Stabilizing Set Based Design of Digital Controllers Based on Mikhailov's Criterion

M.A. Rahman
Department of Computer Science
Tuskegee University
Tuskegee, AL, USA
mrahman@tuskegee.edu

L.H. Keel
Department of Electrical Engineering
Tennessee State University
Nashville, TN, USA
lkeel@tnstate.edu

S.P. Bhattacharyya
Department of Electrical Engineering
Texas A&M University
College Station, TX, USA
spb@tamu.edu

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Abstract—In this paper we develop a systematic approach to the design of a digital controller for a linear plant. The digital controller is parametrized by an unknown or design, real parameter vector $\underline{k}$, which may represent for instance, the coefficients of the digital controller transfer function. The stabilizing set $\mathcal{S}$ in the space of these design parameters is an important and crucial piece of information which is needed to initiate the design process, since every design must reside in this set. In particular multiobjective design is impossible without determining this set. In general this set is not convex or even connected, its computation is difficult and few results are available. In this paper, we use Mikhailov's criterion and the Tchebychev representation of a polynomial, evaluated on the unit circle, to determine an inner approximation $\mathcal{S}_i$ of $\mathcal{S}$, which is convex and described by linear inequalities. Applications of this result to the design of controllers are presented. In particular we show that a geometric approach to gain and phase margin based designs are feasible in this framework. Illustrative examples are presented with a focus on the practically important case of PI and PID controllers. The results should have application to many areas of digital controller design such as hard disk servomechanisms, driverless cars and robots.

Index Terms—digital control, stabilizing set, Mikhailov's criterion

I. INTRODUCTION

Digital control is almost universally employed in all applications of control and in most implementations. For example it is common practice to implement the popular and widely used Proportional-Integral-Derivative (PID) controller which provides robust asymptotic tracking and disturbance rejection of step inputs, digitally. These controllers are used in traditional industries such as Motion Control, Process Control and Aerospace Systems as well as in Driverless Cars and UAV's. Recent progress on PID controllers includes computation of the stabilizing set, using a sweeping parameter, and the determination of subsets achieving various performance objectives [1], [2]. However systematic design of digital controllers significantly lags the corresponding theory for continuous time systems.

In [1], [2] the PID stabilizing set was computed for both continuous and discrete time systems. Even for these special cases the computation in the discrete time case is complicated and the sets are not convex or even connected. In general the computation of stabilizing sets for arbitrary digital controllers remains a difficult open problem which is important for the development of systematic design methods. Indeed determination of the stabilizing sets or regions is the basic problem in the parametric theory of the robust stability and control [4], [5], [6].

In this paper we describe a simple but effective approach to generating stabilizing sets for an arbitrary fixed order digital controller. The result is important for executing methodical multiobjective designs of such controllers. The approach consists of a systematic use of Mikhailov's criterion [3] along with the choice of a set of "frequencies" and the Tchebychev representation [1] of the system characteristic polynomial evaluated on the unit circle.

Mikhailov's criterion which is a statement of the monotonic phase increase of the characteristic polynomial of a discrete time stable system evaluated on the unit circle, is used here, in conjunction with a choice of frequency points, to construct an approximation of the stabilizing set in controller parameter space. This leads to a set of linear inequalities in the controller gains. The feasible solutions of these linear inequalities provide a convex inner approximation to the complete stabilizing set, which in general is neither convex nor even connected. By repeating the algorithm with various sets of frequencies, new inner approximations are obtained. The union of these approximations is also a stabilizing set and constitutes in general a better inner approximation to the stabilizing set. With a stabilizing set in hand design based on gain and phase margin, for example, can be carried out using the loci of constant magnitude and constant phase in the space of controller parameters..

In the following sections, we describe this algorithm and its applications to the design of digital controllers with illustrative examples specialized to the popular PI and PID controllers (see [7], [8], [9], [10], and references therein).

