

HOW AN INQUIRY-ORIENTED TEXTBOOK SHAPED A CALCULUS INSTRUCTOR'S PLANNING

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ABSTRACT

We investigate how an inquiry-oriented, dynamic, open-source calculus textbook shaped one college instructor's planning. We rely on Dietiker et al.'s (2018) curriculum noticing framework to situate the instructor's actions during lesson planning using data from surveys, logs, and interviews. The instructor's planning practices are characterized by intense use of the textbook, including creating additional curricular material related to its content. Our observations suggest that the textbook supported and influenced the instructor in implementing his inquiry-oriented visions and goals while planning his lessons. We conclude by suggesting further investigation of how textbooks shape undergraduate mathematics education and the textbooks' role in shaping undergraduate mathematics planning practices.

Enhancing calculus teaching has been the focus of many mathematics education reforms worldwide (Zuccheri & Zudini, 2014). In the United States, concerns regarding low passing rates, students not working on 'higher-level' problems (Ferrini-Mundy & Graham, 1991), and "widespread weaknesses in numerical and symbolic manipulations" (Ruane, 2001, p. 1) were raised for the first time at the Tulane University Conference in 1986 (Douglas, 1986). In response, the 1990s calculus reform with goals of reconceptualizing the curriculum and teaching of calculus followed. However, despite the large scale and the ample financial support given to these efforts, the work was focused mainly on curriculum development (Rasmussen et al., 2014); researchers missed the opportunity of investigating how the reform worked, and in particular, how teachers used the developed curricula in their work (Larsen et al., 2017).

Recently, there have been increasing calls for infusing inquiry in the undergraduate mathematics curriculum in the United States, with the hope that inquiry-oriented ways of teaching and learning promote student engagement and doing mathematics during instruction (Laursen & Rasmussen, 2019).¹ These calls resemble the calculus reform in the 90s in that considerable work for infusing inquiry has targeted curriculum development of materials that foster inquiry without much research on how such materials are used.² On the other hand, undergraduate curricula that are designed and tested by mathematics education researchers

¹ It is important to note that although inquiry-oriented ways of teaching are often associated with improved student outcomes, the research on the relationship between inquiry and student outcomes has not always been consistent (see Freeman et al., 2014; Johnson et al., 2019; Kirschner et al., 2006; and Kogan & Laursen, 2014).

² For inquiry-oriented curriculum materials, see <http://www.inquirybasedlearning.org/> and <https://www.artofmathematics.org/resources>.

(under the realistic mathematics education or RME paradigm; Freudenthal, 1991) are currently available for a few upper-division mathematics courses and not for calculus (Laursen & Rasmussen, 2019). That is because developing such curricula is research-intensive and time-consuming, as sequences of classroom tasks are created and tested to promote students' engagement with challenging mathematical ideas in an environment where the instructor mostly asks questions to guide students' discovery (e.g., Rasmussen et al., 2018; Wawro et al., 2013). Other than the research needed to develop these curricula, researchers have, for the most part, described how undergraduate students could inquire and learn in environments built on the theoretical advances (e.g., Rasmussen & Kwon, 2007) and the work needed to create instructor support materials or to scale up curriculum development (Larsen et al., 2013; Lockwood et al., 2013). Only a small set of studies have focused on instructor roles, such as their actions that support students' inquiry (e.g., Johnson et al., 2013). This leaves room for investigating processes that could give insight on how instructors enact when using these novel curricula.

We are interested in one specific set of processes regarding instructors' use of such curricula—the work related to their lesson planning. As a precursor to the learning opportunities created for students during instruction with curriculum materials, it is important for these processes to be investigated. It has been theoretically argued (Remillard, 2005) and empirically shown (e.g., Gueudet, 2017; Jones & Pepin 2016) that teachers are not passive consumers or implementers of curriculum materials but active agents and partners in designing the curriculum that students use. We believe that much can be gained from investigating the use of novel curricula, particularly how they shape instructors' planning practices. Without such knowledge, we cannot paint a complete picture of how such reform interventions work, which is needed for authors and designers to improve these curricula.

This study aims at contributing to this body of research. We seek to understand how *Active Calculus* (Boelkins, 2019), a dynamic calculus textbook and its supplemental materials designed to create inquiry opportunities in a first-year calculus course, shaped one instructor's (Casey, a pseudonym) planning practices. The choice of using *Active Calculus* for this investigation was of convenience because this study is a part of a larger study that uses open-source dynamic textbooks in calculus (*Active Calculus*), linear algebra (*A First Course in Linear Algebra*), and abstract algebra (*Abstract Algebra Theory and Applications*) courses to explore students' and instructors' use of such textbooks (Beezer et al., 2018). Prior to participating in the research project, Casey used Stewart's (2016) calculus textbook and primarily relied on lecturing to teach the course. Although he was slowly building more inquiry into his other courses, he found it very time-consuming to design calculus activities that would enable students to explore in class. Casey claimed that Stewart's (2016) textbook was not supporting his inquiry-oriented visions and goals, which included lecturing less frequently and engaging students with activities inside and outside the classroom. Not pleased with how he was teaching the course, Casey decided to take his participation in the project as an opportunity to infuse inquiry-oriented practices in his teaching even though the participants were not asked to change their teaching practices. We seized the opportunity of investigating how Casey used *Active Calculus*, for planning, given that the textbook was designed to support inquiry in the classroom.

For the purposes of this paper, we define planning as the activities instructors engage in to generate a plan that outlines the goals, activities, times, and roles of the teacher and the students in the classroom. In the next section, we provide a review of the literature on undergraduate instructor's use of mathematics curricula for planning.

