Identification of critical pipes of water distribution networks using a hydraulically informed graph-based approach

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ABSTRACT

Water distribution networks (WDNs) are comprised of various components interacting in a complex way. Minor disturbances in them, like a pipe failure, could trigger cascading events causing crucial impacts on the well-being of residents. Therefore, the resilience of WDNs under different failure modes needs to be proactively investigated to ensure that they can cope with disruptive events. This paper introduces a hydraulically informed graph-based approach (HGA) to assess the resilience of WDNs in case of pipe failures. The aim is to rank critical pipes without conducting any hydraulic simulations, solely utilizing topological features and mimicking hydraulic behavior. The suggested method focuses on resistance, capacity, and connectivity of pipes. The results of applying HGA to two WDNs show that more than 95% of critical pipes identified by it are all top-ranked in a hydraulic-based model. The proposed approach can be upgraded to combine multiple and simultaneous failures, which cannot be investigated in acceptable execution time with hydraulic models.

Keywords: Resilience assessment, Complex network analysis, edge betweenness centrality.

INTRODUCTION

Water distribution networks (WDNs) as complex infrastructures can face a wide spectrum of disruptive events such as earthquakes and hurricanes, which threaten their functionality (Hajibabaei et al. 2019). Any loss of WDNs functionality impacts the safety of residents to a particular degree (Marlim et al. 2019). Therefore, water utilities need to identify and prioritize important elements of networks by proactive and/or special protection measures to minimize the impacts of potential disruptions. For this purpose, resilience assessment as a pivotal consideration in the management of urban water infrastructures can be used (Assad et al. 2019). In this context, resilience is defined as "the ability of a system to maintain and adapt its operational performance over design life in the face of adverse conditions" (Herrera et al. 2016).

Pipes are the primary components of WDNs, which fail more frequently compared to other elements (Jun et al. 2008). The critically of pipes can be evaluated by calculating the resilience decrement caused due to their failures. Common approaches for the resilience evaluation of WDNs in case of pipe failures are hydraulic-based. They need hydraulic simulations to model failure scenarios and have been widely applied to identify critical elements of networks (Assad et al. 2019; Jun et al. 2008). Diao et al. (2016) developed a framework to measure the resilience of water systems for several failure modes. They concluded that resilience assessment could be affected by the different details of hydraulic simulation models. Besides, a package including a wide range of metrics and damage scenarios used for the resilience evaluation of WDNs was designed by the U.S. Environmental Protection Agency (EPA) and Sandia National Laboratories (Klise et al. 2018).

One of the drawbacks of the hydraulic-based approaches is the intensive execution time required for large-scale networks. In addition, depending on failure modes, they need detailed data on networks; while many water utilities lack detailed information or even hydraulic models (Chen et al. 2021). Apart from hydraulic modeling, graph-based approaches have also been used for the resilience analysis of WDNs (Pagano et al. 2019). These approaches are based on topological characteristics and require less information and computational efforts than hydraulic-based models. Conventional graph metrics (e.g., node degree, meshedness coefficient, etc.) can provide a simple outline for resilience assessment. However, as Meng et al. (2018) indicated, not all of them are suitable for resilience evaluation since they only rely on topological features. Several studies have suggested graph-based approaches to imitate certain hydraulic attributes like energy losses (Herrera et al. 2016; Lorenz et al. 2021). Pagano et al. (2019) introduced a particular graph measure based on network connectivity to evaluate the effects of pipe failures on resilience. This measure relies on topological characteristics such as the length and diameter of pipes and overlooks the role of water flow in the assessment. Chen et al. (2021) used two standard graph metrics (i.e., betweenness centrality and bridge metric) to quantify pipe criticality in WDNs. They could not find any correlation between the graph metrics and the hydraulic behaviors of pipes. The reason is that the metrics used in the study cannot describe the capacity and the role of redundancy of pipes.

The hydraulic behavior of WDNs cannot be reflected in the current graph-based models because they are mainly based on structural and topological attributes. Moreover, they cannot consider the characteristics of multi-source WDNs. To bridge these gaps, we introduce a hydraulically informed graph-based approach (HGA) to assess resilience and rank critical pipes of WDNs in case of pipe failures. In this context, HGA is a modification added to the standard graph applications of WDNs, wherein weighting functions are proportionally obtained from the hydraulic and structural characteristics to mimic the hydraulic behavior. The suggested HGA is applied to two WDNs and the results are compared with hydraulic simulations.

MATERIALS & METHODS

As a first step in developing and exploring HGA, we focus on the effects of single pipe failures on resilience to evaluate the importance of pipes. The procedure is as follows: First, a hydraulic model is used to assess resilience and rank critical pipes based on their level of failure magnitude. Second, specific graph measures that can be implied for resilience assessment are described. Third, the proposed HGA is explained, and its results are compared with the hydraulic model.

