# Strategic Remote Estimation

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***Abstract*—Motivated by the emerging networked applications that involve *strategic communication* between selfish, rational agents whose objectives are misaligned, this paper studies a strategic variation of the well-known remote estimation problem. The classical remote estimation problem studies the design of a system, consisting of a sensor, an encoder, and a decoder, to observe, transmit and estimate a discrete time stochastic process with minimal transmission energy and estimation error. In order to minimize transmission energy, the encoder makes a real-time (sequential) *to send or not to send* communication decision based on sensor's observation of the stochastic process. In the variation studied in this paper, the encoder aims to render the estimate at the receiver biased in a particular, statistical manner, hence this consideration transforms the classical remote estimation from a team problem to a game. Specifically, we analyze the problem of sending $N$ samples within a time window of length $M$. Assuming quadratic distortion measures and affine strategies for sender and receiver, which were shown to be optimal for Gaussian strategic communication, we first determine the optimal sequential transmission policies for this problem in the context of the Stackelberg equilibrium– where the encoder (sensor) is the leader and the decoder (estimator) is the follower– using dynamic programming. We demonstrate the effectiveness of our approach via numerical results.**

## I. Introduction

Several emerging networks and applications, such as inter-vehicular communication, cyber-physical systems, autonomous systems, and the Internet of Things differ from their classical analogues due to two aspects: i) communication has strict delay and energy constraints; in fact, low complexity real-time communication is often a strict requirement ii) communication occurs between selfish agents with misaligned objectives as opposed to the hidden assumption underlying classical communication theory that is communicating agents form a team sharing an identical objective (minimize some distortion metric or probability of bit/symbol error etc). This paper studies these two aspects of communication at the physical layer, on the particular problem setting of remote estimation.

Remote estimation problems have recently received a revived interest due to the prevalence of cyber-physical systems [1]–[3]. This problem, broadly, pertains to a setting where a random process is observed by a sensor (sender, encoder, decision maker) which samples and transmits its measurement to a remote estimator (receiver, decoder). The objective of the remote estimator is to recover the aforementioned stochastic process with minimal errors. There exists a fundamental trade-off between the number of observations that a remote estimator receives (transmission energy) and the reconstruction error. Remote estimation algorithms optimize this trade-off by sequentially making the communication decisions at the encoder side, based on the realization of the observation.

Most relevant to the proposed scenario is the line of work initiated by Imer and Başar who considered a variation problem over a finite time horizon for a sensor and an estimator, where the sensor is allowed to communicate only a limited number of times in a given time window [2]. By restricting the communication strategies to the class of threshold-based communication strategies, the authors showed that there exists a unique threshold-based communication strategy which can be computed via dynamic programming [4]. The threshold here depends on the realizations, hence varies in time but the mapping of the number of available communication opportunities to the threshold is determined offline. Threshold-based strategy simply refers to using the following decision making procedure: if the magnitude of the sample realization is greater than a pre-specified threshold, it is considered informative and transmitted to the estimator. Otherwise, it is not sent and the estimator also knows that this sample is not sufficiently informative to be communicated and estimates the missing sample accordingly. This intuitive strategy is indeed optimal for Markov, unimodal source processes under mean squared error estimation criteria, as shown in [5]. Most prior work on this problem involve a perfect channel between the sensor and the estimator. Recent efforts have focused on more realistic variations including the packetized channel with queueing [6], with packet drops [7], and real-time encoding and decoding with additive noise channel [8].

