exposed population within a certain impact radius corresponding to certain hazardous materials. The aim is to minimize the total environmental risk posed by hazardous waste at different incident sites. More recently, [62] has discussed that an ethically justified resource allocation during the current pandemic should meet the needs of the highest risk categories. In addition, [48] developed a framework using local COVID-19 data of Los Angeles County to geographically identify the risk level of sub-populations by partitioning based on demographics and vulnerabilities. They assess vulnerabilities based on pre-existing health conditions, barriers to accessing health care, environmental risk, and social vulnerability and find out that the significant presence of racial minorities denotes the most vulnerable neighborhoods. The proposed framework and medical vulnerability have been discussed and evaluated in [54], as well.

Recently, the fairness of algorithmic decision making has been the center of attention of many researchers, who are designing and developing algorithms for different purposes, such as machine learning, ranking, and social welfare [4,5,23,66]. Mitigating the bias of an outcome from a decision model, which is mainly caused by the inherent bias in the data and societal norm, will ensure that the outcome is not favorable or adversarial toward any specific subgroups [4,16,24,31,60,63,66]. One of the critical problems where the fairness of the outcome matters significantly is scarce resource allocation. The notion of fairness in resource allocation has been introduced in [6] and later with a more precise definition in [10].

Fairness was firstly adopted in bandwidth allocation problem for computer network systems [11, 15, 18, 34, 35, 36, 38, 46, 47]. In these settings, the amount of resources requested can be modified by different users. Besides, service allocation mainly covers a group of users and is not necessarily one-to-one allocation. These settings differ from the resource allocation for scarce treatments which is a one-to-one allocation problem.

Now fairness has been addressed in many resource and service allocation methods [25, 37, 40, 41, 59]. The importance of ethical consideration in resource allocation and principal guidelines have been discussed in [62]. Here we highlight a few and emphasize our difference from them. In [25], the authors formalize a general notion of fairness for allocation problems and investigate its algorithmic consequences when the decision-maker does not know the distribution of different subgroups (defined by creditworthy or criminal background) in the population. The distribution estimation is accomplished using censored feedback (individuals who received the resource, not the true number). In our work, we estimate the distribution of different social groups from the data using Bayes rule. Singh [53] considers a fair allocation of multiple resources to multiple users and have proposed a general optimization model to study the allocation. Our proposed method differs from [53] on two aspects. First, [53] assumes multi-resources and multi-type users, whereas we mainly focus on resource allocation problem across different regions and different population subgroups. Second, [53] aims to maximize the coverage, whereas we consider a *fairness-diversity* trade-off by minimizing the diversity and fairness gaps, simultaneously, across different regions. Donahue et al. [20] considers the problem of maximizing resource utilization when the demands for the resource are distributed across multiple groups and drawn from probability distributions. They require equal probabilities of receiving the resource across different groups to satisfy fairness and provide upper bounds on the price of fairness over different probability distributions. In our proposed model, we utilize a similar fairness requirement while requiring diversity (population) consideration across different regions. Thus, our work is an intersection between fairness and diversity. Furthermore, we deal with the one-to-one allocation instead of the coverage problem. One recent research article related to COVID-19 resource allocation designed a vulnerability indicator for racial subgroups that can be used as guidelines for medical resource allocation [48]. The proposed model cannot identify other vulnerable subgroups that are not geographically clustered, thus not able to form spatially concentrated communities. In our paper, we directly identify subgroups' vulnerability using exposure rates estimated from COVID-19 cases and death. We focus on the available data to estimate the necessary parameters and perform an empirical study using the proposed Algorithm 2.

1.2 Summary of Contributions

In this paper, we aim to design a fairness-aware allocation strategy that considers the trade-off between geographical diversity and social fairness (demographic disparity) in allocating resources. We provide a new aspect to the classic allocation problem while seeking a synergistic intervention strategy that prioritizes disadvantaged people in the distribution of scarce resources. The proposed approach is different from the existing works using the maximum utilization [13] or maximum coverage [25, 53] objective. The nature of the treatment allocation problem is not the same as the coverage problems since the former is a one-to-one assignment problem. In contrast, in the latter, the resource can cover more than one user, such as police or doctor allocation [20, 25]. In our work, we aim to study scarce resource allocation considering the trade-off between geographical diversity and social fairness. More specifically, we do not seek to share the vaccines equally among groups of populations. Instead, we aim to emphasize the protection of the vulnerable populations or the ones at high risk, depending on their exposure rate, to prevent the death/spread of the disease. We focus on the available data to estimate the necessary parameters and perform an empirical study using the proposed Algorithm 2. However, our proposed model can be generalized to resource allocation in other scenarios such as disaster relief.

In § 2, we first model the notions of geographical diversity and social group fairness to allocate scarce medical treatments and vaccines. We then formulate the allocation problem into an Integer Program (IP) problem, which incorporates diversity and fairness as constraints, in addition to the capacity constraint, i.e., the number of available resources. Fairness and diversity constraints are bounded by user-defined hyperparameters, ϵ_d, and ϵ_f, which are the allowed diversity and fairness gaps, respectively. To efficiently obtain a feasible solution for the original IP problem, we relax the IP problem to a Linear Programming (LP) problem. Moreover, the fairness and diversity constraints are much more complicated than the capacity constraint. Hence to deal with them efficiently, we use the penalty method, a common practice to solve constrained optimization. The two constraints are combined into a single objective function using a trade-off hyperparameter α. To guarantee that the converted problem is equivalent to the original feasibility problem, we provide theoretical proof. Subsequently, a binary search algorithm is used to obtain a feasible range of α. We evaluate the allocation scenario under different (ϵ_d, ϵ_f) values and provide the corresponding feasible range for α, accordingly. Different levels of the trade-off between diversity and fairness are presented. The proposed framework can be applied at different stages of the pandemic to estimate the exposure rate of population subgroups and obtain a feasible allocation considering both population size and exposed population. In § 4, we evaluate the performance of the proposed model using COVID datasets in Chicago for vaccine and scarce treatment allocations. The results demonstrate the impact of incorporating fairness criteria in the allocation model compared to diverse and uniform allocation. The paper concludes in § 6.

2 Problem Definition

Traditional resource allocation treatments have been widely studied in the optimization literature [14,28,30] where the limited resource is being distributed to areas based on a single objective (e.g., the function of the total population). However, regions might have different needs or require a higher priority in a medical resource allocation setting (e.g.vaccine). An allocation treatment solely based on geographical diversity suggests an equalized distribution among the regions, which may not provide equity in the sense of social fairness. To assure the concept of "equity" as well as "equality", we aim to model and incorporate fairness notion on demographic subgroups as another principal in the allocation decision process. Identifying each region's priority or risk level is critical since it can save many lives and protect vulnerable subgroups. We aim to design a fair and diverse resource allocation framework through modeling the *Social Group Fairness* and *Geographical Diversity* and modeling their trade-off in the optimization setting. The optimization model minimizes the fairness and diversity gaps across different subgroups of different regions while satisfying the capacity constraint to protect a vulnerable population. A trade-off analysis is performed to demonstrate the price of fairness incorporation as the fairness gap (the maximum allocation gap between demographic subgroups) with and without fairness consideration in the allocation. We propose a tuning approach to identify an optimal range for the trade-off hyperparameter(α) in order to provide the best possible allocation solution under different scenarios.

2.1 Notations

We consider a centralized decision maker for allocating available b units of vaccines to a set of centers denoted by M. They include clinics, hospitals, pharmacies, etc. For convenience, we assume the entire area covered by the M centers to be a city, but it can be a county or other administrative district. Let x_j be the decision variable denoting the number of vaccines allocated to the center $j \in M$. Let z_j be the region, such as the list of zip-code areas, that are assigned to be covered by center j. For simplification, we assume there is no overlap between the list of zip-code areas covered by different centers (even if they are close in the distance), i.e., $z_l \cap z_k = \emptyset, \forall l, k \in M$. We also assume that residents can only receive the resources from the center that covers the region where they reside. Such policy is not uncommon in practice, especially in the distribution of scarce resources.

Next, we introduce some key concepts and their notation. If we consider the geographical location of any individual reside to be a random variable, denoted by Z, then $\{j \in M, z_j\}$ are the possible values for Z. Therefore, $P(Z = z_j)$ is the proportion of the population who reside in z_j. Let $U_1, \dots, U_p$ be discrete-valued sensitive variables corresponding to demographic and socioeconomic attributes, and S_i for $i = 1, \dots, p$ be the set of possible levels for each of the sensitive variable. For instance, if $[U_1, U_2, U_3]$ represent three attributes, income, race, gender, respectively, then $S_1 = \{\text{low, medium, high}\}$, $S_2 = \{\text{black, latinx, white, others}\}$, and $S_3 = \{\text{female, male}\}$. The combinations of all the levels of $U_1, \dots, U_p$ is a set denoted by $\mathcal{G}$ and indexed by a set I, i.e., $\mathcal{G} = \{g_i, \text{ for } i \in I\}$. In other words, $\mathcal{G} = S_1 \times S_2 \times \cdots \times S_p$. For any $g \in \mathcal{G}$, it corresponds to a possible combination of levels of $U_1, \dots, U_p$, such as $< low - income, black, female >$, and there can be a group of the population whose values of the sensitive variables $(U_1, \dots, U_p)$ are equal to g.

In short, we call it social group g. Let $s_{i,j}$ be the population size of the social group g_i who reside in region z_j, where $i \in I, j \in M$. Therefore, $s_{i,j} / \sum_{r \in I, k \in M} s_{l,j} = P([U_1, \ldots, U_p, Z] = [g_i, z_j])$. Let E be a binary random variable with $E = 1$ representing the individual is exposed to the infectious disease and $E = 0$ otherwise. So $P(E = 1|g_i)$ is the exposure rate of the social group g_i and $P(E = 1|g_i, z_j)$ the exposure rate of the social group g_i living the area of z_j. It is intuitive to assume these exposure rates depend on the social groups and regions. We will discuss more on some reasonable assumptions on the exposure rates and how to estimate them later.

A key concept in resource allocation is the amount of resource per capita. It is a ratio between the quantity of available resources and the size of the population who will receive the resource. Denote V as the amount of resource one individual receives. One important assumption we make here is that V follows a discrete uniform distribution. There are three parameters involved, the amount of resource X, $\mathcal{Y}$ the population who are to receive the X amount of resource, and size of the population $Y = card(\mathcal{Y})$, the cardinality of $\mathcal{Y}$. Therefore, the mean value of V is $\mathbb{E}(V|X, \mathcal{Y}, Y) = \frac{X}{Y}$ is the resources per capita, and it varies with respect to the three parameters.

