

INEQUALITY, BUSINESS CYCLES, AND MONETARY-FISCAL POLICY

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We study optimal monetary and fiscal policies in a New Keynesian model with heterogeneous agents, incomplete markets, and nominal rigidities. Our approach uses small-noise expansions and Fréchet derivatives to approximate equilibria quickly and efficiently. Responses of optimal policies to aggregate shocks differ qualitatively from what they would be in a corresponding representative agent economy and are an order of magnitude larger. A motive to provide insurance that arises from heterogeneity and incomplete markets outweighs price stabilization motives.

KEYWORDS: Sticky prices, heterogeneity, monetary policy, fiscal policy.

1. INTRODUCTION

WE APPROXIMATE recursive representations of optimal monetary and fiscal policies in an incomplete markets economy with agents who face both aggregate and idiosyncratic risks. Agents differ in wages, exposures to aggregate shocks, holdings of financial assets, and abilities to trade assets. They cannot fully insure themselves because financial markets are incomplete. Firms are monopolistically competitive. Price adjustments are costly. We examine how a Ramsey planner's policies for nominal interest rates, transfers, and flat-rate taxes on labor earnings, dividends, and interest income respond to aggregate shocks.

It is challenging to approximate a Ramsey plan in our setting. The aggregate state in a recursive formulation of the Ramsey problem includes the joint distribution of individual asset holdings and auxiliary promise-keeping variables chosen earlier by a planner. The law of motion for that high-dimensional object must be determined jointly with the optimal policies, and the distributions along the transition path differ substantially from the invariant distribution without aggregate shocks. These aspects render inapplicable common computational strategies that approximate policy functions after summarizing cross-sectional distributions with a small number of moments or that linearize policy functions around some time-invariant distribution.

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We thank the referees as well as Adrien Auclert, Benjamin Moll, and participants at numerous seminars and conferences for helpful criticisms. Bhandari, Evans, and Golosov acknowledge that this material is based upon work supported by the NSF under Grant 36354. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the NSF.

We forge a new computational approach that can be applied to economies with substantial heterogeneity and that does not require knowing long-run properties in advance. Our approach builds on a perturbation theory that constructs a sequence of small-noise expansions with respect to a one-dimensional parameterization of uncertainty along simulations of sample paths of our economy. The procedure is recursive and unfolds over time. At each time period along a simulated sample path, we approximate policy functions by applying a perturbation algorithm at that period's cross-section distribution. We use these approximate decision rules for that period to determine outcomes that include government policy decisions and the cross-section distribution next period. Then we move forward one period and perturb around next period's cross-section distribution to approximate next period's government decision rules and other outcomes. In this way, along an equilibrium sample path we sequentially update cross-section distributions around which we approximate policy functions.

Our perturbation approach requires that each period we compute derivatives of policy functions with respect to all state variables, one of which is a Fréchet derivative with respect to a distribution over a multidimensional vector of agents' characteristics. It is usually hard to compute this Fréchet derivative directly. What makes our approach practical is that, in an interesting class of heterogeneous agent competitive equilibrium models, we can streamline this computationally-intensive step by representing parameters of approximate policy functions as a collection of low-dimensional linear equations that are independent of each other. This is possible because in standard competitive environments, conditional on prices and aggregate quantities, different agents' optimal choices can be solved separately. These systems of linear equations are independent across agents, and thus easily parallelizable. This helps us manage the ample heterogeneity present in our model. Agents' optimal choices can then be aggregated into a set of equilibrium conditions that lead to a low-dimensional fixed-point problem whose solution determines prices and aggregate quantities. A similar computationally convenient linear structure prevails for second- and higher-order expansions, making our approach applicable to many optimal policy problems in which aggregate risks have important effects on equilibrium dynamics.

We apply our approach to a textbook sticky price model (see, e.g., Galí (2015)) augmented with heterogeneous agents. In the tradition of Bewley (1977, 1980), Huggett (1993), and Aiyagari (1994), financial markets are incomplete: Agents can trade only nonstate-contingent nominal debt. Agents' wages are subject to idiosyncratic and aggregate shocks that we calibrate to match U.S. business cycles and cross-sectional properties of labor earnings. We set the initial joint distribution of nominal and real claims, and wages to match cross-sectional moments in the Survey of Consumer Finances. We posit two aggregate shocks: one to productivity, and another to the elasticity of substitution between differentiated intermediate goods that affects firms' optimal markups. We pose two Ramsey problems. In the first, a "purely monetary policy" planner can adjust only nominal interest rates and transfers in response to shocks, while keeping all other tax rates at fixed levels chosen in period 0. This is a common assumption used in New Keynesian models. In addition to interest rates and transfers, our second Ramsey planner has more tools and can adjust tax rates on all sources of income. Since standard calibrations of Bewley–Aiygari economies imply a slow drift toward a long-run distribution with much smaller asset heterogeneity than observed in the U.S. data, we focus our attention on optimal policy responses in the initial 100 periods, when the cross-sectional distribution of earnings and assets is similar to the initial one.

We find that inflation, nominal interest rates, and taxes are substantially more volatile in our calibrated heterogeneous agent (HANK) economy than in its representative agent

(RANK) counterpart. For example, the standard deviation of inflation chosen by the “purely monetary policy” planner is an order of magnitude higher in HANK. Moreover, the magnitudes and signs of correlations between nominal and real variables differ in HANK and RANK. We run diagnostics that show that providing insurance against aggregate shocks to heterogeneous agents accounts for most of the differences between optimal monetary and fiscal policies in the two economies.

To understand how insurance concerns shape optimal policies, consider the optimal monetary response to a one-time positive markup shock that motivates firms to increase their prices. A standard optimal response in New Keynesian models is to stabilize the price level. The planner increases nominal interest rates and in that way decreases firms’ marginal costs by lowering aggregate demand. This response rationalizes the prescription to “lean against the wind” by raising interest rates when firms’ desired markups increase (Galí (2015)). The markup shock also changes relative shares of payments to labor and owners of equity. When firm owners and wage earners are the same people, such movements in factor shares have no welfare consequences, but they can have adverse risk-sharing consequences if different agents have different sources of incomes. When agents are heterogeneous and cannot trade Arrow securities, the Ramsey planner can use monetary and fiscal policy to compensate for missing insurance markets. Since a positive markup shock creates an unexpected drop in wage income and a rise in profits, a Ramsey planner can provide insurance payments to workers by lowering nominal rates to boost wages. A negative markup shock makes the planner want to synthesize insurance payouts to equity owners, making the optimal response a mirror image of the response to a positive markup shock.

Quantitatively, the strength of the insurance motive depends on the correlation between labor and capital incomes: less positive correlations call for more insurance. That the distribution of stock ownership is much more skewed to the right than is the distribution of labor earnings implies that there are potentially large welfare gains from supplying insurance. As a result, in our calibrated economy optimal monetary responses to markup shocks are an order of magnitude larger and opposite in direction from those in a corresponding representative agent economy.

An insurance motive also shapes a Ramsey planner’s responses to TFP shocks. While TFP shocks push profits and wages in the same direction, the consequences of TFP shocks are not shared equally by borrowers and lenders. When agents can trade only nonstate-contingent bonds, a TFP shock changes total output while keeping nominal obligations unchanged. So a negative TFP shock hurts borrowers while a positive TFP shock hurts lenders. A Ramsey planner can provide insurance and improve welfare by lowering (raising) the real return on debt in response to a negative (positive) TFP shock. That optimal response contrasts with the standard New Keynesian prescription of adjusting the nominal interest rate one-for-one with the “natural” rate of interest (i.e., the interest rate that would prevail if nominal prices were perfectly flexible). That we observe substantial dispersion in ownership of nominal claims indicates that the planner’s insurance motive is strong in our calibrated economy.

We also consider a number of extensions and robustness checks to explore the implications of different assumptions about the redistributive objective of the planner, price stickiness, asset trading frictions, and heterogeneity in marginal propensities to consume on optimal policies. In all cases that we considered, insurance considerations account for most differences between optimal policies in HANK and RANK economies.

1.1. *Related Literature*

Our paper contributes to two literatures: one that approximates equilibria of incomplete markets economies with heterogeneous agents, and another that computes Ramsey plans for fiscal and monetary policies.

We compute small-noise expansions around transition paths like those deployed by Fleming (1971), Fleming and Souganidis (1986), and Anderson, Hansen, and Sargent (2012), all of whom study problems with state vectors that are much smaller than ours. That makes direct applications of approaches in those papers computationally impractical for us. Instead, we use functional derivatives techniques¹ to cope with the most computationally intensive step and reformulate the problem of computing approximate policy functions as a manageable collection of low-dimensional linear equations. Our techniques can be used to construct second- and higher-order approximations via a convenient set of recursions.

Relative to popular approaches such as Krusell and Smith (1998) or Reiter (2009), our method brings benefits and costs. In the Krusell and Smith approach, one summarizes the distribution of agents' characteristics with a small number of moments and approximates the law of motion of those moments. In contrast, our method allows for complicated multidimensional distributions that are hard to summarize with few parameters but relies on local expansions with respect to shocks. In Reiter's approach, while policy functions are accurate with respect to idiosyncratic shocks, they are approximated only to the first order with respect to aggregate shocks around the invariant distribution of the no-aggregate-shock economy. In comparison, our method is not constrained to linear approximations, and we repeatedly update the approximation as the state of the economy moves along an equilibrium path. Our method is particularly well suited for economies with possibly non-stationary transition dynamics or when impulse responses depend on past shocks or when higher-order moments of aggregate variables play important roles. Despite its accuracy in approximating optimal responses to aggregate shocks, our method is less accurate in approximating the dependence of optimal policies on idiosyncratic shocks.

We assess the numerical accuracy of our method by computing the competitive equilibrium for a special case of our model that corresponds to an economy studied by Acharya and Dogra (2018) under fixed government policies. An advantage of studying their economy is that we can analytically compute an equilibrium. This allows us to compare both our approximation and Reiter's to a pencil-and-paper answer. Under Acharya and Dogra's calibration, maximum numerical errors in our approximated policy functions are less than 0.05%. Moreover, approximation errors in policy functions of aggregate variables are *two orders of magnitude* smaller than those obtained with Reiter's method. While our method is less accurate in approximating responses to idiosyncratic shocks, those errors average out in the aggregate. (However, second-order errors in approximating responses to aggregate shocks under Reiter's method do not average out). We also show that because errors in approximating responses to aggregate shocks compound over time under Reiter's approach, they adversely affect approximating long-run distributions. Finally, we also illustrate how the drift of the distribution of assets away from the point of approximation leads to errors in impulse responses under a Reiter method that do not emerge with our method.

¹Childers (2018) combined related functional derivative techniques with a Reiter (2009) method, but unlike our approach, his still requires that distributions remain close to the invariant distribution of a no-aggregate-shock economy. We build on and extend Evans (2015).

A substantial literature on optimal monetary and fiscal policies in the Ramsey tradition has mostly studied economies with few if any sources of heterogeneity. For treatments of optimal monetary policies in representative agent New Keynesian models, see Galí (2015) and Woodford (2003).² Bilbiie, Ovidiu, and Ragot (2017), Challe (2017), Bilbiie (2019), and Debortoli and Galí (2017) studied optimal monetary policy in economies with limited heterogeneity and in which a cross-sectional distribution disappears from the formulation of a Ramsey problem and analysis can be done using traditional techniques. Like us, those papers emphasize that uninsurable aggregate shocks create reasons for the planner to sacrifice price stability. Notable recent papers by Nuno and Thomas (2016), LeGrand and Ragot (2017), and LeGrand, Martin-Baillon, and Ragot (2020) develop alternative methods to approximate Ramsey allocations in incomplete market economies with heterogeneity. Nuno and Thomas (2016) studied dynamics of a Ramsey allocation in a small open economy using continuous time methods. LeGrand and Ragot (2017) and LeGrand, Martin-Baillon, and Ragot (2020) truncated idiosyncratic histories and then linearized with respect to aggregate shocks. Their approximations to optimal policies for a standard Aiyagari–Bewley incomplete markets model with nominal rigidities indicate that inflation contributes little to shaping an optimal allocation in the long-run. However, standard calibrations of Aiyagari–Bewley economies like theirs display a much smaller dispersion in the distribution of wealth than those presented in the U.S. data to which we calibrate our model. That means that there are fewer gains that a Ramsey planner can reap from providing insurance using state-contingent movements in inflation in a long-run steady state. More broadly, the scope for insurance depends crucially on the joint distribution of earnings, nominal and real assets. It is important to study optimal monetary policy in models that closely match those distributions in the data because they pin down the heterogeneity in the unhedged exposures to aggregate shocks.