In this paper, we extend the approach of Imer and Başar [2] to strategic scenarios where the objectives of the sensor (sender) and the estimator (receiver) are misaligned. In the classical communication paradigm, the encoder and the decoder share identical objectives (they constitute a team), such as minimizing distortion or probability of bit error. However, in strategic communication, the aforementioned objectives are misaligned, as is the case in several emerging networks such as the Internet of Things and inter-vehicular communication. In game theory parlance, this new consideration substantially transforms the communication from a *team* problem to a *game*. These problems while being central to the field of information Economics, see e.g., [9], [10], have not been analyzed in the engineering literature until very recently. In a recent work [11], this game is analyzed in the context of Stackelberg equilibrium where the sender is the leader and hence committed to its encoding mappings, and the receiver is the follower and acts to minimize its own cost given the encoding map (best response to the encoding strategy). It is shown in [11] that the quadratic-

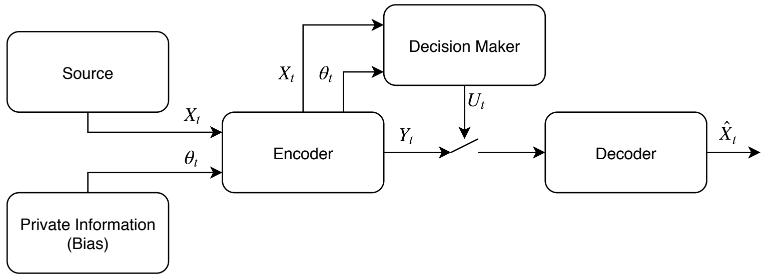


Fig. 1: Problem setting.

Gaussian setting admits an essentially unique equilibrium that is achieved by linear strategies. In this paper, we begin with the linear strategies derived in [11] apriori and numerically derive the optimal communication decision making policies using dynamic programming following the approach taken in [2] for non-strategic version of the problem.

One intriguing question that does not appear in classical (noiseless, non-strategic) remote estimation problem, is on how to account for the change in effective joint statistics of the source and bias variables due to thresholding operation. Since the optimal linear encoding and decoding maps depend on the second order statistics (the marginal variances as well as the correlation of the source and bias information), they also indirectly depend on the threshold. Hence, once need to optimize the parameters of the linear encoding mappings as well as the threshold *jointly* which constitutes a substantial research challenge for this problem. We circumvent this problem by iteratively applying the following two steps until convergence: i) optimize the threshold for a given statistics; and ii) update the statistics for a given threshold. This iterative imposition of person by person optimality conditions is well-understood to converge to a local minima, see e.g. the Lloyd-Max quantization algorithms used in source compression [12].

## II. Preliminaries

### A. Strategic Communication

In this paper, we consider the Stackelberg equilibrium, (or Bayesian perfect subgame equilibirum in the Economics parlance) where the sender is the leader and the receiver is the follower. The game proceeds as follows: the sender plays first and announces an encoding mapping. As a leader in Stackelberg game, the sender is *committed* to its encoding mapping, i.e., the sender cannot change it after the receiver plays. The receiver, knowing this commitment, determines its own mapping that maximizes its pay-off, given the encoding map. The sender, of course, will anticipate this, and pick its map accordingly. Note that there is a natural order in this model of communication: the sender cannot change its mapping after the receiver announces the decoding strategy.

Here, the source $X$ and bias $\theta$ are mapped into $Y \in \mathbb{R}$, via a stochastic mapping $Y = g(X, \theta)$ and the receiver produces an estimate of the source $\hat{X}$ through a mapping $h : \mathbb{R} \to \mathbb{R}$ as $\hat{X} = h(Y)$.

The objective of the receiver is to minimize

$$D_R = \mathbb{E}\left\{\left(X - \hat{X}\right)^2\right\} \tag{1}$$

while that of the sender is to minimize

$$D_S = \mathbb{E}\left\{\left(X + \theta - \hat{X}\right)^2\right\} \tag{2}$$

over the mappings $g(\cdot, \cdot)$ and $h(\cdot)$. We take the source and bias variables jointly Gaussian, i.e., $(X, \theta) \sim \mathcal{N}(0, R_{X\theta})$ where, without any loss of generality, $R_{X\theta}$ is parametrized as

$$R_{X\theta} = \sigma_X^2 \begin{bmatrix} 1 & \rho \\ \rho & r \end{bmatrix}, \tag{3}$$

with $r > \rho^2$. The optimal strategies for this communication game are derived in [11] and reproduced below:

