Topological Acoustics

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Introduction

The field of topology studies the properties of geometric objects that are preserved under continuous deformations, for example, without cutting or gluing. A cup with a handle is topologically equivalent to a donut (or a bagel if you live in New York) because one shape can be deformed into the other while preserving their common *invariant* hole. Exotic topological shapes, such as vortices, knots, and mobius strips, can be globally analyzed using the mathematical tools offered by topology. The connection between topology and acoustics may appear far-fetched, yet recent developments in the field of condensed matter physics and quantum mechanics have been inspiring exciting opportunities to manipulate sound in new and unexpected ways based on topological concepts.

The field of topological acoustics has been inspired by the discovery in condensed matter of *topological insulators*, a class of materials that support highly unusual electrical conduction properties. Like conventional semiconductors, topological insulators are characterized by a gap in electron energy (bandgap) that separates their valence and the conduction bands. For electron energies within this bandgap, topological insulators are not electrically conductive in their bulk, hence their name. However, any finite sample of such materials necessarily supports conduction currents along its physical boundaries; the topological features of the valence and conduction bands ensure the existence of these boundary currents. Therefore, these currents exist independent of the boundary shape or the presence of continuous defects and imperfections that do not affect the bandgap topology. Knowing this feature, we can predict the existence of conduction currents flowing along the boundaries of any finite sample of such materials by simply analyzing the topological features of the bands of the infinite medium (Thouless et al., 1982; Haldane, 1988). As a result, these currents show an unusual robustness to defects and disorder. The electron spin plays a fundamental role in defining the topological response of these materials.

In recent years, there has been a strong interest in exploring analogies for these topological concepts in other realms of physics, in particular, in the context of optics (Raghu and Haldane, 2008; Wang et al., 2009) and acoustics (Fleury et al., 2016; Zangeneh-Nejad et al., 2020). Given that sound does not possess an intrinsic spin, in this quest the role of the electron spin is replaced by the notion of *acoustic pseudospins*. These pseudospins include angular momentum (Fleury et al., 2014), geometrical asymmetries (Xiao et al., 2015; Ni et al., 2018), structured space- and time-dependent material properties (Trainiti et al., 2019; Darabi et al., 2020), and asymmetric nonlinearities (Boechler et al., 2011; Hadad et al., 2018).

These explorations have been enabling new opportunities to route sound in novel and unintuitive ways. For example, topological sound can propagate only in one direction (forward, not backward), and it can take sharp turns following the arbitrary boundaries of an acoustic material just like the boundary currents of topological insulators. These exotic propagation modalities are unaffected by the presence of defects or imperfections that sound may encounter along the way, for example, in the form of localized scatterers or material heterogeneities.

Figure 1a shows one example of an acoustic topological insulator formed by an ordered array of subwavelength resonators whose properties are modulated in space and time with precise patterns to impart angular momentum (Fleury et al., 2016). As a result of the interplay between the array geometry and the angular momentum imparted by the modulation, topological sound propagation is achieved through the pressure fields that travel unidirectionally along the array boundaries (see **Figure 1a**).

In recent years, topological sound has expanded its realms, leading to the exploration of topological features not only in the bands of periodic structures, like the one in **Figure 1a**, but also in real space and parameter space. For example, **Figure 1b** shows the evolution of the eigenvalues of a system as two generic degrees of freedom or

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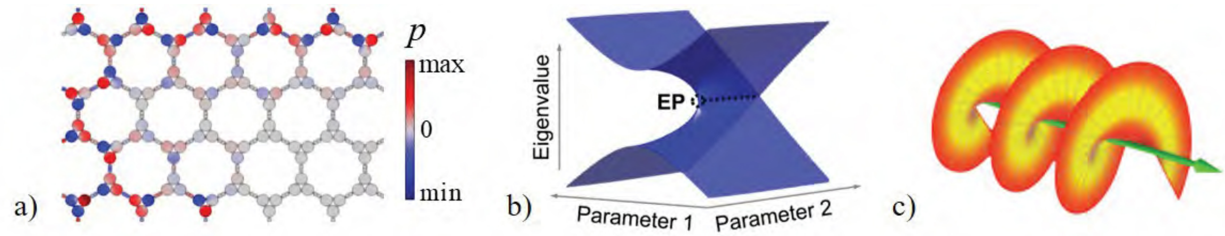


