Compressive Spectral Imaging using Smoothness on Graphs

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Abstract: Compressive spectral imaging reconstruction is performed using smoothness on graphs. In doing so, a highly effective and parallelizable graph-smoothness prior reconstruction algorithm is developed based on simple direct matrix inversion.

1. Spectral Image Estimation using Smoothness on Graphs

Compressive spectral imaging (CSI) systems capture large volumes of spatio-spectral information in a compressed format, enabling such an information to be collected in settings such as autonomous navigation and exploration. To do so, CSI systems take a single spatio-spectrally coded image of the scene, and then iterative algorithms, relying on image priors, can be used to reconstruct the spectral scene [1]. The reconstructed scene is thus the one that best describes our prior knowledge of the scene and simultaneously fits the measurements. To date, many ways to inject prior knowledge into CSI reconstruction algorithms exist, from the most traditional priors based on compressive sensing to the most recent plug-and-play priors based on deep learning [2].

Our goal is to advance on the same front, but from a different perspective. We use ideas from the field of graph signal processing (GSP), and thus assume that the spectral image of interest is the smoothest with respect to a collection of pre-specified graphs. An advantage of this standpoint is that we can search for smooth signals on a graph by minimizing a differentiable function, referred to as the graph Laplacian quadratic form, over the set of signals that satisfy the measurements [3]. To benefit from this, we thus propose a block-based reconstruction algorithm, which approximates the solution to the problem through a series of sub-problems, which can be solved each independently by direct inversion or Gaussian elimination.

1.1. GSP preliminaries

An undirected graph $G = (V, E, w)$ is a triple, consisting of a vertex set $V = \{v_i\}_{i=1}^{n}$, an edge set $E \subset V \times V$, and a non-negative weight function $w : E \mapsto [0, \infty)$ s.t. $w(i, j) = w(j, i)$ for $(i, j) \in E$, and $w(i, j) = 0$ for $(i, j) \notin E$. Also, it is assumed that the graph has no self-loops and thus $(i, i) \notin E$ for $i \in V$. A graph signal x on G is a function $x : V \mapsto \mathbb{R}$ such that the value of x at $i \in V$ is given by $x_i \in \mathbb{R}$. The smoothness of x with respect to G is given by $x \mapsto x^T \mathbf{L}_G x$, where $\mathbf{L}_G \in \mathbb{R}^{n\times n}$ is the graph Laplacian of G with diagonal entries L_{ii} defined by $\sum_{j=1}^{n} w(i, j)$, and off-diagonal entries L_{ij} defined by $-w(i, j)$. Also important is that there are numerous methods of designing graphs from a set of signals so that the subject signals are optimally smooth.

1.2. Block-Based Smoothness on Graphs Algorithm for CSI

We now consider the estimation of a spectral image X of L bands, $X_1, \ldots, X_L \in \mathbb{R}^{n_1\times n_2}$ from a set of non-adaptive measurements $Y \in \mathbb{R}^{n_1\times(n_2+L-1)}$, obeying

$$y = \mathbf{H}x, \tag{1}$$

where $y = \mathrm{vec}(Y)$, $x = (\mathrm{vec}(X_1)^T, \ldots, \mathrm{vec}(X_L)^T)^T$, and $\mathbf{H}$ denotes the discretization of a CASSI system [1]. Since (1) is underdetermined, we cannot simply invert $\mathbf{H}$ to find x, and we thus propose to regularize the system using smoothness on graphs. To do so, x is regarded as a graph signal, residing on the vertex set $V := \{1, \ldots, n_1\} \times \{1, \ldots, n_2\} \times \{1, \ldots, L\}$. To simplify the problem, we now partition the system (1) into K small systems as follows:

1. Define a collection of overlapping subsets $\{S_j\}_{j=1}^{K}$, where $S_j \subset \{1, \ldots, n_1\} \times \{1, \ldots, n_2 + L - 1\}$ indexes a patch Y_{S_j} from Y of size $w_1 \times w_2$.
2. Find the subset $\Omega_j \subset V$, which indexes a spatial-spectral patch X_{Ω_j} of X and maps into Y_{S_j} through $\mathbf{H}$. That is, $y_{S_j} = \mathbf{H}_{S_j,\Omega_j} x_{\Omega_j}$, where $\mathbf{H}_{S_j,\Omega_j}$ is the submatrix of $\mathbf{H}$ with rows and columns indexed by S_j and Ω_j.