The focus of this article is on the following problem. Given a limited amount of resource b, such as vaccines, how should the decision maker allocate the amount of resource x_j to each center j satisfying *geographical diversity* (quantified by $\mathcal{D}(x)$), and *social fairness* (quantified by $\mathcal{F}(x)$).

2.2 Diversity and Fairness Modeling

In this section, we define geographical diversity and social group fairness, which the latter is based on the *equality of opportunity* notion of fairness [24]. To quantify the geographical diversity and social fairness, we introduce $\mathcal{D}(x)$ and $\mathcal{F}(x)$ in Equations (1) and (7).

Definition 1. ***(Geographical Diversity)***
Geographical Diversity of allocation of limited resource to a set of centers M is satisfied if $\forall j \in M$, on average the resource per capita is invariant with respect to the location of the groups of the population, i.e., $\mathbb{E}(V|x_j, z_j, \sum_{i \in I} s_{i,j})$ does not vary with respect to the location z_j.

The notation $\mathbb{E}(V|x_j, z_j, \sum_{i \in I} s_{i,j})$ is simplified and we use z_j to refer to the population who reside in region z_j. Let $s_{i,j}$ be the population size of the social group g_i and $\mathbb{E}(V| \sum_{j \in M} x_j, \sum_{i \in I, j \in M} s_{i,j})$ denote the averaged resource per capita over the entire city under the consideration of the resource allocation plan, and the location is omitted since it is obvious. It is straightforward to formulate the geographical diversity $\mathcal{D}_j(x)$ for $\forall j \in M$ as follows.

$$\mathcal{D}_j(x) = \left| \mathbb{E}(V|x_j, z_j, \sum_{i \in I} s_{i,j}) - \mathbb{E}(V| \sum_{j \in M} x_j, \sum_{i \in I, j \in M} s_{i,j}) \right| = \left| \frac{x_j}{\sum\limits_{i \in I} s_{i,j}} - \frac{\sum\limits_{j \in M} x_j}{\sum\limits_{j \in M} \sum\limits_{i \in I} s_{i,j}} \right|. \quad (1)$$

So if geographical diversity is strictly met, $\mathcal{D}_j(x) = 0$ for all j.

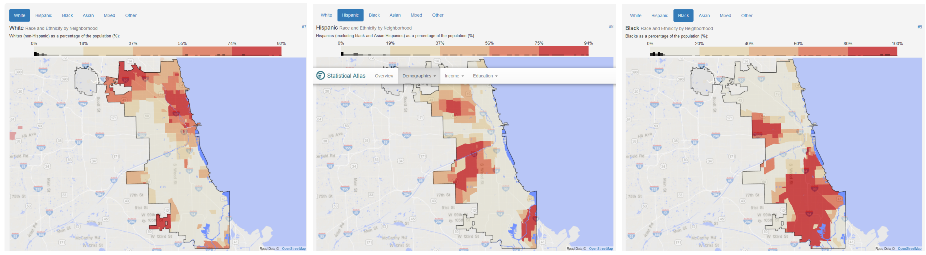
The geometrical diversity represents the conventional or minimal requirement for resource distribution that requires evenly distributing the resources among all geographical regions across the entire city. Next, we introduce the concept of social fairness. In this definition, we emphasize the even distribution of resources among the endangered population, i.e., exposed to the infectious disease, disregarding the social groups.

Definition 2. ***(Social Fairness)***
Social Fairness of allocation of limited resource to a set of centers M is satisfied if $\forall i \in I$, the averaged resource per capita is invariant with respect to the values of $U_1, \ldots, U_p$ of the group of the population, i.e., $\mathbb{E}(V|E = 1, g_i)$ does not vary with respect to the social group g_i.

The notation $\mathbb{E}(V|E = 1, g_i)$ is simplified and we use $E = 1$ and g_i to denote the exposed individuals in social group g_i. Directly translating this definition into a formula, the fairness principle should be

$$\mathcal{F}_i(x) = |\mathbb{E}(V|E = 1, g_i) - \mathbb{E}(V|E = 1)|, \quad \forall i \in I. \quad (2)$$

However, the calculation of $\mathbb{E}(V|E = 1, g_i)$ and $\mathbb{E}(V|E = 1)$ is not straightforward, and we derive them as follows.



(a) White (b) Hispanic groups. (c) African-American

Figure 1: Map of Race and Ethnicity by Neighborhood in Chicago.

We first calculate the resource per capita for the exposed individuals who reside in z_j, disregarding the social group, i.e.,

$$\mathbb{E}(V|E=1,z_j) = \frac{x_j}{\sum\limits_{l\in I} s_{l,j} \times P(E=1|z_j,g_l)}, \quad \forall j \in M \tag{3}$$

in which the denominator is the amount of exposed individuals in z_j. Then,

$$\mathbb{E}(V|E=1,g_i) = \sum_{j\in M} \mathbb{E}(V|E=1,z_j) P(Z=z_j|E=1,g_i) \tag{4a}$$

$$= \sum_{j\in M} \mathbb{E}(V|E=1,z_j) \frac{P(E=1,[U_1,\dots,U_p]=g_i, Z=z_j)}{P(E=1,[U_1,\dots,U_p]=g_i)} \tag{4b}$$

$$= \sum_{j\in M} \frac{x_j}{\sum\limits_{l\in I} s_{l,j} \times P(E=1|z_j,g_l)} \frac{s_{i,j} P(E=1|g_i,z_j)/(\sum_{i',j'} s_{i',j'})}{\sum\limits_{k\in M} s_{i,k} P(E=1|g_i,z_k)/(\sum_{i',j'} s_{i',j'})} \tag{4c}$$

$$= \sum_{j\in M} \frac{x_j}{\sum\limits_{l\in I} s_{l,j} \times P(E=1|z_j,g_l)} \frac{s_{i,j} P(E=1|g_i,z_j)}{\sum\limits_{k\in M} s_{i,k} P(E=1|g_i,z_k)} \tag{4d}$$

In this article, we assume that the chance of exposure of an individual only depends on the individual's social attributes and is independent of the geographical location, i.e., $P(E=1|U_1,\dots,U_p,Z) = P(E=1|U_1,\dots,U_p)$. Equivalently, $\forall i \in I$ and $\forall j \in M$, $P(E=1|g_i,z_j) = P(E=1|g_i)$. This assumption is reasonable since, in many U.S. cities as Chicago, the geometrical locations of the residents are highly correlated with the social and economic status of the residents. For example, in Figure 1, the percentage of the major ethnic groups, White, Hispanic, and African-American, are shown in three heat maps. It is very clear that the geometrical locations of the residents and the racial groups are heavily correlated. Of course, this assumption also makes the rest of the formulation much simpler. Under this assumption, we can simply obtain

$$\mathbb{E}(V|E=1,g_i) = \sum_{j\in M} \frac{x_j}{\sum\limits_{l\in I} s_{l,j} \times P(E=1|g_l)} \frac{s_{i,j}}{\sum\limits_{k\in M} s_{i,k}}. \tag{5}$$

Next, to obtain $\mathbb{E}(V|E=1)$, we need to integrate Equation (5) with respect to g_i, i.e.,

$$
\begin{aligned}
\mathbb{E}(V|E=1) &= \sum_{i\in I}\mathbb{E}(V|E=1,g_i)P([U_1,\dots,U_p]=g_i|E=1)\\
&= \sum_{i\in I}\mathbb{E}(V|E=1,g_i)P(E=1|g_i)\frac{P([U_1,\dots,U_p]=g_i)}{P(E=1)}\\
&= \sum_{i\in I}\mathbb{E}(V|E=1,g_i)P(E=1|g_i)\frac{\sum\limits_{k\in M} s_{i,k}/(\sum_{i',j'} s_{i',j'})}{\sum\limits_{r\in I,k'\in M} s_{r,k'}P(E=1|z'_k,g_r)/(\sum_{i',j'} s_{i',j'})}\\
&= \sum_{i\in I}\left(\sum_{j\in M}\frac{x_j}{\sum\limits_{l\in I} s_{l,j}\times P(E=1|g_l)}\frac{s_{i,j}}{\sum\limits_{k\in M} s_{i,k}}\right)\frac{P(E=1|g_i)\sum\limits_{k\in M} s_{i,k}}{\sum\limits_{r\in I,k'\in M} s_{r,k'}P(E=1|g_r)}\\
&= \sum_{i\in I}\sum_{j\in M}\frac{x_j}{\sum\limits_{l\in I} s_{l,j}\times P(E=1|g_l)}\frac{P(E=1|g_i)s_{i,j}}{\sum\limits_{r\in I,k'\in M} s_{r,k'}P(E=1|g_r)}.
\end{aligned}
$$

Based on the derivations, we can formulate the fairness for $\forall i\in I$,

$$
\mathcal{F}_i(x) = |\mathbb{E}(V|E=1,g_i)-\mathbb{E}(V|E=1)| = \tag{7}
$$

$$
\left|\sum_{j\in M}\frac{x_j}{\sum\limits_{l\in I} s_{l,j}\times P(E=1|g_l)}\frac{s_{i,j}}{\sum\limits_{k\in M} s_{i,k}} - \sum_{i\in I}\sum_{j\in M}\frac{x_j}{\sum\limits_{l\in I} s_{l,j}\times P(E=1|g_l)}\frac{s_{i,j}P(E=1|g_i)}{\sum\limits_{r\in I,k'\in M} s_{r,k'}P(E=1|g_r)}\right|.
$$

Similar to the definition of $\mathcal{D}_j(x)$, if the fairness is strictly met, $\mathcal{F}_i(x)=0$ for all $i\in I$.

3 Fair and Diverse Allocation Optimization

As explained above, if the diversity and fairness requirements are strictly met, all $\mathcal{D}_j(x)$ and $\mathcal{F}_i(x)$ should be 0. However, such constraints are too restrictive and difficult or impossible to satisfy for all regions and social groups. Define $\mathcal{D}(x)=\max_{j\in M}\mathcal{D}_j(x)$ and $\mathcal{F}(x)=\max_{i\in I}\mathcal{F}_i(x)$. $\mathcal{D}(x)$ and $\mathcal{F}(x)$ are auxiliary decision variables that signify the tight upper bounds on the diversity and fairness constraints, i.e., $\mathcal{D}_j(x)\le\mathcal{D}(x)$ and $\mathcal{F}_i(x)\le\mathcal{F}(x)$ for any $j\in M$ and $i\in I$. Ideally, we want to find the feasible solution x such that both upper bounds are equal to zero. In addition, x_j for $j=1,\dots,M$ should satisfy the capacity constraint, i.e., $\sum_{j\in M}x_j=b$. Seeking to achieve geographical diversity and social fairness simultaneously, one can formulate this problem as multi-objective (MO) minimization.

$$
\begin{aligned}
P1:\quad \min_x\quad & (\mathcal{D}(x),\mathcal{F}(x))\\
& \mathcal{D}_j(x)\le\mathcal{D}(x),\quad \forall j\in M\\
& \mathcal{F}_i(x)\le\mathcal{F}(x),\quad \forall i\in I\\
& \sum_{j\in M}x_j=b\\
& x_j\ge 0,\qquad \forall j\in M\\
& x_j\in\mathbb{Z},\qquad \forall j\in M.
\end{aligned}
$$

The integer constraint is because usually, resources such as vaccines are counted in integers, and one individual only receives one vaccination. The integer constraints in $P1$ can be relaxed, particularly in practice when b is large.