Figure 1. *Exotic acoustic phenomena enabled by topological concepts.* ***a:*** *Pressure field* (p) *distribution in a phononic topological insulator formed by an array of subwavelength resonators whose properties are modulated in space and time to impart a pseudospin in the form of angular momentum. The result is a one-way, edge-bound propagation of acoustic pressure (Fleury et al., 2016).* ***b:*** *Topological features around an exceptional point (EP) in the space formed by changing two independent parameters in an acoustic system, for example, a pair of coupled resonators whose resonant features can be controlled by changing two geometrical parameters (Miri and Alù, 2019).* ***c:*** *Pressure distribution (**yellow**, larger pressure fields) forming an orbital angular momentum sound beam. See text for detailed explanations.*

as parameters controlling the system are changed. This may correspond to two coupled acoustic cavities, which we can independently tune through geometric changes. Through proper design, the coupled cavity system can support an *exceptional point* (EP) in the space spanned tuning the two geometric parameters. At the EP, the eigenvalues of the system and the corresponding eigenmodes coalesce and become degenerate. As a result of this degeneracy, the system effectively loses one dimension. This singularity is associated with highly nontrivial topological properties (Xu et al., 2016) that can again provide unusual robustness of the response and at the same time offer opportunities for sensing (Shi et al., 2016; Miri and Alù, 2019).

Finally, topological features can also emerge in real space. **Figure 1c** shows an example of sound propagation with a nonzero orbital angular momentum (OAM) and the pressure distribution of an OAM sound wave traveling in free space. A carefully controlled array of sound emitters can emit such a vortex sound beam whose acoustic phase fronts are characterized by a nonzero topological charge, which can be leveraged to enhance the channel capacity in multiplexing applications and for robust sound propagation (Wang et al., 2018). In this article, we dive deeper into a few applications afforded by topological sound that may be of interest to the acoustics community at large.

Applications

Topological Sound Transport Based on Pseudospin Bias

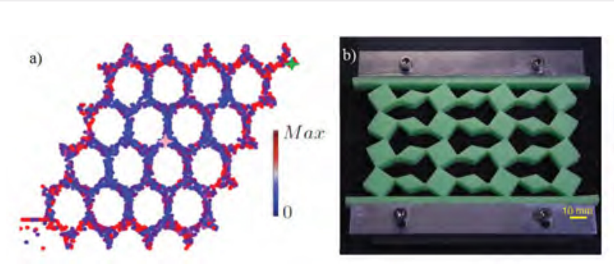
Acoustic waveguides are inherently prone to disorder and imperfections that impact the quality and efficiency of sound transport. Undesired back reflections and scattering can cause interference and distortions that impact several applications. Topological sound has been opening new opportunities for robust information transfer, multiplexing and processing, and data storage and manipulation. The simplest form of pseudospin to enable topological sound relies on geometrical asymmetries, for example, an acoustic array of subwavelength resonators with carefully tailored asymmetries act like a spin on sound waves (Ni et al., 2018). The resulting devices are passive and support topological boundary sound waves somewhat robust to disorder. Their main limitation stems from the fact that these acoustic topological insulators obey time-reversal symmetry, requiring that for any given wave supported in a certain direction and characterized by one pseudospin, the structure also supports an oppositely propagating wave with a reversed pseudospin. In the ideal case, the two modes are orthogonal to each other, but when disorder and imperfections are considered, their asymmetry may couple the two, limiting the overall robustness.

In contrast, topological sound enabled by pseudospins that break time-reversal symmetry, such as angular-momentum bias (Khanikaev et al., 2015) or rotating spatiotemporal modulation patterns (**Figure 1a**) (Fleury et al., 2016; Darabi et al., 2020), provide a stronger form of topological robustness because the corresponding boundary waves are truly unidirectional. The absence of such backward modes and of bulk modes ensures truly robust one-way boundary sound propagation, irrespective of the form of disorder and imperfections.

Figure 2a shows measurements on a practical example of this type of topological insulator for elastic waves, realized by electrically controlling a two-dimensional array of piezoelectric patches, similar to the design in **Figure 1a**, with electrical modulation signals suitably varying in space and time to impart a form of synthetic rotation that induces the desired pseudospin and breaks reciprocity (Darabi et al., 2020). **Figure 2a** shows the measured displacement extracted with a laser vibrometer, demonstrating that signals travel unidirectionally along the array boundaries.