II. NOTATION

Consider the unit circle represented by:

$$z = e^{j\theta}, \quad \text{for} \quad \theta \in [0, 2\pi]. \tag{1}$$

If T is the sampling period, we set

$$\theta = \omega T, \quad \omega = \frac{\theta}{T}, \tag{2}$$

so

$$\theta \in [0, 2\pi] \quad \Leftrightarrow \quad \omega \in \left[0, \frac{2\pi}{T}\right]. \tag{3}$$

We set

$$u_i := -\cos\theta_i \tag{4}$$

where

$$\omega_i = \frac{\theta_i}{T}, \quad \omega_i \in \left[0, \frac{2\pi}{T}\right] \tag{5}$$

and therefore u_i may, with mild abuse of notation, be called a "frequency."

III. PRELIMINARIES

Consider a discrete-time system with monic characteristic polynomial $\delta(z)$ of degree n. The system is asymptotically stable if all roots of $\delta(z) = 0$ lie strictly within the unit circle or equivalently $\delta(z)$ is Schur stable.

Theorem 1 (Mikhailov Criterion):

1. $\delta(z)$ a polynomial of degree n with real coefficients, is Schur stable if and only if $\delta(e^{j\theta})$ turns strictly counterclockwise and runs through $4n$ quadrants as θ ranges increasingly over $[0, 2\pi]$.
2. If the coefficients of $\delta(z)$ are real, then Schur stability is equivalent to the condition that $\delta(e^{j\theta})$ turns strictly counterclockwise and runs through $2n$ quadrants as θ runs over $[0, \pi]$.

Proof: The proof is a statement of the fact that the argument (angle) of $\delta(e^{j\theta})$ increases monotonically and runs through $2\pi n$ radians as θ runs from 0 to 2π. When the polynomial $\delta(z)$ has real coefficients the result follows from the symmetry of the plot of $\delta(e^{j\theta})$ about the real axis. ■

Tchebychev Representation

Let

$$\delta(z) = a_n z^n + a_{n-1} z^{n-1} + \cdots + a_1 z + a_0 \tag{6}$$

denote a polynomial with real coefficients a_i for $i \in \underline{n}$. Then

$$\begin{aligned} \delta(e^{j\theta}) &= \overbrace{(a_n \cos(n\theta) + \cdots + a_1 \cos\theta + a_0)}^{R(\theta)} \\ &\quad + j \overbrace{(a_n \sin(n\theta) + \cdots + a_1 \sin\theta)}^{I(\theta)} \\ &= R(\theta) + jI(\theta) \end{aligned} \tag{7}$$

Letting

$$u := -\cos\theta, \tag{8}$$

it is possible to show (see [11], p.71) that $\cos(k\theta)$ and $\frac{\sin(k\theta)}{\sin\theta}$ are polynomial functions of u, denoted $c_k(u), s_k(u)$:

$$\cos(k\theta) = c_k(u) \quad \text{and} \quad \frac{\sin(k\theta)}{\sin\theta} = s_k(u). \tag{9}$$

Therefore,

$$R(\theta) = \sum_{k=0}^{n} a_k c_k(u) =: \bar{R}(u) \tag{10a}$$

$$I(\theta) = \sqrt{1-u^2} \sum_{k=1}^{n} a_k s_k(u) =: \bar{I}(u). \tag{10b}$$

As θ runs $[0, \pi]$, u runs on the real axis from $[-1, +1]$, and

$$\delta(e^{j\theta})|_{u=-\cos\theta} = \bar{R}(u) + j\bar{I}(u) =: \bar{\delta}(u). \tag{11}$$

The Mikhailov criterion can now be restated as:

Lemma 1: $\delta(z)$ a real polynomial is Schur stable if and only if $\bar{\delta}(u)$ turns strictly counterclockwise and goes through $2n$ quadrants as u runs increasingly over $[-1, +1]$.

