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Design of Distributed Event-Triggered Average Tracking Algorithms for Homogeneous and Heterogeneous Multiagent Systems

Yu Zhao, *Member, IEEE*, Chengxin Xian, *Student Member, IEEE*, Guanghui Wen, *Senior Member, IEEE*, Panfeng Huang, *Senior Member, IEEE*, and Wei Ren, *Fellow, IEEE*

***Abstract*—This article addresses the design problem of distributed event-triggered average tracking (DETAT) algorithms for homogeneous and heterogeneous multiagent systems. The objective of the DETAT problem is to develop a group of distributed cooperative control algorithms with event-triggered strategies for agents to track the average of multiple time-varying reference signals. First, for homogeneous linear multiagent systems, based on sampling measurements and model-relied holding techniques, a class of static-gain DETAT algorithms is proposed with a couple of local event-triggered functions for estimators and controllers, respectively. Compared with the existing distributed average tracking (DAT) algorithms, the static-gain DETAT algorithms greatly reduce the cost over communication networks and the frequency of control protocol updates. Second, to reduce the chattering phenomenon caused by nonsmooth items in static-gain algorithms and requirements of the global information of networks, smooth dynamic-gain DETAT algorithms are introduced based on boundary layer approximation methods and self-adaptive principles. Third, for heterogeneous linear multiagent systems, a new algorithm is established by using the output regulation techniques for the heterogeneous DETAT problem. The outputs of heterogeneous agents can ultimately track the average of multiple time-varying reference signals. To the best of our knowledge, it is the first time to study the DETAT problem for heterogeneous multiagent systems. Finally, some examples are presented to show the validity of theoretical results.**

***Index Terms*—Distributed average tracking (DAT), event-triggered strategy, homogeneous and heterogeneous dynamics, linear multiagent system.**

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Yu Zhao, Chengxin Xian, and Panfeng Huang are with the School of Automation, Northwestern Polytechnical University, Xi'an 710129, China (e-mail: yuzhao5977@gmail.com; xcx@mail.nwpu.edu.cn; pfhuang@nwpu.edu.cn).

Guanghui Wen is with the School of Mathematics, Southeast University, Nanjing 210096, China (e-mail: wenguanghui@gmail.com).

Wei Ren is with the Department of Electrical and Computer Engineering, University of California, Riverside, CA 92521 USA (e-mail: ren@ee.ucr.edu).

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I. Introduction

During the past few decades, the design of distributed cooperative algorithms over communication networks for multiagent systems has become an important research focus due to its wide applications such as cluster battles of unmanned aerial vehicles, saturation attacks of multiple loitering munitions, voltage regulations of smart grids, and industrial transport of logistics systems. As the most fundamental distributed cooperative algorithms, consensus protocols have been investigated deeply [1]–[14]. Also, in leader-following networks, distributed tracking algorithms were studied from lots of research perspectives [15]–[16]. As a novel class of distributed cooperative algorithms, distributed average tracking (DAT) algorithms were raised and studied in [17]–[32]. Different from distributed consensus and distributed tracking algorithms, DAT algorithms may ensure agents to track the average of multiple time-varying reference signals, while each signal is only known by one agent in networks. Therefore, DAT algorithms are more difficult and general.

The objective of DAT problems is to design DAT algorithms for agents to track the average of multiple outside reference signals. The motivation for DAT problems was arising from the coordinated tracking for multicamera systems [26], where multiple nodes equipped with cameras track objects cooperatively. Besides, DAT algorithms had found applications in distributed sensor fusions [1], distributed Kalman filters [17], [18], dynamically merging feature-based maps [19], and distributed formation controllers [31]. Therefore, in order to solve DAT problems, many DAT algorithms were designed. In the literature, some pioneering linear DAT algorithms were proposed in [20]–[24]. A proportional DAT algorithm and a proportional–integral one were proposed in [20] for multiple static signals. Bai *et al.* [21] considered the robustness to initial error issues and extended the proportional–integral algorithm to achieve zero steady-state error for time-varying signals. Furthermore, the proportional DAT algorithm was employed in [22] for discrete-time multiagent systems. In [24], a DAT algorithm based on sliding-mode control methods was developed for Euler–Lagrange multiagent systems. Then, in [25], the DAT problems were investigated for integrator-type dynamics with swarm behavior and experimental validation. Further, some nonlinear DAT algorithms were developed in [26]–[32] to overcome some difficulties that have not been solved by existing linear DAT algorithms in [20]–[25]. To solve DAT problems for multiple dynamic reference signals with bounded derivatives, a novel class of nonlinear DAT algorithms was proposed in [26] based on nonsmooth control approaches for single-integrator-type multiagent systems. Further, in [27], the nonlinear nonsmooth DAT algorithm was extended to double-integrator-type multiagent systems for signals with bounded second derivatives. To extend the high-order linear dynamics of agents, a class of smooth DAT algorithms was proposed in [29], which removed the chattering effect caused by the nonsmooth sign function. Moreover, in [30], the DAT problem for Lipschitz-type nonlinear dynamical systems was solved. It is worth mentioning that both the linear and nonlinear DAT algorithms in [20]–[32] have two common features. First, all these DAT algorithms need continuous communications, which require a large amount of cost for communication networks and is difficult to be utilized in practical engineering. Second, all agent dynamics in existing works [20]–[32] are homogeneous.

To reduce the cost of continuous communications, many researchers have studied an energy-saving mechanism using a triggering strategy that can decide the most optimal instants of data exchange, called event-triggered control [33]. Due to its energy saving effect in discontinuous communication networks, the design of distributed event-triggered cooperative algorithms for multiagent systems has become a hot topic in recent years. In the literature, some distributed event-triggered consensus algorithms were developed for multiagent systems in [34]–[42]. In [37], consensus problems with general linear dynamics were investigated by event-triggered control strategies, which is one of the first attempts to extend the previous work to general linear dynamics. Also, there exist some investigations about event-triggered distributed average consensus (DAC) or DAT algorithms in [43]–[47] for single- or double-integrator dynamics. As is well known, the DAC problem is to estimate the average of time-varying reference signals, while the DAT problem is aiming to have physical trajectories of agents track the target trajectory. The DAT problem is an extension of the DAC problem with more complicated dynamics. Besides, to the best of our knowledge, there is little research about event-triggered DAT algorithms for multiagent systems with homogeneous and heterogeneous general linear dynamics, which may describe many practical systems. Therefore, how to design distributed event-triggered average tracking (DETAT) algorithms for homogeneous and heterogeneous general linear multiagent systems is a theoretically challenging and significant problem in practice.

Motivated by the aforementioned observations, this article aims to design DETAT algorithms for homogeneous and heterogeneous multiagent systems. For homogeneous linear multiagent systems, two types of DETAT algorithms are devised from different corners. First, a static-gain DETAT algorithm is introduced with two local event-triggered functions, which solves the DETAT problem meanwhile reducing the communication frequency. Then, modified by a continuous approximation of sign function and self-adaptive principles, continuous dynamic-gain DETAT algorithms are designed to reduce the chattering phenomenon caused by the nonsmooth item in the static-gain algorithm and to remove the requirement of global information. For heterogeneous linear multiagent systems, by using output regulation techniques, a new DETAT algorithm is established for the DETAT problem with heterogeneous linear multiagent dynamics. The outputs of heterogeneous systems can ultimately track the average of multiple time-varying reference signals.

The main contributions of this article are stated as follows.

1) For general linear multiagent systems, it is the first time to design DAT algorithms under event-triggered communication mechanisms, which are named DETAT algorithms. Compared with the existing DAT algorithms in [20]–[32] and DAC algorithms in [43]–[47], DETAT algorithms proposed in this article may ensure the high-order linear agents to track the target of average signals. Besides, compared with the integrator-type dynamics in [43]–[47], the high-order linear agent dynamics considered in the article are more general and with practical significance. Besides, due to the introduction of event-triggered mechanisms, the communication frequency of the proposed DETAT algorithm is greatly reduced, which may cut down communication costs in real applications.
2) In order to make the DETAT algorithms more applicable in real practices, a class of continuous dynamic-gain DETAT algorithms is extended by using boundary layer approaches and self-adaptive principles. The chattering raised in static-gain DETAT algorithms is greatly reduced. Also, the continuous dynamic-gain DETAT algorithm is fully distributed, which successfully removes requirements of global parameters over the network's topology, such as the eigenvalue of the Laplacian and the scale of the network.
3) For heterogeneous linear multiagent systems, a new DETAT algorithm is constructed. The present results on DAT problems have a common assumption that all agents in networks have homogeneous dynamics, which is an idealized assumption and difficult to apply to real engineering. In general, the dynamics in real multiagent systems are heterogeneous. In this article, by using output regulation techniques, a new DETAT algorithm is designed for heterogeneous multiagent systems. The outputs of heterogeneous agents can track the average signals in a distributed manner. Compared with the previous homogeneous multiagent dynamics [20]–[32], [43]–[47], the dynamics of agents have been greatly expanded, which are closer to reality.

The structure of this article is arranged as follows. Preliminaries are presented in Section II. In Section III, a couple of DETAT algorithms are designed for homogeneous multiagent systems. Further, in Section IV, DETAT problems for heterogeneous systems are investigated. In Section V, some numerical simulations are shown to verify theoretical results. Finally, Section VI concludes the article and looks forward to the future.

II. Preliminaries

A. Notation

Let R^n denote n dimensions as real vectors. Let $R^{n\times n}$ be the set of $n \times n$ dimension real matrices. $\mathbf{1}$ denotes that the elements of column vectors are all 1. I_n represents the identity matrix of dimension n. $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ represent the largest and smallest eigenvalues of a matrix. $\mathrm{diag}(z_1, \ldots, z_p)$ stands for a block-diagonal matrix with diagonal entries z_1 to z_p. For a series of column vectors $x_1, \ldots, x_n$, $\mathrm{col}(x_1, \ldots, x_n)$ represents a column vector by stacking them together. The notation $\otimes$ means the Kronecker product. For a vector $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T \in R^n$, $\|\omega\|$ denotes the Euclidean norm of ω and $\mathrm{sgn}(\omega) = \omega/\|\omega\|$.

B. Graph Theory

An undirected graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ can be described by a node set $\mathcal{V}$ and an edge set $\mathcal{E}$. The adjacency matrix, $A(\mathcal{G}) = [a_{ij}] \in R^{N\times N}$, is defined as $a_{ii} = 0, a_{ij} = 1$, if $(j,i) \in \mathcal{E}$, and $a_{ij} = 0$, otherwise. The incidence matrix of the graph is defined as D [49]. The Laplacian of a graph $\mathcal{G}$: $L = DD^T = \Delta(\mathcal{G}) - A(\mathcal{G})$, is a rank-deficient positive semidefinite matrix. $L = [l_{ij}] \in R^{N\times N}$, where $l_{ii} = \sum_{j=1,j\neq i}^{N} a_{ij}$ and $l_{ij} = -a_{ij}, i \neq j$. And the diagonal matrix $\Delta(\mathcal{G})$ represents of the degree matrix of the graph. The degree of agent i is defined as $d_i = l_{ii}$. If there are paths between any two nodes, then it is said that the undirected graph is connected.

Assumption 1: The undirected communication network is assumed to be connected.

C. Useful Lemmas

Lemma 1: [49] Under Assumption 1, 0 is a simple eigenvalue of L with eigenvector $\mathbf{1}$ and all the other eigenvalues are positive. Moreover, the smallest nonzero eigenvalue λ_2 of L satisfies $\lambda_2 = \min_{z\neq 0, \mathbf{1}^T z=0} \frac{z^T L z}{z^T z}$.

Lemma 2: [50] For nonnegative real numbers a and b and positive real numbers p and q satisfying $(1/p) + (1/q) = 1$, one has $ab \leq a^p/p + b^q/q$.

Lemma 3: [29] Define $M = I_N - \frac{1}{N}\mathbf{1}\mathbf{1}^T$. Then, M satisfies the following properties.

1) 0 is a simple eigenvalue of M with $\mathbf{1}$ as the corresponding right eigenvector and 1 is another eigenvalue with multiplicity $N-1$, i.e., $M\mathbf{1} = \mathbf{1}^T M = 0$.
2) Under Assumption 1, one has $LM = ML = L = L^T$.
3) $M^2 = M$.

Lemma 4: [48] For an undirected and connected graph $\mathcal{G}$ with the associated Laplacian matrix L, there exists a matrix $\Lambda \in R^{N\times N}$ such that $L\Lambda = \Lambda L = M$, where $M = I_N - \frac{1}{N}\mathbf{1}\mathbf{1}^T$.

III. DETAT Algorithms for Homogeneous Linear Multiagent Systems

There are N time-varying reference signals, $r_i(t) \in R^n, i = 1, 2, \ldots, N$, described by the following dynamics:

$$\dot{r}_i(t) = Ar_i(t) + Bf_i(r_i, t) \tag{1}$$

where $r_i(t) \in R^n$ is the state of the ith time-varying signal and $f_i : R^n \times R^+ \to R^p$ represents the reference input of the ith signal, $i = 1, 2, \ldots, N$. A and B are constant matrices with compatible dimensions.

Assumption 2: (A, B) is stabilizable.

Assumption 3: $f_i(r_i, t)$ is bounded, i.e., $\|f_i(r_i, t)\| \leq \overline{f}_i, i = 1, 2, \ldots, N$, for all $t > 0$ and $r_i \in R^n$. Let $f_0 \triangleq \max_{i=1}^{N}\{\overline{f}_1, \ldots, \overline{f}_N\}$.

Consider a homogeneous multiagent system containing N agents, described by the following general linear dynamics:

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t), i = 1, 2, \ldots, N \tag{2}$$

where $x_i(t) \in R^n$ is the state of agent i and $u_i(t) \in R^p$ is the control input.

Assume that agent i can only get the state information of $r_i(t)$ and agent i can receive or send the local information to its neighbor agents, which are denoted by $\mathcal{N}_i, i = 1, 2, \ldots, N$. Besides, let $|\mathcal{N}_i|$ be the number of elements in the set $\mathcal{N}_i$.

On the basis of the network model described above, the main objective of this section is to design a class of DETAT algorithms for agents to track the average of multiple time-varying reference signals generated by (1) in the sense that

$$\lim_{t\to\infty} \left\| x_i(t) - \frac{1}{N}\sum_{k=1}^{N} r_k(t) \right\| = 0, i = 1, 2, \ldots, N$$

and exclude the Zeno behavior.