Resilience assessment and pipe ranking based on hydraulic simulation. Hydraulic simulations are executed using the EPANET2.2 toolkit, based on pressure-driven analysis (PDA). We model pipe failures by adjusting the status of pipes to 'closed' for 24 h in EPANET. For simplicity, this procedure assumes that isolation valves are present to isolate each pipe individually. However, considering a pipe segment-based resilience assessment would not change the overall concept of the proposed methodology. The demand changes over one day are considered by the demand patterns of investigated networks with 24-hourly multipliers.

Hydraulic resilience is evaluated based on the loss of serviceability. Serviceability is the capability of a system to maintain water supply under adverse conditions, and it is a common approach to formulate hydraulic resilience (Herrera et al. 2016). Pipe failures could lead to the loss of serviceability which results in 'water supply failure'. In this study, we measure the criticality of pipes by calculating the corresponding 'supply failure magnitude' (SFM) caused due to their failures. SFM resulting from each pipe failure over one day is calculated using Eq. 1:

$$SFM = \frac{\sum_0^{t=T}(\sum_0^{i=n}(D_{i,t} - S_{i,t}))}{\sum_0^{t=T}(\sum_0^{i=n} D_{i,t})} \quad (1)$$

Where, $D_{i,t}$ is the required demand of node *i* at time *t* (L/s), $S_{i,t}$ is the supplied demand (i.e, outflow) of node *i* at time *t* (L/s), *t* are the time steps (h) in the simulation period T, and n is the number of nodes. In EPANET 2.2, Wagner's equation (Wagner et al. 1988) is applied, and the supplied demand (outflow) is calculated as follows:

$$if\ P_{i,t} \leq P_{min} :\ S_{i,t} = 0$$

$$if\ P_{min} < P_{i,t} < P_{req} :\ S_{i,t} = D_{i,t} \,.\, (\frac{P_{i,t} - P_{min}}{P_{req} - P_{min}})^{\gamma} \quad (2)$$

$if\ P_{i,t} \geq P_{req} :\ S_{i,t} = D_{i,t}$

Where $P_{i,t}$ is the pressure of node *i* at time *t*, P_{min} is the minimum pressure, P_{req} is the required pressure to deliver full demand, and γ is the pressure exponent. In this paper, P_{min} and P_{req} are considered 0 and 30 m, respectively (ÖNORM 2018), and γ is set equal to 0.5 (Gorev et al. 2021).

Graph measures used for the proposed HGA. Urban water networks can be described with a specific branch of mathematics known as graph theory. Accordingly, a WDNs is modeled as a mathematical graph *G* composed of *#N* (vertices/ nodes) connected by a set of *#E* (edges/pipes). In addition, *D* and *s* are subsets of *N*, which represents sinks and source, respectively. Depending on modeling aims, different weights can be assigned to each edge/node. For instance, pipe length divided by diameter can be used as a weighting function for edges, representing the capacity of pipes (Herrera et al. 2016). The following graph measures are the base of the suggested HGA:

Shortest path length (SPL) is utilized to describe the shortest distance between two nodes. In this context, distance refers to the sum of all edge weights in a path that connects two nodes (Dijkstra 1959).

Edge betweenness centrality (*EBC*) of an edge *e* in graph *G* measures how often *e* is part of the *SPL* from the source *s* (e.g., tank) to every node $i \in N$ (Brandes 2008). Sitzenfrei et al. (2020) added a modification to *EBC*, specifically for WDNs analysis, referred to as *demand edge betweenness centrality* (EBC^Q). EBC^Q determines the $SPL_{s,i}$ connecting the source node *s* and every demand node $i \in D$, and adds the demands of node *i* ($Q_i > 0$) to the EBC^Q values of all edges along that path. The EBC^Q of an edge *e* is calculated as follows:

$$EBC^Q(e) = \sum_{s,i \in D} SPL_{s,i}(e) \cdot Q_i \tag{3}$$

The weights used for determining $SPL_{s,i}$ in Eq.3 can be 'static' or 'dynamic'. 'Dynamic weighting function' is a term introduced by Sitzenfrei et al. (2020), in which the weights are modified when iterating through all demand nodes. To apply Eq.3 with dynamic weights for the resilience assessment of WDNs, we added a customized modification to the weighting function of EBC^Q, and we call it '*dynamic* EBC^Q' (i.e., EBC_D^Q). The procedure of EBC_D^Q calculation is shown in Fig. 1. Accordingly, after converting the WDN to a graph, the nodal demands are split into smaller parcels to mimic flow division in loops (e.g., for N1, 1.2 L/s is split to 1 and 0.2 L/s). Next, the first demand parcel of node N1 needs to find its path with the least friction to the source. To do so, *SPL* is determined with the weight (*w*) of hydraulic resistance (*r*). The hydraulic resistance of the pipe *e* (r_e) is proportional to $pf_e \cdot L_e/D_e$; where L_e is the length, D_e is the diameter, and pf_e is the pipe's friction factor calculated based on the diameter and roughness (a_e) according to the