Literature Review

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While there have been studies on undergraduate instructor's use of mathematics curricula, specifically textbooks (e.g., González-Martín, 2015; Mesa & Griffiths, 2012), the research is scant when it comes to use of curricula for planning lessons. Perhaps the closest research that describes university professors' planning has been done by Gueudet (2017), who investigated teachers' documentation work—teachers' activities as they look for resources, modify them and use them to create documents that fulfill a particular aim. Gueudet (2017) considered the role of all resources used by the teachers, including textbooks, as mediation tools that led to developing structured documents and resource systems. Working with six university instructors, she identified the aims of the teaching activities, the associated resources, and how and why instructors used them. Among other findings, she concluded that while novice instructors may be expected to align their teaching with their resources more than experienced instructors, their personal beliefs still play a significant role in shaping their practices. Moreover, while the instructors were selective in using digital resources, none of them used the Internet to search for resources, such as exercises and lesson plans.

In another study, Randahl (2016) used a decision-making lens to investigate the degree to which a college instructor in Norway adopted a popular calculus textbook (Adams, 2006) for teaching calculus to first-year engineering students. Randahl used Speer et al.'s (2010) definition of teaching practices:

Teaching practice concerns teachers' thinking, judgments, and decision-making as they prepare for and teach their class sessions, each involving one or more instructional activities. It includes their planning work prior to classroom teaching, thinking and decision-making during lessons, and their reflections on and evaluations of completed lessons (p.101).

Although Randahl considered planning work as an important part of teaching practices, she mainly relied on classroom observation data to determine the instructor's reliance on the textbook. She concluded that the instructor perceived the textbook as "an important knowledge source" and heavily relied on the textbook for teaching the course, resulting in the textbook guiding instruction (p. 911).

In sum, these studies have addressed how various curriculum materials, including textbooks and other resources, are used by instructors as they develop the documents they need for teaching and as they teach in the classroom. However, these studies do not examine closely how the textbook enters in the process of lesson planning, organizes the mathematical ideas presented in class, and eventually impacts students' opportunities to engage with and learn the content. Such knowledge is necessary for designing and improving textbooks that support instructors in shaping their teaching practices to fulfill their goals. Developing curriculum materials without knowing how instructors use them fails to acknowledge instructors as active agents in implementing the curriculum and designing student mathematical experiences during instruction. Thus, more attention to the activity of lesson planning appears to be necessary. In this case study, we seek to investigate planning processes with a textbook that was designed to engage students with inquiry. We pose the following research question: How does a college instructor new to inquiry in calculus use *Active Calculus*, an inquiry-oriented dynamic textbook, to plan lessons?

Theoretical Underpinnings

Using curriculum materials to plan the teaching of a course is hardly a straightforward task. Remillard (2005) highlighted this complexity, indicating that working with curriculum materials is a multifaceted and dynamic process where teachers "bring their own beliefs and experiences" to create their own meanings and interpret the authors' intentions. Breyfogle and colleagues (2010) have theorized that teachers develop curriculum reasoning (i.e., "the thinking processes that teachers engage in as they work with curriculum materials to plan, implement, and reflect on instruction," p. 308) as they plan their lessons. This type of reasoning allows teachers to identify opportunities offered by curricular materials and also their

limitations. However, for these opportunities to be capitalized, teachers need to first locate them, make sense of the material that offers such opportunities, and then act by incorporating them in their planning practices. Building on Remillard's (2005) notion of participation with curriculum materials by considering the textbook "a dialogic partner" rather than "a fixed object" (p. 522), Dietiker and colleagues (2018) explored teachers' planning processes by introducing the curricular noticing framework. To study how curricular materials mediate planning, they suggest looking into three sets of actions that are necessary for the work teachers do when interacting with curriculum materials: curricular attending, curricular interpreting, and curricular responding. These three sets of activities constitute the curriculum noticing framework.

Curricular noticing refers to how teachers capitalize on the opportunities afforded by the curriculum, both in mathematical and pedagogical ways, to create a teaching plan (Dietiker et al., 2018). The curricular attending phase refers to actions involved in viewing or visually taking in information within the curriculum. This phase comprises all these actions necessary for the teacher to search, recognize, locate, assess, and any other possible way of visually absorbing materials before their interpretation. The curricular interpreting phase includes actions teachers take to make sense, mathematically and pedagogically, of the information they have visually taken in during the attending phase. This phase depends on teachers' prior experiences, their goals and their background knowledge and beliefs. Lastly, the curricular responding phase describes teachers' decisions about and actions towards the curriculum and how it is going to be carried out (enacted) in the classroom (e.g., choosing parts of the curriculum for classroom use, sequencing and adapting tasks). The three phases of interactions between the teacher and the curriculum materials follow one another consecutively but teachers may refer to prior phases as needed while interacting with curricula (see Figure 1).

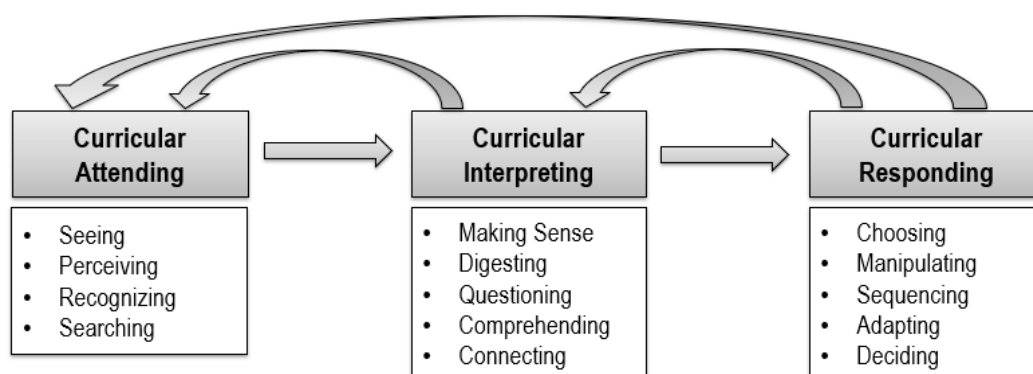


Figure 1: The curricular noticing framework (adapted from Dietiker et al., 2018)

Given that this framework has been developed more recently, we only found three studies that have used it: Males et al. (2015, 2016) and Males and Setniker (2019). Males et al (2015) report on four smaller studies that used the framework to investigate how 62 prospective elementary and secondary mathematics teachers planned their lesson with their curriculum materials. The framework allowed them to fully describe: how teachers identified parts of tasks, elements of task design, and opportunities afforded by the tasks (attending); how they analyzed their curricula with respect to content, practices, and equity (interpreting); and how they created lesson plans, relying on their knowledge and beliefs to make decisions (responding). They concluded that prospective teachers "can learn to notice aspects of curriculum materials in order to make decisions about what to do and how to do it" (p. 94).

