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ORIGINAL PAPER

University instructors' use of questioning devices in mathematics textbooks: an instrumental approach

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Abstract

The goal of this study is to identify the multiple ways in which instructors take advantage of a feature designed into university textbooks that seeks to invite students to get acquainted with the content prior to attending the class in which such content will be discussed. We present an analysis of teacher instrumentation of this feature, which we call questioning devices, using data from 15 instructors who taught calculus, linear algebra, or abstract algebra over one semester. The instructors taught at 14 different universities in the United States. We identified four utilization schemes of the questioning devices in which instructors: completed questioning devices for pre-planning, required students to complete the questioning devices for the purpose of lesson planning, used the questioning devices for the purpose of instruction, and required students complete the questioning devices for the purpose of assessment. These schemes are supported by various operational invariants related to self-perception as competent instructors and implicit theories of teaching and learning. The identified utilization schemes inform textbook developers and author-designers, making them aware of whether these features fulfill their design purposes, and possibly think about changes that might be needed to support instructors in achieving their instructional goals and improve learning. We suggest some further areas of inquiry.

Keywords Questioning devices in textbooks · Instrumental approach · University instructors · Mathematics textbooks · Utilization scheme

1 Rationale and context

Research on textbook use by instructors tends to be broad, attending to the whole textbook (e.g., González-Martín et al., 2018; Mesa & Griffiths, 2012) or to whole documentation systems to which textbooks belong (e.g., Gueudet & Pepin, 2018), rather than to specific elements within the textbook. Understanding how mathematics instructors use specific features embedded in textbooks is important as new technologies are making it possible to create textbooks that are more interactive. Not only can authors produce, distribute, and revise their own textbooks bypassing large publishing companies, but they can also add tools to achieve specific goals—in other words, they are both authors and designers; we hereafter refer to them as author-designers to highlight their dual role in textbook production. As textbook developers and author-designers take advantage of these new technologies, the question becomes: How are particular features of textbooks used by many university instructors as they engage in activities related to mathematics teaching?

In this study, we attempt to answer that question empirically using a subset of data collected from a large-scale study of undergraduate mathematics instructors' textbook use across the U.S. The instructors taught a university course with one of three textbooks—*Active Calculus* (hereafter AC, Boelkins, 2019), *A First Course in Linear Algebra* (hereafter FCLA, Beezer, 2019), and *Abstract Algebra: Theory and Applications* (hereafter, AATA, Judson, 2019). These textbooks are authored in PreTeXt (https://pretextbook.org/), a mark-up language that makes it possible to create textbooks that are open-source and open-access, textbooks that when presented in an HTML format, can include features, such as live computations in Python via Sage cells, hyperlinks to various sections of the

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textbook, automatic and immediate feedback for individual solutions, and boxes that can collect student responses to questions, responses that are then available to instructors in real time.

In this paper, we describe how 14 university faculty used this last feature, which we call *questioning devices*, following Love and Pimm (1996). Questioning devices are content-related questions designed to invite students' reflection and engage them with the mathematics at hand before encountering the material in a class setting. Having students read the textbook ahead of class is an important goal for mathematics instructors, as reading mathematics is considered an important skill for learning mathematics (Axtell & Turner, 2007; Benn, 1997; Inglis & Alcock, 2012; Mesa & Griffiths, 2012; Shepherd et al., 2010; Weber et al., 2008). As with any other design product, however, users will invent other uses that will fit their needs (Rabardel & Waern, 2003; Ruthven, 2018); identifying departures from the intended use and new uses of a feature can inform author-designers as they modify it or envision other features that might be used more efficiently for the new invented uses.

We are interested in questioning devices because they have the potential to influence student interaction with their textbooks in ways that can be supportive of student learning and because they might influence teaching practices. Student engagement has been documented in programs that use interactive technologies (see e.g., Clark-Wilson, 2010; Ruthven et al., 2008, 2009) and that are embedded in active digital curricular programs (Choppin et al., 2014). We attend to how teachers use this feature in their teaching because we have found that students will use their textbooks as directed by their instructors (Mesa et al., 2019). Thus, knowing how instructors themselves use the questioning devices for their teaching activities (planning, class instruction, assessment) can inform whether these features can encourage students to read mathematics in their textbooks.

Focusing on questioning devices contributes to understanding the role of this type of questioning embedded in curriculum materials. While teacher and student questioning in classrooms have been studied in K-12 settings (see DeJarnette et al., 2020 for a comprehensive review) and in post-secondary settings (see e.g., Gerami et al., 2020; Paoletti et al., 2018; Viirman, 2015), how these types of textbook questions are used has not been investigated. There are analyses of examples and exercises in university textbooks, but these focus only on the qualities of those examples and exercises and on the opportunities for learning that they afford students (e.g., Mesa, 2010; Tallman et al., 2016; White & Mesa, 2014) not on how they are used by teachers in their work. This information is useful to textbook developers and author-designers, completing a feedback cycle that can lead to the improvement of features that could have an impact on classroom practice.

As mentioned earlier, authoring textbooks used in PreTeXt has some advantages, one of them being that it allows for multiple renderings of the textbook in multiple formats, online and print (including Braille) without any additional formatting. As open source and open access textbooks they are distributed through licenses that are not subject to the traditional copyright agreements (e.g., Creative Commons, GNU Free Documentation License). Updates are nearly immediately available, and when the textbooks are accessed online, it is possible to track users' viewing activity. For these and other textbook features that can be added to the textbooks, O'Halloran et al. (2018) called them "the latest generation of online mathematics textbooks" (p. 865).