IV. MAIN RESULTS

Consider a parameter dependent characteristic polynomial $\delta(z, \underline{k})$ where the coefficients are real and depend affinely on $\underline{k}$. For example, in the control system Fig. 1, the characteristic polynomial is

$$\delta(z, \underline{k}) = \left(z^2 + K_1 z + K_0\right) D(z) + (K_2 z + K_3)\, N(z) \tag{12}$$

and

$$\underline{k} = [K_0, K_1, K_2, K_3]\,. \tag{13}$$

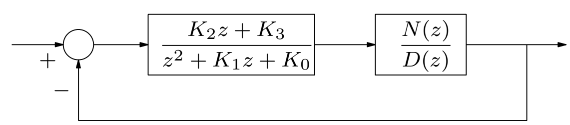

Fig. 1. A unity feedback system

Let $\mathcal{S}$ denote the set of $\underline{k}$ vectors for which the closed-loop system is stable:

$$\mathcal{S} = \{\underline{k} \ : \ \delta(z, \underline{k}) \text{ is Schur}\}\,. \tag{14}$$

In general, $\mathcal{S}$ is not convex or even connected and this poses significant challenges to the design of fixed order controllers in both continuous and discrete time systems.

In the following, we describe an approach to determining convex inner approximations $\mathcal{S}_i$ of $\mathcal{S}$ by exploiting the result stated in Lemma 1 above.

Let

$$U = \{u_1, u_2, \cdots, u_{2n}\} \tag{15}$$

be called an admissible sequence of real numbers if it satisfies

$$-1 < u_1 < u_2 < \cdots < u_{2n-1} < u_{2n} < +1. \tag{16}$$

Then clearly the condition in Lemma 1 for Schur stability holds for $\delta(z)$ if and only if there exists an admissible sequence U satisfying:

$$\bar{\delta}(u_i) \in \text{Quadrant } i, \quad \text{for} \quad i \in \underline{2n}. \tag{17}$$

Applying the above reasoning to the parametrized polynomial $\delta(z, \underline{k})$, we see that (17) becomes

$$\bar{R}(u_i, \underline{k}) + j\bar{I}(u_i, \underline{k}) = \bar{\delta}(u_i, \underline{k}) \in \text{Quadrant } i, \tag{18}$$

for $i \in \underline{2n}$. The condition in (18) stated in terms of the real and imaginary parts:

$$\left(\bar{R}(u_i, \underline{k}), \bar{I}(u_i, \underline{k})\right) \in \text{Quadrant } i, \quad \text{for } i \in \underline{2n} \tag{19}$$

represents a set of $4n$ linear inequalities $L(\underline{k})$ (two for each i). The solution set of $L(\underline{k})$ is a convex polygon and constitutes an inner approximation of $\mathcal{S}$. This idea is used to determine stabilizing sets in the examples that follow.

For the examples that follow, let $P(z)$ and $C(z)$ denote the plant and controller transfer functions written as ratios of real polynomials:

$$P(z) = \frac{N(z)}{D(z)}, \quad C(z) = \frac{N_c(z)}{D_c(z)} \tag{20}$$

Then we write the Tchebychev representation of each of these polynomials

$$D(z)|_{z=-u+j\sqrt{1-u^2}} = R_D(u) + j\sqrt{1-u^2}T_D(u) \tag{21a}$$
$$N(z)|_{z=-u+j\sqrt{1-u^2}} = R_N(u) + j\sqrt{1-u^2}T_N(u) \tag{21b}$$

and

$$D_c(z)|_{z=-u+j\sqrt{1-u^2}} = R_{D_c}(u) + j\sqrt{1-u^2}T_{D_c}(u) \tag{22a}$$
$$N_c(z)|_{z=-u+j\sqrt{1-u^2}} = R_{N_c}(u) + j\sqrt{1-u^2}T_{N_c}(u) \tag{22b}$$

Example 1 (Stabilizing sets $\mathcal{S}_i$):
In this example, we determine a stabilizing set $\mathcal{S}_i$ corresponding to the choice of a specific admissible vector U_i. Let the plant and the fist order controller be

$$\begin{aligned} P(z) &= \frac{N(z)}{D(z)} = \frac{z+0.5}{z^3 - 0.2z^2 - 0.7z - 0.6} \\ C(z) &= \frac{N_c(z)}{D_c(z)} = \frac{K_1}{z+K_0}. \end{aligned} \tag{23}$$