$(2\log_{10}(D_e/a_e) + 1.74)^{-2}$ (Lorenz et al. 2021). This procedure imitates the hydraulic behavior in real WDNs. The gray-colored numbers in Fig.1 show the hydraulic resistance of each edge. After determining the path with the least resistance (i.e., red-colored path), the first demand parcel of node N1 (i.e., 1 L/s) is assigned to the EBC_D^Q values of the corresponding edges. This parcel increases the friction of the red-colored path. Consequently, regarding the conservation of mass and energy in real WDNs, the second demand parcel (i.e., 0.2 L/s in Fig .1c) tends to route through the alternative path to the left branch. Hence, we artificially modify the weights of the red-colored path in the first iteration by applying a function to increase the hydraulic resistance. This function is $1+(Q_1/Q_{max})^2$, where Q_1 is the demand of N1 (1 L/s), and Q_{max} is the maximum nodal demand (i.e., 2.1 L/s). The idea of using this factor is taken from the Darcy-Weisbach equation (Rossman et al. 2020) where friction loss/resistance is proportional to Q^2 (Hajibabaei et al. 2021). This procedure is repeated for all the demand nodes and final values of EBC_D^Q are calculated by summing up the EBC_D^Q of pipes obtained in each iteration (see Fig. 1d). As shown in this example, nodal demands, pipes' resistance, and alternative paths are considered in the EBC_D^Q calculation procedure.

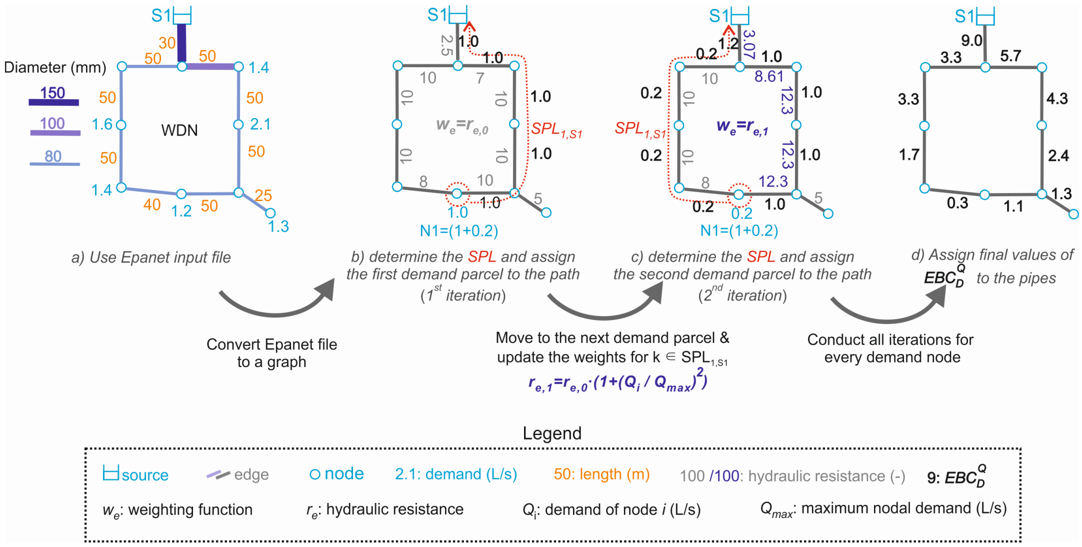

Fig. 1. Using dynamic weights to calculate dynamic demand edge betweenness centrality (EBC_D^Q)

Pipe ranking based on HGA for single-source WDNs. First, according to the procedure described in Fig. 1, EBC_D^Q is calculated under normal conditions and based on the average nodal demand. For this purpose the dynamic weight of $r_{e,j} = r_{e,j-1}.(1 + (Q_i/Q_{max})^2)$ is used; Where $r_{e,j-1}$ is the hydraulic resistance of each e-shortest path between node i and source s in the j^{th} iteration (m/m), Q_i is the demand of node i (L/s), and Q_{max} is the maximum nodal demand that exists in the network (L/s). Second, the failure matrix F ($\#E \times \#E$) for the pipe ranking under single pipe failures is created, where E is the number of edges. The element f_{mn} in F shows the failure consequence of the edge (pipe) n on the edge m. Third, an edge $n \in E$ is removed from the graph of WDN (see Fig. 2), which is considered as the pipe failure. In this situation, two scenarios are possible:

(a) In the first scenario, at least one node becomes disconnected from the source after the edge failure (Fig. 2b). In this case, the total demand that is not fulfilled is equal to the EBC_D^Q routed through the failed edge in the normal condition. Therefore, we allocate the EBC_D^Q of edge *n* in the normal condition $(i.e., EBC_D^Q(n)_{normal})$ to $f_{n,n}$ $(f_{n,n} = EBC_D^Q(n)_{normal})$ as the failure effect of the edge *n* (see Fig. 2b), and assign zero to the other elements (i.e., $f_{m,n}$=0, $m \neq n$). As shown in Fig. 2d, non-zero values on the main diagonal of the failure matrix F are related to the pipes whose failure isolates a part of the WDN from the source (i.e., *p1* and *p5*).