2. ENVIRONMENT

There is a continuum of infinitely lived households indexed by $i \in [0, 1]$. Individual i 's preferences over final consumption good $\{c_{i,t}\}_t$ and hours $\{n_{i,t}\}_t$ are ordered by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_{i,t}, n_{i,t}), \quad (1)$$

where \mathbb{E}_t is a mathematical expectation operator conditioned on time t information, $\beta \in (0, 1)$ is a time discount factor, and u is an infinitely differentiable utility function that is concave in c and $-n$ and satisfies Inada conditions. Partial derivatives are denoted by $u_{c,i,t} \equiv u_c(c_{i,t}, n_{i,t})$, $u_{n,i,t} \equiv u_n(c_{i,t}, n_{i,t})$, and so on. A random variable with subscript t is measurable with respect to time t information.

Agent i supplies $\epsilon_{i,t} n_{i,t}$ units of effective labor, where $\epsilon_{i,t}$ is an exogenous productivity process. A unit of effective labor receives nominal wage $P_t W_t$, where P_t is the nominal price of the final consumption good at time t . Agents trade a one-period risk-free nominal bond. The price of the bond is denoted by Q_t , which equals the inverse of the gross nominal rate between periods t and $t + 1$. We use $P_t b_{i,t}$ to denote the face value of nominal bonds owned by agent i at end of period t , and $P_t d_{i,t}$ to denote nominal dividends

²There are several papers that study optimal monetary and fiscal policies in calibrated representative agent settings. For instance, see Chari and Kehoe (1999) for a neoclassical setup, and see Schmitt-Grohe and Uribe (2004a) and Siu (2004) for optimal responses to government spending shocks in setups with nominal rigidities.

received from intermediate goods producers during period t . In what will serve as our *baseline* specification, we assume that agent i 's dividends in period t are $d_{i,t} = s_i D_t$, where s_i is fixed over time, a specification that restricts agents not to trade equity.

Let $\Pi_t = \frac{P_t}{P_{t-1}} - 1$ denote the net inflation rate. Households receive a uniform lump-sum transfer T_t and face a linear tax Y_t^n on their labor earnings, a tax Y_t^d on their dividends, and a tax Y_t^b on their interest income.³ The budget constraint of household i at date t in units of final goods is

$$c_{i,t} + Q_t b_{i,t} = (1 - Y_t^n) W_t \epsilon_{i,t} n_{i,t} + T_t + (1 - Y_t^d) d_{i,t} + (1 - Y_t^b) \frac{b_{i,t-1}}{1 + \Pi_t}. \tag{2}$$

The government's budget constraint at time t is

$$\bar{G} + T_t + \frac{B_{t-1}}{1 + \Pi_t} = \int \left[Y_t^n W_t \epsilon_{i,t} n_{i,t} + Y_t^d d_{i,t} + \frac{Y_t^b b_{i,t-1}}{1 + \Pi_t} \right] di + Q_t B_t,$$

where \bar{G} is a time-invariant level of nontransfer government expenditures. We denote $Y_t \equiv (Y_t^n, Y_t^d, Y_t^b)$.

A final good Y_t is produced by competitive firms that use a continuum of intermediate goods $\{y_t(j)\}_{j \in [0,1]}$ as inputs into a production function

$$Y_t = \left[\int_0^1 y_t(j)^{\frac{\Phi_t-1}{\Phi_t}} dj \right]^{\frac{\Phi_t}{\Phi_t-1}},$$

where the elasticity of substitution Φ_t is stochastic. Final good producers take the final good price P_t and the intermediate goods prices $\{p_t(j)\}_j$ as given and solve

$$\max_{\{y_t(j)\}_{j \in [0,1]}} P_t \left[\int_0^1 y_t(j)^{\frac{\Phi_t-1}{\Phi_t}} dj \right]^{\frac{\Phi_t}{\Phi_t-1}} - \int_0^1 p_t(j) y_t(j) dj. \tag{3}$$

Outcomes of optimization problem (3) are a demand function for intermediate goods

$$y_t(j) = \left(\frac{p_t(j)}{P_t} \right)^{-\Phi_t} Y_t \tag{4}$$

and a final goods price that satisfies

$$P_t = \left(\int_0^1 p_t(j)^{1-\Phi_t} \right)^{\frac{1}{1-\Phi_t}}.$$

Intermediate goods $y_t(j)$ are produced by monopolists with production functions

$$y_t(j) = [n_t^D(j)]^\alpha [h_t(j)]^{1-\alpha}, \tag{5}$$

³Although Y_t^b multiplies $b_{i,t-1}$, we refer to it as a tax on the interest income because it is equivalent to a tax on the return on a one-period bond. To see this, rewrite the budget constraint using the market value of nominal debt $\tilde{b}_{i,t} = Q_t b_{i,t}$ and notice that $(1 - Y_t^b) \frac{b_{i,t-1}}{1 + \Pi_t} = (1 - Y_t^b) (R_{t-1,t}) \tilde{b}_{i,t-1}$, where $R_{t-1,t} = \left(\frac{1}{Q_{t-1}}\right) \left(\frac{1}{1 + \Pi_t}\right)$ is the real return from holding a nominal bond from $t - 1$ to t .

where $n_t^D(j)$ is effective labor hired by firm j and $h_t(j)$ is an intermediate input measured in units of the final good. Intermediate goods monopolists face downward sloping demand curves $(\frac{p_t(j)}{P_t})^{-\Phi_t} Y_t$ and choose prices $p_t(j)$, while bearing quadratic Rotemberg (1982) price adjustment costs $\frac{\psi}{2} (\frac{p_t(j)}{p_{t-1}(j)} - 1)^2$ measured in units of the final consumption good. Intermediate goods producing firm j chooses prices $\{p_t(j)\}_t$ and factor inputs $\{h_t(j), n_t^D(j)\}_t$ that solve

$$\max_{\{p_t(j), n_t^D(j), h_t(j)\}_t} \mathbb{E}_0 \sum_t S_t (1 - Y_t^d) \left\{ \frac{p_t(j)}{P_t} y_t(j) - W_t n_t^D(j) - h_t(j) - \frac{\psi}{2} \left(\frac{p_t(j)}{p_{t-1}(j)} - 1 \right)^2 \right\}, \tag{6}$$

subject to (4) and (5), where W_t is the real wage per unit of effective labor and S_t is a stochastic discount factor (SDF) process defined recursively via

$$S_t = S_{t-1} Q_{t-1} (1 + \Pi_t) / (1 - Y_t^b), \tag{7}$$

with $S_{-1} = 1$.⁴

In a symmetric equilibrium, $p_t(j) = P_t$, $y_t(j) = Y_t$, $h_t(j) = H_t$, and $n_t^D(j) = N_t$ for all j . Market clearing conditions in labor, goods, and bond markets are

$$C_t = \int c_{i,t} di, \quad N_t = \int \epsilon_{i,t} n_{i,t} di, \quad D_t = Y_t - H_t - W_t N_t - \frac{\psi}{2} \Pi_t^2, \tag{8}$$

$$Y_t = N_t^\alpha H_t^{1-\alpha}, \quad \Pi_t = P_t / P_{t-1} - 1 \tag{9}$$

$$C_t + \bar{G} = Y_t - H_t - \frac{\psi}{2} \Pi_t^2, \tag{10}$$

$$\int b_{i,t} di = B_t. \tag{11}$$

There are aggregate and idiosyncratic shocks. Aggregate shocks are a “markup” shock Φ_t and an aggregate productivity shock Θ_t that follow AR(1) processes:

$$\ln \Phi_t = \rho_\Phi \ln \Phi_{t-1} + (1 - \rho_\Phi) \ln \bar{\Phi} + \mathcal{E}_{\Phi,t},$$

$$\ln \Theta_t = \rho_\Theta \ln \Theta_{t-1} + (1 - \rho_\Theta) \ln \bar{\Theta} + \mathcal{E}_{\Theta,t},$$

where $\mathcal{E}_{\Phi,t}$ and $\mathcal{E}_{\Theta,t}$ are mean-zero random variables that are i.i.d. over time and uncorrelated with each other at all times.

Individual productivity $\epsilon_{i,t}$ follows a stochastic process described by

$$\ln \epsilon_{i,t} = \ln \Theta_t + \ln \theta_{i,t} + \epsilon_{\epsilon,i,t}, \tag{12}$$

$$\ln \theta_{i,t} = \rho_\theta \ln \theta_{i,t-1} + \epsilon_{\theta,i,t}, \tag{13}$$

where innovations $\epsilon_{\epsilon,i,t}$ and $\epsilon_{\theta,i,t}$ are mean-zero, uncorrelated with each other, and i.i.d. across time.

⁴In economies with heterogeneous agents and incomplete markets, a stand must be taken on how firms are valued. To explain our numerical methods most transparently, we chose a simple specification of the SDF that discounts future profits at the after-tax real risk-free rate. Our quantitative results are virtually identical when we use other choices of SDFs: equally and asset-weighted averages of individual intertemporal marginal rates of substitutions as well as a risk-neutral SDF.

We set the initial price level $P_{-1} = 1$. In period 0, agent i is characterized by a triple $(\theta_{i,-1}, b_{i,-1}, s_i)$, where $\theta_{i,-1}$ is agent i 's initial persistent component of productivity, $b_{i,-1}$ denotes the bonds that agent i initially owns, and s_i denotes agent i 's initial ownership of equity. Initial conditions include the set $\{\theta_{i,-1}, b_{i,-1}, s_i\}_i$ for individuals states and a vector (Φ_{-1}, Θ_{-1}) for the aggregate shocks.

2.1. Ramsey Problems

Before diving into details about how we approximate a Ramsey plan in Section 3, it is useful here to provide definitions of a Ramsey plan.

DEFINITION 1: Given initial conditions and a monetary-fiscal policy $\{Q_t, Y_t, T_t\}_t$, a competitive equilibrium is a stochastic process

$$\{ \{c_{i,t}, n_{i,t}, b_{i,t}\}_i, C_t, N_t, B_t, W_t, P_t, Y_t, H_t, D_t, \Pi_t, S_t \}_t$$

that satisfies: (i) $\{c_{i,t}, n_{i,t}, b_{i,t}\}_{i,t}$ maximize (1) subject to (2) and natural debt limits;⁵ (ii) final goods firms choose $\{y_t(j)\}_j$ to maximize (3); (iii) intermediate goods producers' prices and factor inputs solve (6) and satisfy $p_t(j) = P_t$, $y_t(j) = Y_t$, $h_t(j) = H_t$, and $n_t^D(j) = N_t$ for all j ; and (iv) market clearing conditions (8)–(11) are satisfied.