Remark 1: The DETAT problem for homogeneous multiagent systems is to design DAT algorithms based on the distributed event-triggered network communication and ensure the trajectory of the physical agent to track the average of multiple time-varying reference signals. The multireference signals may be various information, such as position state, velocity state, or acceleration state of any target observed or collected by each agent. The target could be an airplane or a vehicle. Besides, due to network communication being discontinuous in real practices, the existing DAT algorithms based on the continuous network communication in [20]–[32] were difficult to be applied. Therefore, the DETAT problem is raised. Different from the traditional equal interval sampling control strategy, the irregular triggering time instants are completely determined by the agent itself and its neighbors. Only the sampled local information is used in triggering functions, which can effectively avoid continuous communication. Therefore, DETAT algorithms may save the communication cost and have great potential in practical engineering fields.

In the following, two kinds of DETAT algorithms are designed consisting of static gain and continuous dynamic gain for homogeneous linear multiagent systems.

A. Static-Gain DETAT Algorithms for Homogeneous Multiagent Systems

First, let $q_i(t)$ and $s_i(t), i = 1, 2, \ldots, N$, be some internal states of static-gain DETAT algorithms to be designed, which satisfy the following relation $s_i(t) = q_i(t) + r_i(t), i = 1, 2, \ldots, N$. Define a model-based event-triggering state $\tilde{s}_i(t)$ of the internal state $s_i(t)$ as follows:

$$\tilde{s}_i(t) = e^{A(t-t_k^i)} s_i\left(t_k^i\right) \quad \forall t \in \left[t_k^i, t_{k+1}^i\right) \tag{3}$$

where t_k^i denotes the kth event-triggering instant of $s_i(t)$. Let $e_i(t) \triangleq \tilde{s}_i(t) - s_i(t), i = 1, 2, \ldots, N$, be estimator measurement errors. To assign the estimator triggering instant t_k^i, an estimator triggering function is designed as follows:

$$T_i(t) = c_1 d_i \|K_1\|^2 \|e_i\|^2 + 2c_2 d_i \|K_1\| \|e_i\| - \frac{c_1}{4} d_i \left\|K_1\left(\tilde{s}_i(t) - \tilde{s}_j(t)\right)\right\|^2 - \mu_i e^{-\nu_i t} \tag{4}$$

where μ_i and ν_i are positive constants. If the triggering condition $T_i(t) \geq 0$ is fulfilled, then $s_i(t)$ updates its current state and transfers its current state to its neighbors, and $e_i(t)$ will be reset to 0. The estimators will update their states as soon as they acquire the latest state information from any neighbors.

Based on the event triggering state $\tilde{s}_i(t)$ and the proposed triggering function (4), an estimator of the DETAT algorithms is designed as follows:

$$\dot{q}_i(t) = Aq_i(t) + Bc_1 \sum_{j\in\mathcal{N}_i} [K_1 (\tilde{s}_i(t) - \tilde{s}_j(t))] + Bc_2 \sum_{j\in\mathcal{N}_i} \text{sgn}[K_1 (\tilde{s}_i(t) - \tilde{s}_j(t))] \quad (5)$$

where c_1 and c_2 are positive static gains and $K_1 \in R^{p\times n}$ is a matrix gain to be designed later.

Then, let $\hat{x}_i(t) = x_i(t) - s_i(t), i = 1, 2, \ldots, N,$ represent tracking errors. Define an event-triggering tracking error of $\hat{x}_i(t)$ by

$$\tilde{x}_i(t) = e^{A\left(t-\tau_k^i\right)}\hat{x}_i\left(\tau_k^i\right) \quad \forall t \in \left[\tau_k^i, \tau_{k+1}^i\right) \quad (6)$$

where τ_k^i denotes the kth event triggering instant of agent i. By denoting $\hat{e}_i(t) \triangleq \tilde{x}_i(t) - \hat{x}_i(t), i = 1, 2, \ldots, N$ to be controller measurement errors, a controller triggering function is given as follows:

$$\Pi_i(t) = \frac{1}{2}\|K_2\|^2\|\hat{e}_i\|^2 - \frac{1}{2}\|K_2\|^2\|\tilde{x}_i\|^2 + 2c_3\|K_2\|\|\hat{e}_i\| - \hat{\mu}_i e^{-\hat{\nu}_i t} \quad (7)$$

with $\hat{\mu}_i$ and $\hat{\nu}_i$ be positive constants. c_3 is a positive static gain and $K_2 \in R^{p\times n}$ is another matrix gain to be designed later. When the triggering condition $\Pi_i(t) \geq 0$ is satisfied, the agent $x_i(t)$ updates its current state and broadcasts its state information to neighbors, and $\hat{e}_i(t)$ will be reset to 0. The agents will update their states once they receive the latest state information from any neighbors.

Based on the event-triggering state $\tilde{x}_i(t)$ and the proposed triggering function (7), a tracking controller of the DETAT algorithms is designed as

$$u_i(t) = c_1 \sum_{j\in\mathcal{N}_i} [K_1 (\tilde{s}_i(t) - \tilde{s}_j(t))] + c_2 \sum_{j\in\mathcal{N}_i} \text{sgn}[K_1 (\tilde{s}_i(t) - \tilde{s}_j(t))] + K_2\tilde{x}_i(t) + c_3\text{sgn}[K_2\tilde{x}_i(t)]. \quad (8)$$

Definition 1: The closed-loop system does not exhibit the Zeno behavior if $T_1 = \inf_k\{t_{k+1}^i - t_k^i\} > 0$ and $T_2 = \inf_k\{\tau_{k+1}^i - \tau_k^i\} > 0, \forall i \in \mathcal{V}$, i.e., an infinite number of communication rounds never happen within a limited period of time.

Algorithm 1: Under Assumptions 1–3, for multiple time-varying reference signals in system (1), the designed static-gain DETAT algorithms with the estimator (5) and the controller (8) can be constructed by the following steps.

1) Solve the following algebraic Riccati equations (AREs):
$$P_mA + A^TP_m - P_mBB^TP_m + I = 0, m = 1, 2 \quad (9)$$
to obtain matrices $P_m > 0$. Choose $K_m = -B^TP_m$.
2) Choose the parameters $c_1 \geq \frac{1}{\lambda_2(L)}$, $c_2 \geq \frac{2\sqrt{N}f_0}{\lambda_2(L)}$, and $c_3 \geq f_0$.
3) Set $\mu_i > 0$, $\nu_i > 0$, $\hat{\mu}_i > 0$, and $\hat{\nu}_i > 0$ to be any positive constants for $i = 1, 2, \ldots, N$, and initialize $\sum_{i=1}^N q_i(0) = 0$.

Theorem 1: Under Assumptions 1–3, Algorithm 1 solves the DETAT problem for the homogeneous multiagent system (2) with time-varying reference signals (1). Furthermore, Zeno behaviors are excluded in estimation and tracking processes.

Proof: See the Appendix A.

Remark 2: Theorem 1 shows that all agents in homogeneous multiagent systems (2) can track the average of time-varying multireference signals (1) by using DETAT algorithms proposed in (4), (5), (7), and (8), which ensure that the estimation error $\xi_i(t)$ and the tracking error $\hat{x}_i(t)$ both asymptotically converge to 0. Besides, Zeno behaviors are excluded during estimation and tracking processes, respectively. A sufficient condition for the existence of (4), (5), (7), and (8) is that the pair (A, B) is stabilizable.

B. Continuous Dynamic-Gain DETAT Algorithms for Homogeneous Multiagent Systems

In the above-mentioned section, the proposed nonsmooth static-gain DETAT algorithm is designed based on the discontinuous function $\text{sgn}(\omega)$, which may inevitably lead to chattering effect in real applications. First, in order to reduce chattering and make the controller easier to implement, the discontinuous function $\text{sgn}(\omega)$ can be replaced by a continuous approximation $\hat{h}_i(\omega) = \frac{\omega}{\|\omega\|+\varepsilon e^{-\varphi t}}$ based on the boundary layer concept, where ε and φ are positive constants. Second, the proposed static-gain DETAT algorithm requires that $c_1 \geq \frac{1}{\lambda_2(L)}$, $c_2 \geq \frac{2\sqrt{N}f_0}{\lambda_2(L)}$, and $c_3 \geq f_0$, which depend on the smallest nonzero eigenvalue λ_2 of L and the number of agents N as well as the upper bounds f_0 of the reference input $f_i(r_i, t), i = 1, 2, \ldots, N$. Since λ_2, f_0, and N are all global variables, it is unrealistic for agents to obtain these global information when the network is large. Therefore, to remove the chattering in controllers and the global information restriction, a group of continuous dynamic-gain DETAT algorithms is designed with a DETAT estimator

$$q_i(t) = Aq_i(t) + B \sum_{j\in\mathcal{N}_i} \alpha_{ij}(t)[K_1 (\tilde{s}_i(t) - \tilde{s}_j(t))] + B \sum_{j\in\mathcal{N}_i} \beta_{ij}(t)\hat{h}_i[K_1 (\tilde{s}_i(t) - \tilde{s}_j(t))]$$

$$s_i(t) = q_i(t) + r_i(t), \quad \sum_{i=1}^N q_i(0) = 0 \quad (10)$$

and a DETAT controller

$$u_i(t) = \sum_{j\in\mathcal{N}_i} \alpha_{ij}(t)[K_1 (\tilde{s}_i(t) - \tilde{s}_j(t))] + \sum_{j\in\mathcal{N}_i} \beta_{ij}(t)\hat{h}_i[K_1 (\tilde{s}_i(t) - \tilde{s}_j(t))] + K_2\tilde{x}_i(t) + c_{ij}(t)\hat{h}_i[K_2\tilde{x}_i(t)] \quad (11)$$

with the dynamic-gain coupling strengths $\alpha_{ij}(t)$, $\beta_{ij}(t)$, $c_{ij}(t)$ satisfy

$$\dot{\alpha}_{ij}(t) = \gamma[(\tilde{s}_i(t) - \tilde{s}_j(t))^T\Gamma_1(\tilde{s}_i(t) - \tilde{s}_j(t)) - \vartheta\alpha_{ij}(t)] \quad (12)$$

$$\dot{\beta}_{ij}(t) = \eta\left[-\chi\beta_{ij}(t) + \frac{\|K_1(\tilde{s}_i(t) - \tilde{s}_j(t))\|^2}{\|K_1(\tilde{s}_i(t) - \tilde{s}_j(t))\| + \varepsilon e^{-\varphi t}}\right] \quad (13)$$

$$\dot{c}_{ij}(t) = \kappa\left[-\varrho c_{ij}(t) + \frac{\|K_2\tilde{x}_i(t)\|^2}{\|K_2\tilde{x}_i(t)\| + \varepsilon e^{-\varphi t}}\right] \quad (14)$$

where $\Gamma_1 \in R^{n\times n}$ is a constant gain matrix, and $\gamma, \vartheta, \eta, \chi, \kappa,$ and ϱ are positive constants. The triggering function for the estimator i is modified as

$$T_i(t) = \sum_{j\in\mathcal{N}_i} (1+2\delta\alpha_{ij}) \|K_1\|^2 \|e_i\|^2 + \sum_{j\in\mathcal{N}_i} 2(1+\delta\beta_{ij}) \|K_1\| \|e_i\| - \frac{1}{4}\sum_{j\in\mathcal{N}_i} \|K_1(\tilde{s}_i - \tilde{s}_j)\|^2 - \mu_i e^{-\nu_i t} \tag{15}$$

where δ is a positive constant. The triggering function for the agent i is modified as

$$\Pi_i(t) = \frac{\delta}{2}\|K_2\|^2\|e_i\|^2 + 2(1+\delta c_{ij})\|K_2\|\|e_i\| - \hat{\mu}_i e^{-\hat{\nu}_i t}. \tag{16}$$

Algorithm 2: Under Assumptions 1–3, the continuous dynamic-gain DETAT algorithms (10)–(16) for multiple time-varying reference signals in (1) are designed in the following three steps.

1) Obtain $K_m = -B^T P_m$, $\Gamma_m = P_m B B^T P_m$, $m = 1, 2$, respectively, by solving AREs in (9).
2) Select γ, η, and κ small enough, such that $\theta_4 \triangleq \max\{\vartheta\gamma, \chi\eta\} \le \frac{1}{\lambda_{\max}(P_1)}$ and $\theta_6 \triangleq \max\{\kappa\varrho\} \le \frac{1}{\lambda_{\max}(P_2)}$.
3) Choose the parameters $\delta, \mu_i, \nu_i, \hat{\mu}_i$, and $\hat{\nu}_i$ to be any positive constants.

Theorem 2: In this section, the DETAT problem for homogeneous multiagent systems is solved by continuous dynamic-gain DETAT algorithms (10)–(16) constructed by Algorithm 2. The estimation error $\xi_i(t)$, the tracking error $\hat{x}_i(t)$, and dynamic gains $\alpha_{ij}(t)$, $\beta_{ij}(t)$, and $c_{ij}(t)$, are uniformly ultimately bounded. Furthermore, Zeno behaviors can be excluded.

Proof: See Appendix B.

Remark 3: Differing from Theorem 1, the DETAT problem can be solved by Theorem 2 without requiring any global information, such as the smallest nonzero eigenvalue λ_2 of L and the number of agents N as well as upper bounds f_0 of the reference input. In other words, continuous dynamic-gain DETAT algorithms in this article are in a fully distributed viewpoint. At the same time, the continuous DETAT algorithms in this section use a continuous approximation of the discontinuous function $\mathrm{sgn}(\omega)$, which will reduce the chattering effect and make the controller easier to implement in real applications.

Remark 4: It is worth noting that the terms $-\vartheta\alpha_{ij}(t)$, $-\chi\beta_{ij}(t)$, and $-\varrho c_{ij}(t)$ that are added into (12)–(14) are essentially motivated by the so-called σ-modification technique in [52]. Using this technique, the estimation error ξ and the tracking error $\hat{x}$ can converge to an arbitrarily small neighborhood of zero by choosing appropriate parameters and the dynamic gains $\alpha_{ij}(t)$, $\beta_{ij}(t)$, and $c_{ij}(t)$ are uniformly ultimately bounded. Furthermore, it should be pointed out that the event-triggered function (16) in the tracking process is simpler than that of in Theorem 1, but the tradeoff is that the trigger frequency will be increased.