A common solution to the multi-objective optimization problem is the weighted sum method, which leads to a simpler minimization problem $P2$. After removing the absolute operation in the constraints, we obtain the relaxed linear programming (LP) problem.

$$
\begin{aligned}
P2: \quad \min_{x} \quad & (1-\alpha)\mathcal{D}(x) + \alpha\mathcal{F}(x) \\
\text{s.t.} \quad & \mathcal{D}_j^{+}(x) \le \mathcal{D}(x), \quad \forall j \in M \\
& \mathcal{D}_j^{-}(x) \ge \mathcal{D}(x), \quad \forall j \in M \\
& \mathcal{F}_i^{+}(x) \le \mathcal{F}(x), \quad \forall i \in I \\
& \mathcal{F}_i^{-}(x) \ge \mathcal{F}(x), \quad \forall i \in I \\
& \sum_{j \in M} x_j = b \\
& x_j \ge 0, \quad \forall j \in M.
\end{aligned}
$$

where $\mathcal{D}_j^{+}(x)$ refers to the positive side of the absolute value function, $\mathcal{D}_j(x)$, and $\mathcal{D}_j^{-}(x)$ refers to the negative side of the absolute function. Similarly, for $\mathcal{F}_j(x)$, we have $\mathcal{F}_j^{+}(x)$ and $\mathcal{F}_j^{-}(x)$. Here $\alpha \in [0, 1]$ is a hyperparameter that controls the trade-off between the importance of fairness and diversity. If $\alpha > 0.5$, the objective function focuses more on achieving fairness, and it focuses more on the diversity if $\alpha < 0.5$.

If we relax the integer constraints of $P1$, based on the multi-objective optimization theory, it is easy to conclude that for any x^* in the *Pareto front* of $P1$, there exists an $\alpha^* \in [0, 1]$ such that x^* is an optimal solution of $P2$. This is because the remaining constraints of $P1$, including $\mathcal{D}_j(x) \le \mathcal{D}(x)$ for all $j \in M$, $\mathcal{F}_i(x) \le \mathcal{F}(x)$ for all $i \in I$, $\sum_j x_j = b$, and $x_j \ge 0$ for all $j \in M$, form a convex polyhedron. Particularly, the constraints $\mathcal{D}_j(x) \le \mathcal{D}(x)$ and $\mathcal{F}_i(x) \le \mathcal{F}(x)$ are all linear in x. Thus, we have the above conclusion.

In practice, the simplex method can be used to find the optimal solution of $P2$ efficiently with common optimization libraries like IBM CPLEX and SciPy.

Algorithm 1 Rounding Heuristic

Find the LP relaxation solution; solve $P3$.
Round x_j to the nearest integer for $j \in M$.
$b' = b - \sum_{j \in M} x_j$.
if $b' < 0$ **then**
    Sort x_j's according to the population size of z_j (from largest to smallest).
    Decrease the x_j's corresponding to the top $[b']$ populated regions by 1.
else if $b' > 0$ **then**
    Sort x_j's according to the exposed population of z_j (from largest to smallest).
    Increase the x_j's corresponding to the top $[b']$ regions by 1.
end if
Return $x_j, \forall j \in M$.

Once the optimal solution of the LP-relaxation problem, $P2$, is obtained for a given α value, we need to round the solution into integers. But the rounded solution is not necessarily feasible for the minimization problem $P2$. To ensure the feasibility of the rounded solution, we employ a heuristic rounding approach in Algorithm 1, which is similar to the one in [56]. The algorithm starts with rounding the LP relaxation solution to the nearest integer values. Next, if the capacity constraint $\sum_{j \in M} x_j = b$ is not satisfied, the algorithm reduces x_j for the top populated region based on the total exceeded amount. If the resource constraint is under-satisfied, the algorithm increases the top exposed populated areas based on the remaining resources. Note that in the Diverse-only ($\alpha = 0$) and Fairness-only ($\alpha = 1$) scenarios, the algorithm only considers the population and exposed population to sort x_j's, respectively.

3.1 Feasibility

As we discussed in § 3, the ideal situation would be minimizing the upper bounds $\mathbb{D}(x)$ and $\mathcal{F}(x)$ to the full extent. An important observation to make here is that the ideal case with zero upper bounds *both* on the fairness and the diversity constraints is almost always impossible due to the underlying biases in data and historical discrimination in society. Therefore, in practice, a small positive upper bound threshold is considered to satisfy fairness and diversity. For example, the fairness requirement can be thought of as the US Equal Employment Opportunity Commission's "four-fifths rule," which requires that "the selection rate for any race, sex, or ethnic group [must be at least] four-fifths (4/5) (or eighty percent) of the rate for the group with the highest rate"[1]. We consider a similar requirement for diversity as well.

The solution obtained under the MO model, $P2$, is unable to guarantee to satisfy these requirements. For example, a solution with zero unfairness (but not satisfactory on the level of diversity) can be on the Pareto front solution of MO – hence might be the optimal output – even though it is not a valid solution.

Subsequently, we introduce two control parameters ϵ_d and ϵ_f, which are the acceptable thresholds for the diversity and fairness requirements, i.e., $\mathcal{D}(x) \leq \epsilon_d$ and $\mathcal{F}(x) \leq \epsilon_f$. The violation threshold parameters $\epsilon_{\mathcal{D}}, \epsilon_{\mathcal{F}} \in [0, 1]$, are user-defined values that determine how diverse and fair the allocation should be. The value $\epsilon_{\mathcal{F}} = 0$ corresponds to a fully fair allocation, whereas $\epsilon_{\mathcal{F}} = 1$ corresponds to a completely fairness-ignorant allocation that solely considers the diversity. These violations must be relatively small by which resources allocated to a particular region and particular group nearly achieve the required level of diversity and fairness, respectively. Note that as quantitative metric, we say that allocation is considered as fair if $\mathcal{F}(x) \leq \epsilon_f$, and allocation is considered as diverse if $\mathcal{D}(x) \leq \epsilon_d$. No feasible solution may exist for the optimization problem depending on the constraints, weights, and available scarce resources. In this case, the decision-maker will have no choice but to relax the constraints. In §4 we will discuss the allocation under different scenario.

Fortunately, as we shall prove in Theorem 1 if the problem has a feasible solution given the fairness and diversity constraints, there must exist an α value under which the optimum solution of $P2$ is feasible and vice versa.

Proposition 1. *Let be $X' = \{x | \mathcal{D}(x) < \epsilon_d, \mathcal{F}(x) < \epsilon_f, \sum_{j \in M} x_j = b, x_j \geq 0\}$:*

1. *If $X' \neq \emptyset$: given $x^* \in X'$, $\exists \alpha^*$ such that x^* is an optimal solution of $P2$.*
2. *If $\nexists\, \alpha^*$ such that x^* is an optimal solution of $P2$ that satisfies both conditions $\mathcal{D}(x) < \epsilon_d$ and $\mathcal{F}(x) < \epsilon_f$, then $X' = \emptyset$.*

We now need to find the α value for which an optimum $P2$ is feasible to satisfy diversity and fairness requirements. A naive approach would be a brute-force search in $[0, 1]$ with a small step size added to α. However, this is not a practical approach since it needs to solve the optimization problem in each iteration. As the step size decreases, the exhaustive search increases, and the number of times it needs to solve $P2$ increases accordingly. Therefore, we first obtain the monotone properties of optimal upper bound $\mathcal{D}(x)$ and $\mathcal{F}(x)$ with respect to α. This result is used to find proper α value later in § 3.2.

Proposition 2. *Let x_1^* and x_2^* be optimum solutions of $P2$ given α_1 and α_2, respectively. It can be shown that if $\alpha_1 \leq \alpha_2$, then $\mathcal{F}(x_2^*) \leq \mathcal{F}(x_1^*)$ and $\mathcal{D}(x_1^*) \leq \mathcal{D}(x_2^*)$.*

3.2 Choosing Trade-off Parameter, α

The parameter α in $P2$ controls the trade-off between the diversity and fairness of the allocation decision. Using the monotone property of $\mathcal{D}(x)$ and $\mathcal{F}(x)$ to α in Proposition 1, we now propose an approach to find a proper α value that satisfies both fairness and diversity constraints for given (ϵ_d, ϵ_f).

Using an example, we show the high-level idea of the tuning algorithm under different scenarios. In Figure 2, the monotonically decreasing red curve corresponds to the fairness constraint, and the monotonically increasing blue curve corresponds to the diversity constraint. The dashed horizontal lines correspond to the thresholds allowed for diversity $\mathcal{D}(x)$ and fairness $\mathcal{F}(x)$ constraints, respectively. For $\alpha < \alpha_1$ we can see that $\mathcal{F}(x)$ exceeds the allowed threshold of ϵ_f. Based on Proposition 1, we should remove the range $[0, \alpha_1)$ from consideration. For $\alpha > \alpha_3$, $\mathcal{D}(x)$ exceeds the allowed threshold of ϵ_d, hence, we can remove the range $(\alpha_3, 1]$ from consideration. For $\alpha = \alpha_2$, both $\mathcal{D}(x) \leq \epsilon_d$ and $\mathcal{F}(x) \leq \epsilon_f$ are satisfied. The color bar below the plot in Figure 2 shows different ranges of α. The green segment corresponds to the range of α values for which the optimum solution of $P2$ satisfies $\mathcal{D}(x) \leq \epsilon_d$ and $\mathcal{F}(x) \leq \epsilon_f$. Consequently, we can prune the infeasible intervals of α by evaluating the violation of diversity and fairness thresholds.

[1]Uniform Guidelines on Employment Selection Procedures, 29 C.F.R. §1607.4(D) (2015).

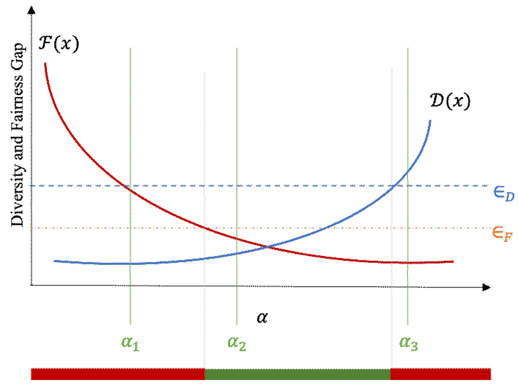


Figure 2: Feasible region with respect to α.