In any given section of his calculus textbook, Boelkins (2019) has three types of questioning devices: motivating questions, preview activities, and activities. Motivating questions constitute a set of one to five questions that open every section, making the learning goals of the section explicit (e.g., "If we know the velocity of a moving body at every point in a given interval, can we determine the distance the object has traveled on the time interval?" Sect. 4.1: determining distance traveled from velocity). The preview activities have between three and eight questions organized around a specific situation. The activities are similar to exercises, except that they are interspersed within the other types of text: explanatory, expository, and examples, and are meant to be assigned as a classroom activity. Figure 1 shows parts of the first preview activity (a) and of the first activity (b) in Sect. 4.1. Of these, only the preview activities are intended to be answered by the students prior to attending class.

In his linear algebra textbook, Beezer (2019) uses two types of questioning devices throughout each section: questions interspersed throughout the explanatory, expository, and examples text; and reading questions; the interspersed questions seek to "inspire further exploration of the mathematics" (Love & Pimm, 1996, see Fig. 2). In his abstract algebra textbook, Judson (2019) uses the same two types of questioning devices as the linear algebra textbook (see Fig. 3a), but he places the reading questions at the end of each chapter, before the exercises and other ancillary material (see Fig. 3b).

When these textbooks are viewed in the HTML format, students can directly type their responses into a box, and those responses are immediately available to their instructors.

In order to facilitate the study of questioning devices embedded in the textbooks, we chose those devices for which three conditions apply: first, the author-designers made explicit their expectation that students will engage with the questioning devices by asking students to use them as they prepare for class and for teachers to assign them to students; second, it is possible for the instructors to collect

Preview Activity 4.1.1. Suppose that a person is taking a walk along a long straight path and walks at a constant rate of 3 miles per hour.

a. On the left-hand axes provided in Figure 4.1.1, sketch a labeled graph of the velocity function $v(t) = 3$.

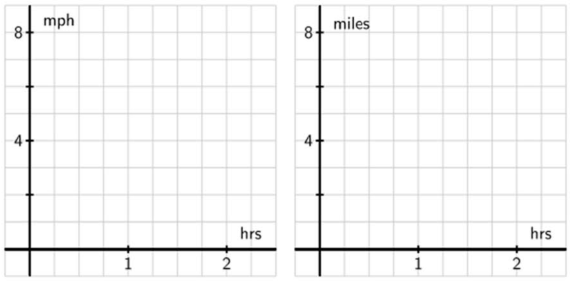

Figure 4.1.1. At left, axes for plotting $y = v(t)$; at right, for plotting $y = s(t)$.

Note that while the scale on the two sets of axes is the same, the units on the right-hand axes differ from those on the left. The right-hand axes will be used in question (d).

Responses→

(a)

Activity 4.1.3. A ball is tossed vertically in such a way that its velocity function is given by $v(t) = 32 - 32t$, where t is measured in seconds and v in feet per second. Assume that this function is valid for $0 \le t \le 2$.

a. For what values of t is the velocity of the ball positive? What does this tell you about the motion of the ball on this interval of time values?

b. Find an antiderivative, s, of v that satisfies $s(0) = 0$.

c. Compute the value of $s(1) - s(\frac{1}{2})$. What is the meaning of the value you find?

d. Using the graph of $y = v(t)$ provided in Figure 4.1.8, find the exact area of the region under the velocity curve between $t = \frac{1}{2}$ and $t = 1$. What is the meaning of the value you find?

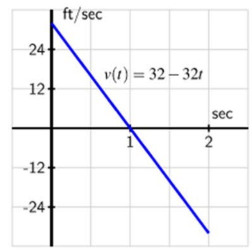

Figure 4.1.8. The graph of $y = v(t)$.

(b)

Fig. 1 Excerpts of two questioning devices in Boelkins's (2019) calculus textbook: **a** preview activity **b** activity, Sect. 4.1: determining distance traveled from velocity

Finally, you may have wondered why we refer to a matrix as *nonsingular* when it creates systems of equations with *single* solutions (Theorem NMUS)! I have wondered the same thing. We will have an opportunity to address this just before Theorem NPNT, and again when we get to Theorem SMZD. Can you wait that long?

NM Reading Questions

1. In your own words state the definition of a nonsingular matrix.
My answer→

2. What is the *easiest* way to recognize if a square matrix is nonsingular or not?
My answer→

3. Suppose we have a system of equations and its coefficient matrix is nonsingular. What can you say about the solution set for this system?
My answer→

Fig. 2 Sample of questioning devices in Beezer's (2019) linear algebra textbook, Section NM Nonsingular Matrices

student responses to those questions in real time, when the textbooks are viewed in HTML format; and third, it is possible to track users' viewing of those questions. These conditions eliminated motivational and other rhetorical questions, leaving us with the *reading questions* in the linear algebra and abstract algebra textbooks and the *preview activities* in the calculus textbook. These are described in the textbooks by two of the author-designers[1] with an explicit indication that the work with these devices needs to be done prior to attending class:

> If you are enrolled in a [linear algebra] course, read the section *before* class and then answer the section's reading questions as preparation for class (FCLA-Preface, emphasis in original)
> Collecting responses to the Reading Questions prior to covering material in class will require students to learn how to read the material. Sections are designed to be covered in a 55-min lecture. Later sections are longer, but as students become more proficient at reading the text, it is possible to survey these longer sections at the same pace. (FCLA-Preface, To the Instructor)
> *Preview activities* are designed to foreshadow the upcoming ideas in the remainder of the section; both the reading and preview activity are intended to be accessible to students *in advance* of class, and to be completed by students before the day on which a particular section is to be considered. (...) there is a great deal of scholarship (…) that shows people learn better

[1] As for the abstract algebra textbook, Beezer who designed the reading questions, indicates that "collecting responses to the Reading Questions prior to covering material in class will require students to learn how to read the material." (Rob Beezer, personal communication, December 29, 2020).

Example 5.18. The group A_4 is the subgroup of S_4 consisting of even permutations. There are twelve elements in A_4:

$$\begin{array}{llll} (1) & (12)(34) & (13)(24) & (14)(23) \\ (123) & (132) & (124) & (142) \\ (134) & (143) & (234) & (243). \end{array}$$

One of the end-of-chapter exercises will be to write down all the subgroups of A_4. You will find that there is no subgroup of order 6. Does this surprise you?