Then

$$\begin{aligned} &R_D(u) = -4u^3 - 0.4u^2 + 3.7u - 0.4, \\ &R_N(u) = -u + 0.5, \\ &R_{D_c}(u) = -u + K_0, \qquad R_{N_c}(u) = K_1, \\ &T_D(u) = 4u^2 + 0.4u - 1.7, \qquad T_N(u) = 1, \\ &T_{D_c}(u) = 1, \qquad T_{N_c}(u) = 0 \end{aligned} \tag{24}$$

and

$$\begin{aligned} \bar{R}(u) &= R_D(u)R_{D_c}(u) + R_N(u)R_{N_c}(u) \\ &\quad - (1-u^2)\left[T_D(u)T_{D_c}(u) + T_N(u)T_{N_c}(u)\right] \\ &= \left(-4u^3 - 0.4u^2 + 3.7u - 0.4\right)K_0 \\ &\qquad + (-u+0.5)K_1 \\ &\qquad + \left(8u^4 + 0.8u^3 - 9.4u^2 + 1.7\right) \end{aligned} \tag{25a}$$

$$\begin{aligned} \bar{I}(u) &= \sqrt{1-u^2}\left[T_D(u)R_{D_c}(u) + T_N(u)R_{N_c}(u)\right. \\ &\qquad \left. + R_D(u)T_{D_c}(u) + R_N(u)T_{N_c}(u)\right] \\ &= \sqrt{1-u^2}\left[\left(4u^2 + 0.4u - 1.7\right)K_0 + K_1\right. \\ &\qquad \left. + \left(-8u^3 - 0.8u^2 + 5.4u - 0.4\right)\right]. \end{aligned} \tag{25b}$$

We selected 4 sets of $U_i = [u_1, u_2, \cdots, u_8]$ as follows:

$$\begin{aligned} U_i = &[-0.99, -0.85, -0.8, -0.6, 0.1, 0.55, 0.65, 0.99] \\ &+ \Delta_i[\ 0, 0, 1, 1, 1, 1, 1, 0\ ] \end{aligned} \tag{26}$$

where

$$[\Delta_1, \Delta_2, \Delta_3, \Delta_4] = [0.15 \quad 0.17 \quad 0.2 \quad 0.25]. \tag{27}$$

A stabilizing set $\mathcal{S}_i$ corresponding to each U_i is shown in Fig. 2. Clearly the union of these sets are also contained in the actual stabilizing set:

$$\underline{\mathcal{S}} := \cup_{i=1}^{4}\mathcal{S}_i \subset \mathcal{S}. \tag{28}$$

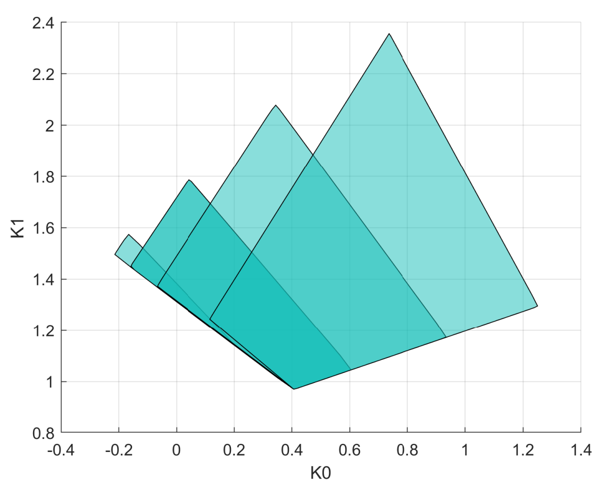

Fig. 2. Stabilizing Set $\underline{\mathcal{S}}$ in $K_0 - K_1$ space

Example 2 (2nd Order Controller Example):
Consider the plant and a 2nd order controller with 3 design parameters:

$$\begin{aligned} P(z) &= \frac{N(z)}{D(z)} = \frac{1}{z^2 - 0.25} \\ C(z) &= \frac{N_c(z)}{D_c(z)} = \frac{b_0}{z^2 + a_1 z + a_0}. \end{aligned} \tag{29}$$