(b) In the second scenario, the edge failure only changes the connectivity between the source and the demand nodes. In this case, the failure consequence is assessed by investigating its impacts on the other edges (pipes). For instance, when the edge *P*2 fails (n = p2, Fig. 2c), the load on it (i.e., EBC_D^Q) is borne by the other edges. These edges can be identified by comparing their $EBC_D^Q(m)_{abnormal}$ with $EBC_D^Q(m)_{normal}$ (see Fig. 2c). In the next step, the extra loads on the paths $(\Delta EBC_D^Q(m) > 0)$ are compared with the optimal capacity C_{opt} of the edges (see also Table 1). As shown in Fig. 2c, the failure effects of edge 2 on the pipes whose excess loads are less than their optimal capacity are neglectable (i.e., $f_{m,2}$=0, *m=1:6*). On the other hand, the excess loads of the red-colored edges are greater than their optimal capacity. Under those circumstances, the failure consequence of the edge *n* on the edge *m* is estimated as follows:

$$f_{m,n} = \frac{\Delta EBC_D^Q(m)_n - C_{opt}(m)}{\delta(m)} \tag{4}$$

Where, $\Delta EBC_D^Q(m)_n$ is the changes of $EBC_D^Q(m)$ due to the failure of the edge *n* (L/s), $C_{opt}\ (m)$ is the optimal capacity of the edge *m* (L/s), and $\delta(m)$ is the overload coefficient of the edge *m* (-). $C_{opt}\ (m)$ is calculated based on the optimal flow velocity $V_{opt}(m)$ of the edge (recommended by the standard codes, Table. 1), and the edge diameter $D(m)$ as follows:

$$C_{opt}(m) = V_{opt}(m)\,.\pi D\ (m)^2/4 \tag{5}$$

Besides, $\delta(m)$ describes the maximum capacity of the edge *m* regarding its optimal capacity ($C_{max}\,/C_{opt}$), and is calculated using Eq. 6.

$$\delta(m) = V_{max}\,/V_{opt}\ (m) \tag{6}$$

Where, V_{max} is the maximum acceptable velocity in WDNs (m/s). In this study, 3 m/s is considered for V_{max} (ÖNORM 2018), and V_{opt} values are determined using Table 1.

Table 1: Suggested values for the optimal velocity in WDNs (ÖNORM 2018)

D (mm)	80	100	125	150	200	250	300	350	400	500	600	700
V_{opt} (m/s)	0.80	0.80	0.80	0.85	0.90	0.95	1.00	1.05	1.10	1.20	1.30	1.40
C_{opt} (L/s)	4.0	6.3	9.8	15.0	28.3	46.6	70.7	101	138	236	368	539