Males et al. (2016) conducted a preliminary comparative study of two high school mathematics teachers' lesson planning with differing years of experience using their curriculum materials. Analyzing portions of think aloud interviews as the teachers planned hypothetical lessons on

slopes, the authors used the framework to identify similarities and differences in the teachers' lesson plans. In exploring practicing teachers' lesson planning compared to prospective teachers using the three phases of the curricular noticing framework, they identified similarities and differences in their planning processes. They identified what these teachers attended to, how they interpreted the material they attended to, and how they responded based on their interpretations. Males et al. (2016) concluded that this strand of research can be extended to "determine how curricular noticing influences the enacted curriculum and eventually students' opportunities to learn" (p. 88). Lastly, Males and Setniker (2019) used the framework to investigate how four prospective mathematics teachers planned hypothetical lessons on slopes using two different sets of curricular materials. Using data from eye-tracking devices teachers wore as they planned their lessons, they found that initiated by paying attention to the curriculum, the preservice teachers simultaneously interpreted and responded to the curriculum. Moreover, the preservice teachers' experience with the materials and the elements of the curriculum materials and their formats seemed to influence their attention, as they seemed to attend more to specific parts of the curriculum materials, mainly "problems, exercises, or examples that included mathematical representations, such as graphs, tables or equations" (p. 163).

In this study, we extend the use of the curriculum noticing framework to the university context. We investigate a college instructor's (Casey) lesson planning as he used a dynamic inquiry-oriented calculus textbook for the first time in his calculus course to plan two lessons, one on applied optimization and one on related rates. Like in prior studies, we rely on interview data but we also observed Casey's lesson planning of actual lessons, meaning that our interview with Casey was not staged. Doing so allowed us to understand what lesson planning of a college instructor might look like in reality while dealing with environmental constraints, such as those imposed by the department or the institution. Similar to Males et al. (2016), we see our research as a step towards making sense of how curricula are enacted in classrooms and how that shapes students' opportunities to interact with and learn the content.

Curricular Context: The *Active Calculus* textbook

Active Calculus (Boelkins, 2019) is an open-source, open-access textbook created in PreTeXt, which allows the textbook to be rendered in various formats such as HTML and print (<https://pretextbook.org>). *Active Calculus* is freely available and can be viewed in PDF and HTML formats on smartphones, tablets, and computers. We call the textbook dynamic because of its open-source nature which gives the instructors the potential to tailor the textbook's content to their needs. When rendered in an HTML format, the textbook includes three interactive features: links to GeoGebra animations (a dynamic mathematics software that brings together geometry, algebra, spreadsheets, graphing, statistics and calculus), WeBWorK exercises (an online homework delivery system that allows students complete their homework online and receive instant feedback about the correctness of their responses, <https://webwork.maa.org>), and preview activities. The preview activities are problems that students are supposed to do ahead of class to preview the content; in the HTML format of the textbook these are configured so that students can type in their answers directly into their textbooks, making their responses immediately available to their instructors to review. To motivate students and connect the new concepts to previously-covered material, each section starts with a set of motivating questions. These types of questions have conceptual characteristics and highlight the significance of upcoming concepts by asking students to elaborate on their thinking and connect previously learned concepts, formulas, and ideas. As Boelkins (2019, Front Matter) points out, motivating questions were created to make clear to the students that the upcoming material is of great interest to the class.

Each section includes a preview activity, which together with a brief introduction, help to "foreshadow the upcoming ideas" in the section and are "intended to be accessible to students

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in advance of class, and to be completed by students before the day on which a particular section is to be considered” (Boelkins, 2019, Front Matter). Sections contain definitions, theorems, or rules, GeoGebra animations, and activities. The activities are often multi-step problems that guide students’ work by asking them to answer various questions (e.g., providing their reasoning, interpreting their solutions, making sense of procedures) as they solve an overarching problem. The sections are wrapped up with a summary of the newly-introduced material, a set of WeBWorK exercises where students can enter their answers and receive immediate feedback without penalty, and a set of non-WeBWorK exercise that “requires the student to connect several key ideas and expects that the student will do at least a modest amount of writing to answer the questions and explain their findings” (Boelkins, 2019, Front Matter).

“To engage in an active, inquiry-driven approach,” the author suggests that students view the textbook as a ‘workbook’ and come to class ready to work on the section’s activities with their peers after completing the preview activities prior to class (Boelkins, 2019, Front Matter). Instructors then are suggested to give short lectures and facilitate discussions as needed to prepare students for activities or wrap up their ideas. Every section of the textbook is accompanied with a YouTube channel with short introductory videos (2–14 minutes). Moreover, the author has designed worksheets for each section (hereafter ‘prep assignments’) for the students’ preparation for the upcoming lesson that he distributes to interested instructors upon request. The prep assignments include an overview of the upcoming section(s), list objectives of the lesson, and may direct students to watch a YouTube video, complete a preview activity, read specific parts of the textbook, state what they have learned, or list their questions. Together, we refer to the YouTube channel and the prep assignments as supplemental materials.

Because the textbook is intended to support students’ learning and make sense of calculus for themselves, proofs of stated theorems or solutions of problems are not included. The author’s goals for learners to “construct solutions and approaches to ideas, with appropriate support through questions posed, hints, and guidance from the instructor and text,” suggests that the author intends to provide an ‘inquiry orientation’ to his textbook. However, one cannot simply claim that using *Active Calculus* would imply teaching with inquiry, as it is the instructor’s decision to adopt the textbook’s suggestions, and therefore plan the “activity-driven approach.” Following that, the instructor can take advantage of the textbook features that promote inquiry-driven approaches and therefore shape their lesson planning in inquiry-oriented ways.