**Theorem 1** ( [11]). *The essentially unique mappings at the Stackelberg equilibrium are given as $g^*(X, \theta) = X + \alpha\theta$ and $h^*(Y) = \kappa Y$, where $\alpha$ and $\kappa$ are:*

$$\alpha = \frac{A-1}{2(r+\rho)}, \quad \kappa = \frac{1+\alpha\rho}{1+\alpha^2 r + 2\alpha\rho} \tag{4}$$

*Costs at this Stackelberg solution are*

$$D_S = \sigma_X^2 \left(1 + \frac{(A-3)(r+\rho)}{A-1}\right) \tag{5}$$

$$D_R = \sigma_X^2 \left(\frac{(r-\rho^2)(A-1)}{A(2r+A\rho+\rho)}\right) \tag{6}$$

*where $A = \sqrt{1 + 4(r+\rho)}$.*

### B. Problem Definition

The problem setting is depicted in Figure 1 where the state of a remote plant is observed over a perfect channel by the sensor, and transmitted to a decoder who recovers the state over a finite time horizon $t = 1, \ldots, M$. The state and bias processes are characterized by one-dimensional, independent, and identically distributed (i.i.d.) stochastic processes denoted by $\{X_t\}$ and $\{\theta_t\}$. We take the joint distribution as jointly Gaussian with zero mean and covariance given in (3), we denote this probability measure as $f(X, \theta)$. The entire system consists of three elements: a sensor, and encoder and a decoder. Here, the sensor and the encoder share identical objectives, while the decoder's objective differs from them.

At each time $t$, the sensors makes perfect (noiseless) measurements of $X_t$. Then it makes a binary decision, denoted here by $U_t$ on whether to transmit or not, where $U_t = 1$ corresponds to transmission and $U_t = 0$ implies non-transmission.

If the sensor decides to transmit its observation, then it sends $X_t$ to the encoder. Otherwise, it transmits a free symbol (denoted by $\epsilon$) representing that there is no transmission. We consider the well-studied two variations for the cost of the sensor. The *soft-constrained* problem variation involves a transmission cost $c$ for each transmission, while there is no limit on the number of total transmissions. A non-transmission decision does not incur any cost. The *hard-constrained* variation limits the number of transmissions to $N$ over the time horizon $M$, that is,

$$\sum_{t=1}^{M} U_t \leq N. \tag{7}$$

After receiving the message $\tilde{X}_t$ from the sensor, the encoder sends a message $Y_t$ to the decoder. In the noise-free channel variation, as also studied in this paper, this message is transmitted to the decoder without any errors.

After receiving the message from the channel the decoder estimates the state process $\{X_t\}$ in minimum mean-squared error sense.

The objective of the team of sensor and encoder is to minimize following cost:

$$D_E(t) = \mathbb{E}\left\{\left(X_t + \theta_t - \hat{X}_t\right)^2\right\} \tag{8}$$

while the objective of the decoder is

$$D_R(t) = \mathbb{E}\left\{\left(X_t - \hat{X}_t\right)^2\right\} \tag{9}$$

We consider two variations of this problem, as done in [2], [8]. In the problem version with a hard constraint, the encoder can use $M$ of the total $N$ communication opportunities. On the version with a soft constraint, there are no hard limits on the number of transmission opportunities, but there is a distortion cost associated with each transmission. We take the channel model as perfect channel, the decoder receives the message transmitted by the encoder (when it makes the *transmit* decision), without any errors or distortions. In both variations of the problem, we assume *threshold-based* decision making strategies apriori, i.e., we assume that the encoder will make the transmit decision if $x, \theta \in \tau$, otherwise it will not transmit. This strategy was first intuitively selected and studied in [2] and later was shown to be optimal for unimodal sources with Markov temporal dependency, under some mild technical assumptions [5].