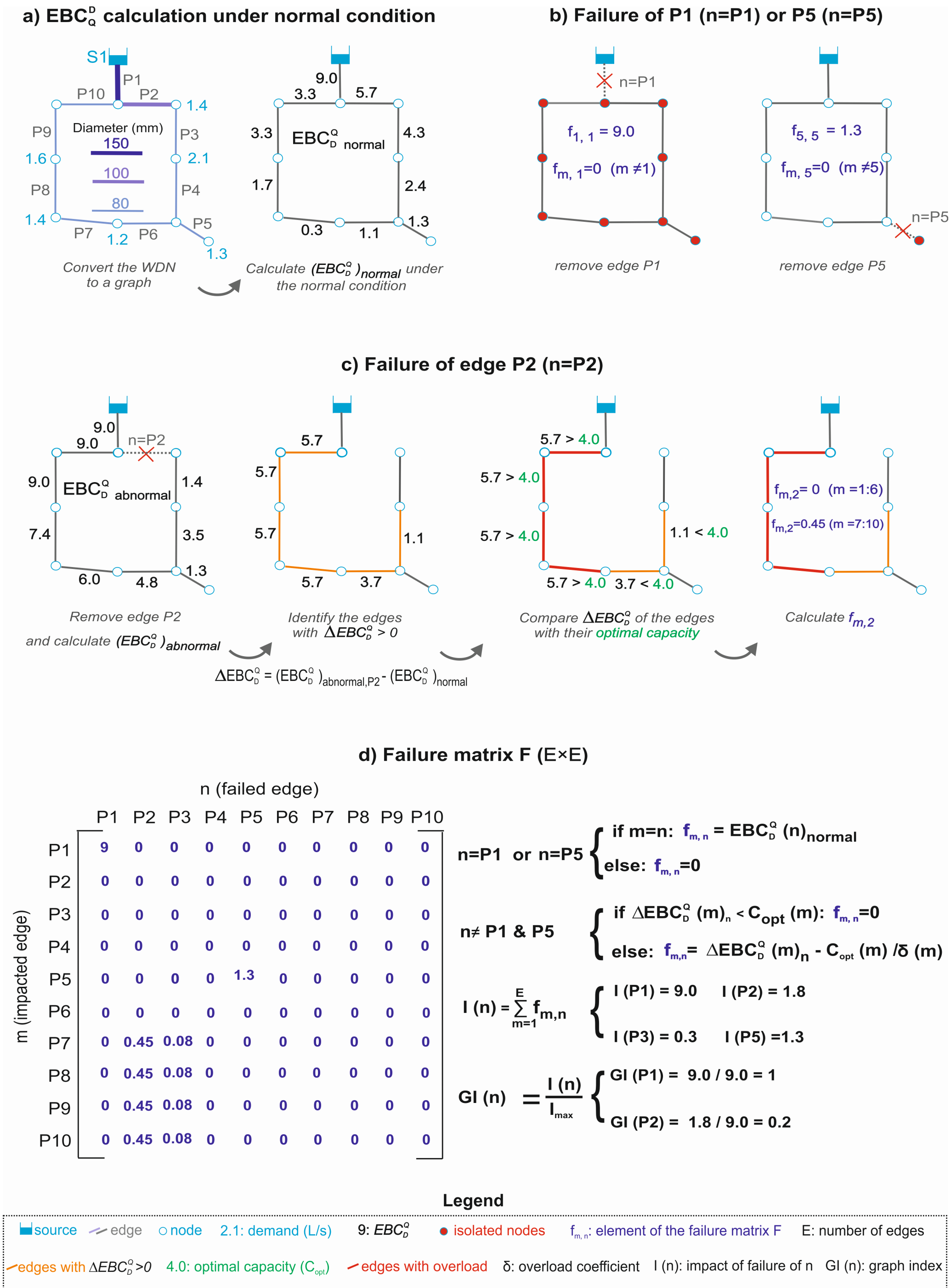

Fig. 2. Pipe ranking under single pipe failures

This procedure is conducted by removing every edge n ($n = 1{:}E$) and calculating its impacts on the WDN using Eq. 7. Thereafter, the maximum value of $I(n)$ is determined ($i.e., I_{max}$), and the graph index (GI) of the failure of edge n is calculated based on Eq. 8. Edges with the positive value of $GI(n)$ are identified and ranked accordingly, and particular attention is given to those with the highest values.

$$I(n) = \sum_{m=1}^{E} f_{m,n} \tag{7}$$

$$GI(n) = \frac{I(n)}{I_{max}} \tag{8}$$

Pipe ranking based on HGA for multi-source WDNs. The first step of pipe ranking for a multi-source WDN is to conduct a graph-based source tracing for every node. As shown in Fig. 3, the nodes in the multi-source WDN are supplied by the sources based on their nodal heads. The nodal head i is estimated by subtracting the energy loss $hf_{i,s}$ along the flow path, from the source s with the head H_s (i.e., $H_{i,s} = H_s - hf_{i,s}$). Head losses in hydraulic models can be determined using the Darcy-Weisbach equation (Rossman et al. 2020). Accordingly, energy loss hf_e, in each pipe e with the diameter D_e, the length L_e, the friction factor f_e, and the water flow Q_e is calculated as follows (in the SI units):

$$hf_e = \frac{f_e\, L_e}{12.1 D_e^5} \cdot Q_e^2 \quad (m) \tag{9}$$

f_e is dependent on the flow regime in pipes, which is determined with the Reynolds number (Rossman et al. 2020). Note that the goal here is to simplify the head loss formula in order to use it as a weighting function for the graph-based source tracing without conducting hydraulic simulations. To do so, if we use the expression of $Q_e = v_e \cdot D_e^2 \cdot \pi/4$ in E.q 9, and assume a constant flow velocity (v_e) in all pipes, the following term is derived for the weighting function:

$$hf_e \approx \mathrm{c} \cdot \frac{L_e}{D_e} \quad (m) \tag{10}$$

Where c with the unit of meter can be interpreted as the value of hydraulic gradient multiplied by the average pipe diameter of WDNs. For instance, hydraulic gradient (friction slope) for optimal WDNs can be assumed to be 20 m/km (i.e., 1/50 m/m) (Sitzenfrei et al. 2020); this means if the average pipe diameter of a network is 0.1 m, the coefficient c is estimated with 1/50.

The term $c \cdot L/D$ in Eq. 10 can be integrated into the weighting function of SPL to estimate the head loss along the flow path (i.e., $hf_{i,S} \approx SPL_{i,s}$). For this purpose, firstly, the $SPL_{i,s}$ is calculated from the node i to every source s with the static weight of $c \cdot L/D$. In the second step, the $SPL_{i,s}$ is subtracted from the corresponding source head ($H_{i,s} = H_s$ - $SPL_{i,s}$). Finally, the nodal head ($H_{i,s}$) resulted from each source are compared, and node i is assigned to the source with a higher value of $H_{i,s}$ (see Fig. 3a). Note that this procedure only presents a simplified assumption to conduct the source tracing, and calculated $H_{i,s}$ with the $SPL_{i,s}$ cannot represent the accurate nodal head.