Then

$$\begin{aligned} R_D(u) &= 2u^2 - 1.25, \quad R_N(u) = 1, \\ R_{D_c}(u) &= 2u^2 - a_1 u + a_0 - 1, \\ R_{N_c}(u) &= b_0, \\ T_D(u) &= -2u, \qquad T_N(u) = 0, \\ T_{D_c}(u) &= -2u + a_1, \qquad T_{N_c}(u) = 0 \end{aligned} \tag{30}$$

and

$$\begin{aligned} \bar{R}(u) &= R_D(u)R_{D_c}(u) + R_N(u)R_{N_c}(u) \\ &\quad - (1-u^2)\left[T_D(u)T_{D_c}(u) + T_N(u)T_{N_c}(u)\right] \\ &= \left(2u^2 - 1.25\right)a_0 + \left(-4u^3 + 3.25u\right)a_1 \\ &\quad + b_0 + 8u^4 - 8.5u^2 + 1.25 \end{aligned} \tag{31a}$$

$$\begin{aligned} \bar{I}(u) &= \sqrt{1-u^2}\left[T_D(u)R_{D_c}(u) + T_N(u)R_{N_c}(u)\right. \\ &\quad \left.+R_D(u)T_{D_c}(u) + R_N(u)T_{N_c}(u)\right] \\ &= \sqrt{1-u^2}\left(-2ua_0 + \left(4u^2 - 1.25\right)a_1\right. \\ &\quad \left.-8u^3 + 4.5u\right). \end{aligned} \tag{31b}$$

By selecting

$$U = [-0.99, -0.8, -0.5, -0.25, -0.03, 0.15, 0.7, 0.99] \tag{32}$$

we have the stabilizing set contained in $\mathcal{S}$ shown in Fig. 3.

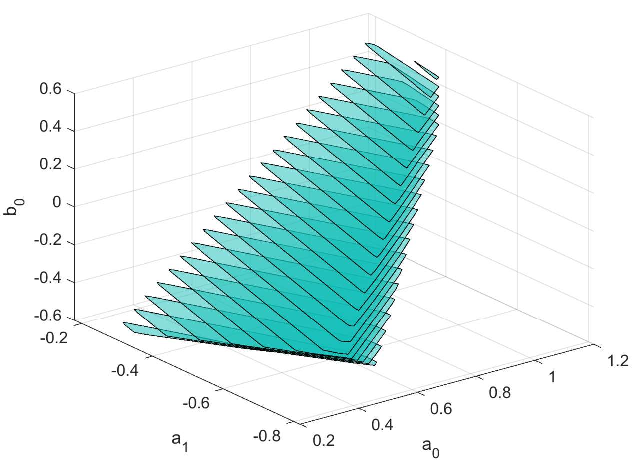

Fig. 3. Stabilizing Set in $a_0 - a_1 - b_0$ space

V. DESIGN WITH GAIN AND PHASE MARGIN REQUIREMENTS

In this section, we introduce a technique to select a set of controller parameters that ensure the closed-loop system satisfying the given gain and phase margin requirements. For simplicity, we discuss here the case of digital PI controller with a phase margin requirement. The problem with the gain margin requirement can be solved similarly.

Let T be the sampling period. For PI controllers, we have

$$\begin{aligned} C(z) &= K_P + K_I T\left(\frac{z}{z-1}\right) \\ &= \frac{(K_P + K_I T)\,z - K_P}{z-1} = \frac{K_1 z + K_0}{z-1} \end{aligned} \tag{33}$$

where

$$\begin{bmatrix} K_1 \\ K_0 \end{bmatrix} = \begin{bmatrix} 1 & T \\ -1 & 0 \end{bmatrix}\begin{bmatrix} K_P \\ K_I \end{bmatrix}. \tag{34}$$