(a)

5.3 Reading Questions

1. Express $(1\,3\,4)(3\,5\,4)$ as a cycle, or a product of disjoint cycles. (Interpret the composition of functions in the order used by Sage, which is the reverse of the order used in the book.)

Click twice to edit...

2. What is a transposition?

Click twice to edit...

3. What does it mean for a permutation to be even or odd?

Click twice to edit...

4. Describe another group that is fundamentally the same as A_3.

Click twice to edit...

5. Write the elements of the symmetry group of a pentagon using permutations in cycle notation. Do this exercise by hand, and without the assistance of Sage.

Click twice to edit...

(b)

Fig. 3 Examples of questioning devices used in Judson's (2019) abstract algebra textbook **a** question embedded in an example **b** reading questions, Section Permutations

when they interactively engage and struggle with ideas themselves, rather than passively watch others. (AC-Preface, emphasis in original)

Thus, these two questioning devices—reading questions and preview activities—are designed by the authors into the textbooks with the explicit goal of making "the text more active for the learner" (Love & Pimm, 1996, p. 403), enticing students to read *before* class, with the goal of making them "learn how to read the material." However, accomplishing these goals will be in part supported by how instructors use these devices in the classrooms; in this study, we seek to document how a group of faculty using these textbooks use these devices in their teaching. For this purpose, we use the instrumental approach, explained next.

2 Theoretical framing

Love and Pimm (1996) note that two pedagogical functions of textbooks, curricular and conceptual (van Dormolen, 1986), are accomplished by organizational devices such as "exposition, explanation, questioning, exercises, examples, and tests… [to] achieve a more active reader of the text and to manifest the aims of the mathematical and cognitive progression" (p. 386). One of these devices, *questioning*, differs from exercises and classroom questions in key ways. Whereas textbook exercises are "the principal means by which the student is encouraged to be an active reader of the text" (Love & Pimm, 1996 p. 387) and tend to progress through various levels of difficulty, questioning devices are statements embedded in explanatory text that may require the reader "to complete some task before the narrative can proceed or… to inspire the student-reader to explore mathematics further" (p. 386). Expository text that includes such questioning is meant to "create problems for students, as they need to learn to interpret signals concerning which activities they are required to undertake" (p. 386). Mason (2020) notes that questioning in classrooms takes the form of "prompts offered to students so as to help them get unstuck or direct their attention in a potentially useful way so that they make mathematical progress" (pp. 705–706). Questioning in textbooks has a different character as they are meant to establish a personal relationship between the reader, the author, and the mathematics (Weinberg & Wiesner, 2011), a relationship that tends to be private. As the increased level of interactive technologies has also reached textbooks, it is now possible for students to type their responses directly in the textbook in response to a question, making those responses immediately available to instructors. Instructors can access students' ponderings, understandings, and reflections on the content, which might inform decisions about teaching.

We use an instrumental approach (Rabardel, 2002; Vergnaud, 1998; Vérillon & Rabardel, 1995) to describe instructors' interactions with these questioning devices in textbooks. An instrument is an entity, "a whole incorporating an artifact (a human-made or a physical object) and one or more utilization schemes" (Rabardel, 2002, p. 65). A utilization scheme is a set of user's stable and structured manners of operating an artifact that encompasses observable elements, such as behaviors, and non-observables ones like goals and rationales. Vergnaud (1998) defines such schemes as the "invariant organization of behavior for a certain class of situations" (p. 229). This organization of behavior encompasses goals and expectations; rules of action (which can be seen as the "generative part of the schemes" p. 229); operational invariants, propositions that are held to be true by the subject when they act, that are used "to infer, from the available and relevant information, appropriate goals and rules"); and possibilities of inferences (p. 229). Vergnaud stated that "situations and schemes are essential in the development of knowledge" (p. 237). Instrumental genesis is the process through which a user transforms an artifact into an instrument by creating utilization schemes associated with that artifact (Rabardel, 2002). The artifacts are more often pre-existing, but they are instrumentalized by a subject—the user. Consequently, different subjects can produce different instruments even though they operate the same artifact. The instrumental genesis process is carried by the subject.

> Because it concerns the two poles of the instrumental entity—the artifact and the utilization schemes, it also has two dimensions and two orientations that are both distinguishable and often related: instrumentalization directed toward the artifact and instrumentation relative to the subject him/herself. (Rabardel, 2002, p. 101)

So whereas instrumentalization processes refer to the "emergence and evolution" of the artifact component of the instrument that may prolong the artifact, instrumentation processes refer to "the emergence and evolution of utilization schemes and instrument-mediated action: their constitution, their functioning, their evolution by adaptation, combination coordination, inclusion and reciprocal assimilation, the assimilation of new artifacts to already constituted schemes, etc." (p. 103). An instrumentation process is both "the action by which a subject acquires an artifact, and the effect of this action on the subject, who develops, from this artifact, an instrument for performing a task" (Trouche, 2020, p. 407). The notion of utilization schemes is central to instrumentation.

In our study, we are interested in the instrumentation of one artifact—questioning devices in textbooks—as instructors describe their use for teaching. More concretely, we are interested in identifying the utilization schemes created by instructors when using questioning devices available in three textbooks designed to increase student interaction with the content. To analyze our data (see Sect. 4.3), we methodologically leaned on Gueudet (2017) and González-Martín et al. (2018), who used the documentational approach to didactics (DAD, Gueudet & Trouche, 2009; Trouche et al., 2020). DAD is derived from the instrumental approach and emphasizes "the variety of artifacts [teachers] consider" (Gueudet & Trouche, 2009, p. 205) using the term *resources* instead of *artifact*; it was developed "to study the interactions between teachers and resources and their consequences, in a context where an abundance of teaching resources is available, on the internet in particular" (Gueudet, 2017, p. 200). Because we investigate the instrumentation of one artifact and not a set of resources, we rely theoretically on the instrumental approach.