Before participating in the study, Casey taught with Stewart (Stewart et al., 2020),³ possibly being one of the most popular calculus textbooks in the United States. Although the resources for Stewart’s calculus textbooks have grown in recent years to include WebAssign (an online homework system), PowerPoint Slides for the instructor, and an online testing platform, these resources are mostly add-ons. Interactive features, such as GeoGebra animations, WeBWorK exercises, are not embedded in the textbook either. Altogether, the content and structure of the sections are mostly similar to the earlier editions. For example, the section on optimization (Stewart, 2016, Chapter 4) still outlines a strategy, provides examples, and states some rules such as the First Derivative Test. The textbook does not include activities to be worked on by the students in class, nor does it prepare students for the upcoming materials, as *Active Calculus* does via motivating questions and preview activities. In short, it seems that Stewart’s textbooks offered fewer opportunities and resources for Casey to change how he planned, and consequently taught, his calculus lessons to incorporate inquiry.

³ *Calculus* by Stewart and colleagues is currently in its 9th edition, with various versions of the original textbook available: *Calculus: Early Transcendentals*, *Essential Calculus*, and *Calculus: Concepts and Contexts*.

Methods

We showcase Casey's planning because he mentioned wanting to implement inquiry into his calculus course for some time after he had seen students benefit in his advanced mathematics courses but had not had the time to create the materials for calculus. Casey taught at a small private university in the Midwestern United States. At the time of the study, he had nine years of teaching experience at the university level, and this was his sixth time teaching calculus. Casey had eight students (three females) in his course, including first-year mathematics and physics majors, second and third-year chemistry majors, and third- and fourth-year biology majors. Most of the students self-identified as Caucasian. The class met four times a week: three 50-minute sessions held in a regular classroom and one 120-minute lab session held in a room with individual computers for students. In the past, Casey used Stewart (2016) and the PowerPoint slides that came with the textbook. He described his teaching as "mainly lecture (...) showing students what to do."

Data Sources

The data we analyzed are from a larger study that investigates students' and instructors' use of open-source dynamic textbooks in calculus, linear algebra, and abstract algebra courses (Beezer et al., 2018). These textbooks are authored in PreTeXt (<https://pretextbook.org/>), a mark-up language that makes it possible to include interactive features such as the possibility of opening and closing textbook elements, live computations in Python via Sage cells, automatic and immediate feedback for individual solutions to problems, and boxes that can collect student responses to questions, responses that are then available to instructors in real time. In addition, the textbooks can be seen on any device and at any time.

The data were collected over the Fall 2019 semester and include three interviews with Casey, audio recordings and fieldnotes of three classroom observations, a teacher survey (collected before teaching started), five teacher logs (short surveys collected throughout the semester), and documents (course syllabus, lesson plans, lecture notes). Of the collected data, we analyzed the first interview, the two documents that he created during that interview (the lesson plan for Wednesday shown in Figure 2 and the prep assignment for Friday shown in Figure 3), and his responses to selected questions about planning in a teacher survey and logs.

The interview, conducted on the 10th week of the term, focused on Casey's planning of one of the lessons we observed that week. During the interview, we video-recorded Casey for 30 minutes as he planned this lesson and created two documents. The first two authors watched the interview (60 minutes long) separately, kept notes and highlighted parts of the transcripts that showcased the three phases of the curricular noticing framework. We were interested in parts that displayed how Casey planned his next lesson while interacting with his curriculum material. After discussing what we found in the interview, the first author employed a thematic analysis of the whole interview, looking for Casey's actions that corresponded to the three phases of the curricular noticing framework. By *actions*, we mean anything that Casey said or did physically (e.g., looking, gesturing), therefore, we coded both the transcript of Casey's spoken words and his physical gestures and body movement as he planned his lessons.

The first author analyzed Casey's responses to 19 questions from the instructor survey that addressed planning, class activities, and his view and evaluation of the *Active Calculus* textbook. From the logs, we analyzed his responses to 10 questions that discussed his planning (e.g., How do you create your lecture notes for a class session?). We employed a thematic analysis for those responses that referred to the use of the curriculum materials for planning (e.g., textbooks, notes). We also checked the recordings and fieldnotes of Casey's

teaching of that lesson to see how much of Casey's planning was enacted in the classroom (for a complete analysis of his planning implementation, see Gerami et al. 2021).

Analysis

Our analysis of Casey's planning consisted of identifying actions that constituted his curricular noticing in the sources described above. The analysis was done in three passes, one for each of the phases of the curriculum noticing framework: attending, interpreting, and responding. To illustrate the analysis, we use an excerpt from Casey's planning on Applied Optimization for the Wednesday class. After presenting the excerpt we show how the analysis for each phase was carried out.

Preparation for Wednesday lesson

[On office computer's screen: the textbook, prep assignment Section:3.4, lesson plan]

- [1] Casey: I already made the prep assignments [looking at the prep assignment] for the students and posted them and in this particular case because we left a bunch of problems dangling, I will work on those first.
- [2] Casey: [*pauses to search the content of all curriculum materials he had on his screen*] Ok, problem 3.3.3 (i.e., Activity 3.3.3) [...] and I feel just by looking at the problems I already know what kind of questions the students are going to have. But I'll go over them and have the solutions ready just in case, and my suspicion is that this will take the first 15-20 minutes of class.
- [3] Interviewer: That was 3.3.2 the one they were working on the board today?
- [4] Casey: We finished 3.3.3 and I assigned them to do 3.3.4 as preparation for the next session. [*Looking at the content of Activity 3.3.4*] It is a standard introductory optimization problem, but I will be very surprised if they made it all the way through. So we will plan on talking about that for a while [*adds the note "Discussion of Activity 3.3.4" to his lesson plan*].
- [5] Casey: [*pauses to search the content of all curriculum materials he had on his screen*] It doesn't look like there is something unusual there ...oh it's a [real] world problem so they will have a lot of issues and then I also assigned the prep problem [Preview Activity[3.4.1, from the next session.
- [6] Casey: [*pauses to search the content of all curriculum materials he had on his screen*]But I know with what I want to start the next class [*scrolling back at Section 3.3: Global Optimization*] finishing Activity 3.3.3. So at this point I am suspecting that they have done those before class. I feel we will spend most of the class answering questions for these two problems going over those [Activity 3.3.4 and Preview Activity 3.4.1] a little more carefully in the morning in class.
- [7] Casey: [*looking at the content of Section 3.4.2: Notes*] So, the next part of the book is kind of a walk through over the steps which is roughly the same steps I use but I may have a handout with some more steps.