Algorithm 2 Tuning α

```
α_l = 0, α_u = 1
while |α_l − α_u| ≥ τ do
    α_m = (α_u + α_l)/2
    x* = argmin P3(α_m)
    if D(x*) ≤ ϵ_d & F(x*) > ϵ_f then
        α_l = (α_l + α_m)/2
    else if D(x*) > ϵ_d & F(x*) ≤ ϵ_f then
        α_u = (α_u + α_m)/2
    else if D(x*) ≤ ϵ_d & F(x*) ≤ ϵ_f then
        return x*
    else
        return x* = ∅
    end if
end while
return x*
```

Algorithm 2 represents the proposed tuning algorithm. The algorithm starts from the middle of the α interval, α_m, and solves $P2$ with that. Then, it splits the α region into half to further prune it until a narrow feasible range of α is obtained. In each iteration, if the diversity threshold is not satisfied by the solution obtained from $P2$, the algorithm prunes the interval larger than α_m. Similarly, if the fairness threshold is not satisfied, the algorithm prunes the interval smaller than α_m. If neither of the thresholds is satisfied with the solution obtained from $P2$ using α_m, a feasible solution does not exist for (ϵ_d, ϵ_f). Finally, when both constraints are satisfied, the algorithm returns a valid solution for $P2$, rounded to obtain an integer solution for $P1$.

3.3 Price of Fairness

We now define the Price of Fairness (PoF) as the difference of the fairness gap calculated for the optimal solution of $P2$ obtained with and without any fairness constraints. As we described in § 2, Equation (7) is the fairness constraint that is defined for each social group. Solving $P2$ with and without the fairness constraints, we can obtain different allocation solutions and consequently different fairness and diversity gaps. This will allow us to compare fair-diverse allocation performance with the Diverse-only allocation to analyze the impact of fairness constraints, namely PoF.

The PoF metric assists the decision-maker on the fairness scheme to be considered, mainly, ϵ_d and ϵ_f. Note that there is a trade-off between diversity and fairness gap in $P2$, when the allocation scheme is more focused on diversity, smaller ϵ_f, the control parameter α become close to 0. Subsequently, $P2$ minimizes the diversity upper bound subject to only the capacity constraint. Comparing the solution of such a problem with the one that focuses on fairness, α close to 1, we expect to see a larger fairness gap and smaller diversity gap. In §4, we evaluate PoF of the allocation solution obtained under different scenarios.

4 Case Study: Resource Allocation for COVID-19 Relief in US cities

In this section, we apply the proposed fair and diverse allocation strategy to the planning of distribution of medical resources for COVID-19 relief among some US cities (Chicago, New York, Baltimore). The city of Chicago is a segregated city with a relatively high Hispanic and Black population [9]. Several research studies have addressed a positive correlation among proportions of Black and Hispanic communities and COVID-19 cases [2, 9, 42, 64]. More specifically, in [9], both positive and statistically significant associations between the proportions of Black and Hispanic population and per capita COVID-19 cases have been identified using data from six segregated cities in the US. In particular, New York City has the highest rate of confirmed cases, followed by Chicago and Baltimore. That is, we present our instance problems based on these cities. Even though our case study is limited to three cities, it includes the first (New York City) and third (Chicago) largest cities in the USA, and presents our approach for cities with higher vulnerable populations (e.g., Baltimore).

We rely on the US census for population and demographic distribution data. For the COVID-19 cases, death tolls, and hospitalization, we separately collect the data provided by the governmental departments of each considered city. The primary sources of each dataset are described in Section 3.1. In the subsequent section, we first perform a descriptive analysis of the data to motivate the necessity of the fair and diverse distribution of medical resources. Next, we evaluate the performance of our proposed *Fair-Diverse* allocation approach and identify a reasonable trade-off parameter α based on the binary search Algorithm 2. Lastly, we will calculate the price of fairness (PoF) to highlight the role of fairness constraints in our optimization setting. The analytical results, optimization models, and algorithms were implemented in Python 3.7 using Docplex and Sklearn packages. The codess are available on github [2].

All experiments have been performed by a Mac-book machine with a 1.8 GHz Dual-Core Intel Core I5 CPU and 8 GB memory.

4.1 Data Description

Population Dataset: The *uszipcode*[3] Python package provides detailed geographic, demographic, socio-economic, real estate, and education information at the state, city, and even zip code level for different areas within the US. Based on the documentation, this package uses an up-to-date database by having a crawler running every week to collect additional data points from multiple data sources. This dataset does not provide the intersectional population. We refer to this dataset as `Pop.` throughout the experiment section.

City of Chicago COVID-19 Database [44] provides daily data on COVID-19 positive-tested cases, death tolls, hospitalizations, and other individuals' attributes (e.g., age, race, gender) to track the pandemic in this city. In our study, we primarily use the *COVID-19 Daily Cases* data to reveal the inequality among different population subgroups. We refer to this dataset as `Chicago-COVID-Cases` throughout the experiment section. The city of Chicago COVID-19 database also provides daily data on *COVID-19 Cases, Tests, and Deaths by ZIP Code* dataset 4.2. We refer to this dataset as `Chicago-COVID-Zipcode` throughout the experiment section.

New York City (NYC) COVID-19 Data Repository[4] consists of different COVID-19 related datasets including daily, weekly, monthly data, data on SARS-CoV-2 variants, the cumulative COVID-19 cases, etc. For our analysis, we use COVID-19 cases and death totals by age, race, and gender since the start of the COVID-19 outbreak in NYC, February 29, 2020. We refer to this dataset as `NYC-COVID-Zipcode` throughout the experiment section.

City of Baltimore COVID-19 data Dashboard[5] provides different statistics and visualization for COVID-19 data in Baltimore. The main source of this data is the Maryland Department of health [45], which is updated daily. This dataset includes COVID-19, the number of cases and death at the zip-code level, and total cases and death tolls by age, race, gender. We refer to this dataset as `BLT-COVID-Zipcode` throughout the paper.

4.2 Descriptive Analysis on COVID-19 risk factor among different population subgroups, and different regions (zip-codes)

Utilizing the `Chicago-COVID-Cases` dataset, we first identify the contribution of each demographic attribute to the total number of COVID-19 cases and deaths in each region of the Chicago area. This analysis reveals critical insights into the fundamental importance and differentiation among demographic subgroups.

Next, we perform a PCA analysis [1] using `Pop.` merged with the `Chicago-COVID-Zipcode` and produced a *Biplot*, as shown in Figure 3, to identify and compare the contribution of different attributes to the COVID-19 death and case numbers. Note that we used cross-validation to tune the number of components of the PCA method. To better visualize the impact of these attributes on different regions, we implement a K-means clustering [32] method to partition our geographical

[2] `https://github.com/nnezam2/Fair-Resource-Allocation/blob/master/README.md`
[3] `https://uszipcode.readthedocs.io/`
[4] `https://github.com/nychealth/coronavirus-data`
[5] `https://coronavirus.baltimorecity.gov/`

areas (Zipcodes) based on the numbers of COVID-19 deaths and cases. We cluster them into three categories and label them as high, medium, and low impacted areas, accordingly.

Our findings in this part show that areas with higher COVID-19 deaths and cases tend to have a higher population, higher elderly population (compared with the young one), more Black Or African American, Latinos population (compared with other races), and lower median income. Furthermore, we believe that the revealed negative correlation between income and COVID-19 cases should raise severe concerns in future decision-making procedures. One justification for that is lower-income individuals are mostly daily-paid and cannot afford living expenses if they are self-quarantined or stop working. Consequently, the pandemic inevitably affects the areas harder with larger lower-income populations. However, we do not have COVID-19 data based on income level. Hence, we do not consider income in this study.

In brief, we can observe notable differences in contribution to the COVID-19 positive cases and death tolls across specific demographic attributes (e.g., older age). This fact reveals the critical role of a *Fair-Diverse* model, which considers not only the overall population size but also the unrepresentative (exposed) population for scarce resource allocation problems.

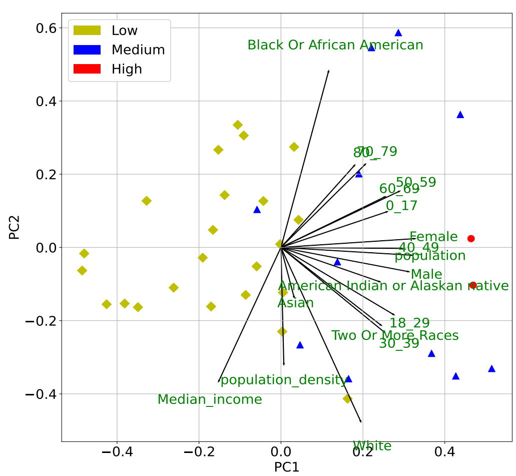


Figure 3: Biplot of demographic features using PCA Analysis with Kmeans clustering -Chicago City regions

Demographic Groups	Exposure rates
Female	0.058617
Male	0.060259
Age_18_29	0.052464
Black_non_latinx	0.037505
Age_30_39	0.067502
Other_race	0.097971
Age_40_49	0.078672
White_non_latinx	0.020698
Age_50_59	0.077986
Age_60_69	0.07898
Age_0_17	0.038741
Age_70_79	0.07275
Two or more race	0.088107
Age_80_	0.078768
Asian_non_latinx	0.020833

Figure 4: Exposure Rates of Population Subgroups-Chicago City

4.3 Estimating the Distribution of Exposed Population

Estimating the marginal distribution of the number of high-risk individuals in each demographic group, i.e., $P(E = 1|g_i)$, requires the data of positive-tested (infected) cases and death tolls. In this article, we intend to propose a guideline for resource allocation of vaccines and scarce treatments for the COVID-19 pandemic. However, we only have access to the count of individuals of each social group who were infected from COVID-19, namely $P(g_i|E = 1)$. The probability $P(E = 1|g_i)$ can be calculated from the Bayes formula

$$P(E = 1|g_i) = \frac{P(g_i|E = 1)P(E = 1)}{P(g_i)}.$$

We will use $P(E = 1|g_i)$, in the §4.4 to form the fairness constraints of the optimization problem, Equation 7, and calculate the exposed population in each zip-code. Table 4 presents the exposure rates for different population subgroups in Chicago City (The exposure rates for other cases (cities) are provided in Appendix 15, 16.

4.4 Fair-Diverse Allocation

To evaluate the proposed *Fair-Diverse* model, we construct *Diverse-only*, *Fair-only*, and equalized importance or *alpha=0.5* models, and compare the allocation solutions as well as the resulted fairness and diversity gaps among them. In our terminology, *Diverse-only* corresponds to an allocation that is merely based on diversity constraint, Equation 1, and is not considering any other (e.g., fairness) measures. Similarly, *Fair-only* corresponds to a model solely based on fairness constraint. Furthermore, *alpha=0.5* refers to a model that has equalized weights on Fairness and Diversity constraints in the optimization setting.