## III. Main Results

### A. Problem with the hard constraint

We follow the steps similar to those in [2] to derive a dynamic programming solution to the hard constrained problem, albeit there are significant modifications due to the strategic aspect of the problem at hand. We start by defining the encoder distortion as a function of the communication opportunities left $s$ and the time remaining $t$, as $e(s,t)$. The optimal value of this function is denoted as $e^*(s,t)$. There are boundaries at $s = t$ and $s = 0$ where the threshold becomes constant. We let $\Gamma^T(X,\theta)$ and $\Gamma^{NT}(X,\theta)$ denote the encoder's distortion when the message is sent and not sent. In the non-strategic (original) variation of the remote estimation problem, these functions are simply $\Gamma^{NT}(X) = X^2$ and $\Gamma^T(X) = 0$. Here, due to the strategic aspect of the problem both functions involve $X$ and $\theta$, and given as follows:

$$\Gamma^T(X,\theta) = ((X+\theta) - \kappa(X+\alpha\theta))^2 \tag{10}$$

$$\Gamma^{NT}(X,\theta) = (X+\theta)^2 \tag{11}$$

We let $\tau_{(s,t)}$ denote the set of $X, \theta$ values that results in the transmission decision when time is $t$ and the number of communication opportunities left is $s$.

Plugging in the expectation expressions and after straightforward algebra (omitted here), we obtain equation (13).

The recursive equation (12) follows from the distortion relations. These recursive equations suggest a dynamic programming solution, as it does in the case of [2] which require the determination of initial conditions. One initial condition pertains to $s = 0$ case: here, we have no more communication opportunities left, and the expected error is the product of the expected error for not transmitting and $t$, i.e., $e^*(0,t) = t\mathbb{E}[\Gamma^{NT}(X,\theta)]$. The other initial condition is the $s = t$ case: here, the encoder always transmits if transmission is beneficial to the encoder, and does not transmit otherwise, i.e., the one step error is simply[1]

$$P_1 = \mathbb{P}\{A\}\mathbb{E}[\Gamma^T(X,\theta)|A] + \mathbb{P}\{B\}\mathbb{E}[\Gamma^{NT}(X,\theta)|B],$$

where $A$ is the event $\Gamma^{NT}(X,\theta) \geq \Gamma^T(X,\theta)$ and $B$ denotes the event $\Gamma^T(X,\theta) \geq \Gamma^{NT}(X,\theta)$. The overall error is simply $e^*(t,t) = tP_1$. Here we note that the encoder transmits messages only if $\Gamma^{NT}(X,\theta) \geq \Gamma^T(X,\theta)$ even in the case of unlimited communication opportunities, because there are realizations of $X$ and $\theta$ for which transmitting a message increases encoder's cost due to the way the receiver estimates the source (more on this in Remark 1). With these conditions the optimal error can be calculated for any state. We also note that $\tau_{(s,t)}$ can be expressed as

$$\tau_{(s,t)} = \left\{(X,\theta)|\Gamma^{NT}(X,\theta) - \Gamma^T(X,\theta) > \beta_{(s,t)}\right\} \tag{14}$$

By applying the first order necessary conditions for optimality on equation (13), we obtain the optimal threshold $\beta^*_{(s,t)}$ as:

$$\beta^*_{(s,t)} = e^*_{(s-1,t-1)} - e^*_{(s,t-1)}. \tag{15}$$

Hence, once the error functions associated with every time and state have been computed, the optimal threshold can be simply computed via (15) as well.

We have implemented the dynamic programming solution described above to obtain numerical simulation results. We

[1]The symbol $\mathbb{P}(A)$ denotes the probability of event $A$.