In the first pass, we identified actions belonging to the attending phase, such as: searching, looking, locating, surveying, and other ways of visually taking in the curriculum materials. In Turn 1, Casey mentioned that the prep assignments were already available for the students while searching for the prep assignment for Section 3.5: Related Rates, assigned to students for Wednesday's lesson. Between Turn 1 and Turn 2, we saw Casey searching the textbook's content, looking at the activities; in Turn 2, he located Activity 3.3.3⁴ ("Ok, problem 3.3.3"). In Turn 4, after recalling what they had done in the class and what he had assigned students to do, Casey searched his previous prep assignment document to check the activity he had assigned to his students, and when he located it, he looked at the textbook for Activity 3.3.4.

⁴ The activity asks students to find the exact absolute maximum and minimum of several functions on a bounded interval (see <https://books.aimath.org/ac/sec-3-3-optimization.html>).

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Between Turn 4 and Turn 5, Casey looked at both the textbook's content and his lesson plan. In Turn 5, Casey searched and located Preview Activity 3.4.1 from the next session, remembering that he also assigned it to the students. He momentarily paused between Turn 5 and 6, compiling his lesson plan to search for the rest of the textbook's content. In Turn 6, Casey scrolled back to Section 3.3: Global Optimization, searching for and locating Activity 3.3.3. He then perceived how his lesson plan was formulating, and he assessed which activities his students would have completed. Finally, in Turn 7, Casey searched and located Section 3.4.2: Notes, while looking back at his lesson plan.

In the second pass, we identified actions belonging to the interpreting phase, such as: digesting, questioning, comprehending, connecting ideas, and making sense of the material. In Turn 2, we saw him looking at Activity 3.3.3 (curricular attending phase) and he started making sense of the material and "by looking at the problems" he noted that he was already aware of the kind of questions the students were going to have. In Turn 4, we observed Casey connecting the ideas of previous optimization problems and making sense of Activity 3.3.4 considering it as a "standard introductory optimization problem," and questioned the difficulty of the activity and whether the students would have reached the final parts of it. In a similar fashion, after attending Activity 3.3.4 he first questioned the material (Turn 5) whether there was something out of the ordinary with the activity, realizing that there was anything that would surprise the students. He then comprehended the material, observing that it is a "world problem" and because of that he inferred that his students will face some issues (Turn 5) working around this problem. In Turn 6, Casey questioned whether time would be sufficient for what he planned to do after revisiting Activity 3.3.4 and Preview Activity 3.4.1 and made sense of this material. Finally, in Turn 7, after locating Section 3.4.2: Notes, he made sense of the material when he said that this next part of the book summarizes the necessary steps for anyone who works on optimization problems, and he connected the ideas of these Notes with "a handout with some more steps" that he considered to give to his students as additional material.

In the third pass over the episode, we identified actions belonging to the curricular responding phase. Within this phase, we looked for actions that suggested that he made curricular decisions based on his interpretation of curricular materials. Such actions include: choosing, manipulating, sequencing, and adapting the materials. In Turn 2, we observed Casey making sense of Activity 3.3.3 and then choosing to incorporate it in his lesson plan and sequencing it for the "first 15-20 minutes of class." In Turn 4, we observed Casey connecting the ideas of previous optimization problems (an action belonging to the curricular phase again) and then choosing Activity 3.3.4, sequencing it after Activity 3.3.3, and adapting his lesson plan to have a discussion with his students about the content of this activity. Similar actions were observed in Turn 5, where Casey chose to incorporate Preview Activity 3.4.1 from the next session in his lesson plan. He again adapted his lesson plan (Turn 6) when he mentioned that he planned to revisit Activity 3.3.4 and Preview Activity 3.4.1, "a little more carefully in the morning in class." Turn 7 shows Casey connecting the ideas of Section 3.4.2: Notes, choosing to adapt the steps for solving any applied optimization problem. He looked for his own handout, which included some additional steps, and thus, manipulated the already existing material.

In the following section, we present our findings based on the three phases of the curriculum noticing framework: attending, interpreting, and responding. Organized by each phase of the framework, we identify Casey's actions that constituted his lesson planning focusing on the use of the *Active Calculus* textbook. The first author coded the whole interview (a total of 55 turns) with the 60% of the turns also checked from the rest of the authors.

Findings

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In the initial survey, Casey reported using the textbook (not *Active Calculus*) and its resources for planning his lessons. Before working with *Active Calculus*, he reported that the homework design was “usually done from the book” using the WebAssign online education platform and that for planning his lessons, he often used the book for guidance, navigation and to order the topics of the session. However, in the logs and interview questions he reported that he consulted the textbook regularly throughout the semester and used all three interactive features of the textbook (GeoGebra animations, preview activities, and WeBWork exercises) for his planning. In the interview, we observed Casey planning the core of his lessons (for Wednesday and Friday) around the textbook’s activities (Preview Activities and Activities). He also mentioned that this was his first time planning a calculus lesson in such a way because he had difficulties finding a textbook “with the right density” of activities per section. We saw that his planning practices involved mainly the textbook because both the lesson plan and the prep assignments’ content that he created and used derived from the textbook and its features such as activities, GeoGebra links, examples, and online introductory videos.