Nonlinearities combined with geometrical asymmetries can also support pseudospins supporting nontrivial topological sound (Hadad et al., 2018). Although these systems are passive and obey time-reversal symmetry (as long as the nonlinearity is instantaneous), the combination of nonlinearities and geometrical asymmetries breaks reciprocity and enables unidirectional sound transport along the boundaries. An extreme example, seen in **Figure 2b**, shows a mechanical metamaterial made from a three-dimensional printed polymer, which supports a topological response at zero frequency. Mechanical nonlinearities are amplified at the small hinges connecting the diamond-shaped regions in **Figure 2b**, and the tilted elements introduce carefully tuned asymmetries that enable nonreciprocal transport of mechanical displacement when a force is applied to the structure from opposite sides. Interestingly, it can be shown that maximum nonreciprocity is achieved at the transition when the metamaterial changes the topological state, as controlled by the underlying geometrical asymmetries (Coulais et al., 2017). This metamaterial supports an unusual mechanical response; it strongly transmits displacement in one direction, but it dampens it in the opposite one.

Figure 2. ***a:*** *Elastic displacement measured with a laser vibrometer over a spatiotemporally modulated array of piezoelectric patches, demonstrating the emergence of a one-way topological boundary propagation of sound (Darabi et al., 2020).* ***b:*** *A topological mechanical metamaterial made of a three-dimensional printed elastic polymer based on asymmetric nonlinearities (Coulais et al., 2017).*



Radio-Frequency Technology Based on Topological Sound

The pseudospins discussed previously can robustly break reciprocity, enabling fundamental functionalities for several electronics and electromagnetics technologies. For example, nonreciprocity can be used to isolate transmitter and receiver modules in our cell phones, an important functionality in modern communication systems to avoid interference between the strong transmitted signals and the stream of weak signals received from the cell phone tower (Kord et al., 2020). Acoustic signals offer several opportunities in this context because of their small wavelengths and lower rate of energy loss compared with electromagnetic components. These properties have been harvested, for example, in surface acoustic wave (SAW) or bulk acoustic wave (BAW) filters used to process the radio-frequency (RF) signal received by antennas in portable communication devices. However, current solutions rely on linear, passive, single-frequency devices that are unsuitable for the next generation of RF systems because they have a limited range of functionalities and require integration with ever more complex electronic components. More desirable features, ideal for agile communication systems with enhanced data rates and serving many users, target narrowband, low-loss filters, with a small size and a tunable center frequency.

Topological acoustics provides fertile ground to advance these technologies and address current technological challenges. For example, topological acoustics reduces scattering and enables devices approaching the theoretical limits of the intrinsic material losses. The natural robustness to defects associated with topological properties can decrease manufacturing costs, reducing the requirements for high fabrication tolerance. Topological properties also allow for phase control and latency, functionalities not available in acoustic devices today.