IV. DETAT Algorithms for Heterogeneous Linear Multiagent Systems

Note that Section III is based on the homogeneous consideration of the multiagent systems. However, agents with heterogeneous dynamics are very common in practices, which also have a wider range of applications. Therefore, in this section, a new DETAT algorithm for heterogeneous linear multiagent systems is developed.

Consider a heterogeneous multiagent system with N nonidentical agents, described by the following dynamics:

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t)$$
$$y_i(t) = C_i x_i(t), \quad i = 1, 2, \ldots, N \tag{17}$$

where $x_i(t) \in R^{n_i}$, $u_i(t) \in R^{p_i}$, and $y_i(t) \in R^n$ are the state, the control input and output of the ith agent, respectively, and $A_i \in R^{n_i\times n_i}$, $B_i \in R^{n_i\times p_i}$, $C_i \in R^{n\times n_i}$ are constant matrices.

Note that the output dimension of each agent in a heterogeneous system is the same. Therefore, the objective of this section is to design a new DETAT algorithm for heterogeneous systems such that the output $y_i(t)$ of each agent can track the average of multiple time-varying reference signals generated by (1) in the sense that

$$\lim_{t\to\infty}\left\| y_i(t) - \frac{1}{N}\sum_{k=1}^{N} r_k(t)\right\| = 0, i = 1, 2, \ldots, N$$

and reduce communication frequency.

To address the DETAT problem for heterogeneous multiagent systems, some important assumptions and lemmas are listed in the following.

Assumption 4: The states of multiple time-varying reference signals $r_i, i = 1, 2, \ldots, N$, are bounded.

Assumption 5: (A_i, B_i) is controllable and

$$\mathrm{rank}\begin{bmatrix} C_i B_i & 0_{n\times p_i} \\ -A_i B_i & B_i \end{bmatrix} = n_i + n, \quad i = 1, 2, \ldots, N.$$

Lemma 5: [48] Based on Assumption 5, the following linear matrix equations:

$$B_i\gamma_{1i} - \psi_i = 0_{n_i\times n}$$
$$B_i\gamma_{2i} - A_i\psi_i = 0_{n_i\times n}$$
$$C_i\psi_i - I_n = 0_{n\times n}$$

have solution triplets $(\gamma_{1i}, \gamma_{2i}, \psi_i)$, respectively.

Lemma 6: [51] Consider the following cascade system:

$$\dot{y}_1(t) = f_1(t, y_1(t), y_2(t)) \tag{18a}$$
$$\dot{y}_2(t) = f_2(t, y_2(t)) \tag{18b}$$

where $f_1(t, y_1(t), y_2(t)) \in R^{n_1}$, and $f_2(t, y_2(t)) \in R^{n_2}$ are piecewise continuous in t and locally Lipschitz in $y(t) = \binom{y_1(t)}{y_2(t)}$. If system (18a), with $y_2(t)$ as the input is input-to-state stable, and the origin of (18b) are globally uniformly asymptotically stable, then the origin of the cascade system (18a) and (18b) is globally uniformly asymptotically stable.

Let $\upsilon_i \in R^n$ be the auxiliary states of the DETAT algorithms for heterogeneous multiagent systems. To avoid using continuous state information, define $\tilde{y}_i(t) = y_i(t_k^{i*})$ and $\tilde{\upsilon}_i(t) = \upsilon_i(t_k^{i*})$, $\forall t \in [t_k^{i*}, t_{k+1}^{i*})$ as sampling information, and the measurement errors $e_{yi}(t) = \tilde{y}_i(t) - y_i(t)$, and $e_{\upsilon i}(t) = \tilde{\upsilon}_i(t) - \upsilon_i(t)$, $i =$

$1, 2, \ldots, N$, where the triggering instant

$$t_{k+1}^{i*} \triangleq \{t > t_k^{i*} | \Pi_{yi}(t) \geq 0 \cup \Pi_{vi}(t) \geq 0\}$$

where

$$\Pi_{yi}(t) = \|e_{yi}(t)\| - \mu_{yi} e^{-\nu_{yi} t} \quad (19a)$$

$$\Pi_{vi}(t) = \|e_{vi}(t)\| - \mu_{vi} e^{-\nu_{vi} t} \quad (19b)$$

with μ_{yi}, μ_{vi}, ν_{yi}, and ν_{vi} are positive constants. When the triggering condition is satisfied, the agent updates its current state and broadcasts its output information to neighbors, and $e_{yi}(t)$ and $e_{vi}(t)$ will be reset to 0.

Then, based on the event-triggering output state $\tilde{y}_i(t)$ and the proposed triggering functions (19), a class of DETAT algorithms for heterogeneous multiagent systems is proposed as follows:

$$\begin{aligned} u_i &= -K_i^* x_i + \gamma_{1i} \dot{v}_i - (\gamma_{2i} - K_i^* \psi_i) v_i \\ \dot{v}_i &= w_i - \tilde{y}_i + s_i - \alpha \sum_{j \in \mathcal{N}_i} (\tilde{y}_i - \tilde{y}_j) - d_i \\ s_i &= q_i + r_i \\ w_i &= A s_i + B c_1 \sum_{j \in \mathcal{N}_i} [K_1 (\tilde{s}_i - \tilde{s}_j)] \\ &\quad + B c_2 \sum_{j \in \mathcal{N}_i} \operatorname{sgn}[K_1 (\tilde{s}_i - \tilde{s}_j)] + B f_i (r_i, t) \\ \dot{d}_i &= \alpha\beta \sum_{j \in \mathcal{N}_i} (\tilde{y}_i - \tilde{y}_j) \end{aligned} \quad (20)$$

where d_i, v_i, $w_i \in R^n$ are the auxiliary states of agent i. Initialize $\sum_{i=1}^N d_i(0) = 0$. α, β are positive coupling gains. The dynamic of q_i is defined in (5) with $c_1 \geq \frac{1}{\lambda_2(L)}$, $c_2 \geq \frac{2\sqrt{N} f_0}{\lambda_2(L)}$. $K_i^* \in R^{p_i \times n_i}$ is the local feedback gain and γ_{1i}, γ_{2i}, and ψ_i are the solutions of the linear matrix equation in Lemma 5. The design of linear matrix equation in Lemma 5 is partly inspired by the design of regulator equations in the output regulation [53].

Next, it is proven that the output $y_i, i = 1, 2, \ldots, N$, of heterogeneous multiagent systems can track the average of multiple time-varying reference signals. According to Theorem 1, the estimator $s_i, i = 1, 2, \ldots, N$, can obtain the average of multiple time-varying reference signals. Therefore, it is sufficient to prove that the tracking process can be implemented.

First, substituting the controller (20) into the original system (17), the closed-loop system is described by

$$\dot{x} = (A^* - B^* K^*) x + B^* \gamma_1 \dot{v} - (B^* \gamma_2 - B^* K^* \Psi) v \quad (21a)$$

$$\dot{v} = w - \tilde{y} + s - \alpha (L \otimes I_n) \tilde{y} - d \quad (21b)$$

$$w = (I_N \otimes A) s + c_1 (L \otimes B K_1) \tilde{s} \quad (21c)$$

$$+ c_2 (I_N \otimes B) H(\tilde{s}) + (I_N \otimes B) F(t) \quad (21d)$$

$$\dot{d} = \alpha\beta (L \otimes I_n) \tilde{y} \quad (21e)$$

where $x = \operatorname{col}(x_1, \ldots, x_N)$, $v = \operatorname{col}(v_1, \ldots, v_N)$, $s = \operatorname{col}(s_1, \ldots, s_N)$, $w = \operatorname{col}(w_1, \ldots, w_N)$, $\tilde{y} = \operatorname{col}(\tilde{y}_1, \ldots, \tilde{y}_N)$, $A^* = \operatorname{diag}(A_1, \ldots, A_N)$, $B^* = \operatorname{diag}(B_1, \ldots, B_N)$, $C^* = \operatorname{diag}(C_1, \ldots, C_N)$, $K^* = \operatorname{diag}(K_1^*, \ldots, K_N^*)$, $\gamma_1 = \operatorname{diag}(\gamma_{11}, \ldots, \gamma_{1N})$, $\gamma_2 = \operatorname{diag}(\gamma_{21}, \ldots, \gamma_{2N})$, $\Psi = \operatorname{diag}(\psi_1, \ldots, \psi_N)$, and $d = \operatorname{col}(d_1, \ldots, d_N)$.

Then, the following closed-loop system can be obtained by variable substitutions:

$$\dot{x} = (A^* - B^* K^*) x + B^* \gamma_1 \dot{v} - (B^* \gamma_2 - B^* K^* \Psi) v \quad (22a)$$

$$\dot{v} = w - \tilde{v} + s - \alpha (L \otimes I_n) \tilde{v} - d - (\tilde{y} - \tilde{v}) - \alpha (L \otimes I_n)(\tilde{y} - \tilde{v}) \quad (22b)$$

$$\dot{d} = \alpha\beta (L \otimes I_n) \tilde{v} + \alpha\beta (L \otimes I_n)(\tilde{y} - \tilde{v}) \quad (22c)$$

where $\tilde{v}$ is an intermediate variable triggered at the same time when y is triggered.

Theorem 3: Suppose that Assumptions 1–4 hold. Let K_i^*, $i = 1, 2, \ldots, N$, be such that $A_i - B_i K_i^*$ is Hurwitz and $(\gamma_{1i}, \gamma_{2i}, \psi_i)$ be defined as in Lemma 5. Initialize $\sum_{i=1}^N d_i(0) = 0$. Choose the parameters $\alpha > 0$, $\beta > 0$. Then the DETAT problem of heterogeneous multiagent systems is solved. Furthermore, the Zeno behavior is excluded.

Proof: See Appendix C.

Remark 5: It should be mentioned that the design of DETAT algorithms for heterogeneous multiagent systems is a new problem that has never been investigated as far as is known. Compared with the existing results in [20]–[32], where the DAT algorithms are established for homogeneous multiagent systems, agents with heterogeneous dynamics in this article are closer to reality.

Remark 6: Note that the proposed DETAT algorithm (20) with signal estimator q_i defined in (5) is also static-gain algorithm. Thus, to eliminate the limitation of global information, one can also replace (5) with adaptive signal estimator (10) with self-adaptive laws defined in (12) and (13).

V. SIMULATIONS

In this section, two examples are shown to illustrate, respectively, Theorems 2 and 3.

Use the first case to illustrate Theorem 2. Consider a linear multiagent system with one hundred time-varying signals, each of which is a linearized model of the longitudinal dynamics of an aircraft [54], [55], described by (1), with $r_i = [r_{i1}, r_{i2}, r_{i3}]^T, i = 1, 2, \ldots, 100$

$$A = \begin{pmatrix} -0.277 & 1 & -0.0002 \\ -17.1 & -0.178 & -12.2 \\ 0 & 0 & -6.67 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 0 \\ 6.67 \end{pmatrix}.$$

The input $f_i(r_i, t)$ of the ith reference signal satisfies Assumption 3, such as $f_1(r_1, t) = 0.5\sin(t)$, $f_2(r_2, t) = e^{-2t}$, $f_3(r_3, t) = -2.5\cos(2t)$, $f_4(r_4, t) = 3\sin(2t)$, and so on. All the initial values of signals $r_i(0)$ and agents $x_i(0)$ are randomly chosen.

To verify the proposed theoretical conclusions, we simulate a Watts and Strogatz small world network with $N = 100$ and average degree $K = 1$, random reconnect probability $\beta = 0$.

By solving AREs in (9), one obtains matrix $P_m, m = 1, 2$

$$P_m = \begin{pmatrix} 2.4142 & 2.4142 & 1.0000 \\ 2.4142 & 4.8284 & 2.4142 \\ 1.0000 & 2.4142 & 2.4142 \end{pmatrix}.$$

Then, one has $K_m = (-1.0000 \ -2.4142 \ -2.4142)$. By using continuous dynamic-gain DETAT algorithms (10)–(16) with parameters chosen as $\gamma = 6, \vartheta = 0.01, \eta = 6, \chi = 0.02, \kappa = 6, \varrho = 0.01, \mu_i = 5, \nu_i = 0.001, \hat{\mu}_i = 1, \hat{\nu}_i = 0.01, \delta = 0.5, \varepsilon = 0.5, \varphi = 0.5$, simulation results are shown in Figs. 1–3. Fig. 1 depicts estimation errors $s_i - \bar{r}, i = 1, \ldots, 100$, which illustrate all estimators can approximately estimate the average value of multiple time-varying reference signals with bounded errors.

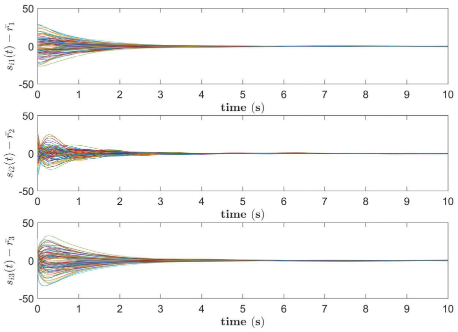

Fig. 1. Estimation errors $s_i - \bar{r}$, $i = 1, \ldots, 100$.

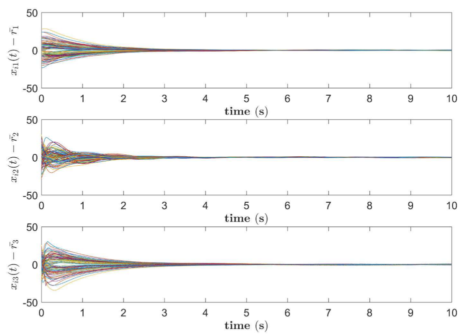

Fig. 2. Tracking errors $x_i - \bar{r}$, $i = 1, \ldots, 100$.

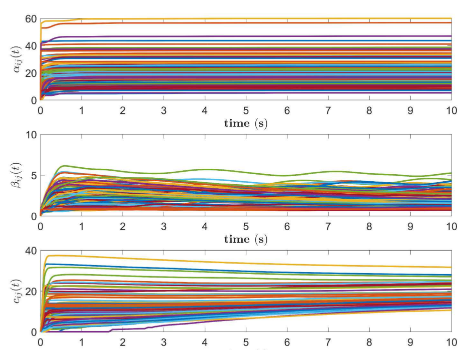

Fig. 3. Dynamic-gain parameters $\alpha_{ij}(t)$, $\beta_{ij}(t)$, and $c_{ij}(t)$.