After conducting the source tracing based on the SPL (Fig 3a), the graph of WDN in Fig. 3b is divided into two parts. The $EBC_D^Q(m)_{normal}$ of each part is calculated using dynamic weights similar to single-source WDNs. After an edge failure, the source tracing needs to be repeated because as shown in Fig. 3c, the supplied nodes by each source could change due to the failure. Thereafter $EBC_D^Q(m)_{abnormal}$ of each part is determined, and edges are ranked similar to the procedure described for single-source WDNs in Fig. 2.

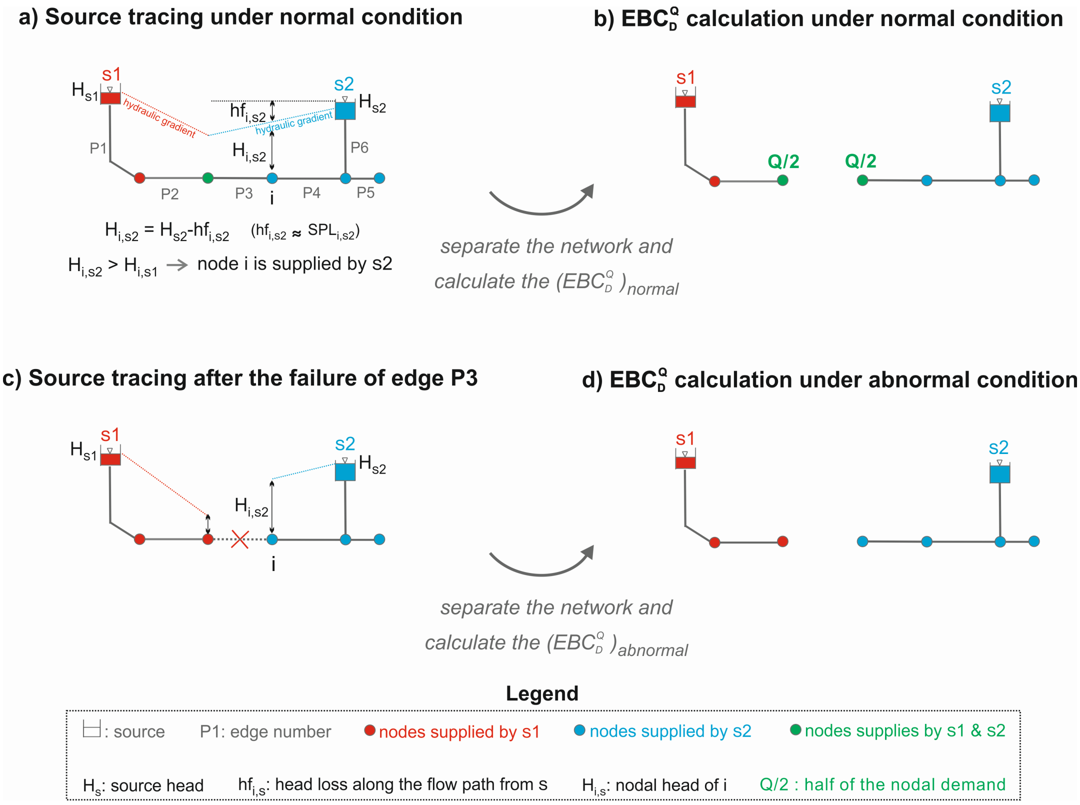

Fig. 3. Source tracing and EBC_D^Q **calculation for a multi-source WDN**

Case study. The case study shown in Fig. 4 is considered for pipe ranking under single pipe failures and evaluating the suitability of the proposed HGA. This case study is a real network consisting of 242 junctions, 268 pipes with an average pipe length of 54 m, and one single reservoir. Because of data protection of the real network, the graph drawing of the layout with force-directed placement is used in Fig. 4.