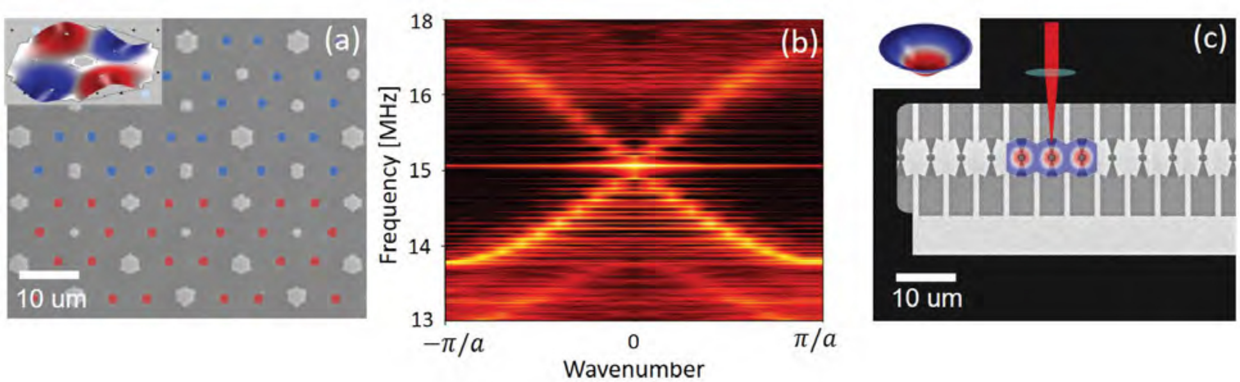
Nanoelectromechanical lattices (NEMLs) of resonators (**Figure 3a**) have demonstrated topologically robust waveguides using two-dimensional periodic arrangements of mechanically coupled, free-standing nanomembranes with circular clamped boundaries (Cha and Daraio, 2018). Such NEMLs form flexural phononic crystals with well-defined dispersion features, which can be used to tailor topological bandgaps (**Figure 3b**) offering a pathway toward the miniaturization of even more complex acoustic topological insulators, like the ones in **Figure 3b**. An additional advantage arising from these miniaturized acoustic devices is the possibility to transduce energy between different physical domains (Hackett et al., 2021). For example, nanomembranes can convert optical, magnetic, or electrical signals into mechanical strains and vice versa. These couplings can, in turn, be used to introduce nonlinearities (**Figure 2a**), modulation, and tunability of the fundamental resonant frequencies and the dispersion of the devices (Cha et al., 2018). The functionalities of these new acoustic devices can extend beyond conventional filtering, enabling complete networks and circuitry transporting pseudospins as a degree of freedom carrying information.

The potential of topological acoustics for RF communication systems opens a path toward a new technological landscape with lower energy consumption, smaller form factors, and larger bandwidths. Such opportunities also come with challenges, including design and fabrication complexity. Miniaturized topological acoustic metamaterials need to rely on advances in multimaterial fabrication capabilities to accomplish design flexibility, nonlinearity and dissipation control, and new strategies to impart the pseudospins of choice.

Information Science Based on Topological Sound

Sound is naturally used to encode and convey information. Human speech supported by sound carries information because our voice varies continuously in time and amplitude. Acoustic cues such as frequency and amplitude modulation allow communicators to derive meaning. Although this form of communication is based on analog signals, most information encoding, transmission, and processing techniques today are carried out in the digital domain, where signaling cues are restricted to discrete values. Modern digital information processing relies on electronic digital logic circuits, whose elementary units are Boolean logic gates and use the binary numbers 0 and 1 to implement Boolean functions such as the NOT, AND, and OR gates. Consequently, processing of sound-encoded

Figure 3. ***a:*** *Scanning electron microscope (SEM) image of a topological waveguide.* ***Red and blue dots****, lattice points of membranes with slightly different geometries. Flexural membrane motions (****inset****) were excited by simultaneously applying a DC/AC voltage (Cha and Daraio, 2018).* ***b:*** *Dispersion of topological edge modes experimentally measured in the geometry of panel* ***a****, where a is the lattice period.* ***Yellow*** *and* ***red****, modal resonances.* ***c:*** *SEM image of a nonlinear nanoelectromechanical lattice (NEML) (Cha et al., 2018).* ***Red arrow****, localized probe exciting the structure.* ***Inset:*** *geometrical nonlinearity induced by electrostatic softening.* ***Red*** *and* ***blue****, field maxima and minima, respectively. See text for further explanation.*



information in conventional electronic systems necessitates the conversion of sound into electrical signals.

Topological acoustics enables new forms of acoustic information processing that rely on integrated circuits. The development of acoustic metamaterials has provided physical platforms for the realization of acoustic Boolean logic gates. By exploiting their unique spectral, refractive, and phase properties, we can tailor the constructive or destructive interference of input and control acoustic waves to achieve Boolean functions of choice (Bringuier et al., 2011). Similarly, interference has been used to demonstrate Boolean logic gates in acoustic metamaterials (Zhang et al., 2016), and acoustic logic elements have been demonstrated in driven chains of spherical particles (Li et al., 2014).

Until recently, all these acoustic information processing elements have made use of the spectral and refractive properties of composite materials. Topological acoustics enables all-acoustic information processing that goes beyond the canonical attributes of sound, that is, frequency, wave vector, and dynamical phase (Deymier and Runge, 2017). The pseudospins enabling topological sound can be used as new degrees of freedom for information transport, realizing a wide range of acoustic Boolean logic elements with enhanced robustness and operating with low-energy requirements.