$$e^*(s,t) = \min_{\tau_{(s,t)}} \left\{ \left(e^*_{(s-1,t-1)} + \mathbb{E}\left[\Gamma^T(X,\theta)|\ (X,\theta)\in\tau_{(s,t)}\right]\right) \int_{\tau_{(s,t)}} \mathrm{d}f + \left(e^*_{(s,t-1)} + \mathbb{E}[\Gamma^{NT}(X,\theta)|\ (X,\theta)\in\tau^c_{(s,t)}]\right) \int_{\tau^c_{(s,t)}} \mathrm{d}f \right\} \tag{12}$$

$$e^*(s,t) = e^*_{(s,t-1)} + \int \Gamma^T(X\theta)\mathrm{d}f + \min_{\tau_{(s,t)}} \left\{ \left(e^*_{(s-1,t-1)} - e^*_{(s,t-1)}\right) \int_{(X,\theta)\in\tau_{(s,t)}} \mathrm{d}f + \int_{(X,\theta)\in\tau_{(s,t)}} \left(\Gamma^T(X,\theta) - \Gamma^{NT}(X,\theta)\right)\mathrm{d}f \right\} \tag{13}$$

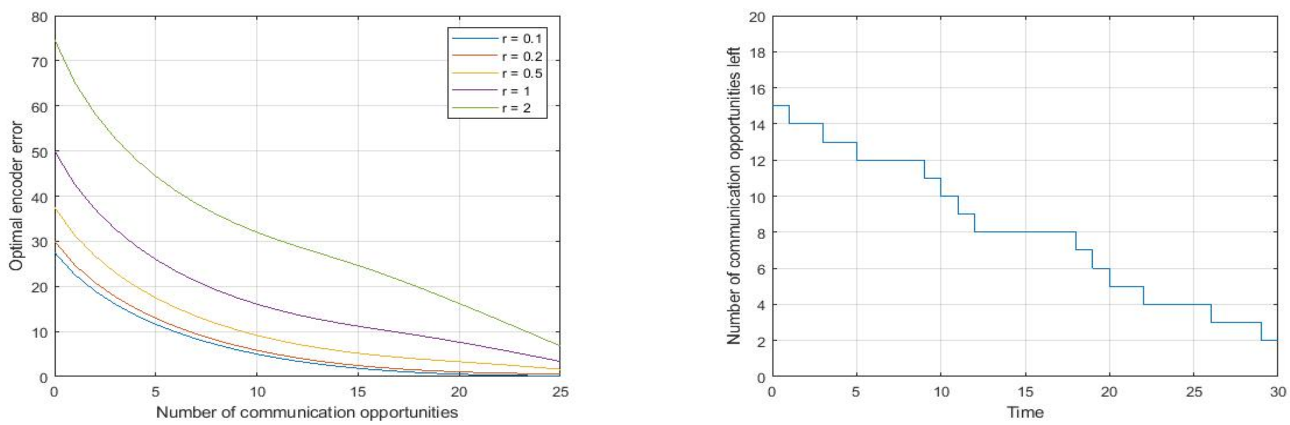


Fig. 2: Encoder distortions for $\rho = 0$ varying $r$ (left), and sample communication path (right).

plot encoder's costs associated with varying initial communication opportunities ($M$) in Figure 2 for $N = 25$ for statistics with varying $r$ and $\rho$ is fixed at $\rho = 0$. As expected, an increase in the number of communication opportunities decreases the expected error. However, as can be seen from Figure 2, while the non-strategic version of the problem results in an expected convex relationship between the number of communication opportunities available $M$ and encoder's cost, as the problem becomes more and more strategic, the numerical results suggest that this complexity disappears, as can be seen for the case $r = 2$.

*B. Problem with the soft constraint*

**Remark 1.** *A rather surprising observation here, that does not appear in the classical remote estimation problem, is that the encoder may not use all of its communication opportunities available. A similar observation is also made for the setting with additive noise channel in [8]. However, here the underlying reason is different than that of in [8]. In this strategic setting, it might be more beneficial not transmit (even if the transmission is available for no cost) and have the decoder estimate the source as its mean, e.g., if the realization of $X$ is close to that of $-\theta$, the encoder distortion $\mathbb{E}\{(X+\theta-\hat{X})^2\}$ would be small given that the decoder will reconstruct $\hat{X} = 0$. This is demonstrated in Figure 2 as there are unused communication opportunities left at the end of the communication window. In the noisy but non-strategic setting analyzed in [8], the underlying reason for this phenomenon is the fact that the received (noisy) signal at the decoder may not be as informative as the information obtained from the thresholding operation (which is sent to the receiver over a noiseless ternary channel).*