We can characterize competitive equilibria by feasibility constraints (7), (8), (9), and (10); consumers' and firms' optimality conditions

$$(1 - Y_t^n)W_t \epsilon_{i,t} u_{c,i,t} = -u_{n,i,t}, \tag{14}$$

$$Q_t u_{c,i,t} = \beta \mathbb{E}_t u_{c,i,t+1} (1 - Y_{t+1}^b) / (1 + \Pi_{t+1}), \tag{15}$$

$$\begin{aligned} 0 = & \frac{1}{\psi} Y_t \left[1 - \Phi_t \left(1 - \frac{1}{1 - \alpha} \left(\frac{1 - \alpha}{\alpha} W_t \right)^\alpha \right) \right] - \Pi_t (1 + \Pi_t) \\ & + \mathbb{E}_t \frac{S_{t+1}}{S_t} \left(\frac{1 - Y_{t+1}^d}{1 - Y_t^d} \right) \Pi_{t+1} (1 + \Pi_{t+1}), \end{aligned} \tag{16}$$

$$\frac{1 - \alpha}{\alpha} W_t = \frac{H_t}{N_t}, \tag{17}$$

and agents' budget constraints that, by using equation (14) to eliminate $(1 - Y_t^n)W_t \epsilon_{i,t}$, we can represent as

$$c_{i,t} - T_t - (1 - Y_t^d) s_i D_t - \frac{(1 - Y_t^b) b_{i,t-1}}{1 + \Pi_t} = \left(\frac{u_{n,i,t}}{u_{c,i,t}} \right) n_{i,t} + \mathbb{E}_t \left(\frac{u_{c,i,t+1}}{u_{c,i,t}} \right) \frac{(1 - Y_{t+1}^b) b_{i,t}}{1 + \Pi_{t+1}}. \tag{18}$$

⁵We impose natural debt limits by imposing for all t ,

$$\lim_{s \rightarrow \infty} \mathbb{E}_t \left(\prod_{k=1}^s Q_{t+k} (1 - Y_{t+k}^b) \right) P_{t+s} b_{i,t+s} = 0.$$

A Ramsey planner orders allocations by

$$\mathbb{E}_0 \int \sum_{t=0}^{\infty} \beta^t \vartheta_i u(c_{i,t}, n_{i,t}) di, \quad (19)$$

where $\vartheta_i \geq 0$ is a Pareto weight attached to agent i and $\int \vartheta_i di = 1$.

DEFINITION 2: Given initial conditions and a time-invariant tax policy satisfying $Y_t = \bar{Y}$ for some \bar{Y} , an *optimal monetary policy* is a stochastic process $\{Q_t, T_t\}_t$ that brings about a competitive equilibrium allocation that maximizes (19). Given initial conditions, an *optimal monetary-fiscal policy* is a stochastic process $\{Q_t, Y_t, T_t\}_t$ that implements a competitive equilibrium allocation that maximizes (19). A maximizing monetary or monetary-fiscal stochastic process is called a *Ramsey plan*; an associated allocation is called a *Ramsey allocation*.

We construct an optimal monetary-fiscal policy and an associated competitive equilibrium by maximizing the welfare criterion (19) subject to constraints (7)–(10) and (14)–(18). Choice of an optimal monetary policy is subject to the constant-tax-rate constraints $Y_t = \bar{Y}$ for all $t \geq 0$.

2.2. Discussion of the Environment

To bring out economic forces that shape how optimal policies respond to aggregate shocks, we use two baselines that differ in whether a Ramsey planner can adjust tax rates. In what we call our *optimal monetary-fiscal policy baseline*, a Ramsey planner can freely adjust the nominal interest rate and all tax rates in response to aggregate shocks. In what we call our *optimal monetary policy baseline*, the Ramsey planner can adjust only the nominal interest rate. Our use of a monetary-policy-only baseline follows a New Keynesian tradition that, in our notation, imposes time-invariant tax rates $Y_t = \bar{Y}$ and assumes that only the nominal interest rate Q_t^{-1} and lump sum transfers T_t can respond to shocks, with lump sum transfers adjusting to satisfy the government's budget constraint. A popular justification for this restriction is that central banks can adjust interest rates fast enough to react to shocks at business cycle frequencies, while institutional constraints prevent governments from adjusting tax rates quickly. Optimal monetary policy depends on \bar{Y} ; following the New Keynesian tradition, our Section 4 quantitative application focuses on a level of \bar{Y} that maximizes welfare (19) under an optimal monetary policy associated with that \bar{Y} .

We extend a New Keynesian model like that of Galí (2015, Chapter 3) to allow for incomplete markets and heterogeneous agents as in the models of Bewley (1977, 1980), Huggett (1993), and Aiyagari (1994). Galí's setup had the advantage that policy prescriptions for the representative agent version are widely understood. That allows us to isolate modifications of those prescriptions that heterogeneity and incomplete markets bring. In our baseline environment, we model heterogeneity with a wage process representative of ones used in the macro-labor literature, for instance, Low, Meghir, and Pistaferri (2010). In Section 6.4, we enrich the baseline process for wage dynamics to allow for some of the diverse responses of labor earnings to recessions documented by Guvenen, Ozkan, and Song (2014).

In our two baseline models, we assume that all agents can freely trade bonds subject to natural debt limits. This means that Ricardian equivalence holds so that optimal timings of transfers are undetermined.⁶ This is a natural baseline since economies with *ad hoc* debt limits often imply Ramsey plans that prescribe a nonstationary optimal fiscal policy that front-loads transfers in order to undo those debt constraints.⁷ We relax that assumption in Section 6.2 by including a subset of liquidity-constrained agents.

In our baseline models, we assume that agents can trade debt but not equity. We relax the nontradability of claims to dividends in Section 6.3 when we introduce mutual funds that hold corporate equity and government debt and that issue mutual fund shares that households trade in a competitive market.

3. APPROXIMATING A RAMSEY PLAN

We approximate Ramsey plans for heterogeneous agent (HA) economies that present a continuation Ramsey planner with a state vector that includes a joint probability distribution of agents' characteristics. For reasons anticipated in Section 1.1, this feature prevents us from approximating with a projection method like that of [Krusell and Smith \(1998\)](#) or a method like that of [Reiter \(2009\)](#) who perturbs around an economy with no aggregate shocks.

A Krusell–Smith approach can work well when the dimension of a state vector is small and when policy functions are nearly affine over sufficiently large parts of the state space to make agents' policy rules aggregate well. Even in simple versions of our problem, the state vector includes a high-dimensional joint distribution, so a Krusell–Smith approach would require tracking too many moments. Reiter designed his approach for situations in which it is easy to compute an invariant distribution when there are no aggregate shocks *and* in which the state vectors stay near the support of that invariant distribution when aggregate shocks are active. These conditions can prevail in some models operating under arbitrarily fixed government policies, but not in our setting.⁸

We propose an alternative method that constructs a stochastic sequence of small-noise expansions along a simulated optimal path. A key step uses functional derivative techniques to characterize how government decisions depend on a high-dimensional state vector that changes over time in response to aggregate shocks. We present fast computational techniques that work at any order of approximation.⁹

⁶In the general formulation of the Ramsey problem, we do not restrict lump-sum transfers T_t to be positive. However, in our Section 4 quantitative application, transfers are always positive, since households are unequal and the planner cares about redistribution.

⁷Bhandari et al. (2017) studied a Ramsey problem with *ad hoc* debt limits, in which a planner who enforces both debt contracts and tax liabilities can time transfers to undo the *ad hoc* debt limits. But sometimes a Ramsey planner can improve outcomes by not enforcing private debt contracts; see Yared (2013) for related results.

⁸The long-run behavior of the state variables in even the simplest Ramsey problems can differ dramatically with and without aggregate shocks in otherwise identical economies. One can readily see this from the classic tax-smoothing model of [Barro \(1979\)](#), in which government debt is the only endogenous state variable. Without aggregate shocks to government expenditures, it stays at its initial level, while with aggregate shocks it follows a random walk; thus, whether aggregate shocks are present has important implications for the long-run distribution of debt. Similarly, [Aiyagari, Marcet, Sargent, and Seppala \(2002\)](#), [Farhi \(2010\)](#), and [Bhandari, Evans, Golosov, and Sargent \(2017\)](#) all studied Ramsey policies and find that the invariant distribution of state variables, while being well-defined in all cases that they consider, is discontinuous with respect to the size of aggregate shocks around a no-aggregate-shock case.

⁹While we compare our techniques to alternatives in detail in Section 3.3, an informal summary might help at this point. Perturbation methods of [Reiter \(2009\)](#) and [Kaplan, Moll, and Violante \(2018\)](#) are exact with

Section 3.1 considers a special case in which the state vector for a continuation Ramsey problem is a joint distribution of agents’ characteristics. This example allows us to describe essential aspects of our approach. Section 3.2 then extends things to settings with additional state variables that appear in the quantitative applications presented in later sections. Section 3.3 discusses numerical accuracy, computational speed, and comparisons with earlier methods for approximating equilibria of HA economies.

3.1. An Enlightening Special Case

In the following case, (i) utilitarian (i.e., equal) Pareto weights are imposed: (ii) equity holdings s_i are uniform across households; (iii) $\alpha = 1$, so that no intermediate goods are used as inputs; (iv) all shocks are i.i.d. These restrictions reduce the size of the state space while keeping it large enough to convey essential features of our technique. The second and third assumptions imply that the Phillips curve constraint (16) is slack in all periods and so can be omitted from the optimal monetary-fiscal policy problem. The last assumption implies that past shocks do not appear as arguments in optimal policy functions.

There are alternative ways to choose state vectors. Our approach works best when state vectors satisfy an independence property that we define below. In the simple economy under study, most popular choices of state variables satisfy this property. We purposefully adopt a recursive formulation that preserves the independence property in more general settings.

Let $M_t \equiv \int u_{c,i,t} di$ be the average marginal utility of consumption at time t , and let $m_{i,t} \equiv u_{c,i,t}/M_t$ be the scaled marginal utility of consumption of agent i at time t . We can interpret $m_{i,t}$ as (an inverse of) a “Negishi” weight that the planner attaches to agent i at time t .¹⁰ Replace (15) with

$$Q_t M_t m_{i,t} = \beta \mathbb{E}_t u_{c,i,t+1} (1 - Y_{t+1}^b) / (1 + \Pi_{t+1}), \quad m_{i,t} = u_{c,i,t} / M_t, \quad M_t = \int u_{c,i,t} di. \quad (20)$$

Let $\beta^t \mu_{i,t}$ be a Lagrange multiplier on constraint (18) for agent i . Following Marcet and Marimon (2019), the monetary-fiscal Ramsey planner’s Lagrangian is

$$\begin{aligned} & \inf \sup \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int \left[u(c_{i,t}, n_{i,t}) + (u_{c,i,t} c_{i,t} + u_{n,i,t} n_{i,t} - u_{c,i,t} (T_t + (1 - Y_t^d) D_t)) \mu_{i,t} \right. \\ & \left. + (1 - Y_t^b) \frac{b_{i,t-1}}{1 + \Pi_t} u_{c,i,t} (\mu_{i,t-1} - \mu_{i,t}) \right] di \end{aligned} \quad (21)$$

subject to $\mu_{i,-1} = 0$; the infimum is with respect to $\{\mu_{i,t}\}_{i,t}$ and the supremum is with respect to the stochastic process

$$\{c_{i,t}, n_{i,t}, b_{i,t}, m_{i,t}, C_t, N_t, D_t, T_t, M_t, Q_t, \Pi_t, Y_t\}_{i,t}$$

subject to constraints (8)–(10), (14), and (20).

respect to the dependence of policy functions on idiosyncratic shocks but only first-order approximate with respect to aggregate shocks, all around a fixed distribution $\bar{\Omega}$. That means that approximation errors are on the order of $\mathcal{O}(\sigma_{\text{agg}}^2 \|\Omega - \bar{\Omega}\|^2)$, where σ_{agg} measures the size of aggregate shocks. Childers (2018) provided a formal treatment. Our approach constructs expansions with respect to both aggregate and idiosyncratic shocks around the time t distribution Ω_t in that period, and can be done to an arbitrary order of approximation. Approximation errors are of the order $\mathcal{O}(\sigma_{\text{agg}}^{n+1}, \sigma_{\text{idiosync}}^{n+1})$ for arbitrary n . The two approaches are therefore complementary and have advantages and disadvantages in different applications.

¹⁰We call $m_{i,t}$ as “Negishi” weights to distinguish from Pareto weights ϑ_i .