In planning his Wednesday lesson, Casey first created a lesson plan for personal use that would map out the sequencing of his teaching (Figure 2). The core of the planning session was creating the prep assignment for the Friday lesson (Figure 3). The prep assignment included examples, GeoGebra links, a lesson overview, and a set of basic learning objectives. We found actions from each phase in the interview. Table 1 shows example actions for each phase of the curricular noticing framework. As mentioned before, actions are behaviors we observed Casey doing (e.g., looking, searching) or behaviors he said he engaged in. In the remainder of this section, we categorize our findings by reporting on the three phases of the curriculum noticing framework: attending, interpreting, and responding.

Curricular Noticing Phases	Example Actions
Curricular Attending (e.g., seeing, locating, recognizing, searching)	• Casey pauses to search the textbook content scrolling down Sections 3.4.1 and 3.4.2.: “So the next part of the book is a walk through the steps” • Casey looks at the content of Section 3.5: “I already know this is going to be a tough one, students are always in trouble with related rates.” • Casey searches and locates a particular GeoGebra link (Oil Slick): “[Here are] a bunch of links that helps you picture things”
Curricular interpreting (e.g., making sense, questioning, connecting)	• Casey makes sense of the content of Activity 3.4.3.: “That is the standard kind of problem I know students have trouble with, there is some Pythagorean Theorem and stuff” • Casey interprets and comprehends the content of Section 3.5.: “So the next part of the book is a walk through the steps [...] roughly the same steps I use.” • Casey makes sense of the content of the Oil Slick GeoGebra link by playing with it.: [nodes his head in agreement]
Curricular responding (e.g., choosing, adapting, deciding)	• Casey includes Activity 3.4.3 in his lesson plan.: “I will put them in groups and see how far they can get.” • Casey chooses a number of GeoGebra links and includes them in the prep assignment: “I will probably have them do something with these as part of the prep assignments.”

	Casey revisits parts of the textbook, chooses the Oil Slick GeoGebra link and adds it in the prep assignment.: “I will ask them something very qualitative about this, not really looking for the right or wrong, just wanting them to think about this relationship.”
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Table 1: Examples of actions of the three Curricular Noticing phases from the planning interview

3.4 Optimization - Discuss problems left from last time: 3.3.3. - Take up preview. - Discussion of Activity 3.3.4. - Discussion of Activity 3.4.1. Discuss problem solving steps. Activity 3.4.2: (a)-(b) Students work, then compare. (c) and discuss. (d) and discuss. Activity 3.4.3: - Students do in pairs. Activity 3.4.4: - Students work in pairs. Activity 3.4.5: - If time allows. -OR- Do in lab. -OR- assign for next time.

Figure 2: Casey’s lesson plan for Wednesday’s lesson

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Calculus 1
Section 3.5 Related Rates

Directions Before Class

Spend at most 60 minutes completing this assignment. As you do so, take notes for yourself that summarize 3 things that you have learned or that you have questions about. If after 60 minutes you have not finished, please stop and write, "I have worked for 60 minutes."

Overview

We conclude our work in Chapter 3 (Using the Derivative) by seeing how the derivative may be employed to *relate the rates* of two different quantities that are related and each changing as time varies. The main idea here is: if two quantities are related to one another, and each is changing as time changes, then the *rates* at which each quantity is changing must be *related*. Hence, we consider a class of problems known as *related rates problems*.

Basic learning objectives

These are the tasks you should be able to perform with reasonable fluency **when you arrive at our next class meeting**. Important new vocabulary words are indicated *in italics*.

- Understand the basic idea of a related rates problem: if the values of two (or more) changing quantities are related, then their respective rates of change must also be related.
- Be able to use basic geometry results such as the Pythagorean Theorem, formulas for area of familiar figures, and trigonometry to establish relationships among quantities of interest.
- Understand the difference between differentiating an equation such as $A = x^2$ with respect to t and differentiating with respect to x . (Remember that this was discussed as *implicit differentiation*.)

Preparation for Class

Reading: Read 3.5 through the bullet points before 3.5.2. You should skim over example 3.5.1 (and its solution), but you can skip 3.5.2. You will answer some questions about the applets before 3.5.2.

Watching: Watch the following additional resources on the web that have been developed to support your learning:

- Screencast 3.5.1: <http://gvsu.edu/s/vH> recaps the key ideas of the section.
- Screencast 3.5.2: <http://gvsu.edu/s/vG> is a straightforward example.

Do the following before class.

1. Work preview problem 3.5.1.
2. List three things you learned or questions you have after looking over the intro to the section.
3. Play around with the Oil Slick applet and get a feel for the relationship between the radius and area. Notice that the radius is increasing at a constant rate. Write a sentence summarizing what's happening to the rate of change of the area.
4. Play around with the Conical Tank applet and get a feel for the relationship between the height and the volume. Notice that the height is increasing at a constant rate. Write a sentence summarizing what's happening to the rate of change of the volume.
5. Consider the lamp post applet. Write a sentence summarizing the relationship between the skateboarder's position and the length of his shadow.

Advanced learning objectives

In addition to mastering the basic objectives, here are the tasks you should be able to perform **after class, with practice**:

- Solve a wide range of related rates problems, including those found in Activities 3.14 - 3.17.

To master the advanced learning objectives.

- There are three automatically graded problems if you need practice.
- Submit written answers to problems 4, 5, 6.

Figure 3: Casey's prep assignment for Friday's lesson

Curricular Attending

Casey mentioned on several occasions that he relied primarily on the textbook and usually searched through his past notes for anything that might be missing. In planning these lessons, Casey was particular about what he searched for and looked at in his curriculum materials.