3 Literature review

Even though university instructors frequently mention that their students do not read their textbook (Mesa & Griffiths, 2012), there is scant literature on how common it is for them to expect their students to read the textbook or do some preparatory work before going to class. Some studies have shown that there are benefits to having students do some reading before class (pre-reading). Rubenstein (1992) reported ways in which pre-reading encouraged students to be more involved and engaged in the classroom, whereas Maida (1995) documented that students felt that pre-reading was helpful in their understanding of the material. We found one empirical study, Axtell and Turner (2007), that investigated which types of reading questions (definitional, computational, or conceptual) assigned to students in introductory math courses to be answered before coming to class were more effective in students' "overnight" retention of material. Axtell and Turner defined definitional questions as those asking students to write out a definition from the reading; computational questions as those questions asking students to mimic an example presented in the text in order to solve a similar problem; and conceptual questions as those that required students to demonstrate an understanding of the theory behind the material. Gathering data over one semester, for two courses (precalculus and calculus, 18 and 27 students respectively), the researchers found, via reading quizzes, that students could reconstruct and retain a fair amount of conceptual knowledge by reading the text before instruction and that they were pretty good at mimicking computational steps to solve problems but not very good at recalling definitions. The researchers suggest that asking conceptual questions is more beneficial for students.

Adding interactive features to textbooks is a recent phenomenon but one that is seen as having much benefit for students. Choppin et al. (2014) referred to curricular programs that employed "interactive applets to introduce mathematical ideas with students" as active, "as they assigned [students] a more 'active' role in manipulating representations of a scenario so as to solve mathematical problems set in that context" (p. 264). They noted that these features were absent from conventional textbooks. These authors suggest that students' interaction with digital systems could provide students with feedback and that the feedback from the digital system, the discussion between students, and the metacognitive scaffolding by the teacher, can generate a rich base for students to engage in formative assessment. However, how instructors plan or use these interactive features for their teaching is not documented.

Studies of instrumented activity with textbooks also focus on the students. Rezat (2013), for example, in a study with sixth- and twelfth-grade German students, identified three utilization schemes that students developed as they were working on their own at home practicing the material: position-dependent practicing, block-dependent practicing, and salience-dependent practicing. In the first, there were two operational invariants characterizing the scheme: that practicing had to use similar tasks as those used by their teacher, and that tasks that were closely placed to each other, were similar. In the second scheme, the operational invariant related to students practicing with the rules provided in boxes that were considered as well-phrased and thus important; in the third scheme, the operational invariant related to students looking for visual resemblance to tasks that their teacher solved. According to Rezat, these three operational invariants provide information about how students think about textbooks and about mathematics learning, specifically about how to use the textbook effectively, which might be both useful, when students identify the key notions to practice and not so useful when students practice without comprehending the material. This study is important because it illustrates a viable approach to the understanding of textbooks in use.

A study that appeared to be related to our work is González-Martín et al.'s (2018) investigation of how collegial instructors approached the teaching of numerical series of real numbers. Using both the documentational approach to didactics (DAD, Trouche et al., 2020) and the anthropological theory of didactics (ATD, Chevallard, 2003), these authors identified the utilization schemes that emerged when collegial instructors used a calculus textbook. They found two utilization schemes, scaffolding mathematical content (series) and organizing students' learning about series by assigning exercises associated with various operational invariants. Some of the operational invariants were related to the textbook (the order of the textbook is good; including series in the Integral Calculus curriculum is debatable; series is a steppingstone for more important parts of the content), some related to student cognition (series is a counterintuitive concept; visualization is good for student learning, helps develop intuition), and some related to the institution (time pressure and student fatigue constrain the teaching approach). These authors noted that personal relationships with the content of series were essential elements in resource utilization schemes. Their approach is related to ours, as they investigated particular content within the textbook, not just an individual feature, and illustrated how to derive utilization schemes.

To summarize, while there is some literature about the importance of reading textbooks for student learning, about how students use specific features for their learning, and about the potential of increasing student engagement with interactive features, not much is said about how instructors use those features in textbooks to promote student engagement with the material. Without this information, it is difficult to know whether the goals of the specific features can be fulfilled. In this study, we contribute to this knowledge, as we seek to describe the different ways in which instructors in our project used the questioning devices in their textbooks. More specifically, we answer the following research question: What are university instructors' utilization schemes of questioning devices available in PreTeXt textbooks for calculus, linear algebra, and abstract algebra?

4 Method

4.1 Participants

This investigation is part of an ongoing, 4-year, large mixed-methods study involving 59 different instructors teaching a course using one of the textbooks (AC, FCLA, AATA) for one semester. Participants were recruited through a nationwide invitation distributed to various networks of university instructors in the United States. Interested participants submitted a letter of interest in the project and a letter of commitment from their department head, indicating that the instructor would be assigned to teach with one of the three textbooks during a specific term. From an initial pool of over 40 applicants, we selected an initial set of 32, seeking to have instructors teaching at institutions from a diverse set of characteristics including control (public or private), teaching and research activity (high or low), size, selectivity, and diversity of the student body. The analysis reported herein, includes a subset of 15 instructors, six teaching linear algebra, four abstract algebra, and five calculus, whose data were collected between September 2018 and December 2019. Table 1 includes characteristics of the sample (gender, state, years of teaching experience) by the textbook they used. The instructors taught at 14 different institutions located in 11 different states and received compensation for participating; the study was approved by their ethics committees.