3.1.1. *Computational Strategy*

In the Online Supplementary Material (Bhandari, Evans, Golosov, and Sargent (2021)) in the Appendix, we show that the solution to (21) can be conveniently split into a set of functions that describe the $t = 0$ choices of the Ramsey planner, and a set of functions for $t \geq 1$ choices. We also show that the $t \geq 1$ allocation is a function of the bivariate distribution over $z_{i,t-1} \equiv (m_{i,t-1}, \mu_{i,t-1})$. We denote this distribution by Ω and use z to denote a typical value in the support of Ω . Working backwards, we solve problem (21) in two steps. First, we solve a typical continuation Ramsey planner’s problem for a $t \geq 1$. Second, we solve a time $t = 0$ Ramsey problem to obtain an allocation $\{c_{i,0}, n_{i,0}\}_i$ and a distribution Ω_0 , both as functions of the initial state $\{\theta_{i,-1}, s_{i,-1}, b_{i,-1}\}_i$ confronting the Ramsey planner. We devote most of the text to the continuation Ramsey plan and focus on how policy functions depend on the cross-section distribution of agent’s characteristics.¹¹

3.1.2. *Computing a Continuation Ramsey Plan*

We use tildes to denote policy functions for a time $t \geq 1$ continuation Ramsey plan. *Aggregate* policy functions determine the time t values of all upper-case choice variables in problem (21). We denote the vector of these functions by $\tilde{X}(\Omega, \mathcal{E})$, where \mathcal{E} is a vector of aggregate shocks. *Individual* policy functions determine all lower-case time t choice variables for the planner in problem (21). We denote individual policy functions by $\tilde{x}(z, \Omega, \epsilon, \mathcal{E})$, where ϵ is a vector of idiosyncratic shocks. Policy functions for individual states \tilde{z} are components of \tilde{x} . We define p to be a matrix that selects \tilde{z} from \tilde{x} so that $\tilde{z} = p\tilde{x}$. The law of motion for the aggregate state is $\Omega' = \tilde{\Omega}(\Omega, \mathcal{E})$.

Consider the subset of first-order optimality conditions for problem (21) for $t \geq 1$. We split these conditions into two groups. The first group consists of optimality conditions for individual choices that connect current period individual and aggregate policy functions \tilde{x}, \tilde{X} ; current period realizations of shocks (ϵ, \mathcal{E}) ; and expectations of current and next period policy functions, $\mathbb{E}[\tilde{x}|z, \Omega]$ and $\mathbb{E}[\tilde{x}(\tilde{z}(z, \Omega, \epsilon, \mathcal{E}), \tilde{\Omega}(\Omega, \mathcal{E}), \cdot, \cdot)|z, \Omega, \epsilon, \mathcal{E}]$. To economize notation, we denote these two mathematical expectations by $\mathbb{E}_-\tilde{x}$ and $\mathbb{E}_+\tilde{x}$, respectively. The first group of conditions can be written as

$$F(\mathbb{E}_-\tilde{x}, \tilde{x}, \mathbb{E}_+\tilde{x}, \tilde{X}, \epsilon, \mathcal{E}, z) = \mathbf{0} \tag{22}$$

for a collection of functions F .¹² The second group of optimality conditions for a continuation Ramsey problem are various aggregate feasibility constraints and first-order conditions with respect to \tilde{X} that connect aggregate functions and averages of individual policy functions. These conditions can be written as

$$R\left(\int \tilde{x} d\Omega, \tilde{X}, \mathcal{E}\right) = \mathbf{0} \tag{23}$$

for some mapping R . The law of motion for measure Ω is

$$\Omega'(z) = \tilde{\Omega}(\Omega, \mathcal{E})(z) = \int \iota(\tilde{z}(y, \Omega, \epsilon, \mathcal{E}) \leq z) d\Pr(\epsilon) d\Omega(y) \quad \forall z \tag{24}$$

¹¹See the Appendix in the Online Supplementary Material for details about the time $t = 0$ Ramsey problem.

¹²Strictly speaking, if \tilde{x} consists of all lower-case choice variables and multipliers in problem (21), then the relevant objects are $\mathbb{E}_-f(\tilde{x})$ and $\mathbb{E}_+g(\tilde{x})$ for some transformations f and g . Our exposition would become more general if we were to extend the definition of \tilde{x} to include variables $f(\tilde{x})$ and $g(\tilde{x})$, for example, by including variable \tilde{u}_c in vector \tilde{x} and its definition $\tilde{u}_c = u_c(\tilde{c}, \tilde{n})$ in mapping F .

where $\iota(\tilde{z} \leq z)$ is 1 if all elements of \tilde{z} are less than or equal to all elements of z and zero otherwise.

At each point in time $t \geq 1$, we use perturbation methods to approximate how continuation Ramsey policy functions depend on ε, \mathcal{E} shocks as the cross-section distribution Ω_t of individual characteristics evolves through a simulated history. From these approximations, we can deduce how the aggregate shock \mathcal{E}_t affects the time $t + 1$ distribution Ω_{t+1} . See the Appendix in the Online Supplementary Material for the list of equations that constitute the F and R mappings for the Ramsey problem (21).

To construct small-noise approximations of policy functions, we consider a family of economies parameterized by a positive scalar σ that multiplies *all* shocks ε, \mathcal{E} , so that policy functions are $\tilde{X}(\Omega, \sigma\mathcal{E}; \sigma)$ and $\tilde{x}(z, \Omega, \sigma\varepsilon, \sigma\mathcal{E}; \sigma)$. Let $\bar{X}(\Omega)$ and $\bar{x}(z, \Omega)$ denote these functions evaluated at $\sigma = 0$. We will often suppress dependence on Ω when it is clear from the context.¹³ We assume that policy functions are smooth enough to justify taking derivatives. We let $\bar{X}_\varepsilon, \bar{x}_\varepsilon(z), \bar{x}_\varepsilon(z)$ be gradients of policy functions with respect to aggregate and idiosyncratic shocks, and \bar{X}_σ and $\bar{x}_\sigma(z)$ denote their derivatives with respect to σ , all evaluated at $\sigma = 0$. Similarly, $\bar{\Omega}_\varepsilon$ refers to the gradient of $\tilde{\Omega}(\Omega, \sigma\mathcal{E}; \sigma)$ with respect to aggregate shocks at $\sigma = 0$. First-order small noise expansions of policy functions are

$$\tilde{X}(\Omega, \sigma\mathcal{E}; \sigma) = \bar{X} + \sigma(\bar{X}_\varepsilon\mathcal{E} + \bar{X}_\sigma) + \mathcal{O}(\sigma^2) \tag{25}$$

and

$$\tilde{x}(z, \Omega, \sigma\varepsilon, \sigma\mathcal{E}; \sigma) = \bar{x}(z) + \sigma(\bar{x}_\varepsilon(z)\varepsilon + \bar{x}_\varepsilon(z)\mathcal{E} + \bar{x}_\sigma(z)) + \mathcal{O}(\sigma^2). \tag{26}$$

Higher-order expansions are constructed analogously.

3.1.3. Zeroth-Order Expansions

Higher-order approximations of policy functions use inputs from lower-order approximations, so we start with a zeroth-order approximation constructed from an economy without shocks. We use bars to denote zeroth-order approximations to functions.

LEMMA 1: *For any Ω and any z , zeroth-order approximations to policy functions satisfy $\bar{z}(z, \Omega) = z$ and, therefore, $\bar{\Omega}(\Omega) = \Omega$.*

PROOF: The first-order condition with respect to $b_{i,t-1}$ in (21) is

$$\mathbb{E} \left[\frac{\tilde{u}_c(z, \Omega, \cdot, \cdot)}{1 + \tilde{\Pi}(\Omega, \cdot, \cdot)} (\mu - \tilde{\mu}(z, \Omega, \cdot, \cdot)) \right] = 0,$$

which implies $\bar{\mu}(z, \Omega) = \mu$ for all z, Ω . To the zeroth-order equation (20) is

$$\bar{Q}(\Omega)\bar{M}(\Omega)m = \beta\bar{m}(z, \Omega)\bar{M}(\bar{\Omega}(\Omega))(1 + \bar{\Pi}(\bar{\Omega}(\Omega)))^{-1}.$$

Since Negishi weights $m(z, \Omega)$ integrate to one, this equation implies

$$\bar{Q}(\Omega)\bar{M}(\Omega) = \beta\bar{M}(\bar{\Omega}(\Omega))(1 + \bar{\Pi}(\bar{\Omega}(\Omega)))^{-1}$$

and, therefore, that $\bar{m}(z, \Omega) = m$ for all z, Ω .

Q.E.D.

¹³For instance, \bar{X} would refer to the function $\bar{X}(\Omega) \equiv \bar{X}(\Omega, \mathbf{0}, 0)$, $\bar{x}(z)$ would refer to the function $\bar{x}(z, \Omega) \equiv \bar{x}(z, \Omega, \mathbf{0}, \mathbf{0}, 0)$, and so on.

The cross-sectional distribution of characteristics Ω stays constant because in a $\sigma = 0$ economy a continuation Ramsey planner wants to keep Negishi weights and Lagrange multipliers on individuals' budget constraints constant over time for all agents. This makes sense because the $\sigma = 0$ economy shuts down all randomness, and so a market structure with a risk-free bond only is enough to support perfect smoothing of agents' quantity choices.

Lemma 1 implies that $\bar{x}(\bar{z}(z, \Omega), \bar{\Omega}(\Omega)) = \bar{x}(z, \Omega)$. Therefore, we can compute \bar{X} and $\bar{x}(z)$ by solving a system of nonlinear equations

$$\bar{F}(z) \equiv F(\bar{x}(z), \bar{x}(z), \bar{x}(z), \bar{X}, \mathbf{0}, \mathbf{0}, z) = \mathbf{0}, \quad \bar{R} \equiv R\left(\int \bar{x}(z) d\Omega(z), \bar{X}, \mathbf{0}\right) = \mathbf{0}. \quad (27)$$

From zeroth-order terms \bar{X} and $\bar{x}(z)$, we construct several objects that we use to compute higher-order terms. Let \bar{R}_x be the derivative of the mapping R with respect to its first argument $\int \bar{x} d\Omega$, and let \bar{R}_X and \bar{R}_ε be derivatives of R with respect to its second and third arguments, respectively, all evaluated at $\sigma = 0$. Similarly, let subscripts $x-$, x , $x+$, X , ε , \mathcal{E} , and z of \bar{F} denote corresponding derivatives of F with respect to each of its arguments evaluated at $\sigma = 0$. From the implicit function theorem, we have $\bar{x}_z(z) = [\bar{F}_{x-}(z) + \bar{F}_x(z) + \bar{F}_{x+}(z)]^{-1} \bar{F}_z(z)$. All of these objects can be constructed from \bar{X} , $\bar{x}(z)$.

Finally, we use $\partial\bar{x}(z, \Omega)$, $\partial\bar{X}(\Omega)$ to denote Fréchet derivatives of $\bar{x}(z, \Omega)$ and $\bar{X}(\Omega)$ with respect to the measure Ω .¹⁴ Fréchet derivatives generalize the notion of gradients to infinite-dimensional variables and capture how changes in the distribution Ω affect policy functions. In principle, these Fréchet derivatives could be calculated from (27), but except for some very simple cases, that approach is impractical because of how the number of unknowns in the operators $\partial\bar{x}(\cdot, \Omega)$ and $\partial\bar{X}(\Omega)$ grows exponentially with the size of Ω . We show that this problem can be overcome when policy functions satisfy an *independence property* that we define in the next corollary.

COROLLARY 1: *Policy function \bar{z} satisfies the independence property: $\partial\bar{z}(z, \Omega) = \mathbf{0}$ for all z, Ω .*

Corollary 1 asserts that at $\sigma = 0$, the Fréchet derivative of policy functions for individual states equals zero. This property eases the task of calculating $\partial\bar{\Omega}$, a key term in our expansions. In the case studied in this section, Corollary 1 and Lemma 1 imply $\partial\bar{\Omega} = I$. More generally, we show that as long as the independence property is satisfied, $\partial\bar{\Omega}$ can be expressed in terms of $\bar{z}_z(z)$, which is easy to compute.

