Stochastic Modeling and Analysis of Public Electric Vehicle Fleet Charging Station Operations

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***Abstract*— The electric vehicle (EV) fleet is gradually growing into a major part of public transportation. Proper planning and operation of EV supply equipment (EVSE) is essential to ensure the efficient and economic operations of the EV fleets. Charging stations (CS) have gained market attention due to their lower cost and versatility. Battery swapping stations (BSS) have also received considerable attention because of their promise to provide fast and sustainable battery replacements. However, their commercial viability is unclear due to their requirement for large capital and infrastructure deployment. In this paper, we develop a stochastic model for interactions between CS/BSS and taxi/bus fleets. The model is based on a realistic abstraction of users' behavior defined by various stochastic processes. It also considers the dynamic impacts of the road congestion. Analytical revenue boundaries are derived and verified by simulations. These simulation results may prove valuable for future studies of public transit.**

***Index Terms*— Electric vehicle, electric vehicle supply equipment, EV charging networks, public transit, smart grids.**

I. Introduction

IN THE past decade, growing concerns about the relationship among climate, pollution, and personal consumption have led to a rapid rise in demand for the electrification of public transportation such as electric buses and taxis. Numerous cities worldwide have already announced commitments to the electrification of public transportation, joining cities in China that have piloted the effort. By the end of 2017, Shenzhen, one of the largest cities in China, completed its transition to all-electric mass transit. With approximately 16,000 buses and 22,000 taxis, it became the first city worldwide to fully electrify its bus fleets. There are more than 30 cities in China committed to fully electrifying their public transportation by 2020. Los Angeles, United States is electrifying its entire vehicle fleet with the goal of 100% coverage by 2030. Denver, United States has also announced to have 100% buses electrified by 2050. Tallinn, Estonia plans to purchase 650 electric buses by 2035 while Paris, France will have 800 by 2024. The United Kingdom government has launched a plan named "road to zero" that calls for all vehicles on the roads to be zero emissions by 2040.

As the number of electric vehicle (EV) fleets continues to grow, it is imperative that businesses and municipalities meet the charging demand and provide easy access to electric vehicle supply equipment (EVSE) services [1]. There are currently two types of EVSE services on the market. In most cases, charging stations (CS) have become the default service. However, due to the limitations of current charging technology and the existing infrastructure, charging speed of CS may be inadequate for EVs that are sensitive to charging time [2]–[5]. As an alternative, battery swapping stations (BSS) provide a fast replacement of fully charged batteries [6]–[8].

A fundamental question remains: How different is the interactive dynamics given a set of EVs and EVSE services and user behaviors? While BSS may provide a fast turn-over rate for large and frequent demands, questions remain: What is the overall quantitative benefit to be generated for users and service providers? Are they worth the additional infrastructure and equipment cost? What is the best size and configuration needed to serve a certain charging demand base? To answer these questions, an accurate and computationally efficient model for the interactions between vehicles and service providers is needed. Various studies have identified the stochastic nature of EV driving characteristics to be critical for EVSE planning and operation. Zhou *et al.* [9] considered the charging characteristics of various EVs and proposed a simulation framework. Bo *et al.* [10] formulated charging using CS and BSS as a constrained Markov decision process and investigated the optimal policy using the Lagrangian method and dynamic programming. The electric taxi routing behavior has also been considered [11]. Chekired and Khoukhi [12] proposed a cloud scheduling algorithm to optimize the waiting time for EV users at public stations. Based on the historical driving data from Denmark and Japan, a method to quantify impact of EV charging load on distribution grids was introduced [13]. Zhang *et al.* [7] presented a stochastic model of taxi and bus fleets and used Monte-Carlo simulations to evaluate the CS/BSS service capacities. Researchers have computed the realistic profitability and sustainability of BSS and CS. A probabilistic evaluation method for the dispatch potential

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of household EVs with the considerations of the multiple travel needs was proposed [14]. Munshi and Mohamed [15] proposed an algorithm to categorize users' charging profiles and determine their flexibility to address the performance of CS for different users. Mak *et al.* [16] provided an overview of BSS infrastructure management and discussed approaches to optimally deploy BSS for an EVSE provider. Yang proposed a charging strategy to maximize electric taxi's profit by choosing appropriate charging stations under uncertain electricity prices and time-varying incomes [6]. Yang *et al.* [17] proposed a route selection and navigation optimization model to minimize EV users' travel costs and to optimize grid load levels. Dai *et al.* [18] constructed stochastic models to estimate uncontrolled BSS energy consumption based on the number of EVs for swapping, start time, travel distance, and charging duration. Environmental variations and traffic conditions were considered in optimizing the energy-efficient driving algorithm [19]. Authors developed an integrated optimization method for customized bus stop deployment, route design, and timetable development [20].

Past developments in EV charging networks call for a comprehensive study regrading abstraction and modeling of EVSE planning and optimization. Considering the impacts of road traffic conditions on EV charging may improve the accuracy of the system model. Furthermore, a comparative study of the operation of various EV charging types and EV classes in a typical public transport setting may provide additional unique insights. In this paper, we propose a discrete stochastic model for public EV transportation service processes that include both the CS and the BSS modes based on past statistical results of EV charging behaviors from the collected operations data and the simulation results [7], [21]. The analytical models are used to calculate the revenue of a stochastic system. The presented results may help with integrating new elements, deriving realistic battery degradation characteristics, and estimating infrastructure cost and planning needs. The proposed model considers the impacts of road traffic conditions in addition to the charging characteristics of taxi and bus EVs during various time periods.

The remaining of this paper is organized as follows: The notation used in the paper is given in Nomenclature. Section II provides the stochastic models of the system elements. The theoretical closed-form solutions for service fleets are presented in Section III. Numerical results and their comparison with simulation are given in Section IV. Use cases for the proposed solution are presented in Section V followed by the conclusion in Section VI. The proofs of lemmas are given in the Appendix.

NOMENCLATURE

C	Battery capacity of EV (kWh).
B	Remaining battery energy of EV (kWh). B_{in} and B_{out} are the battery energy of an EV arriving at and departing from an EVSE station, respectively. $B_{\text{in,normal}}$ represents the battery energy of an EV arriving at an EVSE station during normal hours.
M_{total}	Total driving mileage of an EV (mile).
M_{hired}	Driving mileage of a hired EV (mile).
M_{avai}	Driving mileage of an EV vacant for hire (mile).
N_s	Number of charging or battery swapping services. For a bus: $N_s = N_{s,b}$.
N_{EV}	Number of EVs at EVSE station.
N_{EVSE}	Designed service capacity of an EVSE station.
V_t	EV driving speed at time t (Mph) of a taxi $V_{x,t}$ and a bus $V_{b,t}$,
v_h	Average driving speed of a hired taxi EV (Mph).
v_b	Average driving speed of a bus EV (Mph).
v_a	Average driving speed of a vacant taxi (Mph).
v_m	Average speed of a taxi neither in service nor for hire (Mph).
v_{tr}	Average traffic adjustment.
c_t	Traffic congestion weight at time t.
K_r	Road traffic network capacity.
r_t	Road traffic weight at time t.
σ_x	Standard deviation of a taxi EV speed.
σ_b	Standard deviation of a bus EV speed.
J_t	Energy injected into an EV at an EVSE station at time t (kWh).
$U_{x,t}$	Total income of a taxi at time t (\$).
$U_{b,t}$	Total income of a bus at time t (\$).
R_{unit}	Price of unit energy (\$/kWh).
R_{net}	Net electricity income (\$/kWh).
u_t	Income rate of an in-service taxi at time t (\$/mile).
u_b	Average income rate of a bus.
ψ	Unit income rate of a bus EV (\$/mile).
P_a	Probability of an EV arriving at an EVSE station for service.
P_d	Probability of an EV leaving an EVSE station.
P_s	Probability of taxi being hired for service.
q	Queue length at an EVSE station.
$\mathcal{N}(\mu, \sigma)$	Normal distribution function with the mean μ and the standard deviation σ.
κ	The go-charging factor.
η	Energy consumption rate of an EV (kWh/mile).
ρ	Battery price of a bus EV.
λ	Charging speed of CS.
Ψ	State of charge of an EV (%).
T_{total}	Set of all time slots.
T_h	Time slots when an EV is hired for service.
T_d	Time slots when an EV is driving.
T_c	Time slots when an EV is at EVSE station.
T_s	Time slots when an EV is at an EVSE station for charging/battery swapping.
T_q	Time slots when an EV is queuing at an EVSE station.
$t_{\text{rest,st}}$, $t_{\text{rest,et}}$	Starting and ending times of an EV in rest hours, respectively.