As mentioned in § 3, we propose a diversity and fairness trade-off problem as in $P2$. As long as the resource constraint is satisfied, $P2$ has an optimal solution given an α value. However, the optimal solution obtained from $p2$ might not be feasible given the diversity and fairness requirements ϵ_d and ϵ_f as mentioned in § 3.1. Hence, we apply our proposed binary search algorithm to discover a range for α that results in a feasible solution.

To evaluate the performance of the Algorithm 2,we will consider three US cities and different sensitive attributes (Race, Age, and Gender) and solve each *instance* problem separately. We will then show the allocation results for each problem using the four models mentioned above and discuss α ranges in detail. The total number of zip-code areas varies for each city, with 177 regions in New York City, 58 in Chicago, and 36 in Baltimore. The resulted allocations of each problem is sorted by population size for the top 15 area (zip-codes).

Before discussing the results, we provide the computation time of our proposed algorithms for different test cases. For the City of Chicago, the total computation time of optimization of $P2$ is 2.2 secs, and the overall computation time of binary-search is 10.8 secs. Similarly, for NYC, the total computation time of optimization of $P2$ is 6.1 secs, and the total computation time of binary-search is 76.4 secs. For the City of Baltimore, the total computation time of optimization of $P2$ is 2.1 sec, and the total computation time of binary-search is 4.3 secs.

Note that the computation time of optimization of $P2$ is proportional to the dataset size (number of zipcodes). Subsequently, NYC and the City of Baltimore has the highest and lowest time, respectively. Moreover, the computation time of binary-search is proportional to the computation time of optimization of $P2$ as it aims to solve $P2$ in each iteration. Note that the number of performed iterations of binary-search also impacts the overall computation time of the algorithm. Therefore, modifying threshold hyperparameter ϵ_d, ϵ_f, or τ would affect the reported times. Overall, our proposed algorithms are computationally efficient at the city and state levels. At the national or worldwide level, it will still be efficient but requires an efficient parallel implementation to reduce the computation time further.

City of Chicago

We consider $b = 200000$ units of vaccines as the total available resources to be allocated for the city of Chicago. Using `Pop.`, and `Chicago-COVID-Zipcode` datasets, we evaluate the allocation results of our proposed resource allocation framework at the zip-code level in the city of Chicago.

The first instance problem considers *Race* as the sensitive attribute, and the associated *Fair-Diverse* model captures inequalities across different *racial subgroups*. The results for the top 15 populated areas are reported in Table 1. The baseline values for ϵ_f and ϵ_d are derived from the *alpha=0.5* model and are equal to 0.24 and 0.007. Note that we do not use the tuning algorithm for this *alpha=0.5* model. For *Fair-Diverse* model ϵ_f and ϵ_d are both set to be 0.025 to decrease the fairness gap compared to the baseline value. We obtained a tuned range of [0.54 0.86] for α in this case (midpoint=0.70). Looking at Table 1, the area associated with zip-code "60639" for instance, receives a lower number of vaccines using *Diverse-only* model but higher in both *alpha=0.5* and *Fair-Diverse* model ($\alpha = 0.70$) due to having higher total exposed population. In contrast, the *Fair-only* closes the fairness gap to the full extent ($\epsilon_f = 0$), and as a result, obtains an extreme allocation solution in which only a few areas (zip-codes) receive vaccines. Undoubtedly, this could not be a desirable allocation solution under certain fairness and diversity requirements. Since the table represents the top 15 populated areas, we cannot observe all areas with positive allocation using *Fair-only* model.

The second instance problem considers *Age* as the sensitive attribute, and the associated *Fair-Diverse* model captures inequalities across different *age groups*. The results for the top 15 populated areas are reported in Table 2. The baseline values for ϵ_f and ϵ_d are derived from the *alpha=0.5* problem

Zipcode	Total population	Exposed Population	alpha=0.5	alpha tuned (0.70)	Diverse-only	Fair-only
60629	113046	5802	10329	12125	9497	0
60618	91351	3652	7002	9798	7675	0
60623	91159	4815	8330	9777	7659	0
60639	89452	5145	8174	9594	7515	70901
60647	86586	3824	7912	9287	7274	0
60617	83590	3782	7638	8966	7023	0
60608	81930	3909	6280	8788	6883	0
60625	78085	2945	5986	8031	6560	0
60634	73894	2491	5664	4491	6208	0
60620	72094	2744	6587	7733	6057	0
60641	70992	2973	6455	7614	5964	0
60614	66485	1602	5096	4040	5586	0
60657	65841	1591	5047	4001	5532	0
60640	65412	2047	5014	3975	5496	0
60609	64420	3157	5886	6909	5412	0

Table 1: Resource Allocation (Racial groups): Top 15 populated areas- Chicago

and are equal to 0.012 and 0. Next, we run the binary search algorithm to tune the α value and find a feasible solution for the *Fair-Diverse* model. In this case, ϵ_f and ϵ_d are both set to be 0.003. We obtained a tuned range of [0.57 1] for α in this case (midpoint=0.78). As mentioned previously, the *Diverse-only* model assigns vaccines to areas only based on the total population, and the *Fair-only* model obtains an extreme allocation solution in which only a few areas (zip-codes) receive vaccines. Therefore, none of these models are capable of delivering a fair and diverse allocation solution. Note that in this instance problem, *alpha=0.5* model is not doing any better than the *Diverse-only* model since the weight on the fairness component is not adequate to change the results (diversity-gap is dominant). This result can further reveal the necessity of the proposed tuning algorithm.

Zipcode	Total population	Exposed Population	alpha=0.5	alpha tuned (0.78)	Diverse-only	Fair-only
60629	113046	6367	9204	8989	9204	0
60618	91351	5478	7672	7492	7672	0
60623	91159	5084	7422	7249	7422	70300
60639	89452	5127	7367	7195	7367	0
60647	86586	5121	7312	7141	7312	0
60617	83590	5007	6931	7093	6931	0
60608	81930	4872	6904	6743	6904	0
60625	78085	4698	6569	6415	6569	0
60634	73894	4610	6219	6365	6219	0
60620	72094	4404	6009	6150	6009	0
60641	70992	4302	5955	5816	5955	0
60614	66485	4028	5735	5604	5735	0
60657	65841	4034	5730	5864	5730	0
60640	65412	4200	5644	5776	5644	0
60609	64420	3651	5264	5141	5264	0

Table 2: Resource Allocation (Age groups): Top 15 populated areas-Chicago

Finally, a notable instance occurs when we consider *gender* as the sensitive attribute and the *Fair-Diverse* model attempts to eliminate the unfairness between male and female subgroups. The results for the top 15 populated areas are reported in Table 3. The baseline values for ϵ_f and ϵ_d derived from the *alpha=0.5* problem and are both close to zero (6.04e-05 and 0). It is worth mentioning that this is because of the similar gender population distribution across different regions. Consequently, the fairness and diversity requirements can be satisfied even with *Diverse-only* model. We can still run the binary search algorithm to tune the α value and find a feasible solution for *Fair-Diverse* model by closing the fairness gap further. To do this, the ϵ_f and ϵ_d values in *Fair-Diverse* model should be set to 0 and 0.1. We obtained a tuned range of [0.92 1] for α in this case (midpoint=0.96). Looking at Table 3 and exposure rates (shown in Table 4), we notice that, in this specific instance problem, the exposed population size is, in actuality, aligned with the total population size in different areas. In other words, highly populated areas tend to have higher exposed populations as well. Therefore, the resulted allocations from *Diverse-only*, *alpha=0.5*, and *Fair-Diverse* models are very close and even equal in some cases.

Zipcode	Total population	Exposed Population	alpha=0.5	alpha tuned (0.96)	Diverse-only	Fair-only
60629	113046	5802	9520	9529	9520	0
60618	91351	3651	7695	7703	7695	0
60623	91159	4815	7697	7705	7697	0
60639	89452	5145	7555	7563	7555	0
60647	86586	3824	7295	7302	7295	0
60617	83590	3782	7033	7026	7033	0
60608	81930	3909	6914	6921	6914	79366
60625	78085	2944	6573	6579	6573	0
60634	73894	2490	6209	6203	6209	0
60620	72094	2743	6035	6029	6035	0
60641	70992	2972	5989	5995	5989	0
60614	66485	1602	5567	5562	5567	0
60657	65841	1591	5515	5521	5515	0
60640	65412	2047	5498	5504	5498	0
60609	64420	3156	5424	5430	5424	0

Table 3: Resource Allocation (Gender groups): Top 15 populated areas-Chicago

Figures 5 and 6 present the results for the top 15 populated regions in the Chicago City for `racial` and `age` instance problems. Note that in both instance problems, the total population equals the summation of all associated groups (e.g. Age groups) due to having unknown labels in the data. In Figure 5, for example, "60614" and "60634" regions receive less vaccines both under *Fair-Diverse* and *alpha=0.5* models due to having higher exposed population, Table 1. The *Diverse-only* does not consider the exposed population. Therefore, the associated allocations are higher for these regions. Besides, the allocation obtained from the tuned range of α is significantly different from the allocation obtained with the *alpha=0.5* model since the latter does not satisfy the fairness requirement ϵ_f. Moving to another instance problem, Figure 6 represents the allocation results for the age instance problem. Based on the plot, we can notice that the *alpha=0.5* and *Diverse-only* allocation solutions overlap. This can be justified by the fact that the 50% emphasis on fairness is not sufficient to close the age subgroups disparities, and it requires a higher α value as we obtained through the tuning algorithm. That being said, the tuned α value, in this case, is 0.78, which is substantially higher than 0.5. For example, "60614" and "60609" regions receive less vaccines using *Fair-Diverse* model with α tuning since they have relatively lower exposed population, Table 2. For observing more interactive visualization tools, please check our newly created web application on the Chicago City datasets using Rshiney[6].