Interestingly, the features of topological acoustics offer avenues to go even beyond Boolean logic to pursue sound-based quantum-like information processing. In contrast to conventional computing, where a bit can be in a zero or one state, quantum computing processes a zero and a one at the same time by using a coherent superposition of states. Topological acoustic quantization, for example, based on two opposite pseudospins, coherence and correlations, can be harnessed to overcome stability and scalability challenges in current approaches to massive-data information processing, within the context of the second quantum revolution (Dowling and Milburn, 2003).

The pseudospin degrees of freedom of topological sound offer intriguing opportunities to achieve quantum-like phenomena like entanglement. Entanglement occurs when the state of a composite system composed of subsystems cannot be described in terms of the states of independent subsystems. Entangled superpositions of quantum states exhibit the attributes of *nonlocality* and *nonseparability*. Nonlocality is a unique feature of quantum mechanics that Einstein dubbed a "spooky action at a distance." Nonlocality allows, for example, two photons of light to affect each other instantly, irrespective of their distance of separation. Acoustic waves, because of their nonquantum nature, that is, their "classical character," are limited to local interactions. Nonetheless, nonseparability or *classical entanglement* can be realized in systems supporting classical waves, including sound.

An acoustic wave propagating in a cylindrical pipe can be represented by the product of three functions, each dependent on three degrees of freedom: one variable describing the pipe along its length and radial and angular variables characterizing the pipe through its cross section. In this sense, conventional guided acoustic waves are separable. A nonseparable acoustic wave, in contrast, is represented by a wave function that cannot be factored into a product of functions. Such waves can be created in externally driven systems composed of parallel arrays of waveguides coupled elastically and uniformly along their length (Hasan et al., 2019). These classically nonseparable states are constructed as a superposition of acoustic waves, each a product of a plane wave and a spatial degree of freedom analogous to OAM (**Figure 1c**). The plane wave portion describes an elastic wave propagating along the waveguides, and the spatial degree of freedom characterizes the amplitude and phase profile across the array of waveguides (**Figure 4a**). These nonseparable and therefore nonindependent degrees of freedom are the classical analogue of two correlated qubits. The amplitude of the nonseparable acoustic state is then analogous to the simplest examples of quantum entanglement of two qubits, known as Bell states in quantum mechanics. The displacement fields of the modes supported in the array of coupled waveguides are shown in **Figure 4**, **b-d**. Although **Figure 4**, **b and d**, shows separable OAM and plane wave states, characterized by symmetric patterns, **Figure 4c** shows a nonseparable and largely asymmetric linear combination of waves, with distinct momentum and OAM degrees of freedom. This type of acoustic superposition of states dramatically expands the opportunities for massive information storage and processing (Deymier et al., 2020).

Implementation of quantum-like algorithms necessitates the manipulation of nonseparable classical states, providing the parallelism required to achieve the goals of quantum information science (Jozsa and Linden, 2003). The analogies between quantum mechanics and classical wave physics have been recently exploited to emulate

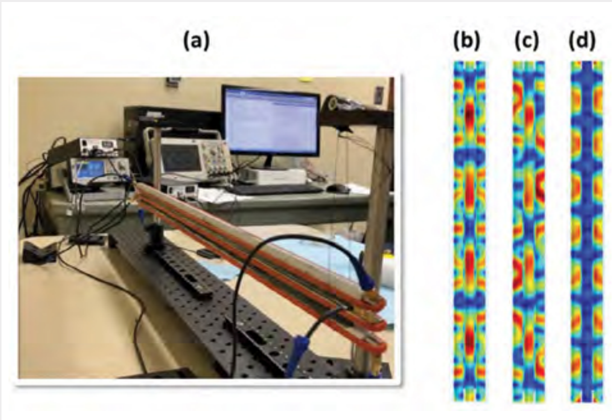


Figure 4. ***a:*** *Parallel array of three elastically coupled waveguides (aluminum rods glued with epoxy), driven at their ends by piezoelectric transducers (black structures at the bottom right of picture), designed to support acoustic analogues of quantum Bell states (Hasan et al., 2019).* ***b*** *and* ***d:*** *Color maps of the magnitude of the displacement field calculated using the finite-element method in the array of coupled waveguides in separable orbital angular momentum (OAM) and plane wave states.* ***Red and blue****, large and low magnitudes of displacement, respectively. Separability is visualized as symmetric color patterns across the array of rods.* ***c:*** *A nonseparable linear combination of waves, each with a different momentum and OAM degrees of freedom. The loss of symmetry in the color pattern across the array of rods is indicative of nonseparability.*