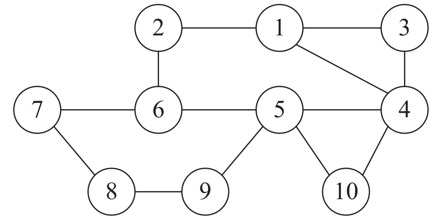

Fig. 4. Communication topology for heterogenous multiagent systems.

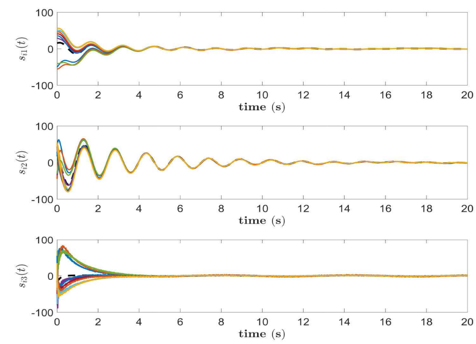

Fig. 5. Estimation process of DETAT algorithms for heterogeneous multiagent systems. Solid lines are state trajectories s_i of ten estimators. The dash line is the average of multiple time-varying reference signals $\bar{r}$.

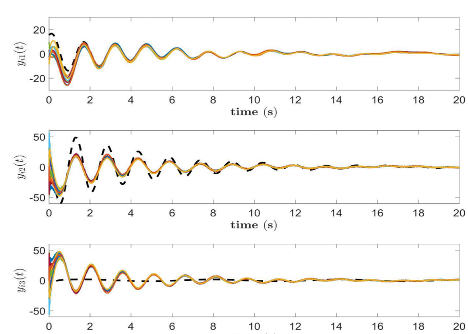

Fig. 6. Tracking process of DETAT algorithms for heterogeneous multiagent systems. Solid lines are output trajectories y_i of ten agents. The dash line is the average of multiple time-varying reference signals $\bar{r}$.

Tracking errors $x_i - \bar{r}, i = 1, \ldots, 100$, are shown in Fig. 2. It can be concluded that the objective of DETAT problem is realized. The dynamic-gain parameters $\alpha_{ij}(t)$, $\beta_{ij}(t)$, and $c_{ij}(t)$ are depicted in Fig. 3, from which it can be seen that $\alpha_{ij}(t)$, $\beta_{ij}(t)$, and $c_{ij}(t)$ are all bounded. The introduction of adaptive gains eliminates the limitation of global information.

The second case is used to illustrate Theorem 3. Consider a heterogeneous multiagent systems with ten heterogeneous agents and ten reference signals. The reference signals are the same as in the first case. In this simulation, the communication graph among the agents is shown in Fig. 4, and the system matrices are described by: $A_i = [0, 1; 0, 0]$, $B_i = [0, 1; 1, -2]$, $C_i = [1, 1; 1, -1; -1, 1], i = 1, 3, 5, 7, 9$, and $A_j = [0, -1; 1, -2]$, $B_j = [1, 0; 3, -1]$, $C_j = [1, 1; 1, -1; -1, 1], i = 2, 4, 6, 8, 10$. The parameters of the control law (20) and the triggering function (19) are chosen as $\gamma_{1i} = [1.5, 0.25, -0.25; 0.5, 0.25, -0.25]$, $\gamma_{2i} = [-0.0183, -0.4563, 0.4563; 1.3227, -0.4788, 0.4788]$, $\psi_i = [0.5, 0.25, -0.25; 0.5, -0.25, 0.25]$, $K_i = [0.9310, 1.1057; -0.3651, -1.2804], i = 1, 3, 5, 7, 9$, and $\gamma_{1j} = [0.5, 0.25, -0.25; 1.0, 1.0, -1.0]$, $\gamma_{2j} = [-0.8204, 0.3901, -0.3901; -1.0378, -0.1879, 0.1879], \psi_j = [0.5, 0.25, -0.25; 0.5, -0.25, 0.25]$, $K_j = [0.0402, 0.6006; 0.4136, -0.3381], j = 2, 4, 6, 8, 10. \alpha = 1, \beta = 0.5, \mu_{yi} = 3, \nu_{yi} = 0.1$. The initial values $x_i(0)$ and $\upsilon_i(0)$ are randomly chosen and $\sum_{i=1}^{N} d_i(0) = 0$.

Trajectories of estimators and the average of multiple time-varying reference signals are shown in Fig. 5, where states of all estimators converge to the average of multiple time-varying reference signals. The tracking process is given in Fig. 6, which shows that outputs of all agents can track the average of multiple time-varying reference signals. Thus, the DETAT problem

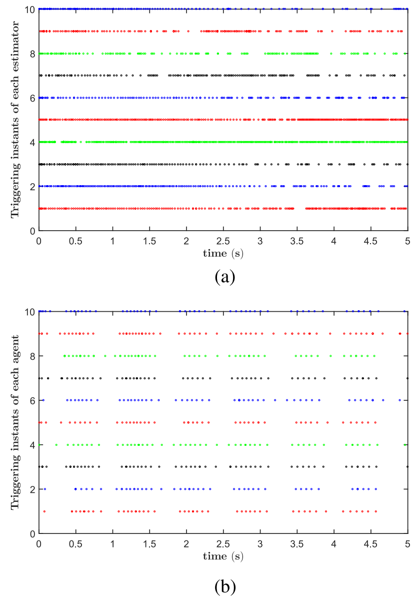

Fig. 7. Triggering instants of DETAT algorithms for heterogeneous multiagent systems. (a) Triggering instants of estimators. (b) Triggering instants of agents.

for heterogeneous multiagent systems is realized. Triggering instants of the first 5 s of estimation and tracking processes are shown in Fig. 7, respectively, where the communication is discrete and the communication frequency is reduced.

VI. Conclusion

In this article, DETAT problems have been studied for homogeneous and heterogeneous multiagent systems. For homogeneous multiagent systems, a couple of DETAT algorithms with static and dynamic gains are designed by using the model-based local sampled state information, respectively, which ensure each agent to track the average of multiple time-varying reference signals. Meanwhile, Zeno behaviors are excluded during estimation and tracking processes. Further, for heterogeneous multiagent systems, a new DETAT algorithm has been developed that makes the output of each agent can track the average of multiple time-varying reference signals. Compared to the previous related works, this article introduces the event-triggering mechanism into the design of DAT algorithms and the agent dynamics is extended to heterogeneous cases. The future works will extend DETAT algorithms to the situation of agent dynamics and reference signals satisfying more general nonlinearities such as Lipschitz-type conditions and communication topologies under directed networks.

Appendix

A. Proof of Theorem 1

The following proof contains two steps. First, it can be proven that for the ith estimator

$$\lim_{t\to\infty}\left\|s_i(t)-\frac{1}{N}\sum_{k=1}^{N}r_k(t)\right\|=0, i=1,2,\ldots,N.$$

Let $\hat{x}(t)=[\hat{x}_1^T(t),\ldots,\hat{x}_N^T(t)]^T$,$\tilde{x}(t)=[\tilde{x}_1^T(t),\ldots,\tilde{x}_N^T(t)]^T$, $s(t)=[s_1^T(t),\ldots,s_N^T(t)]^T$, $F(t)=[f_1^T(t),\ldots,f_N^T(t)]^T$, $\xi(t)=[\xi_1^T(t),\ldots,\xi_N^T(t)]^T$, where $\xi_i(t)\triangleq s_i(t)-\frac{1}{N}\sum_{j=1}^{N}s_j(t)$.

Consider the following Lyapunov function candidate:

$$V_1=\frac{1}{2}\xi^T\left(I_N\otimes P_1\right)\xi. \tag{23}$$

It then follows from Lemma 1 that

$$V_1\geq\frac{1}{2}\lambda_{\min}(P_1)\|\xi\|^2 \tag{24}$$

where $\lambda_{\min}(P_1)$ is the smallest eigenvalue of the positive matrix P_1. It follows from (1), (5), and Lemma 3 that the closed-loop system can be described by

$$\begin{aligned}\dot{\xi}(t)=&\left(I_N\otimes A\right)\xi(t)+c_1(L\otimes BK_1)\tilde{s}(t)\\&+c_2\left(I_N\otimes B\right)H(\tilde{s})+\left(M\otimes B\right)F(t)\end{aligned} \tag{25}$$

where

$$H(\tilde{s})=\begin{bmatrix}\sum\limits_{j\in\mathcal{N}_i}\operatorname{sgn}\left[K_1\left(\tilde{s}_1(t)-\tilde{s}_j(t)\right)\right]\\\vdots\\\sum\limits_{j\in\mathcal{N}_N}\operatorname{sgn}\left[K_1\left(\tilde{s}_N(t)-\tilde{s}_j(t)\right)\right]\end{bmatrix}.$$

The time derivative of V_1 along system (25) can be obtained as follows:

$$\begin{aligned}\dot{V}_1=&\,\xi^T\left(I_N\otimes P_1A\right)\xi+c_1\xi^T(L\otimes P_1BK_1)\tilde{s}\\&+\xi^T\left(M\otimes P_1B\right)F(t)+c_2\xi^T\left(I_N\otimes P_1B\right)H(\tilde{s}).\end{aligned} \tag{26}$$

Because $K_1=-B^TP_1$, one has

$$\begin{aligned}&\xi^T(L\otimes P_1BK_1)\tilde{s}=-\frac{1}{2}\xi^T\left(L\otimes P_1BB^TP_1\right)\xi\\&+\frac{1}{2}e^T\left(L\otimes P_1BB^TP_1\right)e-\frac{1}{2}\tilde{s}^T\left(L\otimes P_1BB^TP_1\right)\tilde{s}.\end{aligned} \tag{27}$$

By noting $a_{ij}=a_{ji}$ and using Lemma 2, one has

$$\begin{aligned}&e^T\left(L\otimes P_1BB^TP_1\right)e\\&=\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}e_i^TP_1BB^TP_1\left(e_i-e_j\right)\\&\leq\frac{3}{2}\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}e_i^TP_1BB^TP_1e_i+\frac{1}{2}\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}e_j^TP_1BB^TP_1e_j\\&\leq2\sum_{i=1}^{N}d_i\left\|B^TP_1\right\|^2\|e_i\|^2\end{aligned} \tag{28}$$

and

$$\begin{aligned}&\tilde{s}^T\left(L\otimes P_1BB^TP_1\right)\tilde{s}\\&=\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}\tilde{s}_i^TP_1BB^TP_1\left(\tilde{s}_i-\tilde{s}_j\right)\\&=\frac{1}{2}\sum_{i=1}^{N}d_i\left\|B^TP_1\left(\tilde{s}_i-\tilde{s}_j\right)\right\|^2.\end{aligned} \tag{29}$$

Under Assumption 3, one has $\|f_i(r_i,t)\| \leq f_0, i=1,2,\ldots,N$, which further implies $\|F(t)\| \leq \sqrt{N} f_0$. It then follows that:

$$\begin{aligned}
&\xi^T (M \otimes P_1 B) F(t)\\
&\leq \left\|\xi^T (M\otimes P_1B)\right\| \cdot \|F(t)\|\\
&\leq \frac{\sqrt{N}f_0}{\lambda_2(L)} \left\|\left(L\otimes B^TP_1\right)\xi\right\|\\
&\leq \frac{\sqrt{N}f_0}{\lambda_2(L)} \sum_{i=1}^N \sum_{j\in\mathcal{N}_i} \left\|B^TP_1(\xi_i-\xi_j)\right\|\\
&\leq \frac{\sqrt{N}f_0}{\lambda_2(L)} \sum_{i=1}^N \sum_{j\in\mathcal{N}_i} \left\|B^TP_1(\tilde{s}_i-\tilde{s}_j)\right\|\\
&\quad + \frac{2\sqrt{N}f_0}{\lambda_2(L)} \sum_{i=1}^N \sum_{j\in\mathcal{N}_i} \left\|B^TP_1\right\| \|e_i\|. \qquad (30)
\end{aligned}$$

Noting the facts that $\xi_i-\xi_j = s_i - s_j, e_i = \tilde{s}_i - s_i$, one gets

$$\begin{aligned}
&\xi^T (I_N\otimes P_1B) H(\tilde{s})\\
&= -\sum_{i=1}^N \sum_{j\in\mathcal{N}_i} \frac{\xi_i^T P_1BB^TP_1(\tilde{s}_i-\tilde{s}_j)}{\left\|-B^TP_1(\tilde{s}_i-\tilde{s}_j)\right\|}\\
&= -\frac{1}{2}\sum_{i=1}^N \sum_{j\in\mathcal{N}_i} \frac{(\xi_i-\xi_j)^T P_1BB^TP_1(\tilde{s}_i-\tilde{s}_j)}{\left\|-B^TP_1(\tilde{s}_i-\tilde{s}_j)\right\|}\\
&= -\frac{1}{2}\sum_{i=1}^N \sum_{j\in\mathcal{N}_i} \frac{(\tilde{s}_i-\tilde{s}_j)^T P_1BB^TP_1(\tilde{s}_i-\tilde{s}_j)}{\left\|-B^TP_1(\tilde{s}_i-\tilde{s}_j)\right\|}\\
&\quad + \frac{1}{2}\sum_{i=1}^N \sum_{j\in\mathcal{N}_i} \frac{(e_i-e_j)^T P_1BB^TP_1(\tilde{s}_i-\tilde{s}_j)}{\left\|-B^TP_1(\tilde{s}_i-\tilde{s}_j)\right\|}\\
&\leq -\frac{1}{2}\sum_{i=1}^N \sum_{j\in\mathcal{N}_i} \left\|B^TP_1(\tilde{s}_i-\tilde{s}_j)\right\| + \sum_{i=1}^N d_i \left\|B^TP_1\right\|\|e_i\|. \qquad (31)
\end{aligned}$$