$t_{\text{busy,st}}$, $t_{\text{busy,et}}$ Starting and ending times of an EV in busy hours, respectively.
Θ Extension factor.
Υ Predetermined routing distance between charging/swapping services.

II. Stochastic Model

In this section, we describe a stochastic model and compare it to the data collected from pilot projects. The model consists of two classes of participants: EV operators and EVSE service providers. EV operators control service fleets such as buses and taxis. Their revenue is the service fee paid by their passengers while their cost are the service charges that include the electricity charges paid to the EVSE service providers, the vehicle maintenance charges, and battery degradation cost. A typical EV operator may be an EV driver or an autonomous vehicle. For simplicity, in this paper, EV drivers and users are referred to as EV operators. The EVSE service providers offer EV charging services with CS or BSS to EV operators. The net profit of the EVSE service providers is generated from the difference between the service charge collected from their EV users and the cost of electricity paid to the utilities. A stochastic model of the EV operation workflow is shown in Fig. 1.

In order to model the EV driving behavior as a statistical sample of a community, we consider the reaction of the EV driver to the battery state of charge (SOC). The decision of an EV driver to use EVSE station for charging/swapping service is based on the battery SOC level monitored by the sensor displayed on the EV dashboard. The SOC level is an indicator of the available energy stored in the battery. The SOC level higher than a certain value indicates that the stored energy is high and, hence, the EV driver is less likely to charge the EV. When the EV battery SOC level falls below a threshold, there is a higher probability that the driver will use charging/swapping service. The remaining battery energy depends on the capacity and the SOC level:

$$B_t = C\Psi_t, \tag{1}$$

where B_t, C, and Ψ_t are the remaining battery energy, the total battery energy capacity, and the battery SOC of the EV at time t, respectively.

The behavior of public EV fleets is modeled as the Markov process consisting of three elements: (a) the operation mode (charging/swapping or driving), (b) the remaining battery energy of EV, and (c) the income of EV fleets. Driver's operation is modeled as the Bernoulli process. The probability that a taxi arrives at and departs from an EVSE station depends on the remaining energy of the EV. The time-varying function of battery energy of an EV is calculated as:

$$B_t = B_{t-1} - \mathbb{1}_{\text{driving}}\eta V_t + \mathbb{1}_{\text{charging}} J_t, \tag{2}$$

where B_t and B_{t-1} are the remaining battery energy at times t and $t-1$, respectively, $\mathbb{1}_{\text{driving}}$ and $\mathbb{1}_{\text{charging}}$ are the operational state indicator functions for an EV in the driving state and in the charging state, respectively, η is the battery energy consumption rate (kWh/mile) for the EV, V_t is the driving speed (mile/hour) of an EV at time t, and J_t is the energy (kWh) injected into the EV at time t. In practice, there is energy losses when an EV stops and waits because there are systems other than the powertrain system that still require energy such as air conditioning, audio, navigation. and control systems. Furthermore, there is energy losses of battery over time. However, these energy losses are relatively small compared to the energy consumption of a conventional fuel vehicle with the engine idling while waiting. Hence, to simplify the model, we assume no energy loss while the EV is waiting. Although there are technologies such as wireless charging and the on-car integrated solar energy system that enable energy to be replenished to EVs while they are moving and waiting, those technologies are still in their early stages of research and development and are not yet ready for practical mass applications. Hence, we may assume that there is no energy injection while the vehicle is waiting. Therefore, $J_t = 0$ when the EV is neither driving nor charging: If the EV stops or is waiting for charging, the EV will not consume energy nor will any energy be injected.

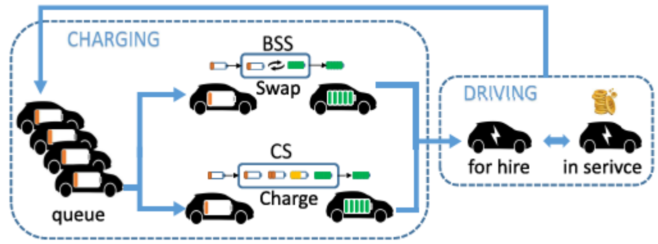


Fig. 1. Illustration of the EV operation workflow.

A. Taxi Fleet

Considering both earnings and operating costs, the income of an taxi EV at time t is given as:

$$U_{x,t} = U_{x,t-1} + \mathbb{1}_{\text{hired}} u_t V_{x,t} - \mathbb{1}_{\text{charging}} J_t R_{\text{unit}}, \tag{3}$$

where $U_{x,t}$ and $U_{x,t-1}$ are the total incomes of a taxi at times t and $t-1$, respectively, $\mathbb{1}_{\text{hired}}$ is the indicator function for the taxi being hired by customers, u_t is the taxi's income rate when it is hired, $V_{x,t}$ and J_t are the taxi's driving speed and the energy injected into the vehicle at time t, respectively, and R_{unit} is the unit energy cost per kWh. Noted that we use the notation of x to represent a "taxi". (Later in the paper we use b to represent a "bus".)

1) Time Segmentation: The taxi driving behaviors are highly correlated with the time of the day. Experiments conducted with four taxis using BSS pilot project[1] in Hangzhou city, Zhejiang Province, China between Feb. 1st and Apr. 30th, 2013 are shown in Fig. 2. The least number of requests for battery swaps is at the beginning of a day. There are two peak times during the day:

Around 5:00 pm and around 10:00 pm. Note that the drivers prefer to swap their batteries at the end of the day even though

[1]This project was the only BSS taxi EV project where the BSS is semi-automatic and the taxis do not support CS mode. The project began in 2011 and was completed in 2016.

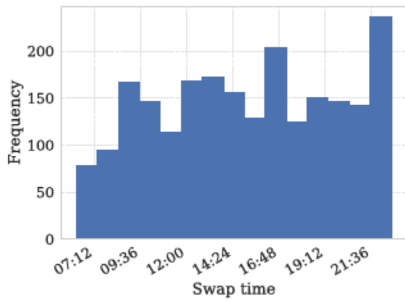


Fig. 2. Histogram of hours when taxis swap batteries during experiments conducted in Zhejiang Province, China.

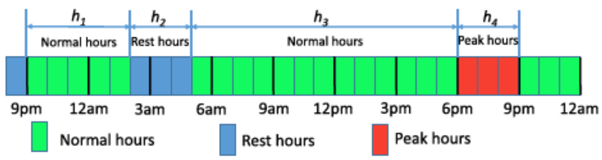


Fig. 3. The illustration of taxi operation hours.

the queue in the evening is longer than in the morning. The peak demand begins at 5:00 pm when taxi drivers go to get a battery fully charged before the rush hours begin. The end time of the taxi's shift and the closing time of the BSS is 10:00 pm. Taxi drivers desire to have a fully charged battery before the following day. This is the reason for the expected second peak for battery swapping services.

Based on the observed behaviors shown in Fig.3, the model for a taxi with 24-hour shift consists of: (a) peak time between 6:00 pm and 9:00 pm when taxi drivers are motivated to drive, (b) rest time between 2:00 am and 5:00 am when driver prefers to charge the vehicle, and (c) normal: the remaining time when taxi drivers operate the EV.

2) Operation Mode: During normal hours, the probabilities that a taxi arrives at and departs from an EVSE station depend on the remaining battery energy of the EV as:

$$P_a = e^{-B} \tag{4}$$
$$P_d = e^{B-C}, \tag{5}$$

where P_a and P_d are the probabilities of the taxi's status changing from driving to charging and from charging to driving, respectively, B is the remaining battery energy, e is the (natural) exponential function, and C is the battery capacity of the vehicle ($B = C$, when the battery is fully charged).