quantum phenomena in classical settings. For instance, optical metamaterials have been able to simulate a quantum algorithm with electromagnetic waves (Cheng et al., 2020). However, these simulations have relied on wave superposition and interference to realize algorithms that do not require entanglement. In contrast to electromagnetic waves, the stronger nonlinearities and robustness arising in topological acoustics offer unique opportunities to realize nonseparable states for algorithms harnessing entanglement to speed up computational tasks beyond Boolean operations.

Sensing with Topological Sound

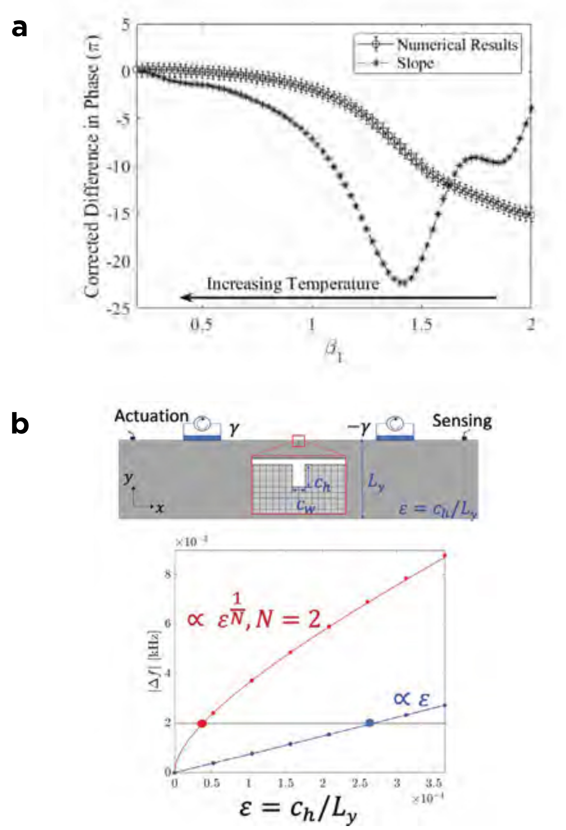
Topological acoustic attributes, such as pseudospin as well as amplitude, wavelength, and the frequency of sound, provide access to the global physical properties of a material or of a system. This allows transduction and encoding of information over a broad range of frequencies and implies the ability of observing features at multiple length scales and resolutions. Thus, the ability to observe and measure these attributes holds the promise for unparalleled sensitivity and resolution in acoustic-based sensing and imaging. For example, the emerging literature on the sensitivity to the *geometric phase* as a form of acoustic pseudospin is already making an impact in the areas of ecological and environmental sciences, aimed at measuring changes in temperature, density, or stiffness of the underlying medium. A recent study (Lata et al., 2020) has exploited the sensitivity to the geometric phase of ground-supported long-wavelength acoustic waves, such as seismic waves, in a forest environment, an acoustic medium where trees act as scatterers.

In the era of climate change, melting permafrost poses significant challenges to local Arctic communities. New technologies are needed to provide reliable ways to monitor and characterize the global properties of permafrost such as temperature and thawing state. This is vital to the management of natural and built environments in Arctic regions. Current techniques relying on data collected through boreholes and drilling sites produce rough permafrost maps and are not suitable for continuous monitoring. Also, remote sensing based on aerial and satellite imaging that indirectly measures ground characteristics through the reflection of electromagnetic waves, for example, using LiDAR technology, require a direct field of view and therefore are not suitable for forested areas. In contrast, the variation of the geometric phase as a function of frequency is experimentally measurable through distributed arrays of ground transducers. These can operate in *active* mode, according to pulse/echo schemes that employ transmitter and receiver transducer pairs, or in a *passive* modality, whereby the transducers receive and correlate the diffuse acoustic field corresponding to the ambient seismic noise. Through geometric phase monitoring, large detectable changes in phase in response to changes in ground stiffness/temperature (up to $3\pi/1°C$ have been predicted for frequencies near resonance of trees (**Figure 5a**).