Suppose that the desired phase margin is ϕ^* and let the corresponding gain crossover "frequency" be u_g. Let

$$|\bar{C}(u)| := |C(z)|^2_{z=-u+j\sqrt{1-u^2}} \tag{35a}$$

$$|\bar{P}(u)| := |P(z)|^2_{z=-u+j\sqrt{1-u^2}} \tag{35b}$$

This consideration leads

$$\begin{aligned} |\bar{C}(u)|^2_{u=u_g} &= \frac{\left(K_0 - K_1 u_g\right)^2 + \left(1-u_g^2\right)K_1^2}{\left(1+u_g\right)^2 + \left(1-u_g^2\right)} \\ &= \frac{1}{|\bar{P}(u_g)|^2} =: M^2 \end{aligned} \tag{36}$$

and

$$\angle\bar{C}(u_g) = \pi + \phi^* - \angle\bar{P}(u_g) =: \Phi_g. \tag{37}$$

After some manipulation of (36), we have

$$\begin{aligned} |\bar{C}(u_g)|^2 &= \frac{\left(K_1+K_0\right)^2(1-u) + \left(K_1-K_0\right)^2(1+u)}{4(1+u)} \\ &= \frac{\left(K_1+K_0\right)^2(1-u)}{4(1+u)} + \frac{\left(K_1-K_0\right)^2}{4} \\ &= \frac{1}{|\bar{P}(u_g)|^2} = M^2 \end{aligned} \tag{38}$$

By letting

$$L_0 := K_1 + K_0, \quad L_1 = K_1 - K_0, \tag{39}$$

we have

$$\frac{L_0^2(1-u)}{4(1+u)} + \frac{L_1^2}{4} = \frac{1}{|\bar{P}(u_g)|^2}. \tag{40}$$

Note that

$$\frac{1+u}{1-u} = \tan^2\left(\frac{\theta}{2}\right). \tag{41}$$

Thus, we have the following conditions:

$$\frac{L_0^2}{4\tan^2\left(\frac{\theta}{2}\right)} + \frac{L_1^2}{4} = \frac{1}{|\bar{P}(u_g)|^2} \tag{42}$$

$$\begin{aligned} \pi + \phi^* - \angle\bar{P}(u_g) &=: \Phi_g \\ &= \tan^{-1}\left(\frac{-L_0}{L_1\tan\left(\frac{\theta}{2}\right)}\right). \end{aligned} \tag{43}$$

(43) can be written as a function representing a line:

$$L_1 = \left(-\frac{1}{\tan\Phi_g\tan\left(\frac{\theta}{2}\right)}\right)L_0. \tag{44}$$

Clearly, the intersection points of the ellipsoid represented by (42) and the straight represented by (44) give the values of K_0 and K_1 such that the closed-loop system gives the specified phase margin. The design parameter set is obtained by finding the points satisfying the two conditions in (42) and (44) that reside inside the stability region $\underline{\mathcal{S}}$.

VI. GAIN-PHASE MARGIN BASED DESIGN

Consider a digital PI or PID controller with transfer functions, respectively,

$$C_1(z) = \frac{K_0 + K_1 z}{z-1} \quad \text{and} \quad C_2(z) = \frac{K_0 + K_1 z + K_2 z^2}{z(z-1)}. \tag{45}$$

In each case, let

$$\underline{k} = (K_0, K_1) \quad \text{or} \quad \underline{k} = (K_0, K_1, K_2) \tag{46}$$

denote the design parameter vector. If a gain α is inserted into the loop at the input of the controller $C_1(z)$ or $C_2(z)$, the effect is to replace the vector $\underline{k}$ by $\alpha\underline{k}$. If $\mathcal{S}_i$ is an inner approximation to the stabilizing set, described by linear inequalities and $\underline{k}^0$ a nominal controller have the following geometry shown in Fig. 4. Clearly, the ray $L = \alpha\underline{k}^0$ enters

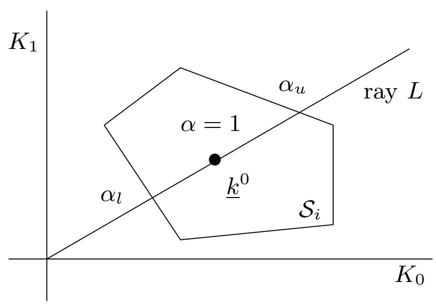

Fig. 4. Ray $\alpha\underline{k}^0$ penetrating the stabilizing set.