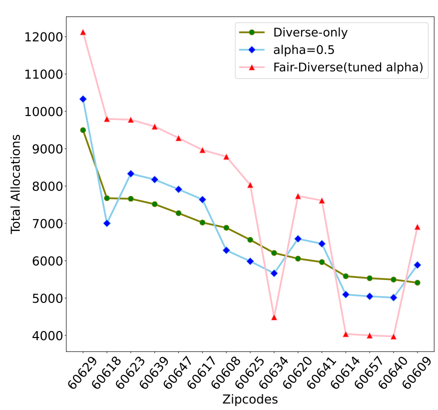


Figure 5: Resource Allocation (Racial groups): Top 15 populated areas-Chicago

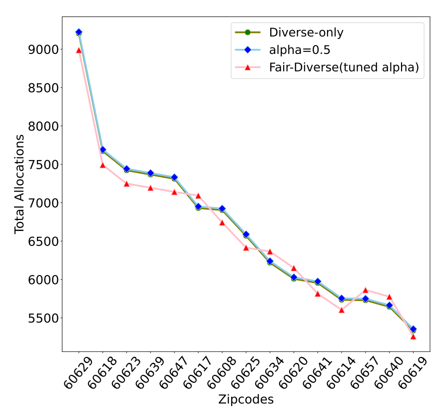


Figure 6: Resource Allocation (Age groups): Top 15 populated areas-Chicago

[6] https://nazanin.shinyapps.io/Fair_Resource_Allocation/

NewYork City (NYC)

Utilizing `Pop.`, and `NYC-COVID-Zipcode` datasets, we assess our proposed resource allocation framework at a zip-code level for New York City. In this problem, we consider the total number of vaccines available b to 500000 since NYC has a higher population than Chicago. Similar to the Chicago case study, the first instance problem for NYC considers *Race*, and the second instance consider *Age* as the sensitive attribute. We ignore the description of the *Gender* instance problem in this section (please see the appendix for the results).

The first instance problem considers *Race* to capture inequalities across different racial subgroups. The results for the top 15 populated areas in NYC are reported in Table 4. The ϵ_f and ϵ_d derived from the *alpha=0.5* model, are 0.10 and 0.0007 respectively. For *Fair-Diverse* model ϵ_f and ϵ_d are both set to be 0.017 to decrease the fairness gap compared to the baseline value. The tuned α range is between 0.66 and 0.69 (midpoint=0.67). Looking at Table 4, the area associated with zip-code "10467", receives a lower number of vaccines using *Diverse-only* model but higher numbers in both *alpha=0.5* and *Fair-Diverse* model ($\alpha = 0.67$) due to having higher total exposed population and more risks. In contrast, the *Fair-only* closes the fairness gap to the full extent ($\epsilon_f = 0$), and as a result, obtains an extreme allocation solution in which only a few areas (zip-codes) receive vaccines. As shown in Table 4, none of the top 15 populated areas will receive vaccines under this extreme condition. As a result, this could not be an applicable allocation solution.

The second instance problem considers *Age* to captures inequalities across different *age groups*. The results for the top 15 populated areas are reported in Table 5. The baseline values for ϵ_f and ϵ_d are derived from the *alpha=0.5* problem and are equal to 0.008 and 0 while the α value range is between zero and one. Next, we run the binary search algorithm to tune the α value and find a feasible solution for *Fair-Diverse* model with ϵ_f and ϵ_d both set to 0.003. We obtained a tuned range of [0.68 1] for α in this case (midpoint=0.84). As mentioned previously, the *Diverse-only* model assigns vaccines to areas merely based on the total population, and the *Fair-only* model obtains an extreme allocation solution in which only a few areas (zip-codes) receive vaccines. Therefore, these models are not capable of delivering a fair and diverse allocation solution. Note that in this instance problem, *alpha=0.5* model is not doing any better than the *Diverse-only* model since the weight on the fairness component is not adequate, which further reveals the necessity of the proposed tuning approach.

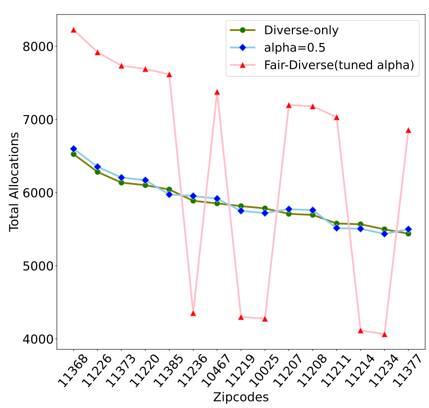


Figure 7: Resource Allocation (Racial groups): Top 15 populated areas-NYC

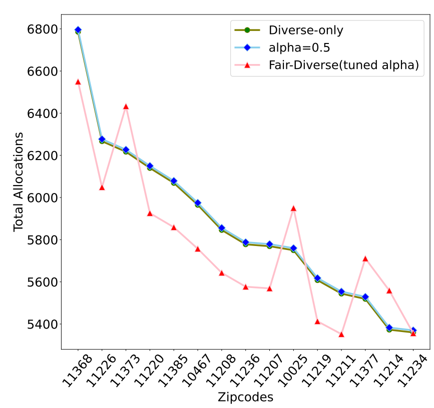


Figure 8: Resource Allocation (Age groups): Top 15 populated areas-NYC

Figures 7 and 8 present the results for the top 15 populated regions in the New York City for `racial` and `age` instance problems. Note that in both instance problems, the total population equals the summation of all associated groups (e.g. Age groups) due to having unknown labels in the data. In Figure 7, "11236" and "11219" zip-codes receive less vaccines both under *Fair-Diverse* and *alpha=0.5* models due to having lower exposed population, Table 4. The *Diverse-only* does not consider the exposed population. Therefore, the associated allocations are higher for these regions. Besides, the allocation obtained from the tuned range of α is significantly different from the allocation obtained with the *alpha=0.5* model since the latter does not satisfy the fairness requirement ϵ_f.

Zipcode	Total Population	Exposed Population	alpha=0.5	alpha_tuned	Diverse-only	Fair-Only
11368	101558	12156	6614	8226	6525	0
11226	97766	7186	6367	7918	6282	0
11373	95500	8765	6219	7735	6136	0
11220	94949	8798	6183	7690	6101	0
11385	94032	7941	5960	7616	6042	0
11236	91632	6051	5967	4354	5888	0
10467	91077	8504	5931	7377	5852	0
11219	90499	5798	5736	4300	5815	0
10025	90025	5950	5706	4277	5784	0
11207	88857	7579	5787	7197	5709	0
11208	88639	8445	5772	7179	5695	0
11211	86805	6049	5502	7031	5577	0
11214	86639	5843	5491	4116	5567	0
11234	85567	5208	5423	4065	5498	0
11377	84635	7242	5512	6855	5438	0

Table 4: Resource Allocation (Racial groups): Top 15 populated areas- NYC

Moving to another instance problem, Figure 8 represents the allocation results for the age instance problem. Based on the plot, we can notice that the *alpha=0.5* and *Diverse-only* allocation solutions overlap. This can be justified by the fact that the 50% emphasis on fairness is not sufficient to close the age subgroups disparities, and it requires a higher α value as we obtained through the tuning algorithm. That being said, the tuned α value, in this case, is 0.84, which is substantially higher than 0.5. For example, "11226" and "11211" regions receive less vaccines using *Fair-Diverse* model comparing with *Diverse-only* or *alpha=0.5* since they have relatively lower exposed population, Table 5.

Zipcode	Total Population	Exposed Population	alpha=0.5	alpha_tuned	Diverse-only	Fair-Only
11368	108904	10192	6786	6550	6786	0
11226	100574	9711	6267	6049	6267	0
11373	99772	9835	6217	6433	6217	0
11220	98531	9403	6140	5926	6140	0
11385	97405	9466	6069	5859	6069	0
10467	95723	9076	5965	5757	5965	0
11208	93812	8674	5846	5643	5846	0
11236	92721	9020	5778	5577	5778	0
11207	92591	8633	5769	5569	5769	0
10025	92284	9549	5750	5950	5750	0
11219	90004	8005	5608	5413	5608	75932
11211	88978	8286	5544	5352	5544	0
11377	88572	8808	5519	5711	5519	0
11214	86223	8725	5373	5559	5373	0
11234	86012	8536	5360	5356	5360	0

Table 5: Resource Allocation (Age groups): Top 15 populated areas- NYC

Baltimore City

Utilizing `Pop.`, and `BLT-COVID-Zipcode` datasets, we empirically tested our proposed resource allocation framework at the zip-code level. In this problem, we modify the total number of vaccines available $b = 100000$ since Baltimore has a relatively lower population than Chicago and New York City. However, similar to other case studies, the first instance problem considers *Race*, and the second instance considers *Age* as the sensitive attributes and ignore the description of the Gender instance problem (please see the appendix for the results).

The *Race* instance problem captures the inequalities across different racial subgroups. The results for the top 15 populated areas in Baltimore are reported in Table 6. The ϵ_f and ϵ_d derived from the *alpha=0.5* model, are 0.046 and 0.016 respectively. For *Fair-Diverse* model ϵ_f and ϵ_d are both set to be 0.025 to decrease the fairness gap compared to the baseline value. The tuned α range is between 0.57 and 1 (midpoint=0.78). Looking at Table 6, the area associated with zip-code "21230", receives a higher number of vaccines using *Diverse-only* model but lower numbers in both *alpha=0.5* and *Fair-Diverse* model ($\alpha = 0.78$) due to having lower total exposed population and less risk level. On the other hand, the *Fair-only* closes the fairness gap ($\epsilon_f = 0$), and obtains an extreme allocation solution in which only a few areas (zip-codes) receive vaccines. As shown in Table 6, four regions among the top 15populated areas receive vaccines under this extreme condition. As a result, this could not be an applicable allocation solution.

The *Age* instance problem captures inequalities across different *age groups*. The results for the top 15 populated areas are reported in Table 7. The baseline values for ϵ_f and ϵ_d are derived from the *alpha=0.5* problem and are equal to 0.014 and 0 while the α value range is between zero and one. Next, we run the binary search algorithm to tune the α value and find a feasible solution for *Fair-Diverse* model with ϵ_f and ϵ_d both set to 0.007. We obtained a tuned range of [0.76 1] for α in this case (midpoint=0.88). As mentioned previously, the *Diverse-only* model assigns vaccines to areas merely based on the total population, and the *Fair-only* model obtains an extreme allocation solution in which only four areas (zip-codes) receive vaccines. Therefore, these models are not capable of delivering a fair and diverse allocation solution. Note that in this instance problem, *alpha=0.5* model is not doing any better than the *Diverse-only* model since the weight on the fairness component is not adequate, which further reveals the necessity of the proposed tuning approach.