For the soft constrained problem, the encoder does not have limited communication opportunities, but there is a penalty, $c$, for sending a message in addition to the distortion. i.e., the overall cost of the encoder is

$$\min_{\tau_t} \{\mathbb{E}[\sum_{t=1}^{T} cU_t + d_t(X_t,\theta_t)]\} \tag{16}$$

where

$$d_t(X_t,\theta_t) = \begin{cases} \Gamma^{NT}(X_t,\theta_t), & \text{if } U_t = 0 \\ \Gamma^T(X_t,\theta_t), & \text{if } U_t = 1 \end{cases} \tag{17}$$

Noting the memoryless nature of the source ($X$) and bias $\theta$, and following the arguments used in [8] for a similar problem, it is straightforward to decouple the objective and transform the problem equivalently to the following one-step (static) problem.

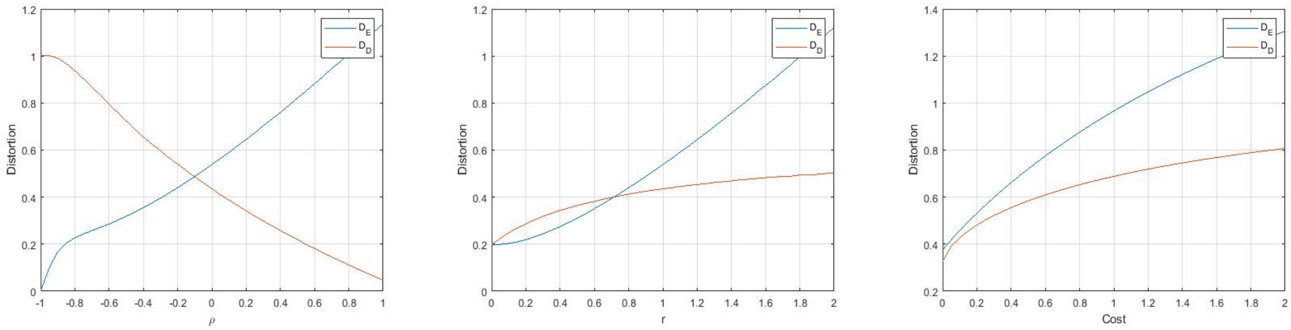


Fig. 3: Encoder and decoder distortions for $r = 1$ and varying $\rho$ (left), $\rho = 0$ and varying $r$ (middle), and varying $c$ with $r = 1, \rho = 0$ (right).

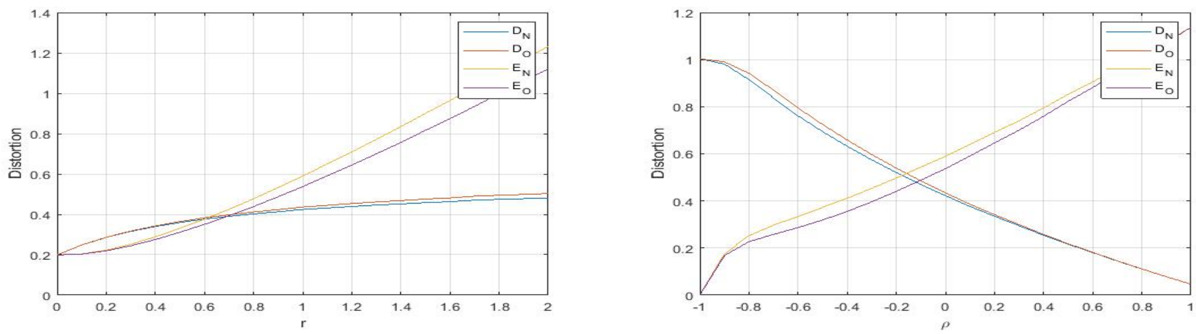


Fig. 4: Distortion costs of the original encoding/decoding parameters and the revised encoding/decoding parameters.