During the break time, taxi drivers charge their EVs when possible. Therefore, the probability P_a of a taxi using CS is: given as:

$$P_a = \begin{cases} 1, & \text{if } t_{\text{rest, st}} < t < t_{\text{rest, et}} \text{ and } B < \kappa C \\ 0, & \text{else,} \end{cases} \tag{6}$$

where $t_{\text{rest, st}}$ and $t_{\text{rest, et}}$ are the starting and ending times of the resting period, respectively, and $\kappa \in (0, 1]$ is the go-charging factor that represents the threshold of the battery SOC above which the driving may not charge the vehicle.

At the onset of the peak time, there is a high probability P_d that drivers will depart from the EVSE station:

$$P_d = e^{t_{\text{busy, st}} - t}, \quad \text{if } t_{\text{busy, st}} < t < t_{\text{busy, et}}, \tag{7}$$

where P_d is the probability that the EV drives away from the EVSE station and $t_{busy,st}$ and $t_{busy,et}$ are the starting and ending times of the busy period.

The operation differs in the BSS mode. Given the nature of the BSS, the battery swapping time is usually constant. Therefore, vehicles always leave after the swapping operation process is completed.

3) Travel Speed: Based on the taxi driving status, the speed of the taxi $V_{x,t}$ is a random variable generated at time t:

$$V_{x,t} = \begin{cases} r_t \max(\mathcal{N}(v_h + v_{tr}, \sigma_x), 0), & \text{in service} \\ r_t \max(\mathcal{N}(v_a, \sigma_x), 0), & \text{vacant, } t_{\text{rest,et}} < t < t_{\text{rest,st}} \\ r_t \max(\mathcal{N}(v_{\text{m}}, \sigma_x), 0), & \text{else,} \end{cases} \tag{8}$$

where $V_{x,t}$ is the driving speed of a taxi, max(.) is the maximum function, and $\mathcal{N}(\mu, \sigma)$ is the normal distribution function with the mean μ and the standard deviation σ. v_h is the average speed when the taxi has customers, v_a is the average speed when the taxi is vacant for hiring, v_m is the average speed when the taxi is not hired during the remaining periods, v_{tr} is the average traffic adjustment, and σ_x is the speed standard deviation. $r_t \in [0, 1]$ is the traffic condition factor that reflects the degree of traffic congestion. Value $r_t = 1$ implies that there is no traffic congestion. If $r_t = 0$, a vehicle is unable to move. Collected traffic data and studies regarding the road traffic congestion modeling are available [21]–[25]. In this analysis, we adapt a time-dependent indicator of traffic road conditions that is inversely proportional to the traffic congestion weight:

$$r_t \propto \frac{K_r}{c_t}, \tag{9}$$

where K_r is the road network capacity and c_t is the traffic congestion weight.

Under the same traffic conditions, we assume that a taxi's speed depends on whether or not the taxi is hired by customers. The probability of a taxi being hired is high during the peak hours, lower during the normal hours, and the lowest during the rest hours.

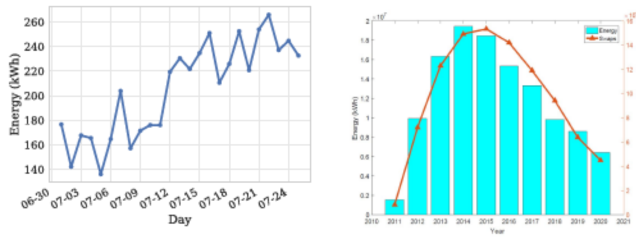
4) Algorithm: The taxi's operation is outlined in Algorithm 1:

Algorithm 1 The Taxi Operation

Input: B_t, $U_{x,t}$
Output: B_{t+1}, $U_{x,t+1}$
 if charging **then** decide whether or not to charge
 else decide whether or not to depart from EVSE station
 Implement (2) and (3)

B. Bus Fleet

Buses have several general differences from taxis: The bus battery usually has larger capacity. A bus follows a



(a) (b)

Fig. 4. Energy consumption of buses using BSS during experiments in Xuejiadao, Shandong Province, China: (a) Average daily energy consumption in July 2017 and (b) Yearly power consumption and the number of swaps between 2011 and 2020.

predetermined route with a known distance and, hence, a bus charges its battery at a bus park based on a schedule. The income of a bus depends on the number of passengers at the time. Only a fraction of buses are driven during the night shift.

An experiment conducted in Xuejiadao, Shandong Province, China collected daily and yearly energy consumption of a bus using BSS shown in Fig. 4. The 3-lane BSS in the experiment serve more than 156 electric buses on 10 routes with 388 average daily swaps. The longest and shortest single laps of the bus line are 77 km and 4.6 km, respectively. The bus battery capacity is 171 kWh. Buses have higher energy consumption rates (approximately 200 kWh per day) when compared to passenger EVs (usually equipped with battery size between 10 kWh and 100 kWh). Collected data indicate that the daily energy consumption of buses grows steadily. The increase between July 11^{th} and 12^{th}, 2017 is possibly due to a longer shift. In 2019, the total energy consumption was 8.59×10^6 kWh, with an average daily power of 24,000 kWh. In 2020, it was 6.42×10^6 kWh, with an average of 17×10^3 kWh per day. The average daily number of battery swapping services was 175 and 123 in 2019 and 2020, respectively.

The income of a bus at time t is:

$$\begin{cases} U_{b,t} = U_{b,t-1} + \psi V_{b,t} - \mathbb{1}_{\text{charging}} J_t R_{\text{unit}} \\ V_{b,t} = r_t \max(\mathcal{N}(v_b + v_{tr},\ \sigma_b),\ 0) \\ \psi = \max(\mathcal{N}(u_b,\ \sigma_b),\ 0), \end{cases} \tag{10}$$

where $U_{b,t}$ and $U_{b,t-1}$ are the bus incomes at times t and $t-1$, respectively, $\mathbb{1}_{\text{driving}}$ and $\mathbb{1}_{\text{charging}}$ are the indicator functions of buses in the states of driving and charging, respectively, ψ is the unit income rate, u_b is the average unit income rate of a bus, σ_b is the standard deviation, J_t is the energy injected into the bus at time t, R_{unit} is the unit energy cost per kWh, r_t is the traffic condition parameter, $V_{b,t}$ is the speed of the bus, and $\mathcal{N}(v_b+v_{tr}, \sigma_b)$ follows the normal distribution where v_b is the average driving speed of the bus and v_{tr} is the average traffic adjustment. Similar to a taxi, a bus has the highest income rate during peak hours, a lower income rate during normal hours, and the lowest income rate during night shifts.

The operation of a bus is described in Algorithm 2:

Algorithm 2 The Bus Operation

Input: B_t, $U_{b,t}$
Output: B_{t+1}, $U_{b,t+1}$
if charging **then** charge until fully charged
else drive until predetermined distance is reached
Implement (2) and (10)

C. Charging and Battery Swapping Stations

A CS service provider offers a number of charging ports with charging services to EVs while a BSS service operator offers lanes with battery swap robots for users to swap their batteries. In the BSS mode, spared batteries are requested for the battery swapping service. We assume that a BSS service operator always has charged batteries and is ready to provide service to EV users. Hence, there is always at least one fully charged battery for the arriving vehicle. It should be noted that while battery backup may improve the service quality of BSS, it also increases the cost of BSS service providers. The optimal solution may be found through proper scheduling of batteries. The operations of CS and BSS are described in Algorithm 3, where $N_{\text{EV,t}}$ is the number of EV charging/swapping at time t, $J_{\text{t+1}}$ is the energy injected into all vehicles at time $t+1$, and R_{net} is the net electricity income (\$/kWh) based on the cost of purchasing electricity from a utility.

Algorithm 3 EVSE Operation

if an EV arrives to EVSE for service **then**
  add the EV to the queue
while there is an open slot for pending vehicles **do**
  move the vehicle to charge/swap
  $U_{t+1} = U_t + N_{\text{EV,t+1}} J_{\text{total}} R_{\text{net}}$

III. Analysis of Operation Efficiency

Given a number of participants (taxi/bus and CS/BSS), the income of taxis and buses in the considered stochastic system may be evaluated using numerical or analytical approaches. In this section, we present the analytical approach for both types of EV fleets.