$\mathcal{S}_i$ at $\alpha = \alpha_l$, the lower gain margin, and exits $\mathcal{S}_i$ at α_u, the upper gain margin. α_l and α_u are easily determined as the smallest and largest values violating the linear inequalities defining $\mathcal{S}_i$. The segment of L inside $\mathcal{S}_i$ represents in $\alpha \in [\alpha_l, \alpha_u]$.

Each controller on this segment corresponding to a fixed phase margin. Thus, one may determine the variation of phase margin with gain margin along this segment of the ray, as illustrated in Fig. 5.

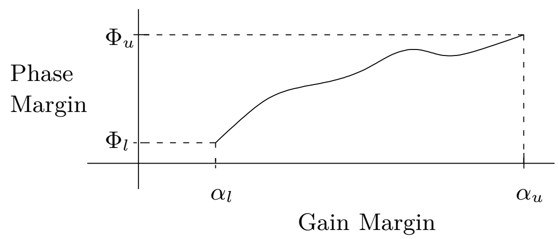

Fig. 5. Gain-phase margin

The above procedure may be repeated with various rays as illustrated in Fig. 6.

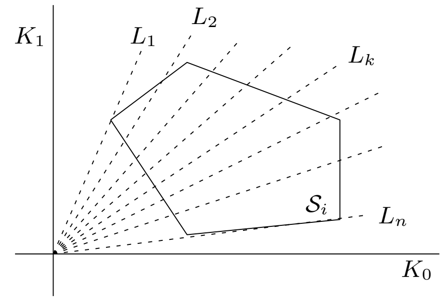

Fig. 6. Family of rays

Therefore, each ray corresponding to a gain-phase margin curve as shown in Fig. 7.

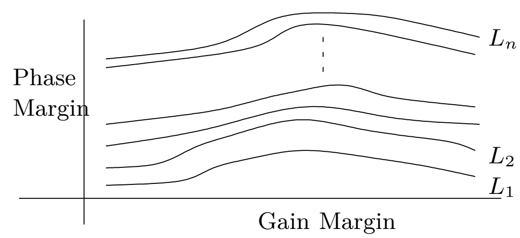

Fig. 7. Gain vs phase margins

This approach to gain-phase margin design avoids the frequency sweeping approach proposed in [2].

Example 3 (Design with Phase Margin Requirement): In this example, we select a set of controller parameters inside the stabilizing set that satisfies the given phase margin requirement. Let the plant and the controller with design parameters be

$$P(z) = \frac{z - 0.1}{z^3 + 0.1z - 0.25}, \quad C(z) = \frac{K_1 z + K_0}{z-1}. \tag{47}$$

Then, using the previous notation:

$$\begin{aligned}
&R_D(u) = -4u^3 + 2.9u - 0.25, \\
&R_N(u) = -u - 0.1, \\
&R_{D_c}(u) = -u - 1, \quad R_{N_c}(u) = -K_1 u + K_0, \\
&T_D(u) = 4u^2 - 0.9, \qquad T_N(u) = 1, \\
&T_{D_c}(u) = 1, \qquad T_{N_c}(u) = K_1
\end{aligned} \tag{48}$$

and

$$\begin{aligned}
\bar{R}(u) &= R_D(u)R_{D_c}(u) + R_N(u)R_{N_c}(u) \\
&\quad - (1-u^2)\left[T_D(u)T_{D_c}(u) + T_N(u)T_{N_c}(u)\right] \\
&= K_0(-u - 0.1) + K_1\left(2u^2 + 0.1u - 1\right) \\
&\quad + 8u^4 + 4u^3 - 7.8u^2 - 2.65u + 1.15
\end{aligned} \tag{49a}$$

$$\begin{aligned}
\bar{I}(u) &= \sqrt{1-u^2}\left[T_D(u)R_{D_c}(u) + T_N(u)R_{N_c}(u)\right. \\
&\quad \left.+ R_D(u)T_{D_c}(u) + R_N(u)T_{N_c}(u)\right] \\
&= \sqrt{1-u^2}\left(K_0 + K_1(-2u - 0.1) - 8u^3\right. \\
&\quad \left.-4u^2 + 3.8u + 0.65\right)
\end{aligned} \tag{49b}$$

We now let $u_g = -0.97$ and the desired phase margin be $\phi^* \approx 60^o$ from the Bode plots of $P(u)$. Fig. 8 shows that the design parameter set satisfying the two conditions in (42) and (44) is found inside the stability region $\underline{\mathcal{S}}$.