¹⁴A Fréchet derivative of some variable $\bar{X}(\Omega)$ is a linear operator from the space of distributions Ω to \mathbb{R} with a property that $\lim_{\|\Delta\| \rightarrow 0} \frac{\|\bar{X}(\Omega+\Delta) - \bar{X}(\Omega) - \partial\bar{X}(\Omega) \cdot \Delta\|}{\|\Delta\|} = 0$. It can be found by fixing a feasible direction Δ and calculating a directional (Gateaux) derivative, since when both derivatives exist, they coincide, $\partial\bar{X}(\Omega) \cdot \Delta = \lim_{\alpha \rightarrow 0} \frac{\bar{X}(\Omega+\alpha\Delta) - \bar{X}(\Omega)}{\alpha}$. Following Luenberger (1997), we refer to $\partial\bar{X}(\Omega) \cdot \Delta$ as a Fréchet derivative of \bar{X} at a point Ω with increment Δ . Think of $\partial\bar{X}(\Omega)$ as a measure and $\partial\bar{X}(\Omega) \cdot \Delta$ as an integral of function Δ with respect to $\partial\bar{X}(\Omega)$.

3.1.4. *First-Order Expansions*

We can now construct a first-order Taylor expansion of equations (22)–(24). As a preliminary step, use Lemma 1 and observe that expansions of $\mathbb{E}_-\tilde{\mathbf{x}}$ and $\mathbb{E}_+\tilde{\mathbf{x}}$ are

$$\begin{aligned} \mathbb{E}_+\tilde{\mathbf{x}} &= \bar{\mathbf{x}}(z) + [\bar{\mathbf{x}}_z(z)\mathfrak{p}\bar{\mathbf{x}}_\mathcal{E}(z) + \partial\bar{\mathbf{x}}(z) \cdot \bar{\Omega}_\mathcal{E}] \sigma \mathcal{E} + [\bar{\mathbf{x}}_z(z)\mathfrak{p}\bar{\mathbf{x}}_\sigma(z)] \sigma \mathcal{E} + \bar{\mathbf{x}}_\sigma(z) \sigma + \mathcal{O}(\sigma^2), \\ \mathbb{E}_-\tilde{\mathbf{x}} &= \bar{\mathbf{x}}(z) + \bar{\mathbf{x}}_\sigma(z) \sigma + \mathcal{O}(\sigma^2). \end{aligned}$$

This implies that the Ramsey planner’s optimality conditions equations (22) and (23) satisfy, up to $\mathcal{O}(\sigma^2)$,

$$\begin{aligned} &\bar{F}(z) + [(\bar{F}_x(z) + \bar{F}_{x^+}(z)\bar{\mathbf{x}}_z(z)\mathfrak{p})\bar{\mathbf{x}}_\mathcal{E}(z)] \sigma \mathcal{E} \\ &\quad + [\bar{F}_{x^+}(z)\partial\bar{\mathbf{x}}(z) \cdot \bar{\Omega}_\mathcal{E} + \bar{F}_X(z)\bar{X}_\mathcal{E} + \bar{F}_\mathcal{E}(z)] \sigma \mathcal{E} \\ &\quad + [(\bar{F}_x(z) + \bar{F}_{x^+}(z)\bar{\mathbf{x}}_z(z)\mathfrak{p})\bar{\mathbf{x}}_\sigma(z) + \bar{F}_\sigma(z)] \sigma \mathcal{E} \\ &\quad + [(\bar{F}_{x^-}(z) + \bar{F}_x(z) + \bar{F}_{x^+}(z))\bar{\mathbf{x}}_\sigma(z) + \bar{F}_X\bar{X}_\sigma] \sigma = \mathbf{0} \end{aligned} \tag{28}$$

and

$$\bar{R} + \left[\bar{R}_x \int \bar{\mathbf{x}}_\mathcal{E}(z) d\Omega + \bar{R}_X \bar{X}_\mathcal{E} + \bar{R}_\mathcal{E} \right] \sigma \mathcal{E} + \left[\bar{R}_x \int \bar{\mathbf{x}}_\sigma(z) d\Omega + \bar{R}_X \bar{X}_\sigma \right] \sigma = \mathbf{0}. \tag{29}$$

The values for $\{\bar{\mathbf{x}}_\mathcal{E}(z), \bar{\mathbf{x}}_\sigma(z), \bar{X}_\sigma, \bar{X}_\mathcal{E}, \bar{X}_\mathcal{E}\}$ are fully characterized by equations (28) and (29) since these equations hold for all \mathcal{E} , \mathcal{E} and σ .

Let’s consider each of these functions in turn. From (28), we immediately get

$$\bar{\mathbf{x}}_\mathcal{E}(z) = -(\bar{F}_x(z) + \bar{F}_{x^+}(z)\bar{\mathbf{x}}_z(z)\mathfrak{p})^{-1} \bar{F}_\mathcal{E}(z).$$

All the terms on the right-hand side are known from the zeroth-order expansion, so we can compute $\bar{\mathbf{x}}_\mathcal{E}(z)$ by matrix inversion. This step is easily parallelizable because the computation can be done separately for each z . Terms $\bar{\mathbf{x}}_\sigma(z)$ and \bar{X}_σ can be computed in a similar way; it is straightforward to verify that they equal zero.

Calculating $\bar{\mathbf{x}}_\mathcal{E}(z)$ and $\bar{X}_\mathcal{E}$ is more challenging. The aggregate shock \mathcal{E} changes next period’s state by $\bar{\Omega}_\mathcal{E}$ and that alters expectations of next period’s policies by $\partial\bar{\mathbf{x}}(z) \cdot \bar{\Omega}_\mathcal{E}$, as can be seen from the first square bracket in (28). Neither $\partial\bar{\mathbf{x}}(z)$ nor $\bar{\Omega}_\mathcal{E}$ is known at this stage. The next theorem and proof use functional derivatives to construct $\partial\bar{\mathbf{x}}(z) \cdot \bar{\Omega}_\mathcal{E}$.

THEOREM 1: *From the zeroth-order expansion, we can construct matrices $A(z)$ and $C(z)$ that satisfy*

$$\partial\bar{\mathbf{x}}(z) = C(z)\partial\bar{X}, \tag{30a}$$

$$\partial\bar{\mathbf{x}}(z) \cdot \bar{\Omega}_\mathcal{E} = C(z)\partial\bar{X} \cdot \bar{\Omega}_\mathcal{E} = C(z) \int A(y)\bar{\mathbf{x}}_\mathcal{E}(y) d\Omega(y). \tag{30b}$$

PROOF: Lemma 1 implies $\partial\bar{\Omega} = \mathbf{1}$. Fréchet derivatives of (22) and (23) with arbitrary increment Δ satisfy

$$(\bar{F}_{x-}(z) + \bar{F}_x(z) + \bar{F}_{x+}(z) + \bar{F}_{x+\bar{x}_z}(z)\rho)\partial\bar{x}(z) \cdot \Delta + \bar{F}_X(z)\partial\bar{X} \cdot \Delta = \mathbf{0}, \tag{31a}$$

$$\bar{R}_x\partial\left(\int \bar{x}(y) d\Omega(y)\right) \cdot \Delta + \bar{R}_X\partial\bar{X} \cdot \Delta = \mathbf{0}. \tag{31b}$$

The first equation yields (30a) with

$$C(z) = -(\bar{F}_{x-}(z) + \bar{F}_x(z) + \bar{F}_{x+}(z) + \bar{F}_{x+\bar{x}_z}(z)\rho)^{-1}\bar{F}_X(z).$$

Since directional and Fréchet derivatives coincide, by fixing a direction Δ and computing the directional derivative (see footnote 14), we obtain

$$\partial\left(\int \bar{x}(y) d\Omega(y)\right) \cdot \Delta = \int (\partial\bar{x}(y) \cdot \Delta) d\Omega(y) + \int \bar{x}(y) d\Delta(y). \tag{32}$$

We want to evaluate the integral on the right side at $\Delta = \bar{\Omega}_\varepsilon$. Differentiating (24) at any $z = (m, \mu)$ and applying Lemma 1 gives

$$\bar{\Omega}_\varepsilon(m, \mu) = -\int_{y_2 \leq \mu} \bar{m}_\varepsilon(m, y_2)\omega(m, y_2) dy_2 - \int_{y_1 \leq m} \bar{\mu}_\varepsilon(y_1, \mu)\omega(y_1, \mu) dy_1,$$

where ω is the density of Ω . The density of $\bar{\Omega}_\varepsilon(m, \mu)$, which is denoted by $\bar{\omega}_\varepsilon(m, \mu)$, is then

$$\bar{\omega}_\varepsilon(m, \mu) = -\frac{d}{dm}[\bar{m}_\varepsilon(m, \mu)\omega(m, \mu)] - \frac{d}{d\mu}[\bar{\mu}_\varepsilon(m, \mu)\omega(m, \mu)].$$

Substitute this equation and (30a) into (32) to get

$$\begin{aligned} \partial\left(\int \bar{x}(y) d\Omega(y)\right) \cdot \bar{\Omega}_\varepsilon &= \int C(y)\partial\bar{X} \cdot \bar{\Omega}_\varepsilon d\Omega(y) - \int \bar{x}(y)\frac{d}{dm}[\bar{m}_\varepsilon(y)\omega(y)]dy \\ &\quad - \int \bar{x}(y)\frac{d}{d\mu}[\bar{\mu}_\varepsilon(y)\omega(y)]dy \\ &= (\partial\bar{X} \cdot \bar{\Omega}_\varepsilon) \int C(y) d\Omega(y) + \int \bar{x}_z(y)\rho\bar{x}_\varepsilon(y) d\Omega(y), \end{aligned}$$

where the second equality follows from integration by parts. Substitute this expression into (31b) and solve for $\partial\bar{X} \cdot \bar{\Omega}_\varepsilon$ to obtain

$$\bar{X}'_\varepsilon \equiv \partial\bar{X} \cdot \bar{\Omega}_\varepsilon = \int A(y)\bar{x}_\varepsilon(y) d\Omega(y), \tag{33}$$

where $A(z) = -(\bar{R}_x \int C(y) d\Omega(y) + \bar{R}_X)^{-1}\bar{R}_x\bar{x}_z(z)\rho$. Together with (30a), we get (30b). *Q.E.D.*

Economic forces drive Theorem 1. In a competitive equilibrium, agents care about the distribution Ω only because it helps them predict aggregate prices and income. That

means that effects $\partial\bar{x}$ on individual variables from a perturbation of distribution Ω can be factored into effects $\partial\bar{X}$ on aggregate variables and a known loading matrix $C(z)$, which captures how individual variables respond to changes in the aggregates. Equation (30a) captures this.

Feasibility and market clearing impose a tight relationship between how individual policy functions respond to aggregate shocks in the current period, $\bar{x}_\varepsilon(z)$, and how aggregates are expected to change next period, \bar{X}'_ε . This relationship sets up a fixed-point problem presented in equation (33). Together with (30a), equation (33) allows us to express the Fréchet derivative $\partial\bar{x} \cdot \bar{\Omega}_\varepsilon$ as a linear function of $\bar{x}_\varepsilon(z)$.