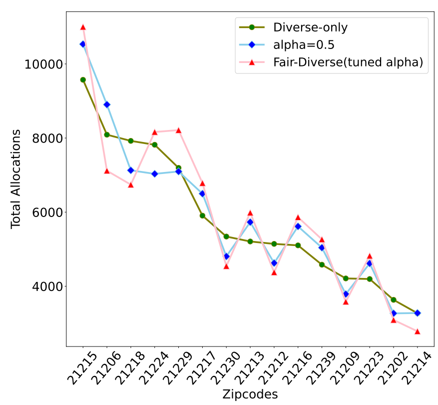


Figure 9: Resource Allocation (Racial groups): top 15 populated areas-Baltimore

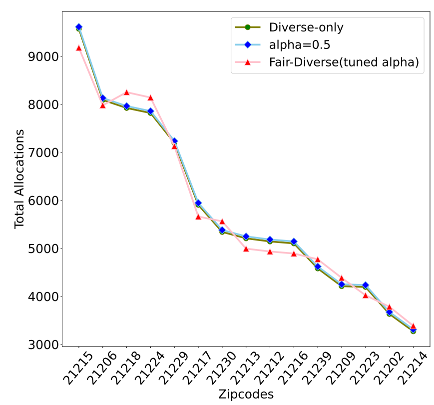


Figure 10: Resource Allocation (Age groups): top 15 populated areas-Baltimore

Zipcode	Total Population	Exposed Population	alpha=0.5	alpha_tuned	Diverse-only	Fair-Only
21215	60161	4893	10534	11000	9572	0
21206	50846	3944	8903	7118	8090	0
21218	49796	3739	7128	6742	7923	0
21224	49134	3782	7033	8163	7818	14868
21229	45213	3555	7097	8214	7194	12266
21217	37111	3050	6498	6785	5905	0
21230	33568	2188	4805	4545	5341	0
21213	32733	2722	5731	5985	5208	0
21212	32322	2176	4626	4376	5143	0
21216	32071	2725	5615	5864	5103	49078
21239	28793	2303	5041	5265	4581	0
21209	26465	1494	3788	3583	4211	11242
21223	26366	2143	4616	4821	4195	0
21202	22832	1724	3268	3091	3633	0
21214	20564	1510	3277	2784	3272	0

Table 6: Resource Allocation (Racial groups): Top 15 populated areas- Baltimore

Figures 9 and 10 show the results for the top 15 populated regions in the Baltimore City for `racial` and `age` instance problems. Note that in both instance problems, the total population equals the summation of all associated groups (e.g. Age groups) due to having unknown labels in the data.

In Figure 9, and zip-codes receive less vaccines both under *Fair-Diverse* and *alpha=0.5* models due to having lower exposed population, Table 6. The *Diverse-only* does not consider the exposed population. Therefore, the associated allocations are higher for these regions. Besides, the allocation obtained from the tuned range of α is significantly different from the allocation obtained with the *alpha=0.5* model since the latter does not satisfy the fairness requirement ϵ_f.

Moving to the next instance problem, Figure 10 represents the allocation results for the age instance problem. Based on the plot, we can notice that the *alpha=0.5* and *Diverse-only* allocation solutions overlap. This can be justified by the fact that the 50% emphasis on fairness is not sufficient to close the age subgroups disparities, and it requires a higher α value as we obtained through the tuning algorithm. That being said, the tuned α value, in this case, is 0.88, which is substantially higher than 0.5. For example, "21217" and "21223" regions receive less vaccines using *Fair-Diverse* model comparing with *Diverse-only* or *alpha=0.5* since they have relatively lower exposed population, Table 7.

Zipcode	Total Population	Exposed Population	alpha=0.5	alpha_tuned	Diverse-only	Fair-Only
21215	60161	4943	9572	9177	9572	13765
21206	50846	4211	8090	7978	8090	0
21218	49796	4213	7923	8251	7923	0
21224	49134	4285	7818	8141	7818	0
21229	45213	3762	7194	7126	7194	0
21217	37111	3040	5905	5661	5905	0
21230	33568	2961	5341	5562	5341	0
21213	32733	2661	5208	4993	5208	7954
21212	32322	2682	5143	4934	5143	0
21216	32071	2614	5103	4892	5103	0
21239	28793	2416	4581	4771	4581	0
21209	26465	2230	4211	4385	4211	2268
21223	26366	2150	4195	4022	4195	0
21202	22832	2048	3633	3783	3633	0
21214	20564	1727	3272	3386	3272	26707

Table 7: Resource Allocation (Age groups): Top 15 populated areas- Baltimore

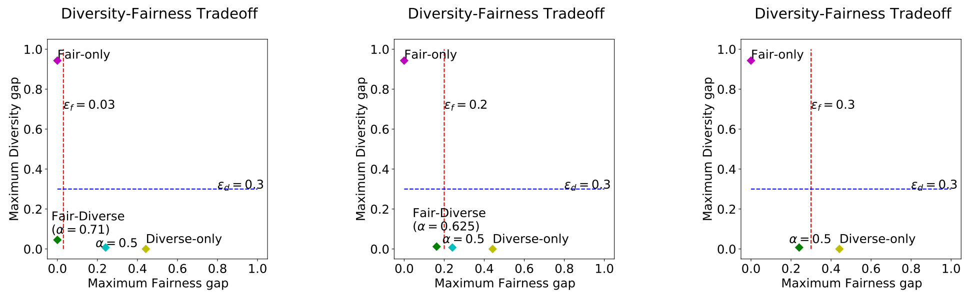
As we discussed in § 3.1, the allocation solution that is obtained from $P2$ does not necessarily satisfy the fairness and diversity requirements (under any α value). To demonstrate the performance of the tuning algorithm, which always returns a range for α under which the optimal solution of $P2$ is feasible, we now study the impact of ϵ_f and ϵ_d on the optimal solution under different models using the city of Chicago case study.

The plots in 11 are based on the *racial* instance problem. These figures reveal that under any fairness and diversity requirements (given ϵ_f and ϵ_d values) the α tuning algorithm returns a feasible solution for *Fair-Diverse* model. Note that, this is not the case for other models (*Fair-only*, *Diverse-only* and *alpha=0.5*) as it can be observed from the Figures 11(a) and 11(b). In other words, the optimal solutions obtained from the *Diverse-only* and *alpha=0.5* models does not satisfy the fairness requirement ($\epsilon_f \leq 0.03$ and $\epsilon_f \leq 0.2$), and the solution obtained from the *Fair-only* model fails to satisfy the diversity requirement ($\epsilon_d \leq 0.3$) in Figures 11(a) and 11(b). However, if we relax the fairness requirement to $\epsilon_f \leq 0.3$, Figure 11(c), the *alpha=0.5* model can indeed achieve a feasible solution. It is worth mentioning that the diversity requirement is easier to achieve compared to the fairness requirement due to the larger inherent disparities in exposed population.

4.5 Price of Fairness

In this section, we will compare the fairness and diversity gaps under different models and population subgroups to discuss the price of fairness (PoF). The results are obtained based on the aforementioned racial group instance problem for the city of Chicago. We present the results under tuned α value (0.71) in this part.

Firstly, Figure 12 reveals that the *Fair-Diverse* model reduces fairness and diversity gaps more compared to *Diverse-only* and *Fair-only* models. Although the *Diverse-only* model eliminates the diversity gap, it fails to decrease the fairness gap. Similarly, the *Fair-only* model eliminates the



(a) (b) (c)

Figure 11: Impact of the ϵ_f and ϵ_d on the diversity-fairness trade-off

fairness gap, but it fails to decrease the diversity gap. Moreover, Figure 13 shows some considerable reduction in the gaps across different population subgroups (racial groups) using the *Fair-Diverse* model. The comparison between the results with the *Diverse-only* and the uniform allocation solution reveals the necessity of the *Fair-Diverse* model in closing the gaps, and therefore, reducing the disparities across different population subgroups.

Lastly, Figure 14 illustrates the trade-off between fairness and diversity gaps considering different α values. Note that the α values represent the midpoint of the feasible range. We can observe that increasing the α value, which is the weight on the fairness component in $P2$, decreases the fairness gap as expected. However, the diversity gap will increase due to the fairness-diversity trade-off.

Finally, as discussed in § 3.3, the Price of Fairness (PoF) can be evaluated using the difference of the fairness gap from the optimal solution of P2 that is obtained with and without any fairness constraints. POF could be defined as the fraction of the fairness gap that is obtained from the (*Diverse-only*) allocation to the allocation solution based on the *Fair-Diverse* model. If an allocation solution decreases the fairness gap more than the *Fair-Diverse* model, the POF is less than one. Otherwise, PoF is > 1, and we will need to balance the fairness and diversity objectives to decide on which allocation to choose. In the case of the Racial instance problem with $\alpha = 0.71$, the PoF equals $\frac{0.4419}{0.025} = 17.67$, which is significantly larger than 1.

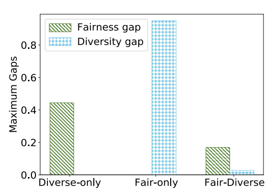


Figure 12: Diversity and Fairness gaps in different models

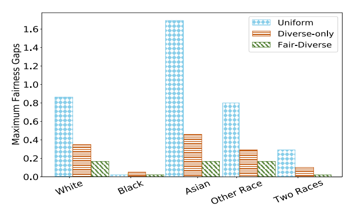


Figure 13: Fairness gaps across different population subgroups

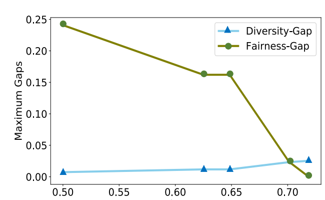


Figure 14: Fairness and Diversity gaps based on different α values

5 Discussion

In this section, we discuss the scope and limitation of the proposed fair-diverse resource allocation method, which is introduced in §2. Through the three case studies in §4.1, we can see that the proposed method applies to the early stage of medical resource allocation when the resource is still considered to be scarce. As long as the required data are available and accurate, the proposed allocation scheme can be applied. We want to point out some of the limitations that the proposed approach can be further improved upon.

First, the exposure rate estimation can be improved. Recall that in §2 the COVID-19 exposure rate for each intersectional group is denoted by $P(e|g_i)$. However, the primary challenge while estimating COVID-19 exposure rates for intersectional subgroups (e.g., <black,female,age30-39>), is the lack of intersectional population data in each Zipcode, denotes by S_{ij} in §2. Given the availability of the required data format, one could use a Poisson regression model to obtain the intersectional exposure rates. Second, the proposed method does not consider the profession of residents in prioritizing the sub-groups. In the U.S., during the early vaccination process, priority was given to the sub-groups based on age, underlying health conditions, and professions. Front-line workers such as health care workers, grocery employees, K-12 educators, etc., whose works are essential to normal social functions were among the first one or two batches of vaccine receivers. In our work, we have estimated the COVID-19 exposure rate from the daily infected cases. It indirectly considers the residents' profession since front-line workers are expected to have a larger chance of being exposed to the virus. But we are not able to directly include the profession of resident as a characteristic to define the sub-group of populations. It is mainly due to the lack of data with job descriptions for residents (or the percentage) in each Zipcode area. We plan to work with city officials or non-profit organizations with more detailed population data and further improve our proposed method.