Then, substituting (27)–(31) into (26), one obtains that

$$\begin{aligned}
\dot{V}_1 &\leq \frac{1}{2}\xi^T\left[I_N\otimes\left(P_1A+A^TP_1\right) - c_1L\otimes P_1BB^TP_1\right]\xi\\
&\quad + c_1\sum_{i=1}^N d_i\left\|B^TP_1\right\|^2\|e_i\|^2 - \frac{c_1}{4}\sum_{i=1}^N d_i \left\|B^TP_1(\tilde{s}_i-\tilde{s}_j)\right\|^2\\
&\quad + \frac{\sqrt{N}f_0}{\lambda_2(L)} \sum_{i=1}^N \sum_{j\in\mathcal{N}_i} \left\|B^TP_1(\tilde{s}_i-\tilde{s}_j)\right\|\\
&\quad + \frac{2\sqrt{N}f_0}{\lambda_2(L)} \sum_{i=1}^N \sum_{j\in\mathcal{N}_i} \left\|B^TP_1\right\|\|e_i\|\\
&\quad - \frac{1}{2}c_2\sum_{i=1}^N \sum_{j\in\mathcal{N}_i} \left\|B^TP_1(\tilde{s}_i-\tilde{s}_j)\right\| + c_2\sum_{i=1}^N d_i\left\|B^TP_1\right\|\|e_i\|. \qquad (32)
\end{aligned}$$

Utilizing $c_2 \geq \frac{2\sqrt{N}f_0}{\lambda_2(L)}$ follows

$$\begin{aligned}
\dot{V}_1 &\leq \frac{1}{2}\xi^T\left[I_N\otimes\left(P_1A+A^TP_1\right) - c_1L\otimes P_1BB^TP_1\right]\xi\\
&\quad + c_1\sum_{i=1}^N d_i\left\|B^TP_1\right\|^2\|e_i\|^2 - \frac{c_1}{4}\sum_{i=1}^N d_i \left\|B^TP_1(\tilde{s}_i-\tilde{s}_j)\right\|^2\\
&\quad + 2c_2\sum_{i=1}^N d_i\left\|B^TP_1\right\|\|e_i\|\\
&\leq \frac{1}{2}\xi^T\left[I_N\otimes\left(P_1A+A^TP_1\right) - c_1L\otimes P_1BB^TP_1\right]\xi\\
&\quad + \sum_{i=1}^N \mu_i e^{-\nu_i t} \qquad (33)
\end{aligned}$$

where one can use the triggering condition (4) to obtain the last inequality. According to Lemma 1 and Assumption 1, one has that $\xi^T(L\otimes P_1BB^TP_1)\xi \geq \lambda_2(L)\xi^T(I_N\otimes P_1BB^TP_1)\xi$. By using $c_1 \geq \frac{1}{\lambda_2(L)}$ and substituting (23) into (33), one has

$$\dot{V}_1 \leq -\theta_1 V_1 + \sum_{i=1}^N \mu_i e^{-\nu_i t} \qquad (34)$$

where $\theta_1 = \frac{1}{\lambda_{\max}(P_1)}$. By using the famous comparison lemma in [51], one obtains that

$$V_1(t) \leq V_1(0)e^{-\theta_1 t} + \sum_{i=1}^N \mu_i\Omega_i(t,\theta_1)$$

where $\Omega_i(t,\theta_i)$ is determined by

$$\Omega_i(t,\theta_i) = \begin{cases} te^{-\theta_i t} & \theta_i=\sigma_i\\ \frac{1}{\theta_i-\sigma_i}\left(e^{-\sigma_i t}-e^{-\theta_i t}\right) & \theta_i\neq\sigma_i\end{cases} \qquad (35)$$

since $\lim_{t\to\infty}\Omega_i(t,\theta_i)=0$. In light of $V_1 \geq \frac{1}{2}\lambda_{\min}(P_1)\|\xi(t)\|^2$, $\xi(t)$ exponentially converges to 0. According to Lemma 3, $\xi_i(t), i=1,2,\ldots,N$, satisfy

$$\lim_{t\to\infty}\xi_i(t) = \lim_{t\to\infty}\left\|s_i(t)-\frac{1}{N}\sum_{k=1}^N s_k(t)\right\| = 0.$$

It follows from (5) and $\sum_{i=1}^N q_i(0)=0$ that $\frac{1}{N}\sum_{i=1}^N q_i(t)=0$. By noting that $s_i(t)=q_i(t)+r_i(t), i=1,2,\ldots,N$, one has $\frac{1}{N}\sum_{i=1}^N s_i(t) = \frac{1}{N}\sum_{i=1}^N r_i(t), i=1,2,\ldots,N$. Therefore, $\lim_{t\to\infty}\|s_i(t)-\frac{1}{N}\sum_{k=1}^N r_k(t)\| = \lim_{t\to\infty}\|s_i(t)-\frac{1}{N}\sum_{k=1}^N s_k(t)\|=0, i=1,2,\ldots,N$, which shows that estimators (5) can obtain the average.

Second, it is proven that

$$\lim_{t\to\infty}\left\|x_i(t)-\frac{1}{N}\sum_{k=1}^N r_k(t)\right\| = 0, i=1,2,\ldots,N.$$

Consider the following Lyapunov candidate:

$$V_2 = \frac{1}{2}\hat{x}^T(I_N\otimes P_2)\hat{x} \qquad (36)$$

with $P_2>0$. By using the controller (8) for (2), one has

$$\begin{aligned}
\dot{\hat{x}}(t) &= (I_N\otimes A)\hat{x}(t) + (I_N\otimes BK_2)\tilde{x}(t)\\
&\quad + c_3(I_N\otimes B)\,\mathrm{SGN}\left[(I_N\otimes K_2)\tilde{x}(t)\right] - (I_N\otimes B)F(t) \qquad (37)
\end{aligned}$$

where

$$\mathrm{SGN}\left[(I_N \otimes K_2)\tilde{x}(t)\right] = \begin{pmatrix} \mathrm{sgn}\left(K_2\tilde{x}_1(t)\right) \\ \vdots \\ \mathrm{sgn}\left(K_2\tilde{x}_N(t)\right) \end{pmatrix}.$$

By taking the time derivative of V_2 along (37), one gets

$$\begin{aligned}\dot{V}_2 &= \hat{x}^T\left(I_N \otimes P_2 A\right)\hat{x} + \hat{x}^T\left(I_N \otimes P_2 B K_2\right)\tilde{x} \\ &\quad + c_3\hat{x}^T\left(I_N \otimes P_2 B\right)\mathrm{SGN}\left[\left(I_N \otimes K_2\right)\tilde{x}\right] \\ &\quad - \hat{x}^T\left(I_N \otimes P_2 B\right)F(t). \end{aligned} \tag{38}$$

By choosing $K_2 = -B^T P_2$, one has

$$\begin{aligned}\hat{x}^T\left(I_N \otimes P_2 B K_2\right)\tilde{x} &= -\frac{1}{2}\hat{x}^T\left(I_N \otimes P_2 B B^T P_2\right)\hat{x} \\ &+ \frac{1}{2}\hat{e}^T\left(I_N \otimes P_2 B B^T P_2\right)\hat{e} - \frac{1}{2}\tilde{x}^T\left(I_N \otimes P_2 B B^T P_2\right)\tilde{x}\end{aligned}$$

and

$$\begin{aligned}&- c_3\hat{x}^T\left(I_N \otimes P_2 B\right)\mathrm{SGN}\left[\left(I_N \otimes B^T P_2\right)\tilde{x}\right] \\ &= -c_3\sum_{i=1}^{N}\left(B^T P_2\hat{x}_i\right)^T\mathrm{sgn}\left(B^T P_2\tilde{x}_i\right) \\ &= -c_3\sum_{i=1}^{N}\frac{\hat{x}_i^T P_2 B B^T P_2\tilde{x}_i}{\|B^T P_2\tilde{x}_i\|} \\ &= -c_3\sum_{i=1}^{N}\frac{\left(\tilde{x}_i - \hat{e}_i\right)^T P_2 B B^T P_2\tilde{x}_i}{\|B^T P_2\tilde{x}_i\|} \\ &= -c_3\sum_{i=1}^{N}\frac{\tilde{x}_i^T P_2 B B^T P_2\tilde{x}_i}{\|B^T P_2\tilde{x}_i\|} + c_3\sum_{i=1}^{N}\frac{\hat{e}_i^T P_2 B B^T P_2\tilde{x}_i}{\|B^T P_2\tilde{x}_i\|} \\ &\leq -c_3\sum_{i=1}^{N}\left\|B^T P_2\tilde{x}_i\right\| + c_3\sum_{i=1}^{N}\left\|B^T P_2\right\|\|\hat{e}_i\|.\end{aligned}$$

It follows from $\|f_i(r_i,t)\| \leq f_0, i = 1, 2, \ldots, N$, that

$$\begin{aligned}&- \hat{x}^T\left(I_N \otimes P_2 B\right)F(t) \\ &= -\sum_{i=1}^{N}\left(B^T P_2\tilde{x}_i\right)^T f_i(t) + \sum_{i=1}^{N}\left(B^T P_2\hat{e}_i\right)^T f_i(t) \\ &\leq \sum_{i=1}^{N}\left\|B^T P_2\tilde{x}_i\right\| f_0 + \sum_{i=1}^{N}\left\|B^T P_2\hat{e}_i\right\| f_0.\end{aligned}$$

Then substitute the above inequality into (38) and use $c_3 \geq f_0$, one has

$$\begin{aligned}\dot{V}_2 &\leq \frac{1}{2}\hat{x}^T\left[I_N \otimes \left(A^T P_2 + P_2 A - P_2 B B^T P_2\right)\right]\hat{x} \\ &\quad + \frac{1}{2}\sum_{i=1}^{N}\left\|B^T P_2\right\|^2\|\hat{e}_i\|^2 - \frac{1}{2}\sum_{i=1}^{N}\left\|B^T P_2\right\|^2\|\tilde{x}_i\|^2 \\ &\quad + 2c_3\sum_{i=1}^{N}\left\|B^T P_2\right\|\|\hat{e}_i\|.\end{aligned} \tag{39}$$

Using (7) and (9), one has

$$\dot{V}_2 \leq -\theta_2 V_2 + \sum_{i=1}^{N}\hat{\mu}_i e^{-\hat{\nu}_i t} \tag{40}$$

where $\theta_2 = \frac{1}{\lambda_{\max}(P_2)}$. Similar to the first step, one obtains $\dot{V}_2 \leq 0$ and $V_2 \geq \frac{1}{2}\lambda_{\min}(P_2)\|\ \hat{x}_i\|^2$. Therefore, the tracking error $\hat{x}_i(t) = x_i(t) - s_i(t)$ exponentially converges to 0.

$$\begin{aligned}&\lim_{t\to\infty}\left\|x_i(t) - \frac{1}{N}\sum_{k=1}^{N} r_k(t)\right\| \\ &= \lim_{t\to\infty}\|x_i(t) - s_i(t)\| + \lim_{t\to\infty}\left\|s_i(t) - \frac{1}{N}\sum_{k=1}^{N} r_k(t)\right\| \\ &= 0.\end{aligned}$$

Thus, the agent can track the average of multiple time-varying reference signals.

In the following, it is shown that Zeno behaviors do not exist in closed-loop systems. The proof is divided into two parts.

First, it can be proven that there is no Zeno behavior in the estimation process. For estimator $s_i, \forall t \in [t_k^i, t_{k+1}^i), t_{k+1}^i < \infty$, it follows from $e_i(t) \triangleq \tilde{s}_i(t) - s_i(t)$ and (5) that the derivative of e_i(t) can be written as

$$\begin{aligned}\dot{e}_i(t) &= A e_i(t) - c_1 B\sum_{j\in\mathcal{N}_i}\left[K_1\left(\tilde{s}_i(t) - \tilde{s}_j(t)\right)\right] \\ &\quad - c_2 B\sum_{j\in\mathcal{N}_i}\mathrm{sgn}\left[K_1\left(\tilde{s}_i(t) - \tilde{s}_j(t)\right)\right].\end{aligned} \tag{41}$$

Consider the norms of (41)

$$\begin{aligned}\|\dot{e}_i(t)\| &\leq \|A\|\|e_i(t)\| + \|c_1 B\sum_{j\in\mathcal{N}_i}\left[K_1\left(\tilde{s}_i(t) - \tilde{s}_j(t)\right)\right]\| \\ &\quad + \|c_2 B\sum_{j\in\mathcal{N}_i}\mathrm{sgn}\left[K_1\left(\tilde{s}_i(t) - \tilde{s}_j(t)\right)\right]\|.\end{aligned} \tag{42}$$

Note that $\lim_{t\to\infty}(\|c_1 B\sum_{j\in\mathcal{N}_i}[K_1(\tilde{s}_i(t) - \tilde{s}_j(t))]\| + \|c_2 B\sum_{j\in\mathcal{N}_i}\mathrm{sgn}[K_1(\tilde{s}_i(t) - \tilde{s}_j(t))]\|) = 0$. Thus, $\|e_i(t)\| \leq e^{\|A\|t}$ when t is large enough, $\forall t \in [t_k^i, t_{k+1}^i)$. It is not difficult to obtain that the triggering function (4) satisfies $T_i(t) \leq 0$ if the following condition satisfies:

$$\|e_i(t)\| \leq \frac{\sqrt{c_1^2 d_i^2\left\|B^T P_1\left(\tilde{s}_i - \tilde{s}_j\right)\right\|^2 + 4c_1 d_i\mu_i e^{-\nu_i t}}}{2c_1 d_i\left\|B^T P_1\right\|}$$

which further yields

$$\|e_i(t)\|^2 \leq \frac{c_1 d_i\left\|B^T P_1\left(\tilde{s}_i - \tilde{s}_j\right)\right\|^2 + 4\mu_i e^{-\nu_i t}}{2c_1 d_i\left\|B^T P_1\right\|^2}.$$

Therefore, a lower bounded T_1 of $t_{k+1}^i - t_k^i$ can be obtained by solving the following inequality:

$$(e^{\|A\|T_1})^2 \geq \frac{c_1 d_i\left\|B^T P_1\left(\tilde{s}_i - \tilde{s}_j\right)\right\|^2 + 4\mu_i e^{-\nu_i T_1}}{2c_1 d_i\left\|B^T P_1\right\|^2}.$$

Further

$$\begin{aligned}&t_{k+1}^i - t_k^i \geq T_1 \\ &\geq \frac{1}{\|A\|}\ln\left(\frac{\sqrt{c_1^2 d_i^2\left\|B^T P_1\left(\tilde{s}_i - \tilde{s}_j\right)\right\|^2 + 4c_1 d_i\mu_i e^{-\nu_i T_1}}}{2c_1 d_i\left\|B^T P_1\right\|}\right).\end{aligned} \tag{43}$$

Therefore, the Zeno behavior does not exist in the estimation process.