A. Taxi

Proposition 1: The expected net income of a taxi is:

$$\mathbb{E}(U_x) = \mathbb{E}(M_{hired}) u_t - \eta \mathbb{E}(M_{total}) R_{unit}, \tag{11}$$

where $\mathbb{E}(.)$ is the expectation function, U_x is the taxi's income, u_t is the taxi's income rate, M_{hired} is the total hired mileages, η is the taxi's battery energy consumption rate, M_{total} is the taxi's total driving mileages, and R_{unit} is the unit energy cost.

The total mileage M_{total} is the sum of the mileage of a hired taxi M_{hired} and a taxi that is available for hire M_{avail}:

$$M_{\text{total}} = M_{\text{hired}} + M_{\text{avail}}. \tag{12}$$

The Proposition 1 is derived based on (3):

$$U_x := \sum_{t \in T_{\text{total}}} U_{x,t} = \sum_{t \in T_h} u_t V_{x,t} - \sum_{t \in T_s} J_t R_{\text{unit}}, \tag{13}$$

where T_{total} is the set of all time slots, T_h is the set of time slots when the taxi is hired, and T_s is the set of time slots when the taxi is in charging service. Therefore, $\sum_{t \in T_h} V_{x,t}$ is the total mileage a taxi traveled with having customers, $\sum_{t\in T_s} J_t$ is the total consumed energy, and R_{unit} is the cost of unit energy.

The energy consumed by a taxi linearly depends on the total traveled mileage (2). Hence,

$$\sum_{t\in T_s} J_t = \eta M_{\text{total}}. \tag{14}$$

Substituting in (13), leads to (11).

Lemma 1: The expected mileage of a taxi having customers is:

$$\mathbb{E}(M_{hired}) = \mathbb{E}(V_{x,t}|hired)P_s\mathbb{E}(T_d) \tag{15}$$

while the expected mileage of a taxi available for hire is:

$$\mathbb{E}(M_{avail}) = \mathbb{E}(V_{x,t}|vacant)(1 - P_s)\mathbb{E}(T_d), \tag{16}$$

where $V_{x,t}$ is the speed of a taxi, P_s is the probability that the taxi is hired for riding service, and T_d is the total driving time of the taxi.

Remark 1: For a given time period T_{total}:

$$T_{total} = T_d + T_c, \tag{17}$$

where T_{total} is the total time under observation, T_d is the total driving time, and T_c is the total time that the EV spent at EVSE that includes the time for charging or battery swapping service and the queuing time.

All variables except $\mathbb{E}(T_d)$ in Proposition 1 and Lemma 1 are known.

Therefore, the main goal in order to calculate $\mathbb{E}(U_x)$ is to estimate $\mathbb{E}(T_d)$.

1) Taxi Using CS: The behaviors of a taxi using CS depend on the time periods. We consider four time periods as shown in Fig. 3: h_1 (normal hours between 9:00 pm and the next day 2:00 am), h_2 (rest hours between 2:00 am and 5:00 am), h_3 (normal hours between 5:00 am and 6:00 pm), and h_4 (peak hours between 6:00 pm and 9:00 pm). The classification of the normal, rest, and peak hours may be adjusted according to study cases.

In the proposed model, there is a high probability of a taxi EV being hired during busy hours. However, a taxi driver may not drive during the peak time due to the limited battery capacity. Therefore, we consider two extreme scenarios in terms of the expected number of charging services $\mathbb{E}(N_s)$ during the peak time: In the optimistic case, a taxi is assumed not to charge during busy hours h_4. Its energy consumption during the period h_4 is carried over to the rest hours h_2 when the hiring rate is the lowest. In the pessimistic case, we assume that a taxi operates normally in the peak hours h_4 and does not leave EVSE early.

Proposition 2: In the optimistic case, the expected number of charging/swap services is:

$$\mathbb{E}(N_s) = \begin{cases} \eta\mathbb{E}(M_1)/\mathbb{E}_x(B), & during\ h_1 \\ \max(\eta\mathbb{E}(M_4 + M_2)/\mathbb{E}_x(B), 1), & during\ h_2 \\ \eta\mathbb{E}(M_3)/\mathbb{E}_x(B), & during\ h_3 \\ 0, & during\ h_4, \end{cases} \tag{18}$$

where N_s is the number of charging services, η is rate of the battery energy consumption for the taxi, M_1, M_2, M_3, and M_4 are the total mileage in each respective period of normal (h_1), rest (h_2), normal (h_3), and peak (h_4) hours, and $\mathbb{E}_x(B)$ is the expected energy consumption of a taxi during each charging session.

The expected energy consumption of a taxi $\mathbb{E}_x(B)$ is:

$$\mathbb{E}_x(B) = \mathbb{E}(B_{\text{out}}) - \mathbb{E}(B_{\text{in}}), \tag{19}$$

where $\mathbb{E}(B_{\text{out}})$ and $\mathbb{E}(B_{\text{in}})$ are the expected battery energy of an EV departing from and arriving at an EVSE station, respectively. $\mathbb{E}(B_{\text{in}})$ is given as:

$$\mathbb{E}(B_{\text{in}}) = \begin{cases} \mathbb{E}(B_{\text{in,normal}}), & \text{for } h_1,\ h_3, and\ h_4 \\ C - \mathbb{E}(M_4), & \text{for } h_2, \end{cases} \tag{20}$$

where C is the battery capacity, $B_{\text{in,normal}}$ is the remaining energy of the battery when the EV arrives at CS/BSS for charging during normal hours, and M_4 are miles of an EV during the peak hours.

Based on the law of large numbers, Proposition 2 is true for a large enough number of EVs. In theoretical analysis we assume that the aggregated energy consumption within each time period is completed within the same time period. This avoids confusion in the analysis of power consumption in different time periods.

Hence, for h_1, $\mathbb{E}(N_s) = \eta\mathbb{E}(M_1)/\mathbb{E}_x(B)$.

Each taxi will charge during the rest hours at the beginning of h_2 based on (4) and (6). Hence, a taxi is charged at least once and the driving mileage of a taxi during h_2 is carried over to h_3. Hence, $\mathbb{E}(N_s) = \eta\mathbb{E}(M_2 + M_3)/\mathbb{E}_x(B)$for h_3. As assumed in the optimistic case scenario, a taxi does not charge during h_4 and the energy used in h_4 was charged during h_2. Therefore, during h_2, $\mathbb{E}(N_s) = \max(\eta\mathbb{E}(M_4)/\mathbb{E}_x(B), 1)$.

Lemma 2:

$$\mathbb{E}(B_{in,normal}) = \sum_{B=0}^{C} Be^{-B}\frac{P(B|driving)}{P_a}, \tag{21}$$

where C is the battery capacity, B is the remaining battery energy, $P(B|driving)$ is the conditional probability of the remaining battery energy B when driving, and P_a is the probability that the taxi uses CS/BSS charging or battery swapping service.

Lemma 3: The fraction $P(B|driving)/P_a$ satisfies:

$$\sum_{B=0}^{C} e^{-B}\frac{P(B|driving)}{P_a} = 1. \tag{22}$$

In the optimistic case scenario, a taxi driver will not charge the EV during peak hours and the energy consumed during the peak hours will be replenished during the rest hours.

Proposition 3: In the pessimistic case scenario, the expected number of charging/swapping services is:

$$\mathbb{E}(N_s) = \begin{cases} \eta\mathbb{E}(M_1)/\mathbb{E}_x(B), & for\ h_1 \\ 1, & for\ h_2 \\ \eta\mathbb{E}(M_2 + M_3)/\mathbb{E}_x(B), & for\ h_3 \\ \eta\mathbb{E}(M_4)/\mathbb{E}_x(B), & for\ h_4. \end{cases} \tag{23}$$

The expected remaining battery energy $\mathbb{E}(B_{x,in})$ when a taxi arrives at EVSE for charging service is:

$$\mathbb{E}(B_{x,in}) = \begin{cases} \mathbb{E}(B_{in,normal}), & \text{for } h_1,\ h_3,\ \text{and } h_4 \\ \kappa C, & \text{for } h_2, \end{cases} \tag{24}$$

where N_s is the number of charging services, η is the battery energy consumption rate of the taxi, κ is the go-charging factor, C is the battery total capacity, $\mathbb{E}(B_{in,normal})$ is the expected battery energy of an EV when arriving at an EVSE station during normal hours, M_1, M_2, M_3, and M_4 are the total mileage in normal (h_1), rest (h_2), normal (h_3), and peak (h_4) hours, respectively, and $\mathbb{E}_x(B)$ is the expected energy consumption of a taxi for each charging session.