$$\min_{\tau} \{\mathbb{E}[cU_t + d_t(X, \theta)]\} \tag{18}$$

We solve this static optimization problem, i.e. find the optimal threshold $\tau$ as a function of $c$, and plot our numerical results in Figure 3 which demonstrates the relationship between the distortion costs associated with the encoder and decoder and the statistical characteristics of $(X, \theta)$, and cost of transmission. These error curves in Figure 3 are similar to those found in the strategic communication setting [11]. The monotonicity of the error curves (and asymmetry around the mean) is demonstrated in the left subfigure in Figure 3, as stated in Remark 1, when $\rho \to -1$, the encoder chooses not to transmit and to force the decoder to estimate $\hat{X} = 0$, and thereby making the encoder distortion $\mathbb{E}\{(X + \theta - \hat{X})^2\} \to 0$ and $\mathbb{E}\{(X - \hat{X})^2\} \to \sigma_X^2$. Hence, as $\rho$ increases $D_E$ also increases while $D_D$ decreases.

### C. Adjusting the statistics due to thresholding

We next address the following concern: once the message $Y$ generated from $(X, \theta)$ pair is thresholded, does the optimal linear mappings change? We note that this problem does not appear in the classical, original variation of the remote estimation problem since there is no *encoding* of the transmitted value in that problem. The noisy channel variation studied in [8] accounts for this effect by simply computing the *conditional* variance of the source given that $U_t = 1$, which is rather straightforward to compute. Here, obviously the problem is more involved. As a practical remedy, even though the thresholding policy generates a non-Gaussian joint distribution of $X$ and $\theta$, we still approximate it as jointly Gaussian, albeit with a different second order statistics, i.e., we recompute $\sigma_X^2, r$ and $\rho$. After the statistics are approximated, we recompute the optimal mappings, i.e., $\alpha$ and $\kappa$ values based on the newly generated $\sigma_X^2, r$ and $\rho$ via (4). We repeat this two step procedure until this process converges.

We implemented the above procedure for our running example of jointly Gaussian $X$ and $\theta$. We plot the revised values of $\alpha$ and $\kappa$ are shown in Figures 5 and 6.

**Remark 2.** *When using thresholding communication and find the new encoding scheme the values of $\alpha$ and $\kappa$ decrease. This is due to the fact that negatively correlated values of $(X, \theta)$ have low error when not transmitted, meaning they will not be in the set that of values that are transmitted. This makes the set of $(X, \theta)$ more positively correlated, decreasing the optimal values of $\alpha$ and $\kappa$.*

We plot our simulation results in Figure 4 for varying $r$ values while keeping $\rho$ fixed at $\rho = 0$ as well as for varying $\rho$ values while keeping $r$ is fixed at $r = 1$. With new values of $\alpha$ and $\kappa$, the costs associated with the encoder and the decoder change as well. This set of $(X, \theta)$ is also more positively correlated than its original (unthresholded) distribution.