Table 1 Characteristics of study participants by textbook

Teacher	Gender[a]	State	Teaching experience[b]
AATA[c]			
T20	M	Texas	20
T21	F	Montana	13
T22	M	Massachusetts	4
T23	M	Colorado	15
AC			
T13	M	New York	17
T16[d]	F	Texas	19
T24	F	Texas	21
T25[d]	M	Kansas	16
T26	F	Massachusetts	20
FCLA			
T10[d]	F	Colorado	13
T15	M	Washington	15
T17	M	Colorado	10
T18	M	Ohio	16
T27	M	North Carolina	29
T28	M	Texas	27

[a] *M* male; *F* female

[b] In years; includes teaching as a graduate student, part-time, and full-time faculty

[c] *AATA* Abstract Algebra Theory and Applications, *AC* Active Calculus, *FCLA* First Course in Linear Algebra

[d] Instructor taught at private religious universities

4.2 Data

We collected various types of data to gather information about how participants used their textbooks for course planning, lesson planning, and lesson enactment. Tracking data confirmed that both instructors and their students spent time on the questioning devices. In addition to the tracking data, we used two types of data collection, a one-time pre-semester survey, and five bi-weekly logs—shorter, themed questionnaires that relied on users' personal recollections based on their records and sought to promote participant reflection. This design made it possible to include many participants teaching at different universities across the country.

The pre-semester survey had 95 questions (open- and closed-ended) that provided information about participants' plans for teaching their courses with the textbooks (e.g., "Please list the goals of this course such as the key ideas, concepts, or skills you want your students to take away from it."), the processes participants typically used to plan lessons and how they used their textbooks and other resources for that purpose (e.g., "How have you used a textbook in the past for designing homework when planning this course?"), how a lesson typically unfolded, and their anticipated use of specific textbook features, including questioning devices. To obtain a holistic view of them as individuals, the survey included questions about their beliefs about mathematics teaching and learning (Clark et al., 2014), attitudes towards technology (LoTi, 2016), and demographic characteristics (e.g., years of experience, gender, ethnicity, etc.).

The bi-weekly logs provided ongoing information about how instructors were using their textbook as the semester progressed; they sought to promote participants' reflections about teaching with the textbook over a 2-week period. We designed five logs with questions that asked participants to recall their actions (e.g., "Please list the goals of your lesson"; "Was the textbook helpful for you to accomplish those goals?" "Please elaborate as much as possible: How do you create your lecture notes for a class session?" "If available for this lesson, how did you use the student responses to the Reading Questions/Preview Activities?"). Several questions were repeated in various logs to track the evolution of uses.

The survey and logs could be answered at any time that was convenient for the participants, removing the logistics of coordinating individual interviews. Asking some targeted questions at various times and having a variety of sources (including recordings and field notes from 18 interviews and six classroom observations) facilitated the triangulation of responses and the formulation of hypotheses about textbook feature use. We also conducted site visits and performed planning and teaching observations, interviews, and student focus groups for a smaller set of six participants.

4.3 Analysis

Our analytical strategy was guided by Gueudet's (2017) study in which she, using DAD, investigated the documentation process followed by six university professors as they planned their lessons. We followed her strategy to identify instructors' utilization schemes for classes of situations (e.g., "preparing and setting up a lecture on complex number" p. 215) as instructors articulated the use of multiple resources and documents (e.g., books, worksheets, notes from the previous year, and students' answers to questions), and her process of identification of rules of actions (e.g., not presenting proofs, assessing students by asking questions) and operational invariants (e.g., "the students must learn to use mathematics textbooks," or "students must learn to write coherent sentences and reasoning" p. 215).

In our case, and due to the open-ended nature of the questions, instructors mentioned questioning devices in their answers to many other questions in addition to the ones we designed to target them. After a systematic search using various keywords (e.g., "read," "reading," "preview," "activities," "prior"), we identified all the questions and responses that referred to either reading questions or preview activities in all the sources. This parsing yielded 95 survey responses to a subset of 11 open-ended survey questions, 265 responses to 40 questions from 10 different logs (five logs per term), and 17 excerpts from interviews.

We created separate documents to analyze each type of data source but used the same process with each. An initial analysis was carried out by the first two authors in several iterations. The log data were organized in various matrices corresponding to the data collection waves; this was necessary because log questions change from term to term, as many questions have to be tailored to the each instructor. We started by reading all responses associated with answers flagged for mentioning questioning devices, either reading questions or preview activities. We sought to identify in each excerpt, (a) goals, subgoals, and expectations (for what purpose are they using the questioning devices?); (b) rules of action (verbs in the sentences that indicate activity or a process); and (c) operational invariants (stated rationales that justify the aims and rules of actions). While Vergnaud (1998) also mentions possibilities of inferences, for our purposes, we assumed that this element was intrinsically present in each utilization scheme because it corresponded to the mental process that allows teachers to infer the best goals and rules of actions for a certain class of situations based on their operational invariants. That is, in the absence of these possibilities to infer, there would be no scheme. In our analysis, and following Gueudet (2017) and González-Martín et al. (2018), we first identified the goals and actions that were stated in the text collected from instructors, and then we inferred the operational invariants when these are not evident, by taking into account all the available data for each instructor and formulating statements that could be implied. By doing this, we assume that the possibilities of inference existed and focus on the outcome of that inference process rather than questioning their existence. After the elements mentioned above were identified by the first two authors, two additional authors (Judson and Quiroz) reviewed the analysis in order to corroborate the identification of the elements of the scheme in the data, and the four of them collaboratively engaged in identifying the utilization schemes that we found. We report only those schemes for which we have evidence from multiple sources. In the next section, as we present our findings, we also illustrate our analysis.

5 Findings

Instructors' instrumentation of questioning devices can be characterized by four utilization schemes that emerged in the instructors' descriptions of various teaching activities, namely pre-planning, planning, instruction in the classroom, and assessment (see Fig. 4). Our analysis indicates that instructors developed more than one utilization scheme. The findings that follow are based on the responses from 13 of the 15 instructors in our study for whom we have evidence in more than three sources.[2] We describe each of the four schemes next, including goals, rules of action, and operational invariants; we inferred this last component when teachers did not explicitly state them.