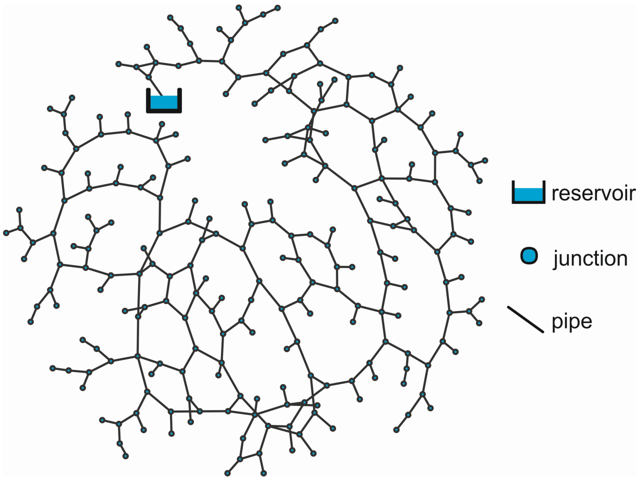

Fig. 4. Layouts of the case study

RESULTS & DISCUSSION

Pipe ranking for the first case study. The existing real network consists of pipes with extra capacity. Consequently, single pipe failures cannot significantly impact its performance. Hence, we redesigned the layout of the real network for two different configurations: a single-source WDN (Fig. 5a) and a two-source WDN (Fig. 5b). The lengths and nodal demands of the optimally designed networks are the same, and the diameters were determined based on the evolutionary algorithm and using the state-of-the-art GALAXY (Wang et al. 2017). More information reading the optimal design procedure can be found in (Sitzenfrei et al. 2020).

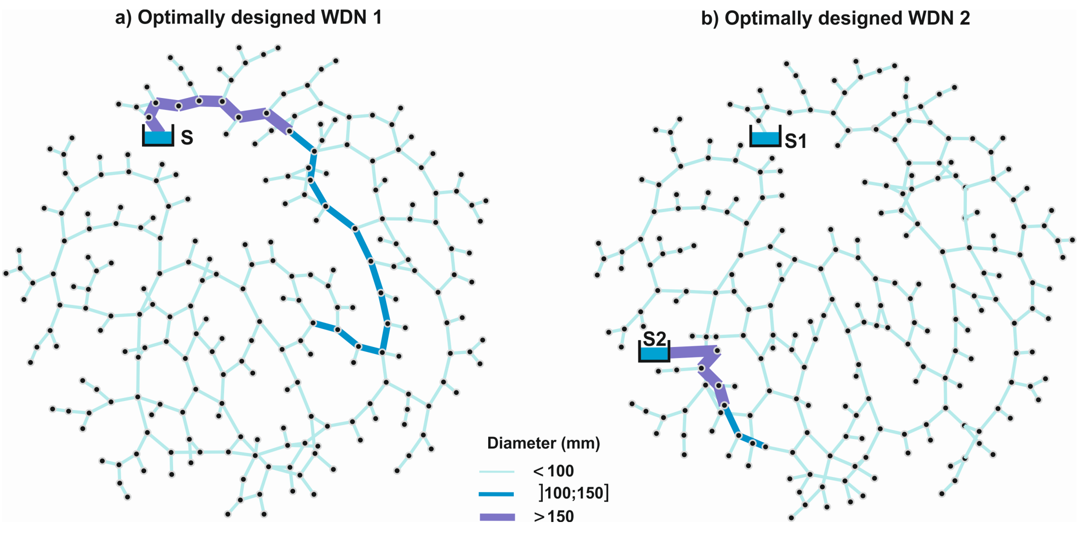

Fig. 5. Optimally designed configurations for the first case study

Fig. 6 shows the comparison of the critical pipes derived based on the hydraulic model (a) with those obtained from HGA (b) for the optimally designed WDN 1. The color and thickness of the edges in this figure represent the pipe criticality. The results of hydraulic simulations show that 46 pipes have a failure magnitude (i.e., SFM) greater or equal to 1% in case of pipe failure. 96% of those pipes (44 out of 46) are recognized by HGA. The neglected pipes by HGA (i.e., pipes P495 and P454 in Fig 6a) have low SFM (i.e., 1%). Besides, the top 13 critical pipes according to the hydric simulations are all the high-ranked pipes based on HGA (with the same order).

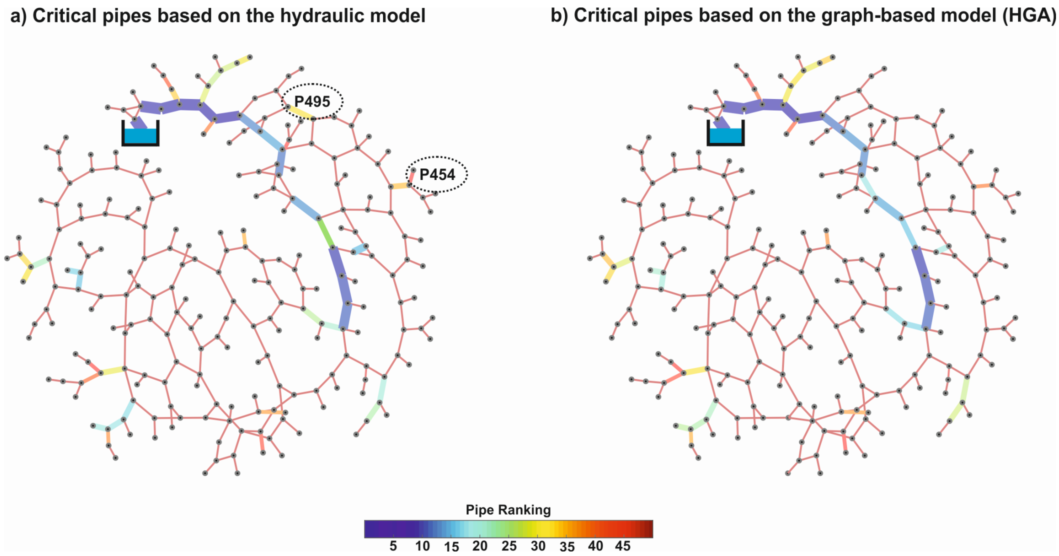

Fig. 6. Critical pipes for optimally designed WDN 1: a) Based on the hydraulic model with the SFM ≥ 1%, b) Based on HGA