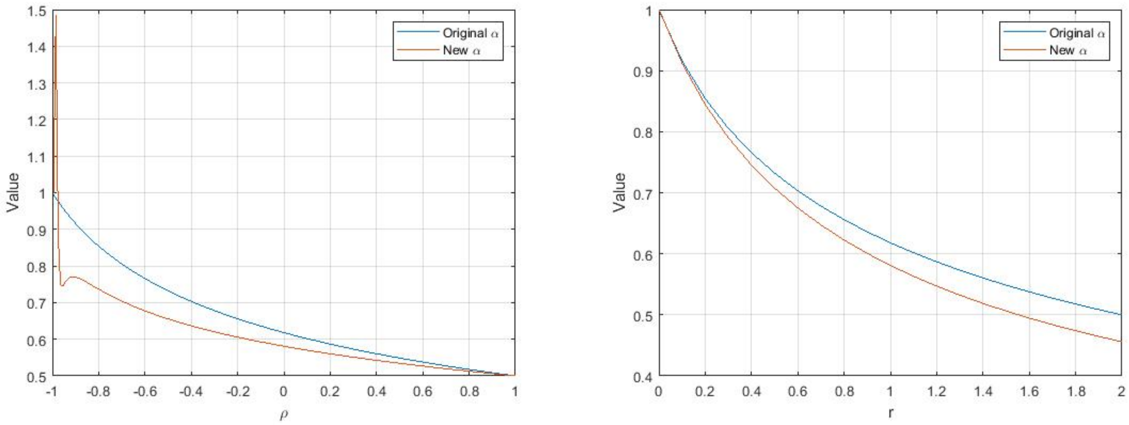


Fig. 5: $\alpha$ value for varying $\rho$ with $r = 1$, $c = 1$

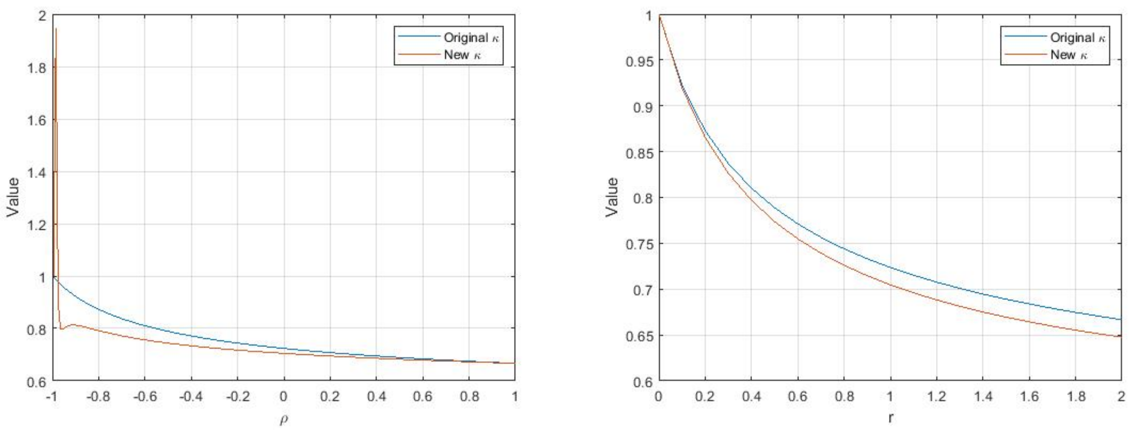


Fig. 6: $\kappa$ value for varying $\rho$ with $r = 1$, $c = 1$

We analyzed in detail the previous section, more positively correlated distributions increase encoder's distortion, $D_E$, while decreasing decoder's distortion $D_D$. This observation is here demonstrated in Figure 4 where it can be seen that $D_N$(New Decoder Error) is smaller than $D_O$(Old Decoder Error). For the encoder more positively correlated messages increase its error and this is also seen in the simulated results with $E_N$(New Encoder Error) is larger than $E_O$(Old Encoder Error).

**Remark 3.** *When using thresholding communication and we find that the new encoding scheme the values of $\alpha$ and $\kappa$ decrease. This is due to the fact that negatively correlated values of $(X, \theta)$ have low error when not transmitted, meaning they will not be in the set that of values that are transmitted. This makes the set of $(X, \theta)$ more positively correlated, decreasing the optimal values of $\alpha$ and $\kappa$.*

## IV. Discussion

In this paper, we have extended the analysis for the remote estimation problem form classical communication to strategic settings where the sender (encoder) and the receiver (decoder) have diverging objectives. Our analysis has uncovered an interesting observation: unlike the classical remote estimation problem, in this strategic variation, the encoder might choose not to utilize all available communication opportunities. Another rather surprising observation pertains to the analysis made to account for the statistics change due to thresholding. This correction of statistics after thresholding, results in positively correlated $X$ and $\theta$, thereby increases the encoder's error $D_E$ while decreases the decoder's error $D_D$.

Extending the analysis to settings with additive communication channels constitutes a part of our future research.


## V. Acknowledgment

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