In this study, we assume that the expected remaining battery energy of an EV after EVSE service are the same for both the pessimistic and the optimistic case scenario. In other words, we assume that all the charging services provide fully charged batteries.

Cases h_1 and h_3 are the same as in Proposition 2. Since we assume that drivers' charging behavior and driving behavior will not be adjusted to optimize for busy hours h_4. $\mathbb{E}(N_s) = \eta\mathbb{E}(M_4)/\mathbb{E}_x(B)$for h_4 during h_4. During the rest period, drivers are still expected to charge at least once.

Lemma 4:

$$\mathbb{E}(B_{x,out}) = \sum_{B=0}^{C} B e^{B-C} \frac{P(B|charging)}{P_d} \tag{25}$$

$$\sum_{0}^{C} e^{B-C} \frac{P(B|charging)}{P_d} = 1, \tag{26}$$

where B_{out} is the battery energy of an EV when leaving CS/BSS after charging or battery swapping service, C is the battery capacity, B is the remaining energy of a battery, $P(B|charging)$ is the conditional probability of the battery remaining energy B when charging, and P_d is the probability that the EV departs from the CS/BSS.

The expected time of an EV spent on charging at CS (excluding the waiting time) T_s is:

$$\mathbb{E}(T_s) = \frac{(\mathbb{E}(B_{\text{out}}) - \mathbb{E}(B_{\text{in}}))\mathbb{E}(N_s)}{\lambda}, \tag{27}$$

where $\mathbb{E}(N_s)$ is the expected number of charging services and λ is the constant charging speed rate of the CS. The charging speed normally is a highly nonlinear function depends on the charging power. In this study, we simplify it to a constant value. Queuing occurs when the number of vehicles is larger than the number of CS/BSS. We consider two extreme cases: In the pessimistic case scenario, all vehicles arrive at the same time with their energy demands. Thus, each vehicle arriving to an occupied CS/BSS will be placed in a queue. In the optimistic case scenario, vehicles arrive uniformly during each time frame. Neither scenario is likely to occur. However, both provide lower and upper bounds for the income.

The expected charging time for a taxi is:

$$\mathbb{E}(T_c) = \mathbb{E}(T_s) + \mathbb{E}(T_q) = \Theta\mathbb{E}(T_s), \tag{28}$$

where T_c is the time that the EV spent at the CS/BSS, T_s is the EVSE service time, T_q is queuing time, and Θ is the extension factor function. The value of Θ reflects the efficiency and profitability of the system. The derivation of (28) is given in the Appendix.

Proposition 4: In the optimistic case, the extension factor $\Theta = 1$.

This implies that an EV will not spend time on queuing at EVSE nor will an EVSE station need to wait for customers. Hence, the system is economically and efficiently optimized.

Proposition 5: In the pessimistic case, the extension factor Θ is:

$$\Theta = \begin{cases} \frac{1}{2}(1 + \frac{N_{EV}}{N_{EVSE}}), & \text{if } N_s \le 1 \\ \frac{1}{2\mathbb{E}(N_s)}[(1 + \frac{N_{EV}}{N_{EVSE}}) + (1+q)(\mathbb{E}(N_s) - 1)], & \\ & \text{if } N_s > 1, \end{cases} \tag{29}$$

where N_{EV} and N_{EVSE} are the number of taxis and EV charging ports in CS, respectively, N_s is the number of charging service times, and q is the length of the queue at the CS.

The length of the CS queue is:

$$q := q_{cs} = \max(\frac{N_{\text{EV}}}{N_{\text{EVSE}}}, 2\frac{N_{\text{EV}}}{N_{\text{EVSE}}} - \frac{\lambda}{\eta\mathbb{E}(V_x)}), \tag{30}$$

where q_{cs} is the CS queue length, λ is the CS charging speed rate, η is the energy consumption rate, and $\mathbb{E}(V_x)$ is the average driving speed of the taxi.

The total driving time $\mathbb{E}(T_d)$ in each time period h_1, h_2, h_3, and h_4 may be calculated using (12), (17), (18)–(23), (27), and (28). In the optimistic case, taxis arrive uniformly within the time period. Therefore, if the total charging time is shorter than the length of the period, the vehicle will not wait (each taxi arrives to an idle CS). Substituting $\mathbb{E}(T_d)$ in Lemma 1, $\mathbb{E}(U_x)$ may be calculated using Proposition 1.

The validity of Proposition 5 may be proved by considering two cases: $N_s < 1$ and $N_s \ge 1$.

If $N_s \le 1$, no taxi will make recurring trip to the CS. The first taxi in the queue will experience no waiting time, the second taxi will have to wait for the first taxi to complete its charging service, and the last taxi will need to wait for all the previous taxis. Hence, the average T_c is the total charging and waiting time of all vehicles in the CS divided by the number of taxis per CS.

The extension factor is:

$$\begin{aligned} \Theta &= (\sum_{i=1}^{N_{\text{EV}}/N_{\text{EVSE}}} i)/(N_{\text{EV}}/N_{\text{EVSE}}) \\ &= \frac{1}{2}(1 + \frac{N_{\text{EV}}}{N_{\text{EVSE}}}). \end{aligned} \tag{31}$$

If $N_s > 1$, a queue extension should be considered for the case when the next group of vehicles arrives and vehicles from previous group have not yet completed their service. The number of vehicles in this case is $\frac{\mathbb{E}_x(B)}{\eta\mathbb{E}(V_x)}/\frac{\mathbb{E}_x(B)}{\lambda}$. The length of the queue is:

$$\begin{aligned} q &= \frac{N_{\text{EV}}}{N_{\text{EVSE}}} + \max(0, \frac{N_{\text{EV}}}{N_{\text{EVSE}}} - \frac{\mathbb{E}_x(B)}{\eta\mathbb{E}(V_x)}/\frac{\mathbb{E}_x(B)}{\lambda}) \\ &= \max(\frac{N_{\text{EV}}}{N_{\text{EVSE}}}, 2\frac{N_{\text{EVSE}}}{N_{\text{EVSE}}} - \frac{\lambda}{\eta\mathbb{E}(V_x)}). \end{aligned} \tag{32}$$

Hence, the extension factor Θ is:

$$\Theta = \frac{1}{2\mathbb{E}(N_s)}[(1+\frac{N_{\text{EV}}}{N_{\text{EVSE}}}) + (1+q)(\mathbb{E}(N_s)-1)]. \quad (33)$$

A queue extension reflects the waiting time in the system. It is the number of additional vehicles in a queue in case when a vehicle A completes charging and drives away while the queue length is L. When the vehicle A empties the battery and decides to return to CS, the queue may have been reduced to L' thanks to the work preformed by the CS. However, by the time A returns, the queue length may become $L+1$. The queue extension term takes this effect into account.

2) Taxi Using BSS: Computations for a taxi using BSS can be obtained using minor modifications. In case of BSS, $\mathbb{E}(B_{\text{out}}) = C$. Since the swapping time is fixed and independent of the desired energy, (28) becomes:

$$\mathbb{E}(T_c) = \Theta\mathbb{E}(N_s)T_s, \quad (34)$$

where $\mathbb{E}(T_c)$ is the expected time of the taxi for swapping and queuing, Θ is the extension factor function, $\mathbb{E}(N_s)$ is the expected number of swapping services, and T_s is the time duration of a battery swapping service. Unlike in the case of CS, the BSS service time T_s is a constant and predetermined by the BSS system.