Second, it is shown that there is no Zeno behavior in the tracking process. For agent i, consider the evolution of $\hat{e}_i(t)$, $\forall t \in [\tau_k^i, \tau_{k+1}^i)$, $\tau_{k+1}^i < \infty$. Combining $\hat{e}_i(t) \triangleq \tilde{x}_i(t) - \hat{x}_i(t)$ and (37) yields that

$$\dot{\hat{e}}_i(t) = A\hat{e}_i(t) - BK_2\tilde{x}_i(t) - c_3 B\text{sgn}\left[K_2\tilde{x}_i(t)\right] + Bf_i(t).$$

Take the norms of both sides

$$\left\|\dot{\hat{e}}_i(t)\right\| \le \|A\| \, \|\hat{e}_i(t)\| + \|BK_2\tilde{x}_i(t)\| + \|c_3 B\text{sgn}\left[K_2\tilde{x}_i(t)\right]\| + \|Bf_i(t)\|. \quad (44)$$

The previous proof of Lyapunov stability shows that $\hat{x}_i(t)$ is bounded. Note that the interval between two consecutive triggering events is bounded. Therefore, $e^{A(t-\tau_k^i)}$ is bounded for any $t \in [\tau_k^i, \tau_{k+1}^i)$. Then $\forall t \in [\tau_k^i, \tau_{k+1}^i)$, $\tilde{x}_i(t) = e^{A(t-\tau_k^i)}\hat{x}_i(\tau_k^i)$ is also bounded. Since $\|f_i(t)\| \le f_0, i = 1, 2, \ldots, N$, it follows from (44) that

$$\left\|\dot{\hat{e}}_i(t)\right\| \le \|A\| \, \|\hat{e}_i(t)\| + \varpi_i \quad (45)$$

where ϖ_i denotes the upper bound of $\|BK_2\tilde{x}_i(t)\| + \|c_3 B\text{sgn}[K_2\tilde{x}_i(t)]\| + \|Bf_i(t)\|$, $\forall t \in [\tau_k^i, \tau_{k+1}^i)$. Consider a function $\ell : [0, \infty) \to R_{\ge 0}$, satisfying

$$\dot{\ell} = \|A\| \, \ell + \varpi_i, \quad \ell(0) = \left\|\hat{e}_i\left(\tau_k^i\right)\right\| = 0. \quad (46)$$

Then, $\|\hat{e}_i(t)\| \le \ell(t - \tau_k^i)$, where $\ell(t)$ is the analytical solution to (46), given by $\ell(t) = \frac{\varpi_i}{\|A\|}(e^{\|A\|t} - 1)$. Thus, the triggering function (7) satisfies $\Pi_i(t) \le 0$, if one has the following condition $\|\hat{e}_i(t)\| \le \frac{-2c_3 + \sqrt{4c_3{}^2 + 2\hat{\mu}_i e^{-\hat{\nu}_i t}}}{\|B^T P_2\|} \le \frac{\sqrt{2\hat{\mu}_i}}{\|B^T P_2\|} e^{\frac{-\hat{\nu}_i}{2} t}$, which further yields:

$$\|\hat{e}_i(t)\|^2 \le \frac{2\hat{\mu}_i}{\|B^T P_2\|^2} e^{-\hat{\nu}_i t}. \quad (47)$$

Then, the interval between two triggering instants τ_k^i and τ_{k+1}^i for agent i can be lower bounded by the time for $\ell^2(t - \tau_k^i)$ evolving from 0 to the right-hand side of (47). Thus, a lower bounded T_2 of $\tau_{k+1}^i - \tau_k^i$ can be obtained by solving the following inequality:

$$\frac{\varpi_i{}^2}{\|A\|^2}\left(e^{\|A\|T_2} - 1\right)^2 \ge \frac{2\hat{\mu}_i}{\|B^T P_2\|^2} e^{-\hat{\nu}_i\left(\tau_k^i + T_2\right)}.$$

Further, one has

$$\tau_{k+1}^i - \tau_k^i \ge T_2 \ge \frac{1}{\|A\|} \ln\left(1 + \frac{\|A\|\sqrt{2\hat{\mu}_i e^{-\hat{\nu}_i\left(\tau_k^i + T_2\right)}}}{\varpi_i \|B^T P_2\|}\right). \quad (48)$$

Therefore, the interval between two consecutive triggering instants is strictly positive in a finite time and the Zeno behavior does not exist in the tracking process.

B. Proof of Theorem 2

Similar to the proof of Appendix A, the proof process is divided into two parts.

First, for the estimation process, consider the Lyapunov candidate

$$V_3 = \frac{1}{2}\xi^T(I_N \otimes P_1)\xi + \sum_{i=1}^N \sum_{j\in\mathcal{N}_i}\left(\frac{\tilde{\alpha}_{ij}^2(t)}{8\gamma} + \frac{\tilde{\beta}_{ij}^2(t)}{4\eta}\right) \quad (49)$$

where $\tilde{\alpha}_{ij} = \alpha_{ij} - \overline{\alpha}, \tilde{\beta}_{ij} = \beta_{ij} - \overline{\beta}, \overline{\alpha}$ and $\overline{\beta}$ are two constants, satisfying $\overline{\beta} \ge \frac{2\sqrt{N}f_0}{\lambda_2(L)}, \overline{\alpha} \ge \frac{4}{\lambda_2(L)}, \overline{\alpha} \ge \overline{\beta}, \overline{\alpha} \ge \frac{1}{\delta}$. It is easy to verify that V_3 is positive definite. The closed-loop system consisting of (1) and (10) satisfies

$$\dot{\xi}(t) = (I_N \otimes A)\,\xi(t) + (L_\alpha \otimes BK_1)\,\tilde{s}(t) + (I_N \otimes B)\,\hat{H}_\beta(\tilde{s}) + (M \otimes B)F(t) \quad (50)$$

where $L_\alpha = (\alpha_{ij} \cdot a_{ij})$

$$\hat{H}_\beta(\tilde{s}) = \begin{bmatrix} \sum\limits_{j\in\mathcal{N}_1} \beta_{1j}\hat{h}_1\left[K_1\left(\tilde{s}_1(t) - \tilde{s}_j(t)\right)\right] \\ \vdots \\ \sum\limits_{j\in\mathcal{N}_N} \beta_{N_j}\hat{h}_N\left[K_1\left(\tilde{s}_N(t) - \tilde{s}_j(t)\right)\right] \end{bmatrix}.$$

The time derivative of V_3 along the trajectory of (50) can be obtained as

$$\begin{aligned}\dot{V}_3 &= \xi^T(I_N \otimes P_1 A)\xi + \xi^T\left(L_\alpha \otimes P_1 BK_1\right)\tilde{s} \\ &+ \xi^T(M \otimes P_1 B)F(t) + \xi^T\left(I_N \otimes P_1 B\right)\hat{H}_\beta(\tilde{s}) \\ &+ \sum_{i=1}^N \sum_{j\in\mathcal{N}_i} \frac{\tilde{\alpha}_{ij}}{4}\left[-\vartheta\alpha_{ij}(t) + \left\|K_1\left(\tilde{s}_i(t) - \tilde{s}_j(t)\right)\right\|^2\right] \\ &+ \sum_{i=1}^N \sum_{j\in\mathcal{N}_i} \frac{\tilde{\beta}_{ij}}{2}\left[-\chi\beta_{ij}(t) + \frac{\left\|K_1\left(\tilde{s}_i(t) - \tilde{s}_j(t)\right)\right\|^2}{\left\|K_1\left(\tilde{s}_i(t) - \tilde{s}_j(t)\right)\right\| + \varepsilon e^{-\varphi t}}\right].\end{aligned} \quad (51)$$

Similarly as in the proof of Theorem 1, one has

$$\begin{aligned}&\xi^T\left(L_\alpha \otimes P_1 BK_1\right)\tilde{s} \\ &\le -\frac{1}{4}\sum_{i=1}^N \sum_{j\in\mathcal{N}_i} \alpha_{ij}\left(\tilde{s}_i - \tilde{s}_j\right)^T \Gamma_1\left(\tilde{s}_i - \tilde{s}_j\right) \\ &+ \sum_{i=1}^N \sum_{j\in\mathcal{N}_i} \alpha_{ij}\left\|B^T P_1\right\|^2 \|e_i\|^2\end{aligned}$$

and

$$\begin{aligned}&\xi^T(M \otimes P_1 B)F(t) \\ &\le \frac{\sqrt{N}f_0}{\lambda_2(L)}\sum_{i=1}^N \sum_{j\in\mathcal{N}_i}\left\|B^T P_1\left(\tilde{s}_i - \tilde{s}_j\right)\right\| \\ &+ \frac{2\sqrt{N}f_0}{\lambda_2(L)}\sum_{i=1}^N \sum_{j\in\mathcal{N}_i}\left\|B^T P_1\right\| \|e_i\|\end{aligned}$$

and

$$\begin{aligned}&\xi^T\left(I_N \otimes P_1 B\right)\hat{H}_\beta(\tilde{s}) \\ &= -\frac{1}{2}\sum_{i=1}^N \sum_{j\in\mathcal{N}_i} \beta_{ij}\frac{\left(\tilde{s}_i - \tilde{s}_j\right)^T P_1 BB^T P_1\left(\tilde{s}_i - \tilde{s}_j\right)}{\left\|-B^T P_1\left(\tilde{s}_i - \tilde{s}_j\right)\right\| + \varepsilon e^{-\varphi t}}\end{aligned}$$

and

$$\begin{aligned}&\frac{1}{2}\sum_{i=1}^N \sum_{j\in\mathcal{N}_i} \beta_{ij}\frac{\left(e_i - e_j\right)^T P_1 BB^T P_1\left(\tilde{s}_i - \tilde{s}_j\right)}{\left\|-B^T P_1\left(\tilde{s}_i - \tilde{s}_j\right)\right\| + \varepsilon e^{-\phi t}} \\ &\le -\frac{1}{2}\sum_{i=1}^N \sum_{j\in\mathcal{N}_i} \beta_{ij}\frac{\left\|B^T P_1\left(\tilde{s}_i - \tilde{s}_j\right)\right\|^2}{\left\|B^T P_1\left(\tilde{s}_i - \tilde{s}_j\right)\right\| + \varepsilon e^{-\varphi t}} \\ &+ \sum_{i=1}^N \sum_{j\in\mathcal{N}_i} \beta_{ij}\left\|B^T P_1\right\| \|e_i\|.\end{aligned}$$

Substituting all the above inequalities into (51) yields

$$\begin{aligned}\dot{V}_3 \leq\ & \xi^T\left(I_N \otimes P_1 A\right)\xi - \frac{\alpha_{ij}}{4}\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}\left(\tilde{s}_i - \tilde{s}_j\right)^T\Gamma_1\left(\tilde{s}_i - \tilde{s}_j\right)\\ &+ \sum_{i=1}^N\sum_{j\in\mathcal{N}_i}\alpha_{ij}\left\|B^T P_1\right\|^2\|e_i\|^2 + \sum_{i=1}^N\sum_{j\in\mathcal{N}_i}\beta_{ij}\left\|B^T P_1\right\|\|e_i\|\\ &+ \frac{\sqrt{N}f_0}{\lambda_2(L)}\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}\left\|B^T P_1\left(\tilde{s}_i - \tilde{s}_j\right)\right\|\\ &+ \frac{2\sqrt{N}f_0}{\lambda_2(L)}\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}\left\|B^T P_1\right\|\|e_i\|\\ &- \frac{1}{2}\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}\beta_{ij}\frac{\left\|B^T P_1\left(\tilde{s}_i - \tilde{s}_j\right)\right\|^2}{\left\|B^T P_1\left(\tilde{s}_i - \tilde{s}_j\right)\right\| + \varepsilon e^{-\varphi t}}\\ &+ \sum_{i=1}^N\sum_{j\in\mathcal{N}_i}\frac{\tilde{\alpha}_{ij}}{4}\cdot\left[-\vartheta\left(\tilde{\alpha}_{ij} + \bar{\alpha}\right) + \left(\tilde{s}_i - \tilde{s}_j\right)^T\Gamma_1\left(\tilde{s}_i - \tilde{s}_j\right)\right]\\ &+ \sum_{i=1}^N\sum_{j\in\mathcal{N}_i}\frac{\tilde{\beta}_{ij}}{2}\cdot\left[-\chi\left(\tilde{\beta}_{ij} + \bar{\beta}\right) + \frac{\left\|K_1\left(\tilde{s}_i - \tilde{s}_j\right)\right\|^2}{\left\|K_1\left(\tilde{s}_i - \tilde{s}_j\right)\right\| + \varepsilon e^{-\varphi t}}\right].\end{aligned} \tag{52}$$

Due to $\tilde{\alpha}_{ij} = \alpha_{ij} - \overline{\alpha}$, $\tilde{\beta}_{ij} = \beta_{ij} - \overline{\beta}$, one has $\sum_{i=1}^N\sum_{j\in\mathcal{N}_i} -\frac{\vartheta}{4}(\tilde{\alpha}_{ij}^2 + \tilde{\alpha}_{ij}\overline{\alpha}) \leq \frac{\vartheta}{4}\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}(\frac{\overline{\alpha}^2}{2} - \frac{\tilde{\alpha}_{ij}^2}{2})$, $\sum_{i=1}^N\sum_{j\in\mathcal{N}_i} -\frac{\chi}{2}(\tilde{\beta}_{ij}^2 + \tilde{\beta}_{ij}\overline{\beta}) \leq \frac{\chi}{2}\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}(\frac{\overline{\beta}^2}{2} - \frac{\tilde{\beta}_{ij}^2}{2})$.