The preceding analysis puts us in a position to compute the coefficients \bar{x}_ε and \bar{X}'_ε . Setting the first square brackets in (28) and (29) to zero and using the definition of \bar{X}'_ε from (33), we obtain the following system of linear equations in the unknowns $\bar{X}_\varepsilon, \bar{x}_\varepsilon(z)$ for all z :

$$(\bar{F}_x(z) + \bar{F}_{x+}(z)\bar{x}_z(z)\rho)\bar{x}_\varepsilon(z) + \bar{F}_{x+}(z)C(z)\bar{X}'_\varepsilon + \bar{F}_x(z)\bar{X}_\varepsilon + \bar{F}_\varepsilon(z) = \mathbf{0}, \quad (34a)$$

$$\bar{R}_x \int \bar{x}_\varepsilon(y) d\Omega(y) + \bar{R}_x\bar{X}_\varepsilon + \bar{R}_\varepsilon = \mathbf{0}. \quad (34b)$$

This linear system allows us to split one large problem of simultaneously finding $\bar{x}_\varepsilon(z)$ for all z into a large number of small problems that independently characterize $\bar{x}_\varepsilon(z)$ for each z . Thus, we use equation (34a) to calculate matrices $D_0(z)$ and $D_1(z)$, which define the affine function

$$\bar{x}_\varepsilon(z) = D_0(z) + D_1(z) \cdot [\bar{X}_\varepsilon \quad \bar{X}'_\varepsilon]^\top.$$

We can substitute this function into equations (33) and (34b) to compute \bar{X}_ε and \bar{X}'_ε . Values of $\bar{x}_\varepsilon(z)$ can be found either from the previous equation or from (30a). This completes calculations comprising first-order terms.

3.1.5. Higher-Order Expansions

Because a generalization of Theorem 1 applies to higher-order expansions, our approach preserves linear and parallelizable structures when used to construct second- and higher-order expansions. The independence property, $\partial\bar{z}(z, \Omega) = \mathbf{0}$, allows a counterpart to equation (30a) to hold for all higher-order Fréchet derivatives. This enables us to compute higher-order analogues of $\partial\bar{x} \cdot \bar{\Omega}_\varepsilon$ explicitly as weighted sums of higher-order coefficients $\bar{x}_{\varepsilon\varepsilon}, \bar{x}_{\varepsilon\sigma}, \bar{x}_{\sigma\sigma}, \dots$, with weights known from lower-order expansions. We then can form higher-order analogues of equations (34). The structure of these equations allows us again to split one large system of equations into a large number of low-dimensional linear problems that can be solved fast and simultaneously. Formal proofs and constructions involve much additional notation, but the steps mirror those in Section 3.1.4. The Appendix in the Online Supplementary Material provides details.

3.2. Approximations More Generally

To use our small-noise expansion method to approximate a Ramsey plan for the Section 2 economy, we modify two features of the Section 3.1 computations. First, now the optimality condition (16) typically binds and cannot be omitted. We add this constraint to our Lagrangian formulation (21), so its multiplier Λ now becomes an aggregate state

variable for the continuation Ramsey problem. Second, since shocks are persistent, policy functions also depend on previous period values of aggregate shocks $\Theta = (\Theta, \Phi)$ as well as idiosyncratic shocks θ . Thus, now $z = (m, \mu, s, \theta, \vartheta)$ is the individual state, Ω is a measure over z , and the aggregate and individual policy functions are functions $\tilde{X}(\Omega, \Lambda, \Theta, \mathcal{E})$ and $\tilde{x}(z, \Omega, \Lambda, \Theta, \mathcal{E}, \varepsilon)$, respectively. Zeroth-order terms have nontrivial (deterministic) transition paths that can be computed with a shooting algorithm.

With persistent shocks, two ways to perturb policy functions yield approximation errors of the same orders of magnitude. One is to scale $\{\sigma\mathcal{E}, \sigma\varepsilon\}$ and expand with respect to σ around current values of (Θ, θ) and Ω . Since to the zeroth-order $\bar{\theta}(z, \Omega) \neq \theta$, it is no longer true that $\bar{\Omega}(\Omega) = \Omega$, so Lemma 1 does not apply.¹⁵ However, functional derivative techniques used to prove Theorem 1 still apply, and we can construct required Fréchet derivatives along the transition path. Tractability is preserved because policy functions still satisfy the independence property, that is, $\partial\tilde{x}(z, \Omega) = \mathbf{0}$ for all z, Ω . The law of motion for exogenous variables does not depend on the distribution Ω , so adding those variables to vector z leaves the independence property intact.

An alternative approach is to scale as $\{\sigma\mathcal{E}, \sigma\varepsilon, \sigma\Theta, \sigma\theta\}$ and then to expand around $\sigma = 0$. Since θ is a component of the vector z of individual characteristics z and, therefore, Ω are now also functions of the scaling parameter σ . A zeroth-order approximation satisfies Lemma 1, but expansions of policy functions involve additional Fréchet derivatives including $\partial\tilde{X} \cdot \bar{\Omega}_\sigma$ and $\partial\tilde{x}(z) \cdot \bar{\Omega}_\sigma$. These derivatives are easy to compute using techniques deployed in Theorem 1.

Although the two approaches imply errors of the same orders of approximation, one approach can be better than the other depending on circumstances. We use the first approach in our application, but in some cases, the second approach maybe can be faster to implement as it does not require computing a transition path. The Appendix in the Online Supplementary Material provides explicit formulas and extensions of Theorem 1 for both approaches.

3.3. Accuracy and Comparisons

Our method builds on perturbation techniques widely used in computational economics (see, e.g., Judd and Guu (1993), Judd and Guu (1997), and Schmitt-Grohe and Uribe (2004b)). We perturb “around the current state” in a way closely related to practices of Fleming (1971), Fleming and Souganidis (1986), Anderson, Hansen, and Sargent (2012), Bhandari et al. (2017), and Phillips (2017). In all of those applications, the state vector has low dimension, and approximations do not require computing high-dimensional Fréchet derivatives. Our approach is designed to apply even when the underlying state is a complicated, high-dimensional object, as typically occurs in HA economies.

To our knowledge, ours is the first method that incorporates effects of the complete current state on continuation Ramsey plans in HA economies. Our approach applies to HA economies for which equilibrium dynamics can be written in the form given by equations (22)–(24), a large class of economies.¹⁶

¹⁵When persistence of the idiosyncratic shocks ρ_θ is close to one, we can recover Lemma 1 if we approximate ρ_θ by $\rho_\theta(\sigma) = 1 - \sigma\rho$ for some $\rho \geq 0$ and expand $\rho_\theta(\sigma)$ with respect to σ .

¹⁶HA economies with inequality constraints, such as ones with additional ad hoc debt limits, can be written in this form by including appropriate complementary slackness conditions. Inequality constraints often imply policy functions with kinks. Although such kinks violate the smoothness assumption that we imposed on equations (25) and (26), we foresee no impediments to extending our method to such structures in our future work.

To assess accuracy, we approximate a competitive equilibrium for a given monetary-fiscal policy within an environment that we have simplified enough to allow us to compute an equilibrium analytically. We then compare that analytical solution to our approximation by varying key parameters likely to affect approximation quality.

In particular, we follow Acharya and Dogra (2018) and assume that labor is supplied inelastically at $n_{i,t} = 1$; preferences are given by $U(c_t, n_t) = -\exp(-\gamma c_t)$; equity holdings are uniform across consumers; there are no aggregate shocks; idiosyncratic shocks $\epsilon_{i,t}$ are i.i.d. normal; government spending and all tax rates equal zero; and interest rates are set according to a Taylor rule $Q_t^{-1} - 1 = a_0(1 + \Pi_t)^{a_1}$ for coefficients a_0 and a_1 chosen to make steady-state inflation equal zero.

Under these assumptions, household income $W_t \epsilon_{i,t} + T_t + D_t$ is normally distributed. Together with the CARA utility function, that means that the consumption-saving problem can be solved analytically. One can then derive explicit expressions both for steady-state aggregate quantities and for deterministic transition paths from given initial conditions.

Acharya and Dogra (2018) called this a Pseudo Representative Agent New Keynesian economy (abbreviated as PRANK) and drew from it useful insights about how more complicated HANK models work. They showed that their PRANK economy has a unique steady state in which all aggregate variables, including output, inflation, and real interest rates, are constant. In a steady state, individual assets follow a random walk, so the dispersion of asset holdings across agents grows without limit. The availability of explicit expressions for policy functions along the transition path means that there are also expressions for how this PRANK economy is affected by an unanticipated aggregate shock.

We list equilibrium conditions and calibrated parameter values in the Appendix in the Online Supplementary Material. We start at the steady state and study equilibrium responses to a one-time, unanticipated 1.23% shock to aggregate productivity in period t that then decays deterministically.¹⁷ We compare our second-order approximation to the exact solution. We report two comparisons: one in which the shock occurs in period $t = 1$ and another one in which it occurs in period $t = 250$. In both cases, shocks arrive when all aggregate variables are at the same steady-state values; the two cases differ only in spreads of asset distributions at the time of the shock.

The solid lines in Figure 1 show exact and approximate impulse responses of output, inflation, and asset inequality measured by the standard deviation of individual wealth. They are almost identical in both experiments. The PRANK economy is engineered to make impulse responses of output and inflation be independent of the asset distribution, so dynamics of output and inflation are the same in the top and bottom rows of Figure 1. This is not the case for other items of potential interest such as dynamics of asset inequality, as can be seen in the rightmost panels in Figure 1.

By comparing the two experiments, we can evaluate the precision of our approximation and how it deteriorates with the time horizon. In the PRANK economy, we can calculate distribution Ω_t exactly for all t and the corresponding impulse responses. Our approach approximates policy functions and distributions for $t = 1, 2, \dots$, so approximation errors can accumulate over time as we compute responses far into the future. Individual wealth follows a random walk process so approximation errors accrued by our method endure forever, making this environment a worst-case test bed for our approximation method. Despite that, we find that our approximate distribution is very close to the exact distribution even at $t = 250$ (see the Appendix in the Online Supplementary Material) and that we capture responses of asset inequality to an aggregate shock in that period very well.

¹⁷This corresponds to a one standard deviation shock to productivity in our baseline economy.

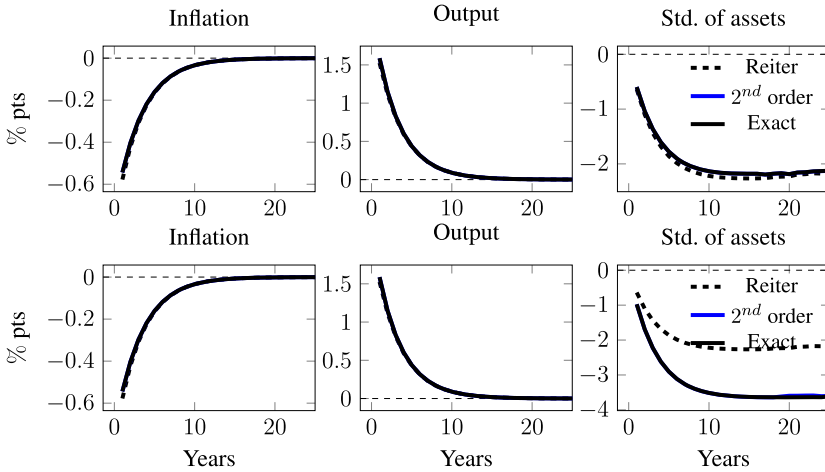


FIGURE 1.—Comparisons of impulse responses to a 1% TFP shock at $t = 1$ in the top panel and $t = 250$ in the bottom panel across approximation methods. The bold lines are the exact solution (“Exact”) and our method applied to second-order (“2nd order”). The dashed lines are responses under the Reiter method.