The extension factor Θ follows Proposition 5. The difference between CS and BSS is the queue length. In the BSS mode, the queue length is:

$$q := q_{x,bss} = \max(\frac{N_{\text{EV}}}{N_{\text{EVSE}}}, 2\frac{N_{\text{EV}}}{N_{\text{EVSE}}} - \frac{\mathbb{E}_x(B)}{\eta\mathbb{E}(V_x)T_s}), \quad (35)$$

where $q_{x,bss}$ is the queuing function of the taxi in the BSS mode, N_{EV} and N_{EVSE} are the numbers of taxis and battery swapping lanes, respectively, $\mathbb{E}_x(B)$ is the expected energy consumption of a taxi for each BSS session, and T_s is the time that a taxi spends in the battery swapping service.

B. Bus

The approach to analyze a bus is similar, albeit simpler to the taxi case.

Proposition 6: The expected income of a bus is:

$$\mathbb{E}(U_b) = \mathbb{E}(M_{total})(\mathbb{E}(u_b) - \eta R_{unit}), \quad (36)$$

where $\mathbb{E}(U_b)$ is the expectation of the income of a bus, M_{total} is the mileage of a bus, u_b is the bus income rate (income per mile), η is the battery energy consumption rate of a bus, and R_{unit} is the cost of unit energy.

Unlike the taxi, the income rate of the bus u_b is a random variable with different distributions in h_1, h_2, h_3, and h_4.

Lemma 5: The expectation $\mathbb{E}(M_{total})$ of the total driving mileage of a bus M_{total} is:

$$\mathbb{E}(M_{total}) = \mathbb{E}(V_{b,t})\mathbb{E}(T_{d,b}), \quad (37)$$

where $\mathbb{E}(V_{b,t})$ is the expectation of the driving speed of a bus at time t, and $\mathbb{E}(T_{d,b})$ is the driving time of a bus.

As the bus follows predetermined schedules, the expected number of charges in each time period $\mathbb{E}(N_{s,b})$ is:

$$\mathbb{E}(N_{s,b}) = \frac{\mathbb{E}(M_{\text{total}})}{\Upsilon}, \quad (38)$$

where Υ is the predetermined routing distance between charge/swapping services. The routing distance is planned and preordained based on the route conditions and the bus transportation capability.

The battery energy B_{out} of a bus at BSS is equal to the bus battery capacity C.

The time that each bus spends in charging using BSS is:

$$\mathbb{E}(T_{s,b}) = \frac{\eta\Upsilon\mathbb{E}(N_{s,b})}{\lambda}. \quad (39)$$

The pessimistic and the optimistic case queuing scenarios for a taxi may be also applied to a bus:

$$\mathbb{E}(T_{c,b}) = \Theta\mathbb{E}(T_{s,b}), \quad (40)$$

where $\mathbb{E}(T_{c,b})$ is the expected staying time of a bus at CS/BSS, Θ is the extension factor, and $\mathbb{E}(T_{s,b})$ is the expected charging service time of a bus at the CS/BSS.

Although there are significant differences in the vehicle arrival models for taxi and bus, the optimistic and pessimistic queuing models are the same at CS/BSS because the optimistic and pessimistic queues are the same regardless of the model that ignores the distribution of vehicle arrivals. While the queuing models are similar for taxis and buses, different Θ shall be considered. In the optimistic case, similar to the taxi $\Theta = 1$, a bus will always have an EVSE station available when it arrives, and there will be no queue at the BSS. In the pessimistic case under the CS mode, Θ remains the same as in the taxi case in Proposition 5. However, in the pessimistic case of the BSS mode, the queue length q for the bus is:

$$q := q_{b,bss} = \max(\frac{N_{\text{EV}}}{N_{\text{EVSE}}}, 2\frac{N_{\text{EV}}}{N_{\text{EVSE}}} - \frac{\Upsilon}{\mathbb{E}(V_b)T_{\text{s,b}}}). \quad (41)$$

$\mathbb{E}(V_b)$ may be found in each time period using (17), (36), (37), (38), (39), and (40).

C. Saturation Analysis

Due to the random nature of the vehicle routing, it is necessary to consider the limits of the service capacity of the service provider. The service provider cannot offer any charging service beyond the length of a certain time period. Hence, for a taxi and a bus in each period i:

$$\sum_{\text{EV/EVSE}} T_{s,i} = T_{s,i}\frac{N_{\text{EV}}}{N_{\text{EVSE}}} \le |h_i|. \quad (42)$$

If the sum $T_{s,i}$ is larger than the length of the period, we consider that the service capacity has been reached. Note that time is divided into four periods and the saturation is bounded within each period.

In practice, the segregation of time periods can be fuzzy. A driver's driving and charging behavior is also a gradual adjustment process and does not exactly follow the directive obedience of the time period. A driver may schedule the charging activities in advance according to the time periods. However, a charging process will not be terminated when it enters into another time period. Thus, in deployed systems and simulations, these four periods may overlap.

The charging time of one period can be "compensated" by a neighboring period that is less saturated.

TABLE I
SIMULATION PARAMETERS

Parameter	Value
C, taxi	30 kWh
C, bus	313 kWh
N_{EVSE}, CS	10
N_{EVSE}, BSS	10
N_{EV}, taxi	100
N_{EV}, bus	20
κ	0.9
v_{h}	30 Mph
v_{a}	10 Mph
v_{m}	1 Mph
v_{tr} in normal hours	0 Mph
v_{tr} in rest hours	3 Mph
v_{tr} in busy hours	-3 Mph
σ_x	10 Mph
R_{unit} for CS	0.6 \$/kWh
R_{unit} for BSS	1 \$/kWh
v_b	25 Mph
σ_b	1.5 Mph
u_t	2 \$/mile
Υ	145 mile
T_s for BSS	10 min
η for taxi	0.34 kWh/mile
η for bus	2 kWh/mile
Charging power	40 kW
Simulation time	30 days
Simulation steps	1 min

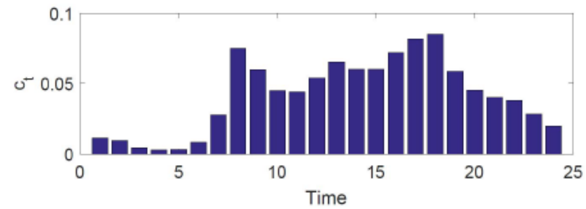


Fig. 5. Average hourly road traffic congestion weight used in simulations.

IV. SIMULATION RESULTS

The proposed stochastic model is validated by adjusting the model parameters in a case study with Monte-Carlo simulations of daily operations of 100 taxis and 20 buses using 10 CS and 10 BSS. The configuration of the simulation parameters is given in Table I. Parameters values are based on statistics of common service vehicles and service rates [26], [27]. The simulation scenario covers a 30-day period with time increments of 1 minute. In this paper, we set $\kappa = 0.9$. Usually, drivers do not wait until the EV battery is completely depleted before they recharge their EVs. Note that there are thresholds above which the drivers may become anxious and find an EVSE station for charging. The value of κ indicates the usage level of battery energy and represents such thresholds. In practice, the value of κ varies among drivers and highly depends on the battery capacity. Parameter κ is not a constant but rather a function that follows a complex irregular distribution as described in studies dedicated to modeling user charging behaviors [11], [14], [28]. We aim to understand the dynamics of the charging behaviors of electric public transport and do not consider the influence of the users. Hence, we have adopted a constant value of κ. Our choice is based on the observation that both the taxi and bus drivers wish to minimize the number of trips to recharge in order to increase their operating income.

The hourly normalized traffic congestion weights shown in Fig. 5 are based on the road traffic monitoring data from the State of Nebraska, the United States [21]. In the simulation study, we do not consider the difference in traffic conditions between weekends and weekdays and the difference between the urban and suburb areas. We assume that all taxis and buses are operated on weekdays in urban areas.

The comparison results of the incomes for an EV under the CS and the BSS modes are shown in Fig. 6. The revenue growth significantly slows down during the rest hours before the dawn while it grows slightly during the peak hours in the evening. The profitability of the BSS mode is significantly higher than for the CS mode. The income difference is caused by the different number of vehicles that are driving. While CS service consumes plenty of vehicles' time in waiting, BSS are more available leading to significantly longer driving time for EVs. The result is expected since the time spent at CS/BSS is very important for an EV fleet.