3. Given a suitable collection of graphs $G_j = \{\Omega_j, E_j, w_j\}$ on Ω_j such that X at Ω_j is smooth on G_j, we can find a local estimate $\hat{x}^{(j)}$ of X at Ω_j by solving the problem:

$$\hat{x}^{(j)} = \arg \min_{z \in \mathbb{R}^{|\Omega_j|}} z^T \mathbf{L}_{G_j} z \text{ s.t. } y_{S_j} = \mathbf{H}_{S_j, \Omega_j} z, \; j = 1, \ldots, K. \tag{2}$$

4. Based on the local estimates $\hat{x}^{(j)}$ of x at Ω_j, the spectral image estimate $\hat{x}$ of x is defined by $\hat{x}_i = \sum_{j=1}^{K} \frac{1_{\{i \in \Omega_j\}} \hat{x}^{(j)}_{\tau_j(i)}}{\sum_{j=1}^{K} 1_{\{i \in \Omega_j\}}}$, for all $i \in V$, where $\tau_j : \Omega_j \mapsto \{1, \ldots, |\Omega_j|\}$ is such that $\hat{x}^{(j)}_{\tau_j(i)}$ is the entry of $\hat{x}^{(j)}$ associated with $i \in \Omega_j$.

1.3. *The Choice of the Collection of Graphs*

Like Gaussian mixture models [4], a suitable collection of graphs may be learned from either examples or from side information. Here, however, we do not yet concern ourselves with this problem. Instead, to illustrate the concept, we use the simplifying assumption that the graphs belong to the family of k-nearest-neighbor (knn) graphs, and the k-nn graphs are then learned from the local statistics of the signal of interest itself. Specifically, let G_j be a k-nn graph on the vertex set $V := \Omega_j$, and assume that there is access to a graph signal $\tilde{x} : \Omega_j \mapsto \mathbb{R}^b$ such that the value of $\tilde{x}$ at $i \in \Omega_j$ is given by a vectorized spatio-spectral patch of X centered at i whose size is b. The edge set E_j of G_j is defined by $\cup_{i \in \Omega_j} \{(i, i_p) : p = 1, \ldots, k\}$, where $\|\tilde{x}_i - \tilde{x}_{i_1}\|_2 \leq \ldots \leq \|\tilde{x}_i - \tilde{x}_{i_k}\|_2$, and the weight function $w(r,s) = \exp(-\|\tilde{x}_r - \tilde{x}_s\|_2^2 / \sigma^2)$ for $(r,s) \in E_j$, where $\sigma^2 = \text{mean}(\{\|\tilde{x}_r - \tilde{x}_s\|_2^2\}_{(r,s) \in E_k})$.

1.4. *Numerical Experiment*

We now test our method on reconstructing the spectral image of a butterfly from its compressive snapshot as displayed in Fig. 1. To do so, let $w_1 = w_2 = 21$, and $S_j = \{(i_y, i_x) : \; j_x \leq i_x \leq j_x + w_2 - 1, \; j_y \leq i_y \leq j_y + w_1 - 1\}$, where i_x, i_y denote the (y, x) coordinates of i. We define $\{S_j\}_{j=1}^{K}$ such that there is at most 75 percent overlap between any two pair of adjacent subsets S_{j_1} and S_{j_2}, and $\cup_j S_j = \{1, \ldots, n_1\} \times \{1, \ldots, n_2 + L - 1\}$, and set as the number of neighbors and signals $k = 27$ and $b = 3^3$, respectively. Figure 1 shows our graph-based estimate, and a total-variation estimate, included for completeness.

Fig. 1. From left to right, simulated CASSI snapshot; RGB rendering of the spectral scene of interest, consising of 31 bands of size 256 by 256; RGB renderings of the reconstructed scenes using total variation and the proposed graph-smoothness prior; and single point reconstructed spectral signatures. For TV, PSNR=20.12 and SAM=18.98. For for graph-smoothness prior, PSNR=30.76 and SAM=14.82.

1.5. *Conclusion*

As observed in Fig. 1, when prior knowledge in the form of a graph is available, not only can we obtain highly accurate estimates, but we also can design simple reconstruction algorithms. As of a future step, to move this idea forward it is important to learn those graphs from data.

References

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