Since $\overline{\beta} \geq \frac{2\sqrt{N}f_0}{\lambda_2(L)}$, one has the following inequality:

$$\begin{aligned}\dot{V}_3 \leq\ & \frac{1}{2}\xi^T\left[I_N \otimes \left(P_1 A + A^T P_1\right) - \frac{\overline{\alpha}}{4}L \otimes P_1 B B^T P_1\right]\xi\\ &+ \frac{\overline{\alpha}}{2}\sum_{i=1}^N\left[\sum_{j\in\mathcal{N}_i}\left(1 + \delta\alpha_{ij}\cdot\frac{2}{\overline{\alpha}\delta}\right)\left\|B^T P_1\right\|^2\|e_i\|^2\right.\\ &+ \sum_{j\in\mathcal{N}_i}2\left(\frac{\overline{\beta}}{\overline{\alpha}} + \delta\beta_{ij}\cdot\frac{1}{\overline{\alpha}\delta}\right)\left\|B^T P_1\right\|\|e_i\|\\ &\left.- \frac{1}{4}\sum_{j\in\mathcal{N}_i}\left\|B^T P_1\left(\tilde{s}_i - \tilde{s}_j\right)\right\|^2\right] + \frac{\bar{\beta}}{2}\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}\varepsilon e^{-\varphi t}\\ &+ \frac{\vartheta}{4}\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}\left(\frac{\overline{\alpha}^2}{2} - \frac{\tilde{\alpha}_{ij}^2}{2}\right) + \frac{\chi}{2}\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}\left(\frac{\overline{\beta}^2}{2} - \frac{\tilde{\beta}_{ij}^2}{2}\right).\end{aligned} \tag{53}$$

By using $\overline{\alpha} \geq \frac{4}{\lambda_2(L)}$, $\overline{\alpha} \geq \overline{\beta}$, $\overline{\alpha} \geq \frac{1}{\delta}$, (9), and (15), one obtains

$$\begin{aligned}\dot{V}_3 \leq\ & -\frac{1}{2}\xi^T\xi + \frac{\overline{\alpha}}{2}\sum_{i=1}^N\mu_i e^{-\nu_i t} + \frac{\bar{\beta}}{2}\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}\varepsilon e^{-\varphi t}\\ &+ \frac{\vartheta}{4}\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}\left(\frac{\overline{\alpha}^2}{2} - \frac{\tilde{\alpha}_{ij}^2}{2}\right) + \frac{\chi}{2}\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}\left(\frac{\overline{\beta}^2}{2} - \frac{\tilde{\beta}_{ij}^2}{2}\right).\end{aligned} \tag{54}$$

Substituting (49) into (54) yields

$$\begin{aligned}\dot{V}_3 \leq\ & -\theta_3 V_3 + \frac{\overline{\alpha}}{2}\sum_{i=1}^N\mu_i e^{-\nu_i t} + \frac{\overline{\beta}}{2}\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}\varepsilon e^{-\varphi t}\\ &+ \sum_{i=1}^N\sum_{j\in\mathcal{N}_i}\left(\frac{\vartheta}{8}\overline{\alpha}^2 + \frac{\chi}{4}\overline{\beta}^2\right)\\ &+ \sum_{i=1}^N\sum_{j\in\mathcal{N}_i}\left[\left(\frac{\theta_3 - \vartheta\gamma}{8\gamma}\right)\tilde{\alpha}_{ij}^2 + \left(\frac{\theta_3 - \chi\eta}{4\eta}\right)\tilde{\beta}_{ij}^2\right]\end{aligned} \tag{55}$$

where $\theta_3 \triangleq \min\{\frac{1}{\lambda_{\max}(P_1)}, \vartheta\gamma, \chi\eta\}$. Furthermore

$$\begin{aligned}\dot{V}_3 \leq\ & -\theta_3 V_3 + \frac{\overline{\alpha}}{2}\sum_{i=1}^N\mu_i e^{-\nu_i t} + \frac{\bar{\beta}}{2}\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}\varepsilon e^{-\varphi t}\\ &+ \sum_{i=1}^N\sum_{j\in\mathcal{N}_i}\left(\frac{\vartheta}{8}\overline{\alpha}^2 + \frac{\chi}{4}\overline{\beta}^2\right).\end{aligned} \tag{56}$$

Let $\zeta_1 = \sum_{i=1}^N\sum_{j\in\mathcal{N}_i}(\frac{\vartheta}{8}\overline{\alpha}^2 + \frac{\chi}{4}\overline{\beta}^2)$, by using the comparison lemma in [51], one obtains that

$$\begin{aligned}V_3(t) \leq\ & \left[V_3(0) - \frac{\zeta_1}{\theta_3}\right]e^{-\theta_3 t} + \frac{\zeta_1}{\theta_3} + \frac{\overline{\alpha}}{2}\sum_{i=1}^N\mu_i\Omega_i\left(t, \theta_3\right)\\ &+ \frac{1}{2}\overline{\beta}\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}\int_0^t\varepsilon e^{-\theta_3(t-\tau)-\varphi\tau}d\tau\end{aligned} \tag{57}$$

where $\Omega_i(t, \theta_3)$ is given by (35). Then, $\lim_{t\to\infty}\Omega_i(t, \theta_3) = 0$. Therefore, V_3 exponentially converges to the residual set as

$$D_1 \triangleq \left\{\xi, \tilde{\alpha}_{ij}, \tilde{\beta}_{ij} : V_3 \leq \frac{\zeta_1}{\theta_3}\right\}.$$

It implies that ξ and $\tilde{\alpha}_{ij}, \tilde{\beta}_{ij}$ are uniformly ultimately bounded.

Next, if $\theta_4 \triangleq \max\{\vartheta\gamma, \chi\eta\} \leq \frac{1}{\lambda_{\max}(P_1)}$, one can rewrite (54) into

$$\begin{aligned}\dot{V}_3 \leq\ & -\theta_4 V_3 + \frac{1}{2}\theta_4\xi^T(I_N \otimes P_1)\xi - \frac{1}{2}\xi^T\xi\\ &+ \frac{\overline{\alpha}}{2}\sum_{i=1}^N\mu_i e^{-\nu_i t} + \frac{\bar{\beta}}{2}\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}\varepsilon e^{-\varphi t} + \zeta_1\\ &+ \sum_{i=1}^N\sum_{j\in\mathcal{N}_i}\left[\left(\frac{\theta_4 - \vartheta\gamma}{8\gamma}\right)\tilde{\alpha}_{ij}^2 + \left(\frac{\theta_4 - \chi\eta}{4\eta}\right)\tilde{\beta}_{ij}^2\right]\\ \leq\ & -\theta_4 V_3 - \frac{1}{2}\left(1 - \theta_4\lambda_{\max}(P_1)\right)\xi^T\xi\\ &+ \frac{\overline{\alpha}}{2}\sum_{i=1}^N\mu_i e^{-\nu_i t} + \frac{\bar{\beta}}{2}\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}\varepsilon e^{-\varphi t} + \zeta_1.\end{aligned} \tag{58}$$

Obviously, it follows from (58) that $\dot{V}_3 \leq -\theta_4 V_3 + \frac{\overline{\alpha}}{2}\sum_{i=1}^N\mu_i e^{-\nu_i t} + \frac{\bar{\beta}}{2}\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}\varepsilon e^{-\varphi t}$, if $\xi^T\xi \geq \frac{2\zeta_1}{1-\theta_4\lambda_{\max}(P_1)}$. Then, in light of $V_3 \geq \frac{\lambda_{\min}(P_1)}{2}\|\xi\|^2$, one has that if $\theta_4 < \frac{1}{\lambda_{\max}(P_1)}$ then ξ exponentially converges to the bounded set

$$D_2 \triangleq \left\{\xi : \|\xi\|^2 \leq \frac{2\zeta_1}{1 - \theta_4\lambda_{\max}(P_1)}\right\}.$$

Second, for the tracking process, consider the Lyapunov candidate

$$V_4 = \frac{1}{2}\hat{x}^T (I_N \otimes P_2)\hat{x} + \frac{1}{2\kappa}\sum_{i=1}^{N}\tilde{c}_{ij}^2 \tag{59}$$

where $\tilde{c}_{ij} = c_{ij} - \bar{c}$, and $\bar{c}$ is a constant satisfying $\bar{c} \geq f_0, \bar{c} \geq \frac{1}{\delta}$. It is not difficult to see that V_4 is positive definite. Substitute (11) into (2). It follows that $\hat{x}_i(t)$ satisfies the following closed-loop dynamics:

$$\begin{aligned}\dot{\hat{x}}(t) =& (I_N \otimes A)\hat{x}(t) + (I_N \otimes BK_2)\tilde{x}(t)\\ &+ (I_N \otimes B)\hat{H}_c[(I_N \otimes K_2)\tilde{x}] - (I_N \otimes B)F(t)\end{aligned} \tag{60}$$

where

$$\hat{H}_c[(I_N \otimes K_2)\tilde{x}] = \begin{pmatrix} c_{1j}\frac{-B^TP_2\tilde{x}_1}{\|-B^TP_2\tilde{x}_1\|+\varepsilon e^{-\varphi t}} \\ \vdots \\ c_{Nj}\frac{-B^TP_2\tilde{x}_N}{\|-B^TP_2\tilde{x}_N\|+\varepsilon e^{-\varphi t}}\end{pmatrix}.$$

The derivative of V_4 is given by

$$\begin{aligned}\dot{V}_4 =& \hat{x}^T (I_N \otimes P_2A)\hat{x} + \hat{x}^T (I_N \otimes P_2BK_2)\tilde{x}\\ &- \hat{x}^T (I_N \otimes P_2B)\hat{H}_c\left[\left(I_N \otimes B^TP_2\right)\tilde{x}\right]\\ &- \hat{x}^T (I_N \otimes P_2B)F(t)\\ &+ \tilde{c}_{ij}\left[-\varrho c_{ij}(t) + \frac{\|K_2\tilde{x}_i(t)\|^2}{\|K_2\tilde{x}_i(t)\| + \varepsilon e^{-\varphi t}}\right].\end{aligned} \tag{61}$$

For the third term of (61), one has

$$\begin{aligned}&- \hat{x}^T (I_N \otimes P_2B)\hat{H}_c\left[\left(I_N \otimes B^TP_2\right)\tilde{x}\right]\\ =& - c_{ij}\sum_{i=1}^{N}\left(B^TP_2\hat{x}_i\right)^T\frac{B^TP_2\tilde{x}_i}{\|B^TP_2\tilde{x}_i\| + \varepsilon e^{-\varphi t}}\\ \leq& -c_{ij}\sum_{i=1}^{N}\frac{\left\|B^TP_2\tilde{x}_i\right\|^2}{\|B^TP_2\tilde{x}_i\| + \varepsilon e^{-\varphi t}} + c_{ij}\sum_{i=1}^{N}\left\|B^TP_2\right\|\|\hat{e}_i\|.\end{aligned} \tag{62}$$

From Appendix A and the above inequality, the derivative of V_4 satisfies

$$\begin{aligned}\dot{V}_4 \leq& \frac{1}{2}\hat{x}^T\left[I_N \otimes \left(A^TP_2 + P_2A - P_2BB^TP_2\right)\right]\hat{x}\\ &+ \frac{1}{2}\sum_{i=1}^{N}\left\|B^TP_2\right\|^2\|\hat{e}_i\|^2 + (f_0 + c_{ij})\sum_{i=1}^{N}\left\|B^TP_2\right\|\|\hat{e}_i\|\\ &+ \bar{c}\sum_{i=1}^{N}\varepsilon e^{-\varphi t} + \varrho\sum_{i=1}^{N}\left(\frac{\bar{c}^2}{2} - \frac{\tilde{c}_{ij}^2}{2}\right).\end{aligned} \tag{63}$$

By using the fact $\bar{c} \geq \frac{1}{\delta}$, one gets

$$\begin{aligned}\dot{V}_4 \leq& \frac{1}{2}\hat{x}^T\left[I_N \otimes \left(A^TP_2 + P_2A - P_2BB^TP_2\right)\right]\hat{x}\\ &+ \bar{c}\sum_{i=1}^{N}\left[\frac{\delta}{2}\left\|B^TP_2\right\|^2\|\hat{e}_i\|^2 + 2(1 + \delta c_{ij})\left\|B^TP_2\right\|\|\hat{e}_i\|\right]\\ &+ \bar{c}\sum_{i=1}^{N}\varepsilon e^{-\varphi t} + \varrho\sum_{i=1}^{N}\left(\frac{\bar{c}^2}{2} - \frac{\tilde{c}_{ij}^2}{2}\right).\end{aligned} \tag{64}$$

According to the triggering condition (16), by substituting (59) into (64), one has

$$\dot{V}_4 \leq -\theta_5 V_4 + \bar{c}\sum_{i=1}^{N}\hat{\mu}_i e^{-\hat{\nu}_i t} + \bar{c}\sum_{i=1}^{N}\varepsilon e^{-\varphi t} + \varrho\sum_{i=1}^{N}\frac{\bar{c}^2}{2} \tag{65}$$

where $\theta_5 \triangleq \min\{\frac{1}{\lambda_{\max}(P)}, \varrho\kappa\}$. Let $\zeta_2 = \varrho\sum_{i=1}^{N}\frac{\bar{c}^2}{2}$, and using the comparison lemma in [51], one obtains that

$$\begin{aligned}V_4(t) \leq& \left[V_4(0) - \frac{\zeta_2}{\theta_5}\right]e^{-\theta_5 t} + \frac{\zeta_1}{\theta_5} + \bar{c}\sum_{i=1}^{N}\hat{\mu}_i\Omega_i(t, \theta_5)\\ &+ \bar{c}\sum_{i=1}^{N}\int_0^t \varepsilon e^{-\theta_3(t-\tau)-\varphi\tau}d\tau\end{aligned} \tag{66}$$

where the function $\Omega_i(t, \theta_5)$ is defined by (35). Therefore, V_4 exponentially converges to the residual set as

$$D_3 \triangleq \left\{\hat{x}_i, \tilde{c}_{ij} : V_4 \leq \frac{\zeta_2}{\theta_5}\right\}.$$

It implies that $\hat{x}_i$ and $\tilde{c}_{ij}$ are uniformly ultimately bounded.