When considering the road traffic congestion, the BSS profitability of buses remains significantly higher than of the CS. However, the BSS mode for taxis is only marginally more profitable than the CS mode. As the congestion becomes severe during the congested hours, the EV speed will significantly decrease, which in turn will cause a significant reduction in the profitability of the taxis. However, simulation results show that the decline in profitability does not increase the willingness of the taxis to opt for a battery swapping service. The reason may be a tangible income gained by taking passengers during the slow traffic time compared to zero revenue when the roads to BSS are congested.

The simulation results are consistent with the results from the BSS taxi EV pilot project in Hangzhou City, China. Since the costs of BSS construction and operation are usually higher than of CS, the investment benefits of the BSS taxi project are lower than the CS taxi project. Based on the analysis of the model and the operation of the deployed project, the BSS taxi project should reduce the BSS construction and operating costs if they are to continue operating. The main costs are the labor required for the BSS operation and the land requirement for the BSS.

Most taxi EVs are charged at night and begin operation early in the morning even though during the evening peak hours the EV user's demand is most likely to be at its highest. It is also observed that even if taxis are stuck in traffic at peak hours, they will not choose to charge during peak hours unless necessary. Hence, the BSS mode has lower impact on the taxi operations than the CS mode. For the buses

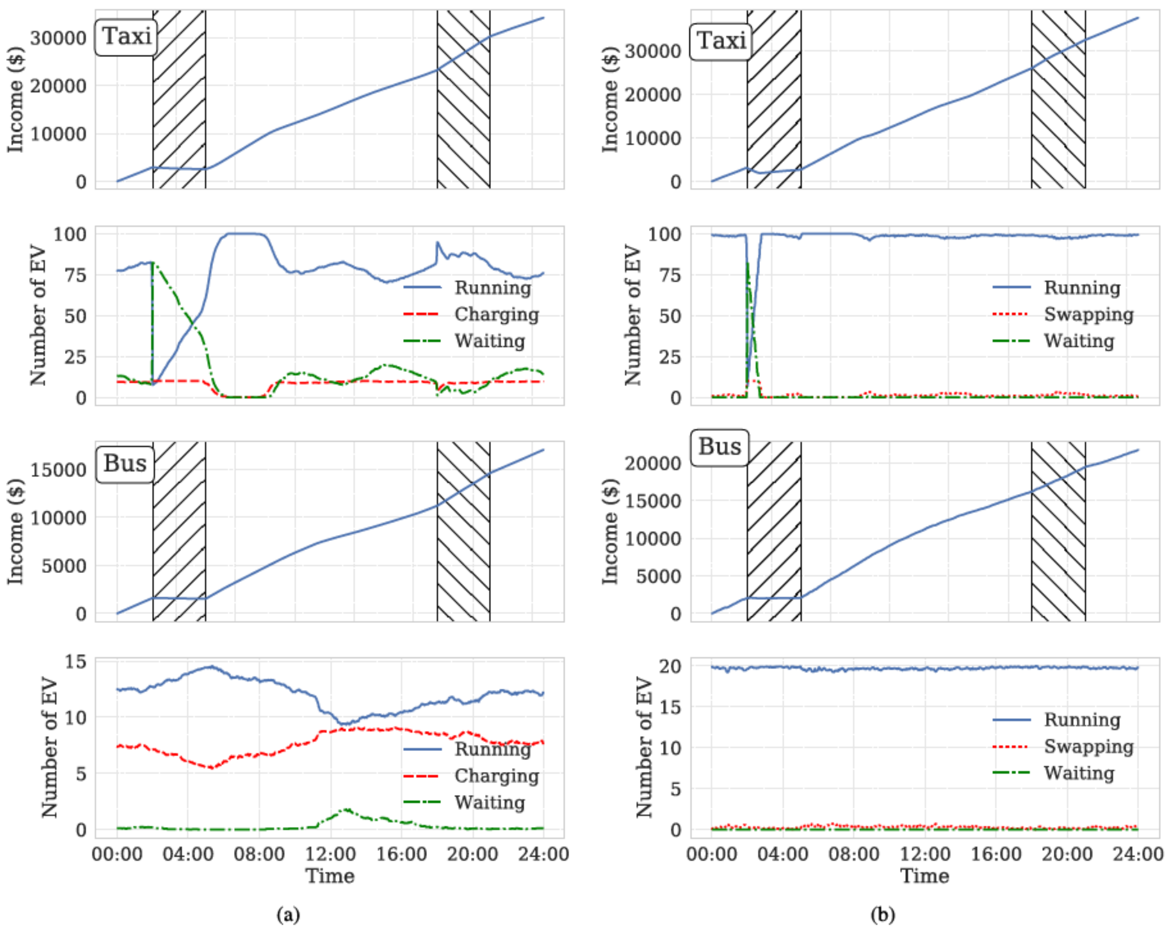


Fig. 6. Average simulation results with consideration of road traffic congestion for a 30-day period with 100 taxis and 20 buses using (a) 10 CS and (b) 10 BSS. Taxi's (top) and bus' (bottom) income and the number of taxis in driving, charging, and waiting. The left and right shaded regions indicate the rest and the busy periods, respectively.

incomes, there are more vehicles running during the morning and evening peak hours than during the midday break. After a day of operation, some bus EVs will have to recharge during the evening peak hours, resulting in fewer vehicles operating in the evening peak hours than in the morning rush hours. This indicates that the transportation capacity of bus EVs in the CS mode is not fully utilized during the evening peak hours. Therefore, the operating efficiency of bus EVs may be improved by rational scheduling and dispatching and, thus, increasing the transportation capacity. BSS work very well for bus operations and the bus workload are hardly affected by the battery swapping service.

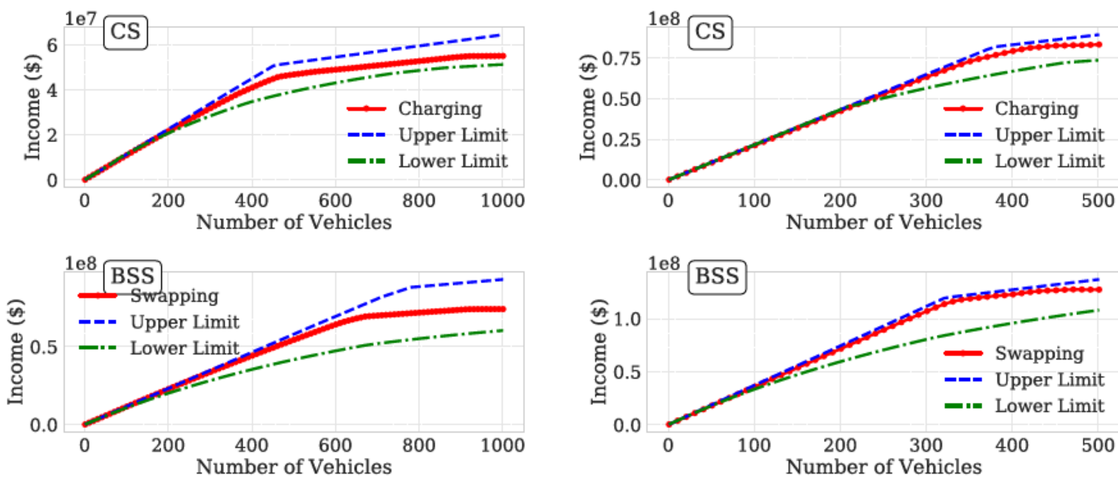
The analytical bounds provide a range of possible ranges for EV incomes with given queuing conditions. The lower and upper boundaries are the EVs with the pessimistic and optimistic scenarios, respectively. In the case when the number of EVs is less than the number of CS/BSS, the optimistic arrival situation applies to both boundaries because there are always available CS/BSS.

The simulation results and analytical boundaries are shown in Fig. 7. The simulation result lies between the lower and upper boundaries. When approaching saturation, the simulation result is closer to the lower boundary, which is reasonable as the charging queue becomes longer. The simulated saturated income is shown in Table II. The lower saturated boundary for buses using BSS is larger than the upper boundary for CS mode. The boundaries for taxi EVs using BSS are larger than that of CS. However, the difference is not strongly meaningful since the income is directly related to the setting of the service price.