5.1 Instructors completed questioning devices for pre-planning

In this utilization scheme, the reading questions provide the instructor an opportunity to verify his or her own knowledge of the material:

> T15: I used the reading questions to *check my understanding after reading the chapter* (goal). First *i skimmed the reading questions, read the chapter thoroughly and then answered the reading questions to make sure i understood what i had read* (rule of action). They definitely helped *reiterate the material (goal)* in the chapter, and a few times *i had to return back to an example in the book to help me through the problem* (rule of action).[3]

Clear goals are stated: check his own understanding of the content to be taught and reinforce the material; various rules of actions are stated: skimming the questions, reading the chapters, and then answering the questions; these actions do support both goals; the instructor also revisits the section when something is not clear. The operational invariant is not explicitly stated, but it is plausible to assume that it is related to ensuring his personal competence with the material; to demonstrate to himself that he is qualified to teach the course. This appears to be especially important as this instructor was teaching with this textbook for the first time.

5.2 Instructors required students to complete the questioning devices for the purpose of lesson planning

In this utilization scheme, instructors assign reading questions or preview activities with a twofold goal. Instructors want first, to connect students with the material (e.g., "I assign the reading questions with hopes that they will help engage the students with material" T15) to enhance their knowledge, and second, to identify students' understandings so they can adapt lesson plans to fit students' needs (e.g., "I read the student responses to the reading assignment and I rewrite my notes to address some of their main concerns" T20). To accomplish these goals, several rules of action are

[2] There was insufficient evidence for T17 and T24 to identify a utilization scheme.

[3] We maintained participants' choice of wording and capitalization in all their quotes.

Goals	Rules of Action	Operational Invariants
Utilization scheme 1: Instructors completed the questioning devices (QDs) for lesson pre-planning T15 (LA)		
- To check own understanding of the content. - To reinforce the material to be used in class.	- Skimming the QDs to familiarize with them. - Reading the textbook chapter concerning the class. - Answering QDs and other types of questions or exercises. - Revisiting the section when something is not clear.	It is important to me as a teacher to know what I will be teaching. I want to appear competent in front of my students. *I used the reading questions to check my understanding after reading the chapter. First I skimmed the reading questions, read the chapter thoroughly and then answered the reading questions to make sure I understood what I had read. They definitely helped reiterate the material in the chapter, and a few times i had to return back to an example in the book to help me through the problem* (T15)
Utilization scheme 2: Instructors required students to do the questioning devices for the purpose of lesson planning T10 (LA), T13 (AC), T15 (LA), T18 (LA), T20 (AA), T23 (AA), T25 (AC)		
- To engage the students with the material. - To identify students' understandings so they can adapt lesson plans to fit students' needs.	- Assigning QDs to be due before or at the beginning of class. - Collecting student responses to the QDs and reading them before the class. - Adapting their lessons according to students' responses to QDs.	Students learn better when struggle or engage with the material before class (e.g., reading or solving a problem) *... with the completion of the reading questions before we cover the chapter that I can use this information to engage the students more in content and ask questions on the material before covering it thoroughly in class* (T18) *I feel like the reading questions are a great way to push the students toward preparing for class. Many students gained a great deal by thinking about those questions beforehand* (T15) [*The students*] *do the reading questions. They haven't done it yet. ... I want them to read the section before I talk about it* (T21, Interview) Students learn more when lessons are adapted to students' needs. *I read[s] the student responses to the reading assignment (reading questions) and rewrite[s] [his lesson] notes to address some of their main concerns* (T20) *I have them do the Preview activity before each unit, which they submit to me our Classroom management system due an hour before class. That way I can review some of them and discuss with them any problems I see before we get started* (T13)
Utilization scheme 3: Instructors used the questioning devices (QDs) for the purpose of instruction T21 (AA), T26 (AC)		
- To use QDs to ensure that students are learning the material. - To use QDs for discovering students' reasoning about the content as well as difficulties in understanding it. - To facilitate students' understanding during class.	- Incorporating QDs as part of the work done in class. - Discussing QDs with individual students, in class. - Discussing QDs with groups of students, in class.	Lesson time is effective for student learning when questioning devices are done in class. [*The students*] *are answering reading questions and when I discuss/prove theorems in class from the book I often reference the number so they know where to refer to in the book* (T21) Students working collaboratively, discussing in groups, and sharing thoughts in class about QDs are beneficial practices for students' understanding. *The preview activity 1.3.1 (which we did during class) was helpful. I think I accomplished the conceptual goal through whole class discussions in which students shared their thinking, added on, asked questions etc* (T26) Students will not do the work by themselves outside the class. *I don't know if* [the students] *will do any prep for* [the class] (T26)
Utilization scheme 4: Instructors required students complete the questioning devices (QDs) for the purpose of assessment T10 (LA), T15 (LA), T18 (LA), T22 (AA), T23 (AA), T24 (AC), T27 (LA), T28 (LA)		
- To force students to do some reading of the textbook section ahead of time. - To give students a grade based on the correctness of their answers to QDs. - To identify students' struggles with the content.	- Assigning QDs as an assignment. - Quizzing students about QDs in class. - Grading QDs for correctness after class.	Activities such as attesting students' work, providing feedback, and grading assignments, help students to envision learning outcomes and advance in their learning process. *The reading questions helped me formulate student learning outcomes* (T28) The instructor is responsible for making sure and knowing that students are learning the material. *I use the text, the reading exercises, previous texts/notes to construct the set of lecture notes that I use in the class. From these notes and the activities we complete in class (usually taken from these texts), I construct quizzes or other assessments of student learning* (T18)

Fig. 4 Utilization schemes identified. Text in italics corresponds to illustrative quotes that supported the operational invariant for any given utilization scheme

put in motion: instructors assign questioning devices to be due before or at the beginning of class, often setting the due date to a time that will allow them to make adjustments to the lesson plan; they use the responses to adapt their lessons by reviewing the responses in class (e.g., "We go over the reading questions together. If the students are having difficulties, we spend more time discussing the reading questions" T10, "reading [the responses] 15 min prior to class showed me that I should discuss the questions with the class to start the lesson" T23) or converting the preview activity problems into a short discussion at the beginning of the lesson (T13). Instructor T20 collected responses to the reading questions the day before class, which provided enough time to make major modifications to the lesson if needed. Instructor T25 collected preview activities at the beginning of class and used the first few minutes of class to discuss student questions.