Topological acoustic attributes may also be employed for monitoring any type of built or natural structures in the broader context of acoustics-based nondestructive testing, which is a multibillion industry. For example, recent findings in the field of topological physics have revealed how enhanced sensing may be achieved by exploiting the unprecedented sensitivity around EPs to perturbations associated with small changes in physical properties

(Chen et al., 2017; Hodaei et al., 2017; Miri and Alù, 2019). The degeneracy at EPs emerges in physical systems characterized by underlying symmetries; breaking these symmetries as a result of external perturbations produces shifting and splitting of the coincident resonant frequencies of a cavity where EPs are formed. The shifts and splits of these resonances can be exploited for the detection and possibly for the quantification of such perturbations (**Figure 1b**). Many conventional sensors rely on the detection of shifts in resonances that are typically linearly proportional with respect to the perturbations that cause them.

Figure 5. ***a:*** *Permafrost monitoring using geometric phase: difference in geometric phase for a model forest of uniformly dispersed trees, and the local slope versus ground stiffness (*β_1*; corresponding temperature range from 0 to −12°C) (Lata et al., 2020).* ***b:*** *Schematic for crack detection through EP evaluation.* ***Top:*** *elastic domain with microscopic crack monitoring.* ***Bottom:*** *variation of* Δf *in terms of crack depth for EP perturbation (**red**) and traditional single mode shift (**blue**) showing the different orders (Rosa et al., 2021).*



In contrast, the separation of resonances around EPs is *superlinear* and, therefore significantly more sensitive to changes. A new class of sensing concepts may emerge by not solely relying on these pronounced shifts but also exploiting the underlying nontrivial topological features. These concepts may find applications in temperature, flow, and pressure sensing, among others (Xiao et al., 2019; Kononchuk and Kottos, 2020). The generation of EPs can occur in systems obeying parity-time (PT) symmetry, which feature balanced distributions of gain and loss (Bender and Boettcher, 1998).

In the context of active sensing, gain and loss can be introduced in acoustic platforms in the form of arrays acting as transmitters and receivers, which are properly placed within a medium to be monitored (Fleury et al., 2015). The medium may be subjected to property changes due to material degradations, the onset of damage or environmental changes (e.g., temperature, pressure).

Recently, the ultrasonic detection of a crack developing in a metallic structural component (**Figure 5b**) has been observed through transducer arrays that both actively monitors the propagation of an ultrasonic wave and implements gain and loss along the wave path to induce an EP (Rosa et al., 2021). The crack perturbs the EP symmetry, inducing two resonant peaks separated by a frequency interval ($\Delta f \propto \epsilon^{1/2}$). Here, ϵ is a small perturbation quantifying the crack depth (**Figure 5c**, *red line*). The spectral shifts that would be observed in a conventional sensor not involving an EP only vary linearly with (**Figure 5c**, *blue line*). Given a specific Δf, for example, that defines the resolution of a detection device translates into the ability of the EP sensor to detect smaller cracks. For Δf = 2 Hz as the available resolution, for example, this translates into the ability to detect cracks that are 85% smaller than those detectable through conventional sensors. It should be mentioned here that there is an ongoing debate regarding the actual superiority of EP sensors compared with other sensing techniques because a superlinear frequency splitting does not necessarily translate into enhanced sensor precision in the presence of realistic noise (Langbein, 2018; Wiersig, 2020). This debate is driving additional explorations devoted to reducing the effects of noise while maintaining the attractive sensitivity properties associated with EPs.

Outlook

In this article, we have offered an overview of the powerful opportunities offered by topological concepts in acoustics to manipulate and control sound in fundamentally new ways. This emerging area of research takes inspiration from groundbreaking advances in condensed matter physics, quantum mechanics, and photonics and leverages the properties of acoustic metamaterials to enable new forms of sound transport. Pseudospins emerging from geometrical asymmetries, external bias, spatiotemporal modulation, and nonlinearities can be leveraged to enable topological sound, benefiting a broad range of applications from sound transport robust to defects, noise, and disorder to multiplexing, information processing, data storage and manipulation, and sensing. We expect the field of topological acoustics to open disruptive directions for sound control, with an impact on basic science and applied technologies.

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