Parameters in the proposed model may be adjusted to better match the profiles of the actual environment. For example, the adjustment should be made according to the weather and season in the user's region. In cold regions, the battery performance is often downgraded due to the low temperature and, hence, the value of battery capacity may be decreased accordingly. In regions where the heating and cooling systems are in place, the rate of battery consumption shall increase.

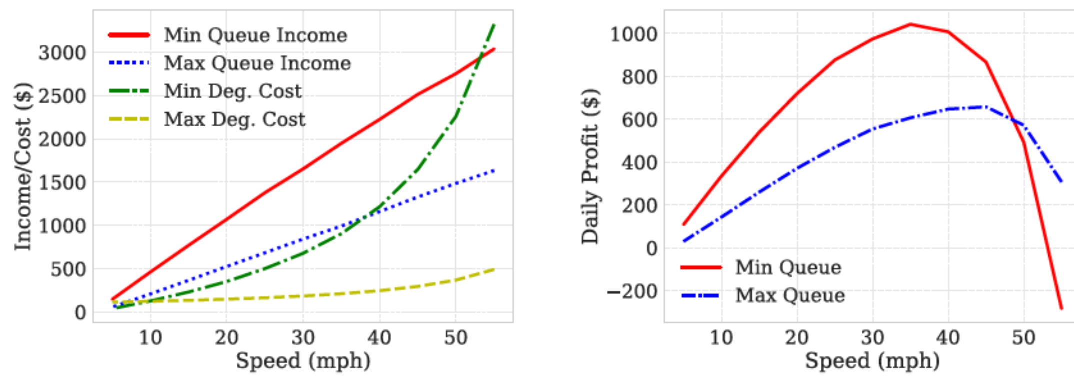
V. Use Cases

The presented analytical solution may be used to evaluate CS and BSS operations with no need to perform extensive numerical simulations. It may also assist in evaluations of larger systems that include CS/BSS. We present an example of finding the optimal driving speed of a bus with given EVSE infrastructure. The faster a bus drives, the more frequently the EV will need to be charged and, therefore, the number of saturated buses decreases. This is the maximum number of



(a) (b)

Fig. 7. Comparison of analytical boundaries and simulation results in terms of income of vehicles and number of vehicles for (a) taxi and (b) bus: CS (top) and BSS (bottom).



(a) (b)

Fig. 8. Income of each bus in a fleet: (a) daily operating income and battery degradation cost; (b) daily profit using BSS.

TABLE II
COMPUTED SATURATED INCOME OF TAXIS AND BUSES WITH EVSE ($)

	Lower Bound	Upper Bound	Simulation
Taxi/BSS	6.216×10^7	9.572×10^7	7.423×10^7
Taxi/CS	5.523×10^7	6.726×10^7	5.522×10^7
Bus/BSS	1.180×10^8	1.436×10^8	1.270×10^8
Bus/CS	7.615×10^7	8.949×10^7	8.337×10^7

vehicles that CS/BSS may serve and the income of the service provider no longer increases.

A faster driving speed requires a higher discharge rate and the battery cycle life decreases with increasing charge/discharge rate [29], [30]. With faster driving, each bus gets additional income per unit time. Therefore, an optimization problem is to find the speed that maximizes profit of a single bus considering the cost of battery degradation:

$$\begin{aligned} &\underset{v_b}{\text{argmax}} \frac{U_b}{N_{\text{EV,sat}}(v_b)} - \frac{\rho}{L(v_b)} N_{s,b} \\ &\text{s.t. } 0 < v_b \leq v_{\text{b,max}}, \end{aligned} \tag{43}$$

where v_b is the bus driving speed, U_b is the income of a bus, $N_{\text{EV,sat}}(v_b)$ is the number of saturated buses given v_b, ρ is the price of the battery for a single bus, $L(v_n)$ is the cycle life of a battery with given v_b, and $N_{s,b}$ is the number of charging services per day. We assume a linearly decreasing relationship between L and v_b. We use the setup described in Section IV and BYD 40' electric transit bus [31] as a reference for battery cost. The daily profit of a single bus, its revenue, and its battery degradation cost for using BSS as a function of speed is shown in Fig. 8. While revenue increases linearly with speed, the cost of battery degradation increases non-linearly especially at upper boundary and high speeds. The optimal speed is 35 mph (assuming the minimum queue/upper bound income) or 45 mph (assuming the maximum queue/lower boundaries income).

VI. CONCLUSION

In this paper, we developed an analytical model to evaluate stochastic interactions between service fleets (taxi/bus) and EVSE (CS/BSS). The model considered the behavior of public transport and the impact of road traffic congestion. The highly stochastic dynamics of vehicles charging activities in the developed model was evaluated by finding the upper and lower boundaries of the operating revenues under various assumptions regarding the behavior of individual users and overall arrivals. The stochastic model was verified via Monte-Carlo simulations. The analytical approach provided further insight into the stochastic model and may facilitate evaluations using various CS/BSS features. The results of the proposed models were consistent with the operational experiences of the pilot projects.

Appendix

A. Proofs of Lemmas

Proof of Lemma 1: The Lemma is proved using Wald's equation [32] when considering time slots when a taxi is either hired or available:

$$\begin{aligned}\mathbb{E}(M_{\text{hired}}) &= \mathbb{E}(V_{x,t}|\text{hired})\mathbb{E}(\sum_{t=0}^{T_d} \mathbb{1}_{\text{hired}})\\ &= \mathbb{E}(V_{x,t}|\text{hired})\mathbb{E}(T_d)P_s\\ \mathbb{E}(M_{\text{avail}}) &= \mathbb{E}(V_{x,t}|\text{avail})\mathbb{E}(\sum_{t=0}^{T_d} \mathbb{1}_{\text{avail}}t)\\ &= \mathbb{E}(V_{x,t}|\text{avail})\mathbb{E}(T_d)(1-P_s). \end{aligned} \tag{44}$$

□

Proof of Lemma 2: The proof is based on the proposition of expectation and the Bayes theorem. Let the driver's state s_t be a Markov process. Then, $s_t = d$ (driving) or $s_t = c$ (charging or swapping). When a driver decides to charge EV at time t, $s_t = c$ and $s_{t-1} = d$:

$$\begin{aligned}\mathbb{E}(B_{\text{in,normal}}) &= \mathbb{E}(B|\text{decide to charge})\\ &= \sum_{B=0}^{C} B \cdot P(B|s_t = c, s_{t-1} = d)\\ &= \sum_{B=0}^{C} B\frac{P(s_t = c|B, s_{t-1} = d)P(B|s_{t-1} = r)}{P(s_t = c|s_{t-1} = d)}\\ &= \sum_{B=0}^{C} Be^{-B}\frac{P(B|s_{t-1} = d)}{P(s_t = c|s_{t-1} = d)}\\ &= \sum_{B=0}^{C} Be^{-B}\frac{P(B|\text{driving})}{P(\text{decide to leave CS/BSS})}. \end{aligned} \tag{45}$$

$P(s_t = c|s_{t-1} = d)$ is a constant. For simplicity, we also assume that $P(B|s_{t-1} = d)$ is a constant for all B. □

Proof of Lemma 3: From the proof of Lemma 2,

$$P(B|s_t = c, s_{t-1} = d) = e^{-B}\frac{P(B|s_{t-1} = d)}{P(s_t = c|s_{t-1} = d)}. \tag{46}$$

Since $B \in (0, C)$,

$$\sum_{B=0}^{C} P(B|s_t = c, s_{t-1} = d) = 1. \tag{47}$$

□

Proof of Lemma 4: The proof is identical to proofs of Lemma 2 and Lemma 3. □

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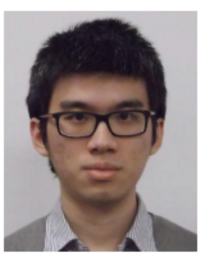

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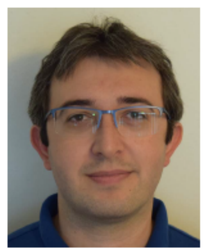

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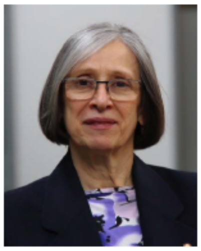

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