Because instructors did not always make explicit their motives or justifications for goals and rules of actions, we infer that the operational invariants associated with their adjustments to the lessons are related to making sure that students understand the material. Otherwise, there would be no reason to discuss the related ideas in class. Therefore, the operational invariants driving these goals and rules of actions in this utilization scheme seem to be instructors' belief that students' learning benefits from engaging more with the material and doing something with it before class. Regarding this belief, T18 expressed that he hoped "with the completion of the reading questions before we cover the chapter that I can use this information to engage the students more in content and ask questions on the material before covering it thoroughly in class." In addition, instructors state that students learn more when lessons are adapted to students' needs. For instance, T20 expresses that he "read[s] the student responses to the reading assignment (reading questions) and rewrite[s] [his lesson] notes to address some of their main concerns." In instrumenting the questioning devices in this way, instructors gain knowledge of students' thinking and difficulties tied to a particular lesson that they use in their planning and teaching.

5.3 Instructors used the questioning devices for the purpose of instruction

In this utilization scheme, the instructors' main goal is to use questioning devices to ensure that students are learning the material and discover their reasoning about the content and difficulties understanding it. Instructors also seek to facilitate students' understanding during the class.

The main rule of action we identified in this scheme is incorporating the questioning devices as part of the class work ("We ended up doing the preview question together in class. I think that was good, some students were lost trying to label the different pieces" T26). Incorporating questioning devices occurred in different forms, some instructors printed the preview activity on a sheet of paper, brought them to class for students to work on, and guided students to an understanding of the questions and correct responses; others projected them on a large screen. This guidance happened in class while visiting individual students or in a large class discussion. For instance, T26 expressed that "I don't lecture. I print out the activities I like … I decide (using the book AC) when to bring in definitions or state conjectures of theorems that students discovered."

We inferred three operational invariants driving this utilization scheme. First is instructors' belief that the lesson time is effective for student learning when questioning devices are done in class. For instance, T21 has effectively combined the work that students do when answering reading questions with her work proving theorems in class "[The students] are answering reading questions and when I discuss/prove theorems in class from the book I often reference the number [of the reading question] so they know where to refer to in the book." Second is the belief that students working collaboratively, discussing in groups, and sharing thoughts in class about questioning devices are beneficial practices for students' understanding. For instance, T26 says, "the preview activity 1.3.1 (which we did during class) was helpful. I think I accomplished the conceptual goal through whole class discussions in which students shared their thinking, added on, asked questions etc." The third inferred operational invariant relates to instructors' concern that students will not do the work by themselves outside of class. T26 expresses this concern when she says, "I don't know if they [students] will do any prep for [the class]." She emphasizes that preparing for class is important ("I told the students that they had to prep for tomorrow" T26), yet some of her students indicate not knowing where to find the information, even though she has put the information online. When we asked T26 about what modification she would make in her teaching if she had to use the textbook again, she replied: "work more explicitly with students on how to use the book effectively (during class)." This response makes evident a process of instrumentation, as she realized that it was difficult to motivate students to do the work on their own.

5.4 Instructors required students complete the questioning devices for the purpose of assessment

Instructors primarily use the questioning devices for assessment purposes in this utilization scheme. A main goal in this scheme is to "force [the students] to do some reading" (T10) of "the section ahead of time" (T27) and to give students a grade based on the correctness of their answers (T28). Another goal includes identifying student struggles with the content. For instance, T15 expresses that they start the class by "discussing common problems or clarifications."

The rules of action associated with this scheme include assigning the reading questions to ensure that the students read the material and to either quiz students in class ("I call on randomly selected students to provide answers to the reading questions" T27) or grade the responses for correctness at a later time ("I look for 5 problems that I assigned as practice, and 5 problems that I assign as graded" T22). We inferred two operational invariants associated with this scheme; first, instructors believe that attesting students' work, providing feedback, and grading assignments, help students to envision learning outcomes and advance in their learning process. Second, instructors believe they are responsible for making sure that students are learning the material. For example, T18 explains how he used reading questions (along with other textbook features and material) to create assessments of student learning:

> I use the text, the reading exercises, previous texts/notes to construct the set of lecture notes that I use in the class. From these notes and the activities we com-

plete in class (usually taken from these texts), I construct quizzes or other assessments of student learning.

Instructors, in criticizing the design of the questioning devices for lacking an automatic way to check for correctness, revealed how important it was for them to give feedback and to attest that students had learned the material. For example, T24 explained why she did not use the preview activities this way:

> I assign readings in the homework but I did not use Preview Activities. Some of those activities have good questions and I was thinking about assigning them as a homework but students have to get some sort of feedback. Either access to solutions (possibly of similar problems) or automated grading with some sort of explanation.

Similarly, T23 provided evidence of this reason when he criticized the reading questions by saying, "the reading questions really need a way for the instructor to respond to the student's submissions. A right/wrong marking, but also a textbox with a written response would be very helpful." In this response, T23 provides evidence of a process of instrumentation because the questioning devices' limitations are triggering a process of evolution in the instructor's utilization scheme, inasmuch he would add that feature to the textbook, and consequently, he would change the way he operates questioning devices.