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Casey mentioned in the interview that to plan his lessons he used the following resources: *Active Calculus* textbook, its resources, the prep assignment of the previous lesson, the Boelkins's provided prep assignments, past lecture notes, Stewart's Calculus (2016), and power-point slides. In his planning session, we saw him attending to all of them except the Stewart's Calculus textbook and its accompanying power-point slides. To prepare the lesson on optimization, Casey, at times, he only looked into the parts of the textbook's activities that he considered essential for the lesson. On multiple occasions, we observed Casey locating among his available resources the material he was searching for, showing awareness of the textbook's content.

For his planning, Casey searched *Active Calculus* textbook for all the new sections, activities, preview activities, examples, and GeoGebra links and located parts of the textbook that he may spend more time on with his students. He searched for the prep assignments that the textbook's author shared with him to find the content he was planning to teach. We also saw Casey reading and consulting the preface of the textbook (<https://books.aimath.org/ac/preface-for-instructors.html>), especially the "Instructors read this!" section that has suggestions for instructors. This section suggests instructors to develop instructional sequences by working on the preview activities and activities. Moreover, Casey searched and used the textbook's accompanying YouTube channel. In the following curricular phases we see how Casey interpreted and responded after attending to this part of the textbook.

Curricular Interpreting

We saw Casey making sense of the material he attended to and making connections between curricular materials for his planning. For example, we observed Casey interpreting the content of the author's prep assignment and connecting his interpretation of the assignment's content to his own prep assignment. He skimmed the content of the activities and brought his understanding and knowledge to make decisions. For example, he made assumptions regarding the kind of questions that his students may have and the possible difficulty they would face when using the derivative to find absolute maxima and minima of the functions in an activity (Activity 3.3.3), "I feel just by looking at the problems I already know what kind of questions the students are going to have."

In a similar fashion, we observed Casey making sense of Related Rates activities (Section 3.5) when he mentioned being aware of the difficulties his students face when working on those problems, "I already know this is going to be a tough one, students are always in trouble with related rates," and mentioned that he would be prepared for a lot of questions from his students. When skimming through activities, he interpreted their content by making sense of the nature of these activities. For example, when he referred to Activity 3.3.4, which asks students to find the maximum possible volume of a cardboard, he identified it as "a standard introductory optimization problem," acknowledging that the problem may be difficult for his students. Casey pointed out that he would have been "very surprised if they [would have] made it all the way through" Activity 3.3.4, which we interpreted as an instance of Casey's interpretation of the content and using his experience to anticipate what would happen with a particular task.

Similar processes for interpreting the material took place while Casey made sense of various activities and links (e.g., Activities 3.4.2, 3.4.3. and 3.4.5; GeoGebra links; screencasts). He recognized opportunities embedded in the textbook's activities and GeoGebra links, thinking they might "provoke discussion among his students" or be redundant, questioning whether he should add Activity 3.4.3 (an optimization word problem) to his lesson plan, because it repeated the textbook's content (i.e., finding maximum possible area and perimeter) and be somehow similar to the previous problem.

Casey expressed his assumptions (i.e., questioned whether he should include the material) regarding the difficulty of the activities, GeoGebra links, and examples' content. For example, in Section 3.5 he assumed that "students are always in trouble with related rates." He therefore used the content of the section and his anticipation regarding student difficulties with the concepts for what could be interpreted as a "good starting point" for the students. Casey wondered about the amount of class time that he would need, should he choose to include these parts of the textbook (e.g., "[Activity 3.3.3] would take about 15-20 minutes of class time"). Content and how it was presented, especially regarding examples and the embedded GeoGebra links, were also a key consideration for what to include or not in the prep assignments. For example, when Casey made sense of the content of a GeoGebra link that described how a circular oil slick's area was growing as its radius was increasing, he considered asking a "qualitative" question about the problem, which he did during the responding phase.

Curricular Responding

Casey made many decisions related to the inclusion of textbook features in creating the lesson plan and the prep assignment based on information about the work done during the prior class session (e.g., he said that Activity 3.3.3 was "left dangling"). He reassessed the class time needed to cover the Activity 3.3.3 and the activities that followed after. On multiple occasions Casey sequenced his materials by breaking activities into parts so that they could be worked on by students in pairs or be discussed with the whole class. He weighed the content and the work he had assigned students regarding a new activity, Activity 3.3.4 about minimizing the distance between three points on a right triangle and decided to include a whole-class discussion to address it. We observed Casey choosing to add discussions to his lesson plan or adding content from the textbook (examples, activities) based on what he believed his students needed at the time.

Activity 3.4.4. Consider the region in the x - y plane that is bounded by the x -axis and the function $f(x) = 25 - x^2$. Construct a rectangle whose base lies on the x -axis and is centered at the origin, and whose sides extend vertically until they intersect the curve $y = 25 - x^2$. Which such rectangle has the maximum possible area? Which such rectangle has the greatest perimeter? Which has the greatest combined perimeter and area? (Challenge: answer the same questions in terms of positive parameters a and b for the function $f(x) = b - ax^2$.)

Figure 4: Applied optimization in the *Active Calculus* textbook, Activity 3.4.4.

After attending and interpreting an activity that Boelkins suggested (e.g., preview activity 3.5.1), Casey thought the activity was a "good starting point" and included it in his own prep assignment. Casey included the Oil Slick Geogebra link in the prep assignment, and because he wanted students to consider the relationship between two rates, he added a "qualitative" question to the assignment: "get a feel for the relationship between the radius and area. Notice that the radius is increasing at a constant rate. Write a sentence summarizing what's happening to the rate of change of the area" (Figure 3, Friday Prep assignment). Using the information gained from attending to and interpreting Boelkins's prep assignment, he excluded sections that he felt would not go well with his lesson (i.e., estimating zeros of functions using Newton's Method) but included almost all the textbook material, providing directives to students about what to work on (e.g., WeBWork problems prior to the homework) and how ("skim Examples 3.5.1").

This phase of the curriculum noticing framework includes decisions about how the teacher responds to the curriculum and how these responses are enacted in the classroom. During the Wednesday class, we observed Casey following through with the plan. Starting with

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Activity 3.3.3, he answered students' questions about the activity and then assigned students to new teams to work on the board on the next applied optimization problem, Activity 3.4.3. Similarly, and as planned in his lesson plan (Figure 2), students worked in pairs with Activity 3.4.4 (Figure 4).