Furthermore, estimating the intersectional population size could not be an appropriate implementation approach. Regarding the data mentioned above limitation issue, we solved the allocation problem for age, race, and gender as the sensitive attributes separately in §4.4. However, our model is able to return a fair-diverse allocation using the intersectional subgroups, given that the real intersectional population is available for each area. Applying the same framework to the intersectional data could be a future direction of this work.

Another future direction with respect to COVID-19 exposure rates could incorporate a learning approach to obtain the rates. Several successful time-series techniques such as traditional ARIMA models or more novel approaches such as RNN-LSTM can be applied to the historical data. Time-series analysis would enable one to predict more precise exposure rates for future time windows.

Moreover, we only considered a single treatment, mainly vaccination. However, the proposed approach can be generalized to incorporate multiple treatments. Our proposed fair-diverse allocation can be utilized at different stages of a pandemic considering the updated exposure ratio.

In this paper, we mainly focused on the resource allocation to the centers and facilities. Although designing allocation policy at a granular level of individuals from each center or facility is also critical, high-level policies have been shown to be more effective and feasible for deployment than individual levels ones. There are multiple barriers to the individual-level deployment of the guidelines. For example, vaccine hesitancy [22,57,58] is a widespread problem for many people, and enforcing it could violate social values such as Freedom of choice(FoC) [21,52]. Moreover, designing strategies at the individual level requires an extensive amount of data and medical considerations as vaccines(e.g., COVID19 vaccines) can cause a severe allergic reaction, and people with pre-existing allergies should avoid taking them [19,29,50].

Last but not least, applications of our proposed framework are beyond the allocation policy design use case. The solutions obtained for the allocation problem could help identify the vulnerable regions to incentive vaccination and for advertisement programs purposes. This will help to maximize the vaccination rate within higher-risk communities in the future.

6 Conclusion

In this paper, we propose the idea of fairness in scarce resource allocation problems (e.g., vaccine distribution) in terms of disparities across various population subgroups in different regions. To do so, we consider diversity and fairness components to design a Fair-Diverse allocation. We first formulate a general multi-objective (MO) problem $P1$, and propose the weighted sum method with the LP relaxation to simplify it to $P2$. We then solve the LP-relaxed problem, $P2$, based on the fairness and diversity requirements as described in § 3.1. For this purpose, we propose a binary search approach, Algorithm 2, to find an optimal range for the trade-off parameter α in $P2$ such that the obtained solution is feasible.

Moreover, we have empirically analyzed our proposed methodologies in §4 using COVID-19 datasets in three major and segregated US cities (New York City, Chicago, and Baltimore). We designed three instance problems for each city based on different demographic attributes, race, age, and gender. We then implemented our fair-diverse model and compared it with other models (Diverse-only, Fair-only, and $\alpha = 0.5$) to investigate the fairness criteria in the solution and highlight the necessity behind our approach. We have also discussed the fairness and diversity requirements, ϵ_d and ϵ_f, and compared the Fair-Diverse allocation with other allocation solutions under different thresholds. Lastly, we have examined the price of fairness based on the associated gaps across different models.

In brief, our empirical results reveal the paramount role of fairness criteria in decision-making problems involving scarce resource allocation (e.g., a vaccine allocation). While certain minorities and population groups are more vulnerable to the COVID-19 virus, a Diverse-only (population-based) vaccine allocation can lead to higher fatality rates by neglecting the vulnerability of various population subgroups. We require to ensure a fair and diverse vaccine allocation to induce lower mortality rates across different regions. That is, we aim to find a *decent balance* between the diversity and fairness measures in other geographical areas and allocate the resources accordingly.

7 Acknowledgements

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APPENDIX

A Proof of Proposition 1

Proposition 1. *Let be* $X' = \{x | \mathcal{D}(x) < \epsilon_d, \mathcal{F}(x) < \epsilon_f, \sum_{j \in M} x_j = b, x_j \geq 0\}$*:*

1. *If* $X' \neq \emptyset$*: given* $x^* \in X'$*,* $\exists \alpha^*$ *such that* x^* *is an optimal solution of* $P2$*.*
2. *If* $\nexists\, \alpha^*$ *such that* x^* *is an optimal solution of* $P2$ *that satisfies both conditions* $\mathcal{D}(x) < \epsilon_d$ *and* $\mathcal{F}(x) < \epsilon_f$*, then* $X' = \emptyset$*.*

Proof. We first argue that the feasibility problem is equivalent to the following optimization model:

$$
\begin{aligned}
&P':\\
&\quad \min \mathcal{D}(x)\\
&\quad \text{s.t.}\\
&\quad \mathcal{F}(x) = \epsilon\\
&\quad \mathcal{D}_j^+(x) \leq \mathcal{D}(x), \quad \forall j \in M\\
&\quad \mathcal{D}_j^-(x) \geq \mathcal{D}(x), \quad \forall j \in M\\
&\quad \mathcal{F}_i^+(x) \geq \mathcal{F}(x), \quad \forall i \in I\\
&\quad \mathcal{F}_i^-(x) \leq \mathcal{F}(x), \quad \forall i \in I\\
&\quad \sum_{j \in M} x_j = b\\
&\quad x_j \geq 0, \qquad \forall j \in M
\end{aligned}
$$

Consider $\epsilon \in [0, \epsilon_{\mathcal{F}}]$. Let x^* be the optimal solution of P'. Hence, $\mathcal{F}(x^*) = \epsilon \leq \epsilon_{\mathcal{F}}$ satisfies the fairness constraint. Now, if $\mathcal{D}(x^*) \geq \epsilon_{\mathcal{D}}$, we conclude that for ϵ there does not exist a feasible solution that satisfies the diversity constraint. Otherwise, that feasible solution would minimize P'.

Introducing a Lagrangian multiplier λ, we now define the dual Lagrangian transformation of P' to be:

$$
\begin{aligned}
&P'':\\
&\quad \min \mathcal{D}(x) + \lambda(\mathcal{F}(x) - \epsilon)\\
&\quad \text{s.t.}\\
&\quad \mathcal{D}_j^+(x) \leq \mathcal{D}(x), \quad \forall j \in M\\
&\quad \mathcal{D}_j^-(x) \geq \mathcal{D}(x), \quad \forall j \in M\\
&\quad \mathcal{F}_i^+(x) \geq \mathcal{F}(x), \quad \forall i \in I\\
&\quad \mathcal{F}_i^-(x) \leq \mathcal{F}(x), \quad \forall i \in I\\
&\quad \sum_{j \in M} x_j = b\\
&\quad x_j \geq 0, \qquad \forall j \in M
\end{aligned}
$$

Now, we argue that given $x^* \in X'$ there exists an α^* such that x^* is an optimum of $P2$. Let λ^* be the multiplier when x^* is an optimum of P'', then $\forall x'' \neq x^*$:

$$
\begin{gathered}
\mathcal{D}(x^*) + \lambda^*(\mathcal{F}(x^*) - \epsilon_{\mathcal{F}}) \leq \mathcal{D}(x'') + \lambda^*(\mathcal{F}(x'') - \epsilon_{\mathcal{F}})\\
\implies \mathcal{D}(x^*) + \lambda^*\mathcal{F}(x^*) - \lambda^*\epsilon_{\mathcal{F}} \leq \mathcal{D}(x'') + \lambda^*\mathcal{F}(x'') - \lambda^*\epsilon_{\mathcal{F}}\\
\implies \mathcal{D}(x^*) + \lambda^*\mathcal{F}(x^*) \leq \mathcal{D}(x'') + \lambda^*\mathcal{F}(x'')
\end{gathered}
$$

Thus, x^* is an optimum of $P2$ where $\lambda^* = \frac{(1-\alpha)}{\alpha}$.

On the other hand, if $\exists \alpha^*$ under which x^* is optimal of $P2$, and $\mathcal{D}(x^*) < \epsilon_d$ and $\mathcal{F}(x^*) < \epsilon_f$, then $x^* \in X'$ and $X' \neq \emptyset$. This means if such α^* does not exist, there is no feasible solution under ϵ_d and ϵ_f. $\square$

B Proof of Proposition 2

Proposition 2. *Let x_1^* and x_2^* be optimum solutions of $P2$ given α_1 and α_2, respectively. It can be shown that if $\alpha_1 \leq \alpha_2$, then $\mathcal{F}(x_2^*) \leq \mathcal{F}(x_1^*)$ and $\mathcal{D}(x_1^*) \leq \mathcal{D}(x_2^*)$.*

Proof. Let $\beta = \frac{\alpha}{(1-\alpha)}$. Given x_1^* and x_2^* corresponding to β_1 and β_2, where $\beta_2 < \beta_1$:

$$\mathcal{D}(x_1^*) + \beta_1 \mathcal{F}(x_1^*) \leq \mathcal{D}(x_2^*) + \beta_1 \mathcal{F}(x_2^*)$$
$$\mathcal{D}(x_2^*) + \beta_2 \mathcal{F}(x_2^*) \leq \mathcal{D}(x_1^*) + \beta_2 \mathcal{F}(x_1^*)$$

Adding the above Equations, we will have:

$$\beta_1 \mathcal{F}(x_1^*) + \beta_2 \mathcal{F}(x_2^*) \leq \beta_1 \mathcal{F}(x_2^*) + \beta_2 \mathcal{F}(x_1^*)$$
$$\implies (\beta_2 - \beta_1)(\mathcal{F}(x_2^*) - \mathcal{F}(x_1^*)) \leq 0$$

which implies $\mathcal{F}(x_2^*) \leq \mathcal{F}(x_1^*)$. The monotonicity proof for $\mathcal{D}(x)$ is the same with $\beta = \frac{(1-\alpha)}{\alpha}$. If $\alpha_2 < \alpha_1$ then $\mathcal{D}(x_1^*) > \mathcal{D}(x_2^*)$. □

C Exposure Rates

Demographic Groups	Exposure rates
Female	0.095383
Male	0.09804
Age_0_4	0.035865
Age_5_12	0.057509
Age_13_17	0.061328
Age_18_24	0.081399
Age_25_34	0.106335
Age_35_44	0.107127
Age_45_54	0.106302
Age_55_64	0.126374
Age_65_74	0.130066
Age_75+	0.173563
Asian/Pacific-Islander	0.070976
Black/African-American	0.063082
Hispanic/Latino	0.207524
White	0.049382

Figure 15: Exposure Rates-New York

Demographic Groups	Exposure rates
White	0.050315
Black or African American	0.083406
American Indian	0.06383
Asian	0.020939
Hawaiian or pacific Islander	0.030405
Other race	0.213587
Male	0.082152
Female	0.085886
Age_0_9	0.042091
Age_10_19	0.056692
Age_20_29	0.092926
Age_30_39	0.120977
Age_40_49	0.087818
Age_50_59	0.086137
Age_60_69	0.099737
Age_70_79	0.087173
Age_80+	0.07707

Figure 16: Exposure Rates-Baltimore