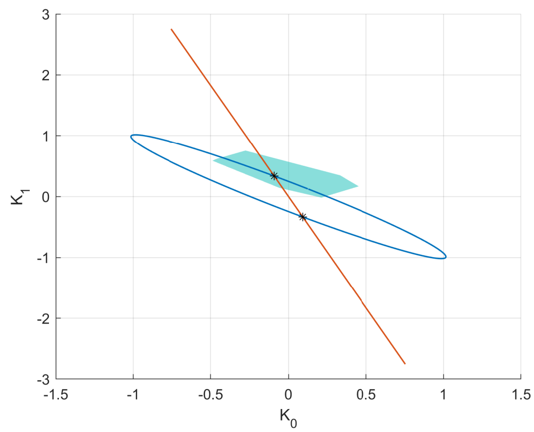

Fig. 8. Selecting PI gains satisfying phase margin requirement

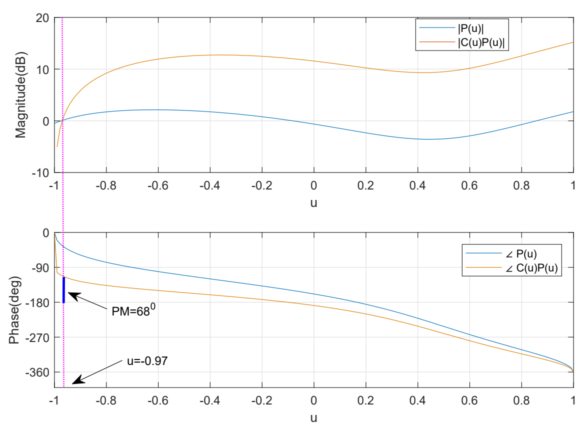

Fig. 9. Bode plots verifying the satisfaction of the phase margin requirement

The Bode plots in Fig. 9 verifies that the resulting closed-loop system satisfies the desired phase margin. Fig. 10 shows the step response of the closed-loop system. This “confirms” the rule of thumb that good phase margin corresponds to small overshoot.

VII. CONCLUDING REMARKS

In this paper, we described an approach to generating inner approximations of the stabilizing set for a digital control system. The approach uses the Mikhailov criterion and results in linear inequalities in the controller gains. The inner approximation thus obtained is therefore a convex set. For continuous-time systems, a similar idea was used in [12]. However, the issue of the performance was not addressed. Moreover, the Tchebychev representation used here may also be used to [12] to make the search interval

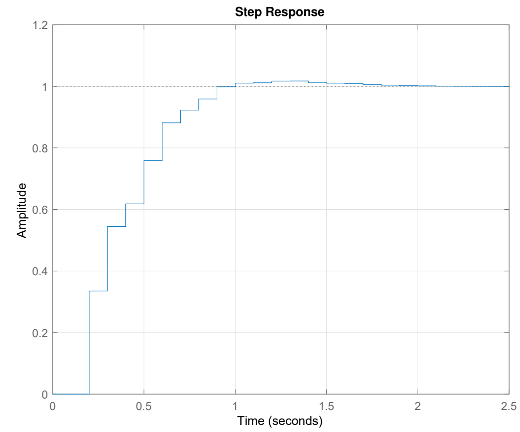

Fig. 10. Step response of the closed-loop system

to be finite. The method is applicable to arbitrary fixed order controllers. In contrast to this, the signature method developed in [1], [2] applies only to PID controllers. We applied this method to generate examples of stabilizing sets for digital PID controllers and to design for prescribed gain crossover frequency and phase margin. We expect this approach will be useful in other fixed order digital controller design problems.

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