To document the accuracy, we compute several types of approximation errors. For the individual policy functions, we compute: (i) % gap in the approximated and true policies (ii) % gap in the approximated and policy rules implied by the Euler equation (or the Euler equation errors) and (iii) dynamic Euler equation errors from Den Haan (2010). The first two errors are acquired from the formulas¹⁸

$$E_{c,t}^{\text{pol}}(b, \epsilon) \equiv 100 \times \frac{|c_t^{\text{True}}(b, \epsilon) - c_t^{\text{apprx}}(b, \epsilon)|}{C_t}, \tag{35}$$

$$E_{c,t}^{\text{EE}}(b, \epsilon) \equiv 100 \times \frac{|c_t^{\text{imp}}(b, \epsilon) - c_t^{\text{apprx}}(b, \epsilon)|}{C_t}, \tag{36}$$

where $c_t^{\text{True}}(b, \epsilon)$ are constructed using the exact solution and $c_t^{\text{imp}}(b, \epsilon)$ is defined as

$$c_t^{\text{imp}}(b, \epsilon) = -\frac{1}{\alpha} \log \left(\frac{\beta}{Q_t} \mathbb{E}_t \left[\frac{1}{1 + \pi_{t+1}} \exp(-\alpha c_{t+1}^{\text{approx}}(b', \epsilon')) \right] \right).$$

The dynamic Euler equation errors are constructed by simulating a panel $\{\tilde{b}_{i,t}, \tilde{c}_{i,t}\}_{i,t}$ in the following recursive fashion. For some agent i , fix an initial value for assets $\tilde{b}_{i,-1}$ and a history of shocks $\{\epsilon_{i,t}\}$. At any period t , apply the function $c_t^{\text{imp}}(\cdot)$ to the pair $(\tilde{b}_{i,t-1}, \epsilon_{i,t})$ to construct $\tilde{c}_{i,t}$, and then use the budget constraint to construct $\tilde{b}_{i,t}$. The dynamic Euler equation errors are computed using the analogue of expression (36) in which we compare \tilde{c}_i to c_t^{apprx} , where the later is computed for the same sequence of shocks and initial assets. These errors have the advantage of allowing for the possibility of small errors accumulat-

¹⁸As CARA preferences feature an aversion to absolute risk and, possibly, negative consumption, we report the absolute errors scaled by average consumption.

TABLE I
APPROXIMATION ERRORS.

Maximum Errors (%)	Ind. Consumption	Agg. Output	Inflation	Interest Rate
<i>2nd Order</i>				
$\gamma = 1, \sigma_\epsilon = 0.50$	0.0039	4.2e-6	3.1e-5	4.3e-5
$\gamma = 1, \sigma_\epsilon = 0.75$	0.0134	2.6e-5	1.5e-4	2.2e-4
$\gamma = 1, \sigma_\epsilon = 1.00$	0.0328	8.2e-5	4.9e-4	6.9e-4
$\gamma = 3, \sigma_\epsilon = 0.5$	0.0453	0.0011	0.0024	0.0034
<i>Reiter-Based</i>				
$\gamma = 1, \sigma_\epsilon = 0.50$	0.0374	0.0616	0.0337	0.0505
$\gamma = 1, \sigma_\epsilon = 0.75$	0.0466	0.0610	0.0335	0.0501
$\gamma = 1, \sigma_\epsilon = 1.00$	0.0492	0.0602	0.0329	0.0493
$\gamma = 3, \sigma_\epsilon = 0.5$	0.0896	0.2252	0.1327	0.1991

Note: Percentage errors in policy functions in response to an one standard deviation unanticipated shock to aggregate TFP at date $t = 1$. The values reported are the maximum errors across states (b, ϵ) and time t relative to the true solution.

ing into large errors over time. For aggregate variable X_t , we simply report

$$E_{X,t} = 100 \times \frac{|X_t^{\text{True}} - X_t^{\text{apprx}}|}{X_t^{\text{True}}}.$$

We will often report maximum errors where the max is taken over state space (b, ϵ) as well as t .

For brevity, we summarize the % gap in the approximated and true policies errors and details of computational speed of our approach in Table I. (See the Appendix in the Online Supplementary Material for Euler equation and dynamic Euler equation errors.) The errors reported are all for a quadratic approximation. Percent errors relative to the true solution for the individual consumption policies are less than 0.05% and vary between 0.004%–0.033% as we double volatility of idiosyncratic shocks, while percent errors for aggregate output, inflation, and the interest rate range from $4.3 \times 10^{-5}\%$ to 0.0007%. In terms of percent errors, increasing the coefficient of relative risk aversion to 3 increases approximation errors by roughly the same amount as does doubling the volatility of the idiosyncratic shocks. Since our approach is easily parallelizable and sidesteps the computationally intensive step of computing Fréchet derivatives, it works quickly and allows us to simulate transition dynamics of a path of 100 periods in 1.5 seconds on a dual AMD EPYC 7351 processor with 32 cores.

3.4. Comparison to Reiter's Method

We can also compare our method to a widely-used approach of Reiter (2009). Our method with order n yields approximation errors that scale with the size of both idiosyncratic and aggregate shocks, that is, $\mathcal{O}(\sigma_{\text{agg}}^{n+1}, \sigma_{\text{idiosync}}^{n+1})$. An approximation like Reiter's delivers policy functions that are linear in aggregate shocks and globally accurate with respect to idiosyncratic shocks around a fixed distribution $\Omega_t = \bar{\Omega}$. Therefore, errors in such an approximation scale with both the size of the aggregate shock and the distance of the current distribution from the point of approximation $\mathcal{O}(\sigma_{\text{agg}}^2, \|\Omega - \bar{\Omega}\|^2)$. We use the PRANK setting to show the trade-offs involved in these two types of errors. All of our comparisons assume $n = 2$.

The bottom four rows of Table I present the percentage errors of the Reiter method¹⁹ relative to the analytic solution, allowing us to compare our method directly to Reiter's approach. Both approaches yield accurate approximations of the analytic solution with small percentage errors range from 0.04% to 0.9%. Despite that, we observe that our method yields errors for the individual consumption policy rules that are consistently smaller than those of the Reiter approach while errors for the aggregate variables are *two orders of magnitude smaller* than the Reiter approach. Our approach is less accurate with respect to the idiosyncratic risk, but those errors partially average out for the aggregates. Meanwhile, second-order errors in aggregates variables under the Reiter approach propagate down to the individual policy rules.

To extract long run consequences of these approximation errors, we augment the PRANK economy with a nondegenerate stochastic process for aggregate TFP. In line with standard calibrations, aggregate shocks are much less volatile than idiosyncratic shocks (standard deviation of innovations about 1% for aggregate TFP versus 50% for individual productivities). We then simulate a long sequence of aggregate shocks and compare distributions of assets at $t = 250$ associated with our method and Reiter's. We find that the obtained asset distributions are visibly different under the two methods, with the standard deviation of assets being more than 1% larger under Reiter's approach. To understand the source of these differences, we drop the second-order terms with respect to aggregate risk from our expansions and recompute the long run distribution associated with using this inferior approximation. This distribution, which has error of the order $\mathcal{O}(\sigma_{\text{agg}}^2, \sigma_{\text{idiosync}}^3)$, is almost identical to the distribution generated by Reiter's method. That finding prompts us to conclude that ignoring higher-order effects of aggregate shocks can inject long-run drifts into approximation errors. We report details about this and some related experiments in the Appendix of the Online Supplementary Material.

Consequences of ignoring movements of Ω_t away from $\bar{\Omega}$ can be gleaned from responses of the standard deviation of assets in Figure 1. While the PRANK economy is constructed so that responses of output and inflation do not depend on the asset distribution, the responses of other moments, including those that describe dispersion of individuals' asset holdings, do depend on it. This implies that Reiter's approximation of these impulse responses deteriorates progressively as the distribution of assets drifts away from the point of approximation. In this example, movement of the distribution away from the point of approximation is due to idiosyncratic income risk.

4. CALIBRATION

To isolate key trade-offs that a continuation Ramsey planner faces, we start from a baseline economy that is close to specifications commonly used in the New Keynesian literature. We start with an initial calibration that ignores important features, including the ample heterogeneities in marginal propensities to consume and effects of recessions on labor earnings that have been documented to prevail in U.S. data. We incorporate these as extensions in Section 6.

¹⁹Conventional applications of Reiter's method requires expanding policy functions around the invariant distribution in the economy without aggregate shocks. Since individual assets follow a random walk, no such distribution exists in the PRANK economy. In our application of Reiter's method, we used initial distribution Ω_0 as the point of expansion.

Preferences and Technology Parameters

We assume $u(c, n) = \frac{c^{1-\nu}}{1-\nu} - \frac{n^{1+\gamma}}{1+\gamma}$ and set $\nu = 3$, $\gamma = 2$. This yields a Frisch labor supply elasticity of 0.5. We calibrate to annual data and set the discount factor $\beta = 0.96$. We set $\bar{\Theta} = 1$ and $\bar{\Phi} = 6$ to attain average markups of 20%. We abstract from the use of intermediate goods in production and set $\alpha = 1$. We choose the cost of nominal price changes ψ to match the slope of the aggregate Phillips curve. [Sbordone \(2002\)](#) estimated the slope of the U.S. Phillips curve in quarterly data to be about 6%. We convert that to an annual frequency by multiplying by 4. To a first-order approximation the slope of the Phillips curve in our model is $(\Phi - 1)/\psi$, which implies $\psi = 21$.

Idiosyncratic and Aggregate Uncertainty

We assume that all shocks are Gaussian and set the standard deviations of $\varepsilon_{\epsilon,i,t}$ and $\varepsilon_{\theta,i,t}$ to 8.7% and 10.3% and the autocorrelation $\rho_{\theta} = 0.992$ to match evidence on individual wage dynamics from [Low, Meghir, and Pistaferri \(2010\)](#).

We calibrate the stochastic process for the markup shocks so that movements in the labor share of output are consistent with movements in the U.S. corporate sector's labor share (Table 1.14, NIPA) over the period 1947–2016. Calibrated values for $(\rho_{\Phi}, \sigma_{\Phi})$ are (0.85, 4.6%).²⁰ We calibrate the stochastic process for aggregate labor productivity $\log \Theta_t$ so that output per hour is consistent with detrended U.S. nonfarm real output per hour (BLS) over the period 1947–2016. Calibrated values for $(\rho_{\theta}, \sigma_{\theta})$ are (0.73, 1.23%).

Initial Conditions

A common approach in the heterogeneous agent macro-labor literature is to specify government policy (\bar{G}, Y_t, Q_t) and study long-run allocations in an associated competitive equilibrium. A deficiency of some current workhorse models is that their invariant distributions understate wealth inequality.²¹ As we shall see, asset inequality has important implications for optimal policy responses. We calibrate initial conditions $\{\theta_{i,-1}, b_{i,-1}, s_i\}_i$ to be consistent with empirical distributions of wages, nominal claims, and claims to real firm profits. In Section 6.1, we show that drift from this distribution is slow and that its presence matters very little for optimal policy responses. We use the 2007 wave of the Survey of Consumer Finances (SCF) as our benchmark for earnings and asset inequality. We adopt a procedure proposed by [Doepke and Schneider \(2006\)](#) to map household-level direct and indirect holdings of financial assets to the joint distribution of claims to nominal debt and claims to equity.²² Table II reports summary statistics for our sample. Of

²⁰There is substantial variety in how macro and financial economists have modeled and calibrated markup shocks. In the DSGE literature, for instance, [Smets and Wouters \(2007\)](#), [Justiniano, Primiceri, and Tambalotti \(2010\)](#), and [Galí, Gertler, and Lopez-Salido \(2007\)](#) used ARMA(1, 1) processes and estimate the quarterly persistence to be in the range of 0.90–0.95. In the finance literature, for instance, [Greenwald, Lettau, and Ludvigson \(2014\)](#) estimated factor share shocks with a monthly persistence of 0.995. Our calibrated value for $\rho_{\Phi} = 0.85$ lies within the range of these estimates.

²¹Incorporating one or more of the popular “fixes” to obtain a sufficiently skewed invariant distribution of wealth—for instance, allowing persistent shocks to discount factors ([Krusell and Smith \(1998\)](#)), bequests ([De Nardi \(2004\)](#)), entrepreneurial choice ([Cagetti and De Nardi \(2006\)](#)), or persistent idiosyncratic differences in returns to financial assets ([Benhabib, Bisin, and Luo \(2019\)](#)), and then computing a Ramsey allocation for such an economy would be interesting but is not something that we do in this paper.

²²The Appendix in the Online Supplementary Material contains details the sample restriction as well as how we apply the [Doepke and Schneider \(2006\)](#) procedure.

TABLE II
FIT OF THE INITIAL DISTRIBUTION.