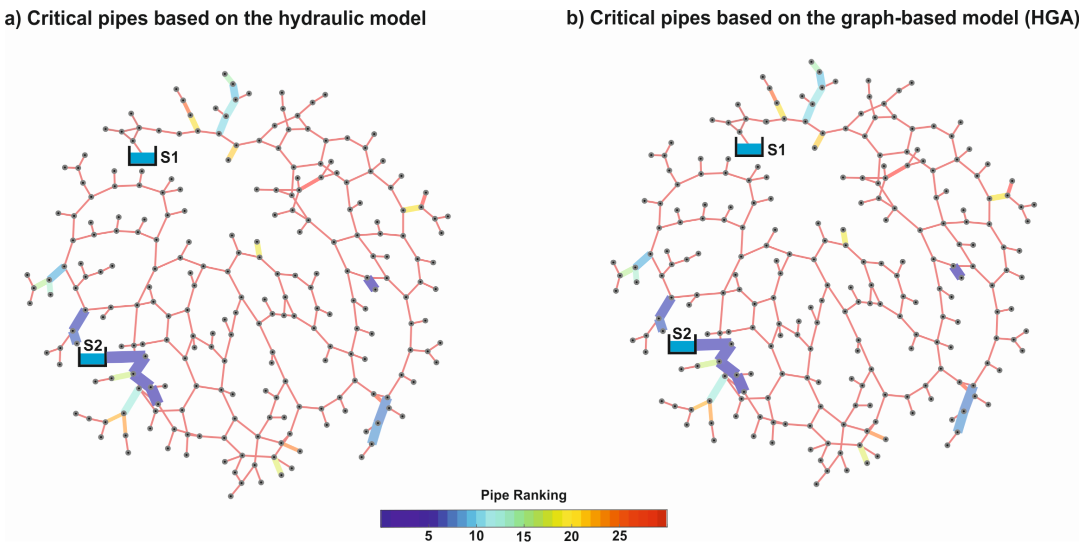

Fig. 7. Critical pipes for optimally designed WDN 2: a) Based on the hydraulic model with the SFM ≥ 1%, b) Based on HGA

The resilience analysis for the optimally designed WDN 2 with two sources shows that 30 pipes have SFM ≥ 1% if they fail individually. As shown in Fig. 7, the HGA can identify all of these pipes with some minor differences in their ranking compared to the hydraulic model. The second source of this network (S2) adds an extra capacity to the system, so the pipe rankings are changed compared to the single-source WDN. As shown in Fig. 7, the main pipes connecting to S1 are not as critical as they are in Fig. 6. The reason is that if S1 is isolated from the network (due to pipe failures), the provided extra capacity through the pipes connecting to S2 is sufficient to supply the WDN. In contrast, in the case of main pipe failures close to S2, the extra capacity between S1 and the nodes is insufficient to support S2. This issue cannot be addressed by the conventional graph measures (e.g., meshedness coefficient) in the literature. However, the suggested HGA in this study can properly address the issue by conducting the source tracing and considering the effects of overloaded edges on resilience. In addition, pipe ranking for this WDN based on HGA takes 46 s in a rapid prototyping environment in Matlab (with only limited considerations regarding code efficiency), while resilience analysis based on the hydraulic simulations (PDA approach) requires 140 s on a desktop computer (Intel® Core™ i7–8,700 CPU @ 3.2 GHz).

SUMMARY & CONCLUSION

In this study, a new hydraulically informed graph-based approach (HGA) was suggested to rank the critical pipes of water distribution systems due to single pipe failures. The base of the proposed approach is a graph measure, referred to as dynamic edge betweenness centrality. This measure considers the connectivity, capacity, and redundancy between a source(s) and pipes. In addition, it imitates the hydraulics of networks by modifying the weights of edges in the calculation procedure. HGA was successfully applied to two WDNs, one with a single source and another with two sources, and the results were compared to those obtained from the hydraulic simulations. Analyzing the results showed that HGA can provide promising results for the pipe rankings. This graph-based method provides good insight to evaluate the criticality of pipes by requiring less information than conventional hydraulic models. Therefore, it can be used by utilities where detailed network information is not available. This research will be extended in the future to consider the combination of multiple and simultaneous failures (like failures due to earthquakes), which cannot be investigated in short execution times with hydraulic models. In addition, HGA will be further investigated by applying it to WDNs with different levels of complexity.

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