6 Discussion

We found that questioning devices were used by university instructors for multiple goals, primarily to motivate their students to read and engage with the textbook, especially before coming to class. Such goals seem to be espoused by instructors' implicit proposition that students' learning benefits from engaging with the material before class—possibly by creating some cognitive dissonance that can be worked out in class; such proposition guides their lesson design. This is consistent with the author-designers' intentions in including the questioning devices in their textbooks and for making them interactive so that responses could be immediately available to instructors. Questioning devices used in this way led to changes in plans related to content coverage. But questioning devices were not just used for lesson preparation but also for class instruction, assessment, and testing personal knowledge. When used as part of the class activity, instructors addressed their uncertainty about students working on the questioning devices prior to attending class; by using them in class, instructors can attest that the content was collectively addressed, possibly complying with departmental guidelines or mandates. Instructors also used them for assessment, some of them wanting an automatic way to verify the correctness of the responses for grading purposes. This unanticipated use seems to put the questioning devices in the same category as exercises. Finally, using the questioning devices to test personal knowledge serves the purpose of affirming self-competence in teaching; we surmise that such use could inform instructors about potential misconceptions or misunderstandings that may emerge after a first reading of the narrative text. It is not clear that when answering the questions, the instructors in our study made note of the difficulties; the activity was self-centered (to produce a mathematically correct lecture) rather than oriented towards understanding how students might be thinking about the content. In sum, we identified different utilization schemes in our study, some anticipated and some not, with most of our instructors developing one or more of those. All in all, the questioning devices did accomplish the goal of engaging students with the content, some instructors made changes to their lessons, and all developed utilization schemes that were consistent with their envisioned practice. This implies that questioning devices are just one small element that can contribute to instructional change.

In this study, we inferred the operational invariants as teachers' propositions regarding what constitutes their teaching and learning theories, however, it was not really apparent what those theories are; as researchers, we have made plausible inferences based on all the data gathered for each instructor. The design of the large study of which these data emerged was oriented towards understanding general aspects of textbook use and how it intervened in planning and in classroom teaching; the large number of participants limited the possibilities for in-depth interviews with all of them, which we addressed by sending the logs (open-ended questionnaires) throughout the semester. It is not clear though, that an interview could have yielded better results as it is in general difficult for university faculty to articulate their teaching and learning theories; we believe this difficulty might relate to how they have developed those theories. In the United States, and as part of their graduate studies, instructors are required to teach lower division courses, but they are rarely required to take courses in pedagogy (Ellis, 2015). It is plausible that their theories have been developed through their practice, rather than through academic training or reflection on their work. Instructors' teaching and learning experiences, as the main sources informing their implicit theories, make it more difficult for those to be easily expressed.

Because for many instructors, this type of textbook was new, we anticipate that over time, and as they continue using the textbook, they may adjust their utilization schemes based on new experiences and as they develop knowledge of what is possible to do with these devices. We plan further analysis of data from instructors who took part in the study more than once, to identify changes or new utilization schemes that might be informed by continuation of their experience.

We did not analyze the students' utilization schemes in this study, and thus it is not possible to articulate how the students' utilization schemes might influence teachers' work or vice versa. There might be an interdependence between teachers' and students' instrumental genesis that may be further explored by following them in tandem for a longer period of time. Likewise, we did not yet analyze the content of the student responses nor instructors' interpretations of those responses, an analysis that would allow us to understand pedagogical knowledge (especially knowledge of students and content) that can be promoted with these devices. This is a matter for future work.

We note that our approach is narrower in scope than the DAD. Our setting does not involve networks of teachers responding to a mandate to integrate technology in the curriculum (Trouche et al., 2020). Our study is more naturalistic as it documents how a feature is instrumented by instructors in their daily work and it observes that work at a different scale, with a close up of an individual feature. We believe that this approach, better suited for our context, can be generative for similar analysis of feature use in textbooks.

Readers may note that an instrumental approach could help identify instrumentalization processes. We have evidence that two abstract algebra instructors (T20 and T23) modified the reading questions. T20 for example, said: "For this section, I asked if the group of integers modulo *n* is a subgroup [of] the integers, and I also asked if the group of integers is a subgroup of the group of nonzero rationals under multiplication" which indicates his need for students to explore specific examples with the material they read. T23 went farther, as he used PreTeXt to place the reading questions after each section (they are at the end of the chapter): "I added new reading questions to the end of each section I planned to cover. This also involved splitting up two sections into four sections so I could put reading questions after each." However, we did not see much evidence that instructors added new questions nor that they changed their textbooks. Making these changes currently requires technical skill and knowledge of PreTeXt which very few instructors in our sample had.

Questioning devices, as designed into PreTeXt textbooks, can support user interactivity, potentially expanding our conceptualization of textbook questions. Questions that the authors intersperse in the narrative tend to be of a rhetorical nature—not because the authors do not want the readers to think about those questions, but because there is no way for readers to share their answers. PreTeXt makes it possible for students to record their responses directly into the textbook and for authors and instructors to see those responses. This possibility can expand authors' and instructors' strategies for engaging readers into a continued conversation. Clearly this is a powerful possibility for instructors, who need to address student thinking daily. PreTeXt also opens the possibility for instructors to create new questions, perhaps some that authors have not anticipated, and questions that may better fit the group of students they encounter. A user-friendly interface for using PreTeXt would allow any instructor to alter the text at will—adding and modifying questioning devices to fit their needs. This is a key area of development that would popularize the idea of teacher? interactivity in textbooks.

Until now, the main influence that authors had when writing a textbook related to curriculum—to what could be taught in the classroom and in what order. Now with the availability of PreTeXt and its potential for incorporating questioning devices into the textbook, the new author-designer can not only state what content is important to include in a textbook, but they now have an opportunity to influence teaching. Perhaps through this work, more opportunities are created to understand student thinking and how it can influence instruction, with the textbook as a viable vehicle.

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