Overall, we observed Casey's planning practices including actions from the whole spectrum of the curricular noticing framework. Furthermore, as Dietiker et al. (2018, p. 527) anticipated, we observed that even though "this framing presupposes the interactions unfolding in a linear fashion," Casey's engagement in the phases of interpreting and responding prompted engagement in other phases defined by the framework. Casey looked back at his lesson plan (attending) after connecting the ideas of a section (interpreting) that illustrated the steps that should be executed in any applied optimization problem and choosing (responding) to manipulate the material by incorporating additional steps.

Discussion and Conclusion

In this study, we aimed to understand how *Active Calculus*, a dynamic calculus textbook designed to support inquiry-oriented ways of teaching and learning, was used by a calculus instructor to plan lessons. We found that Casey, through the creation of two documents—his lesson plans and prep assignments for the students—reflected on the resources he had at hand, namely the textbook, Boelkins's prep assignments, and his previous prep assignments, searching for specific content and features that he could use. For the first time in his teaching career, Casey planned his lessons with an inquiry-oriented calculus textbook. He asserted that the textbook changed the way he planned each lesson "in terms of what [to do] in class but even in terms of the large layout [of content and its ordering]." Casey's decisions were mainly about selecting and sequencing textbook activities and interactive features in both documents and how he and his students would interact with them (e.g., in whole-class discussion or group work). Casey closely followed the textbook and its supplemental materials in designing these documents and consequently embedded the author's inquiry-oriented intentions for textbook use.

Our observations suggest that Casey was happy to work with *Active Calculus*, a textbook designed to promote inquiry, because it supported him in implementing his inquiry-oriented visions and goals while planning his lessons. He used the textbook's motivating questions, the activities to be worked on by the students in class, and the preview activities that would prepare students for the upcoming material, ideas, and concepts. His own experience in the classroom reinforced that the textbook was supporting his vision and he indicated that he found students asking insightful questions related to the topic of the day much earlier "within three minutes of class time." Thus, seeing that students were engaging differently with the content was another confirmation that his shift towards inquiry was a good one.

Casey's planning was supported by the textbook and its supplemental materials. As we described, Casey closely followed Boelkins's prep assignments, his recommendations for structuring each lesson, and the textbook presentation and activities for students. Overall, these materials afforded him time to think through the available information, tweaking the details not aligned with his goals and visions. As this was Casey's first time teaching calculus with *Active Calculus* or with inquiry, it is possible that as a first-time user, Casey relied on the textbook and the author's suggestions more, while building up knowledge, confidence and experience working with the textbook (Mesa & Griffiths, 2012).

Our study extends previous use of the curriculum noticing framework. Whereas Males and colleagues (Males et al., 2015, 2016; Males & Setniker, 2019) have focused on curriculum noticing of prospective teachers using staged interviews, we use the framework to understand the planning of actual lessons by an experienced teacher. Casey's experience and prior

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knowledge played a key role in his planning practices, as he appraised the textbook and Boelkins's prep assignment and used them to plan his lessons and create his prep assignment. Casey exhibited having "curriculum vision" needed for planning with the new textbook, which is defined by Drake and Sherin (2009) as "an understanding of the mathematical and pedagogical goals of the curriculum materials," and its content, design, structure, and philosophy (p. 333). As a result, Casey's curriculum vision aided him in recognizing opportunities embedded in the curriculum materials (Dietiker et al., 2018) and deciding to incorporate them in his teaching. Thus, it should be expected that an experienced teacher may exhibit more nuanced and purposeful actions during each phase of the curricular noticing framework.

Our study has limitations. First, as a case study, this investigation does not suggest that every instructor new to inquiry or new to *Active Calculus* would use the textbook for planning lessons in the way Casey did or rely on it to the extent described in this paper. Because our study may not be representative of planning practices in undergraduate calculus courses (e.g., those at larger universities or with more students), more research is needed to better understand instructor use of curriculum materials, specifically those that are novel. Second, we did not capture the full extent to which *Active Calculus* influenced lesson planning because we only analyzed the planning of two days of Casey's calculus course. However, because our observations took place during the 10th week (out of 16 weeks), it is safe to assume that the observed planning practices were representative of how other lessons were planned and that these practices did not substantially change after our visit or were different from how he planned before the visit.

Next Steps and Implications

Some questions are left for further investigation. First, although we briefly mentioned and described how an instructor's planning practices were shaped by an inquiry-oriented textbook, we did not explicitly investigate changes of those practices using different curricula (e.g., Stewart's Calculus vs. *Active Calculus*) over time: How did the instructor's relationship with the curriculum materials of an inquiry-oriented calculus textbook evolve compared to when using conventional textbooks? To what extent is the relationship specific to calculus? Answering these questions would allow us to parse out better the role of this textbook in planning. Seeing the role of the textbook in providing a wealth of activities for Casey to choose from and teach with, it would be appropriate to exclusively explore the content and the influence of activities or other specific textbook elements (e.g., GeoGebra links, preview activities) on instructors' teaching. Third, knowing that as instructors "gain experience teaching with a particular textbook, the mediation of the textbook with instruction changes" (Mesa & Griffiths, 2012, p. 100), we wonder whether and how Casey's use of the textbook would be different as he uses it more. This would allow us to identify elements of inquiry that are more challenging to sustain over time through curriculum use alone.

Our study suggests that investigating how textbooks influence and shape undergraduate mathematics education is an important and promising area of research. These detailed analyses allows us to see how textbooks can shape planning processes and alter practice in ways that increase student engagement in the classroom. Our study shows that a textbook that is oriented towards inquiry can support and even influence instructors in planning a lesson with inquiry characteristics. Such research would also allow textbook authors, designers, and developers to better design textbooks that support teaching and learning at the tertiary level.

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