Next, denote $\theta_6 \triangleq \max\{\varrho\kappa\} \leq \frac{1}{\lambda_{\max}(P_2)}$, (66) can be rewritten into

$$\begin{aligned}\dot{V}_4 \leq& -\theta_6 V_4 + \frac{1}{2}\theta_6\hat{x}^T (I_N \otimes P_2)\hat{x} - \frac{1}{2}\hat{x}^T\hat{x} + \zeta_2\\ &+ \bar{c}\sum_{i=1}^{N}\hat{\mu}_i e^{-\hat{\nu}_i t} + \bar{c}\sum_{i=1}^{N}\varepsilon e^{-\varphi t} + \sum_{i=1}^{N}\left[\left(\frac{\theta_6 - \varrho\kappa}{2\kappa}\right)\tilde{c}_{ij}^2\right]\\ \leq& -\theta_6 V_4 + \frac{1}{2}(1 - \theta_6\lambda_{\max}(P_2))\hat{x}^T\hat{x} + \zeta_2\\ &+ \bar{c}\sum_{i=1}^{N}\hat{\mu}_i e^{-\hat{\nu}_i t} + \bar{c}\sum_{i=1}^{N}\varepsilon e^{-\varphi t}.\end{aligned} \tag{67}$$

If $\hat{x}^T\hat{x} \geq \frac{2\zeta_2}{1-\theta_6\lambda_{\max}(P_2)}$, then $\dot{V}_4 \leq -\theta_6 V_4 + \bar{c}\sum_{i=1}^{N}\hat{\mu}_i e^{-\hat{\nu}_i t} + \bar{c}\sum_{i=1}^{N}\varepsilon e^{-\varphi t}$. In light of $V_4 \geq \frac{\lambda_{\min}(P_2)}{2}\|\hat{x}\|^2$, if $\theta_6 < \frac{1}{\lambda_{\max}(P_2)}$, then $\hat{x}$ exponentially converges to the bounded set

$$D_4 \triangleq \left\{\hat{x} : \|\hat{x}\|^2 \leq \frac{2\zeta_2}{1 - \theta_6\lambda_{\max}(P_2)}\right\}.$$

Therefore, one has the ultimate bounded convergence of tracking errors according to the above two parts. The DETAT tracking errors $x_i(t) - \frac{1}{N}\sum_{i=1}^{N}r_i(t)$ exponentially converge to the bounded set

$$\begin{aligned}D_5 = \Bigg\{& x_i(t) - \frac{1}{N}\sum_{i=1}^{N}r_i(t) : \left\|x_i(t) - \frac{1}{N}\sum_{i=1}^{N}r_i(t)\right\|\\ &\leq \left(\frac{2\zeta_2}{1 - \theta_6\lambda_{\max}(P_2)}\right)^{\frac{1}{2}} + \left(\frac{2\zeta_1}{1 - \theta_4\lambda_{\max}(P_1)}\right)^{\frac{1}{2}}\Bigg\}.\end{aligned}$$

Next, it is shown that the Zeno behaviors are excluded for closed-loop systems in estimation and tracking processes.

For the estimator i, following from (1) and (10), the derivative of e_i can be written as

$$\begin{aligned}\dot{e}_i(t) =& Ae_i(t) - BK_1\sum_{j\in\mathcal{N}_i}\alpha_{ij}[(\tilde{s}_i(t) - \tilde{s}_j(t))]\\ &- B\sum_{j\in\mathcal{N}_i}\beta_{ij}\hat{h}_i[K_1(\tilde{s}_i(t) - \tilde{s}_j(t))].\end{aligned} \tag{68}$$

Taking the norms of both sides

$$\|\dot{e}_i(t)\| \le \|A\| \, \|e_i(t)\| + \left\| BK_1 \sum_{j\in\mathcal{N}_i} \alpha_{ij} \left[(\tilde{s}_i(t) - \tilde{s}_j(t))\right] \right\| + \left\| B \sum_{j\in\mathcal{N}_i} \beta_{ij} \hat{h}_i \left[K_1 \left(\tilde{s}_i(t) - \tilde{s}_j(t)\right)\right] \right\|. \quad (69)$$

Since α_{ij}, β_{ij} are all bounded, so one can suppose $\alpha_{ij} \le \alpha_{ij}^*, \beta_{ij} \le \beta_{ij}^*, \forall (i,j) \in \mathcal{E}$. The rest of the proof is similar to Theorem 1. The details are omitted here.

Similarly, the Zeno behavior in the tracking process is excluded. The details are skipped for brevity.

C. Proof of Theorem 3

To obtain the conclusion in Theorem 3, the proofs in detail are composed of three steps.

Step 1): Introducing a new variable $\varsigma = y - \upsilon$ to show that the auxiliary state υ is able to converge to y asymptotically.

It follows from the system (21a) that $\dot{x} - \Psi\dot{\upsilon} = (A^* - B^*K^*)(x - \Psi\upsilon)$. Due to $A^* - B^*K^*$ is Hurwitz, thus $\lim_{t\to\infty} x - \Psi\upsilon = 0$. Then, it can obtain that $\lim_{t\to\infty} \varsigma = y - \upsilon = 0$.

Step 2): It is shown that the following nominal system is asymptotically stable at the origin:

$$\begin{aligned} \dot{\upsilon} &= w - \tilde{\upsilon} + s - \alpha \left(L \otimes I_n\right) \tilde{\upsilon} - \bar{d}, \\ \dot{\bar{d}} &= \alpha\beta \left(L \otimes I_n\right) \tilde{\upsilon}. \end{aligned} \quad (70)$$

Let $\xi = \upsilon - (M \otimes I_n)s$ and $\bar{w} = \bar{d} - (M \otimes I_n)s$. According to the triggering function $\Pi_{\upsilon i}(t)$, it is easy to obtain that $\|e_{\upsilon i}(t)\| \le \mu_{\upsilon i} e^{-\nu_{\upsilon i} t}$, which further yields $\lim_{t\to\infty} \|e_\upsilon\| = 0$. Since the estimator s can achieve consistency, $(M \otimes I_n)s = 0$, following from the analysis of [44, Th. 4.2], it can obtain that

$$\lim_{t\to\infty} \left\| \upsilon_i - \frac{1}{N} \sum_{k=1}^{N} s_k \right\| = 0, i = 1, 2, \ldots, N.$$

Step 3): Consider that the original closed-loop system (22) is equivalent to adding $-\tilde{y} + \tilde{\upsilon} - \alpha(L \otimes I_n)(\tilde{y} - \tilde{\upsilon})$ to the nominal system (70). According to the triggering function $\Pi_{yi}(t)$, it is easy to obtain that $\|e_{yi}(t)\| \le \mu_{yi} e^{-\nu_{yi} t}$, which further yields $\lim_{t\to\infty} \|e_y\| = 0$. Similarly, $\lim_{t\to\infty} \|e_\upsilon\| = 0$. Then, it can get from Step 1) that $\lim_{t\to\infty} \varsigma = y - \upsilon = 0$. Hence, according to Lemma 6, the closed-loop system (22) is globally uniformly asymptotically stable.

Therefore, it can be concluded that $\lim_{t\to\infty} y_i - \upsilon_i = 0$ and $\lim_{t\to\infty} \upsilon_i - \frac{1}{N}\sum_{k=1}^{N} s_k = 0$. Because the estimator s_i can obtain the average of multiple time-varying reference signals, one obtains

$$\lim_{t\to\infty} \left\| y_i - \frac{1}{N} \sum_{k=1}^{N} r_k \right\| = 0, i = 1, 2, \ldots, N$$

which means the DETAT problem of the heterogeneous multiagent system is realized.

Then, it is proven that there is no Zeno behavior in the closed-loop system (21).

The derivative of e_{yi} with (17) and (20) over the interval $[t_k^{i*}, t_{k+1}^{i*})$ can be written as

$$\begin{aligned} \dot{e}_{yi} &= -C_i(A_i - B_iK_i^*)(x_i - \psi_i\upsilon_i) - \dot{\upsilon}_i \\ \dot{\upsilon}_i &= w_i - \tilde{y}_i + s_i - \alpha \sum_{j\in\mathcal{N}_i} (\tilde{y}_i - \tilde{y}_j) - d_i \\ s_i &= q_i + r_i \\ \dot{d}_i &= \alpha\beta \sum_{j\in\mathcal{N}_i} (\tilde{y}_i - \tilde{y}_j). \end{aligned} \quad (71)$$

Note that $e_{yi}(t_k^{i*}) = 0$, the solution of $e_{yi}(t)$ is obtained as

$$\begin{aligned} e_{yi}(t) = &- \int_{t_s^{i*}}^{t} C_i \left(A_i - B_iK_i^*\right) \left[x_i(\tau) - \psi_i\upsilon_i(\tau)\right] d\tau \\ &+ \int_{t_s^{i*}}^{t} \Bigg\{ w_i(\tau) + y_i(t_k^{i*}) - s_i(\tau) \\ &+ \alpha \sum_{j\in\mathcal{N}_i} \left[y_i(t_s^{i*}) - y_j(t_m^{j*})\right] \quad + d_i(\tau) \Bigg\} d\tau \end{aligned} \quad (72)$$

where t_s^{i*} and t_m^{j*} are the latest triggering instants of agent i and agent j, respectively. For agent i, define t_{s+1}^{i*} as the next triggering instant and $t' = t - t_s^{i*}$. It is shown that no Zeno behavior is exhibited by proving $t_{s+1}^{i*} - t_s^{i*} > 0$.

Since states of multiple time-varying reference signals $r_i, i = 1, \ldots, N$ are bounded and outputs of agents $y_i, i = 1, \ldots, N$ can track the average of time-varying signals, thus $y_i, i = 1, \ldots, N$ are ultimately uniformly bounded. Further, s_i, $x_i - \Psi_i\upsilon_i$, υ_i, and d_i are bounded. Define $h_1, h_2, h_3, h_4, h_5, h_6 > 0$ such that $\|y_i\| < h_1, \|x_i - \psi_i\upsilon_i\| < h_2, \|d_i\| < h_3, \|\upsilon_i\| < h_4, \|s_i\| < h_5, \|w_i\| < h_6, i = 1, \ldots, N$.

Define $f(\varepsilon) = [(2\alpha N + 1)h_1 + \|C_i(A_i - B_iK_i^*)\|h_2 + h_3 + h_4 + h_5 + h_6]\varepsilon$. Note that $f(\varepsilon) > 0$ if and only if $\varepsilon > 0$. It follows that $\|e_{yi}(t)\| \le f(\varepsilon)$, $\forall t \ge t_s^{i*}$. The time interval $t_{s+1}^{i*} - t_s^{i*}$ is greater than or equal to the implicit solution of the equation $f(\varepsilon^*) = \mu_{yi} e^{-\nu_{yi}(\varepsilon^* + t_s^{i*})}$. The right-hand side of the above equation is always strictly positive, which implies that $t_{s+1}^{i*} - t_s^{i*} \ge \varepsilon^* > 0$ and the interevent time intervals for agent i are strictly positive, which implies that no Zeno behavior is exhibited.

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Yu Zhao (Member, IEEE) received the B.S. degree from Inner Mongolia University, Hohhot, China, in 2009, and the Ph.D. degree from the College of Engineering, Peking University, Beijing, China, in 2015.

He is currently a Tenure Track Associate Professor with the Department of Control Theory and Control Engineering, School of Automation, Northwestern Polytechnical University, Xian, China. His research interests include coordination control of autonomous intelligent systems and analysis and synthesis of complex networks with applications to aerospace engineering.

Dr. Zhao was the recipient of the Best Student Paper Award in the eighth Chinese Conference on Complex Networks in 2012. He has been named the Excellent Talents in Shaanxi Province in China in 2017. He was awarded the Excellent Paper Award of Natural Science in Shaanxi Province in 2020.

Chengxin Xian (Student Member, IEEE) received the B.S. degree in 2019 from the School of Automation, Northwestern Polytechnical University, Xian, China, where he is currently working toward the Doctoral degree with the School of Automation.

His research interests include event-based control, distributed optimal control, and average tracking of multiagent systems.

Guanghui Wen (Senior Member, IEEE) received the Ph.D. degree in mechanical systems and control from Peking University, Beijing, China, in 2012.

He is currently a Professor with the Department of Systems Science, Southeast University, Nanjing, China. His current research interests include autonomous intelligent systems, complex networked systems, distributed control and optimization, resilient control, and distributed reinforcement learning.

Dr. Wen was the recipient of the Best Student Paper Award in the sixth Chinese Conference on Complex Networks in 2010. He has been named a Highly Cited Researcher by Clarivate Analytics since 2018. He was awarded a National Natural Science Fund for Excellent Young Scholars in 2017. Moreover, he was a recipient of the Australian Research Council Discovery Early Career Researcher Award in 2018 and a recipient of the Asia Pacific Neural Network Society Young Researcher Award in 2019. He is a reviewer for *American Mathematical Review* and is an Active Reviewer for many journals. He currently serves as an Associate Editor for the IEEE JOURNAL OF EMERGING AND SELECTED TOPICS IN INDUSTRIAL ELECTRONICS, the IEEE TRANSACTIONS ON SYSTEMS, MAN AND CYBERNETICS: SYSTEMS, the IEEE OPEN JOURNAL OF THE INDUSTRIAL ELECTRONICS SOCIETY, and the *Asian Journal of Control*.

Panfeng Huang (Senior Member, IEEE) received the B.S. and M.S. degrees from Northwestern Polytechnical University, Xian, China, in 1998 and 2001, respectively, and the Ph.D. degree from the Chinese University of Hong Kong, Hong Kong, in automation and robotics in 2005.

He is currently a Professor of the School of Astronautics and Director of Research Center for Intelligent Robotics, Northwestern Polytechnical University. His research interests include space robotics, tethered space robotics, intelligent control, machine vision, space teleoperation.

Dr. Huang was awarded a National Science Fund for Distinguished Young Scholars in 2017.

Wei Ren (Fellow, IEEE) received the Ph.D. degree in electrical engineering from Brigham Young University, Provo, UT, USA, in 2004.

He is currently a Professor with the University of California, Riverside, CA, USA. Prior to joining UC Riverside, he was a Faculty Member with Utah State University and a Postdoctoral Research Associate with the University of Maryland, College Park. His research focuses on distributed control of multiagent systems and autonomous control of unmanned vehicles.

Dr. Ren was a recipient of the IEEE Control Systems Society Antonio Ruberti Young Researcher Prize in 2017 and the National Science Foundation CAREER Award in 2008. He is an IEEE Control Systems Society Distinguished Lecturer.