Moments	
Fraction of pop. with zero equities	30%
Std. share of equities	2.63
Std. bond	6.03
Gini of financial wealth	0.82
Std. ln wages	0.80
Corr(share of equities, ln wages)	0.40
Corr(share of equities, bond holdings)	0.62
Corr(bond, ln wages)	0.33

Note: Moments correspond to SCF 2007 wave after scaling wages, equity holdings, and debt holdings by the average yearly wage in our sample. The share of equities refers to the ratio of individual equity holdings to the total in our sample; the weighted sum of shares equals one. Financial wealth is defined as the sum of nominal and real claims.

particular relevance to results presented in Section 5 is the fact that earnings and assets are positively correlated and that inequality in asset holdings is much larger than earnings inequality.

We calibrate government expenditures \bar{G} to be consistent with the ratio of nontransfer government expenditures to tax revenues. To obtain tax revenues, we model a stylized U.S. tax system. To be consistent with estimates of consolidated federal and state-level average marginal tax rates calculated in [Bhandari and McGrattan \(2019\)](#), we assume that tax rates on labor income, dividends, and interest income are time invariant and set to $(\bar{Y}^n, \bar{Y}^d, \bar{Y}^b) = \bar{Y}^{\text{US}} = (0.38, 0.34, 0)$.²³ We set \bar{G} so that on average the ratio of non-transfer government expenditures to total tax receipts equals 50%, also estimated by [Bhandari and McGrattan \(2019\)](#).

Continuation Ramsey responses depend on the joint distribution of assets and after-tax incomes. For our baseline simulations, we choose Pareto weights so that average optimal levels of taxes are similar to U.S. data. In particular, we assume that Pareto weights are described by

$$\vartheta_i \propto \exp(\delta_1 \theta_{i-1}) + \exp(\delta_2 s_{i-1}) + \exp(\delta_3 b_{i-1}), \quad (37)$$

where $(\theta_{i-1}, s_{i-1}, b_{i-1})$ are the three dimensions of initial heterogeneity and $\delta = (\delta_1, \delta_2, \delta_3)$ are parameters that we set so that in the nonstochastic economy setting $\bar{Y} = \bar{Y}^{\text{US}}$ is optimal. In Section 6.1, we discuss how outcomes vary with alternative choices of Pareto weights.

5. OPTIMAL MONETARY AND MONETARY-FISCAL POLICIES

The Ramsey planner sets a common lump-sum transfer for all agents as well as the nominal interest rate and tax rates on labor income, dividends, and bond income. The

²³To arrive at the estimate of the marginal tax rate on capital income, we combine the [Bhandari and McGrattan \(2019\)](#) estimates of the effective marginal tax rate on corporate business income, on distributed dividends, and the schedule of marginal tax rates on noncorporate business income into a single number by using the [Barro and Redlick \(2011\)](#) procedure. We use the same steps to combine the schedule of marginal tax rates on wage income into the flat tax rate used above. We set the tax rate on bond income to zero in order to represent the observation that most of government bonds are held through tax-deferred accounts.

TABLE III
RAMSEY ALLOCATION: MOMENTS.

	RANK					HANK				
	Std. Dev (%)	Correlations				Std. Dev (%)	Correlations			
		i_t	Π_t	W_t	$\ln Y_t$		i_t	Π_t	W_t	$\ln Y_t$
Nominal rate i_t	0.87	1				1.82	1			
Inflation Π_t	0.03	-0.01	1			0.46	-0.94	1		
Labor share W_t	1.18	-0.09	-0.32	1		2.13	-0.78	0.78	1	
Log output $\ln Y_t$	0.92	-0.98	-0.09	0.24	1	0.88	-0.31	0.10	0.12	1

Note: Moments are computed using allocations under RANK (left) and HANK (right) optimal monetary policies.

government acquires revenues directly through taxes and indirectly through nominal interest rates and inflation. It spends these revenues on servicing government debt, paying for exogenous expenditures \bar{G} , and financing transfers. Ricardian equivalence prevails, disarming effects that might otherwise be produced by altering the timing of transfers.²⁴ Explicit taxes and an implicit tax via inflation generate dead-weight losses. Average levels of taxes and transfers depend on inequalities in incomes from labor and assets and on Pareto weights.

We compare continuation Ramsey plans in our baseline HANK setting to continuation Ramsey plans in a representative agent version of our model (abbreviated as RANK). In the RANK economy, we keep all the parameters the same, except we assume that agents are identical with initial conditions being equal to the average levels in our baseline HANK calibration, and there are no idiosyncratic shocks. When we compute optimal monetary policy in the RANK economy, we set $(\bar{Y}^n, \bar{Y}^d, \bar{Y}^b) = (-1/\bar{\Phi}, 0, 0)$, which corresponds to the nonstochastic optimum.

Table III reports several statistics that summarize stochastic properties of optimal monetary policies in the HANK and the RANK models (stochastic properties of optimal monetary-fiscal policies are similar and are omitted). We refer to these statistics as cyclical properties of optimal policies. To compute these moments, we run 1000 simulated paths, each 50 periods long, and for each simulated path we compute the covariance matrix for output, inflation, wages per effective hour, and nominal rates. We then average the covariance matrix across simulations.²⁵ As one can see from Table III, inflation and nominal interest rates are much more volatile in HANK, and the comovement of labor share with output is lower. Covariances of inflation with output and labor share have different signs in HANK and RANK settings.

5.1. Sources of Welfare Gains

We first want to understand what drives differences in optimal policies between representative and heterogeneous agent economies. In the representative agent economy, an optimal policy is determined by a trade-off between maintaining price stability and reducing the deviation in output from its efficient level, the so-called “output gap.” Two

²⁴Extending the model as we do in Section 6.2 to introduce liquidity-constrained households rearms the timing of transfers as a government instrument. In that section, we shall explore optimal timing of transfers.

²⁵We also experimented with running longer simulations, such as 100 periods, and did not find substantial differences in the results.

additional considerations affect optimal policy in heterogeneous agents settings. Government policies can redistribute resources across ex ante different agents and increase welfare that the planner measures using a Pareto-weighted sum of agents life-time utilities. The planner can also increase welfare by providing better insurance ex post. To understand whether differences in policy responses between HANK and RANK economies are driven by redistribution or insurance concerns, we run diagnostics.

The first diagnostic uses a method proposed by Bhandari, Evans, Golosov, and Sargent (2021) to separate welfare changes resulting from switching from some policy A to another policy B into three components: *aggregate efficiency*, that measures welfare effects of changes in the level of aggregate resources induced by the policy switch; *redistribution*, that measures welfare effects from changes in expected shares of resources received by ex ante different agents; and *insurance*, that measures effects of changes in the ex post volatility of consumption. The Bhandari et al. (2021) decomposition is similar in spirit to the approaches developed by Benabou (2002) and Floden (2001) but, unlike those papers, can be applied to a much larger class of heterogeneous agent environments that includes the one studied here.

In order to construct our decomposition, we need counterparts of optimal RANK policies for our heterogeneous agents setting. Average levels of taxes and transfers are very different in optimal policies for HANK and RANK economies. In the RANK economy, labor tax rates are negative and are financed by lump-sum taxes in order to offset markups. In the HANK economy, labor tax rates are positive and finance transfers. Since our focus is on cyclical properties of optimal policies, we construct a RANK-equivalent policy as follows. We set the average level of tax rates to be the same as under the HANK optimum, but choose stochastic processes for *deviations* of policy variables from their means to be the same as in a RANK optimum. The allocation in the heterogeneous agent economy with RANK-equivalent policies closely mimics the cyclical properties of the RANK economy reported in Table III. We use it as “policy A ” in our Bhandari et al. (2021) decomposition. “Policy B ” is the Ramsey optimal allocation in a HANK economy.

By construction, policy B has higher welfare than policy A . Table IV decomposes this welfare gain into aggregate efficiency, insurance, and redistribution components and reports decompositions for both optimal monetary and monetary-fiscal policies, as well as for several extensions that we consider in Section 6. This table shows several insights that carry through all of our extensions: the insurance component is positive and greater than 100%; the redistribution component is small; and the aggregate efficiency component is negative. This means that essentially all the welfare gains from optimal HANK policies arise from the additional insurance that they provide. Provision of insurance comes at the cost of sacrificing price stability, which creates deadweight losses and lowers total aggregate resources available for consumption. This explains why the aggregate efficiency component is negative. Finally, cyclical variations in monetary and fiscal policies contribute little to redistribution. Most of the redistribution is done by setting average tax rates appropriately. Deviations of tax rates from those average levels mostly provide insurance, not additional redistribution.

A second diagnostic test helps us to distinguish between insurance against aggregate and idiosyncratic shocks. In the Appendix of the Online Supplementary Material, we study how optimal policies are affected when we switch off idiosyncratic shocks, and also when we allow agents to trade a full set of Arrow securities. We find that the latter experiment accounts for nearly all the differences in cyclical properties of optimal policies reported in Table III. While switching off idiosyncratic shocks has little effect optimal policies, adding Arrow securities contingent on aggregate shocks brings optimal policies

TABLE IV
WELFARE DECOMPOSITION.

	Efficiency	Redistribution	Insurance
Baseline			
(a) Optimal monetary policy	-122	9	213
(b) Optimal monetary and fiscal policy	-16	1	115
Extensions			
(c) Liquidity frictions	-78	-4	182
(d) Mutual fund	-154	-12	266
(e) Heterogeneous labor income exposures	-327	-7	334
Alternative Pareto Weights			
(f) High labor tax	-180	-125	405
(g) High bond tax	-115	52	163
(h) High dividend tax	-165	-1	266

Note: We decompose welfare differences between optimal HANK and optimal RANK policies using the Bhandari et al. (2021) procedure. For all cases, the point of comparison (optimal RANK policy) is set so that expected levels of policy variables equal their optimal HANK counterparts in the absence of aggregate risk and stochastic processes for deviations of the policy variables from their means are optimal in the representative agent version. Lines (a) and (b) report results for our baseline calibration applied to both monetary and monetary-fiscal policies. Lines (c), (d), (e) report our decomposition of the optimal monetary policy for extensions that we describe in Sections 6.2, 6.3, 6.4, respectively. Lines (f), (g), (h) consider alternative specifications of Pareto weights discussed in Section 6.1.

close to those in the RANK economy. From these findings, we conclude that the planner's desire to replace the missing insurance markets for aggregate risk explains the differences in the optimal HANK and RANK policies.

5.2. Policy Responses to Aggregate Shocks

To gather further insights into how the Ramsey planner sets optimal policies in HANK settings, we focus on studying policy responses to specific shocks. We summarize optimal policies with implied nonlinear impulse response functions. We define an impulse response of variable X_t to unexpected shock \mathcal{E}_k of size Δ in a particular period $k \geq t + 1$ (often $k = t + 1$) as

$$\mathbb{E}[X_t | \Omega_{-1}, \Theta_{-1}, \Phi_{-1}, \mathcal{E}_k = \Delta] - \mathbb{E}[X_t | \Omega_{-1}, \Theta_{-1}, \Phi_{-1}, \mathcal{E}_k = 0],$$

where Ω_{-1} , Θ_{-1} , Φ_{-1} are time 0 states for the Ramsey planner and conditional mathematical expectations are taken over the ensemble of paths generated by iterations on the optimal policy functions. We approximate conditional expectations by taking averages over $N = 1000$ simulations of sample paths. Below, we typically set Δ to be one standard deviation of \mathcal{E} . Impulse response functions are state-dependent and nonlinear in sizes of shocks. We report impulse responses for several values of the shock arrival date k .

5.2.1. Monetary Policy Responses to a Markup Shock

We describe an optimal monetary policy response to a negative innovation in $\mathcal{E}_{\phi,t}$. Because this shock increases the desired markup $1/(\Phi_t - 1)$, we call it a positive markup shock. Figure 2 plots optimal responses of the nominal interest rate, inflation, the real pre-tax wage per unit of effective labor, and real output to a positive markup in period one.

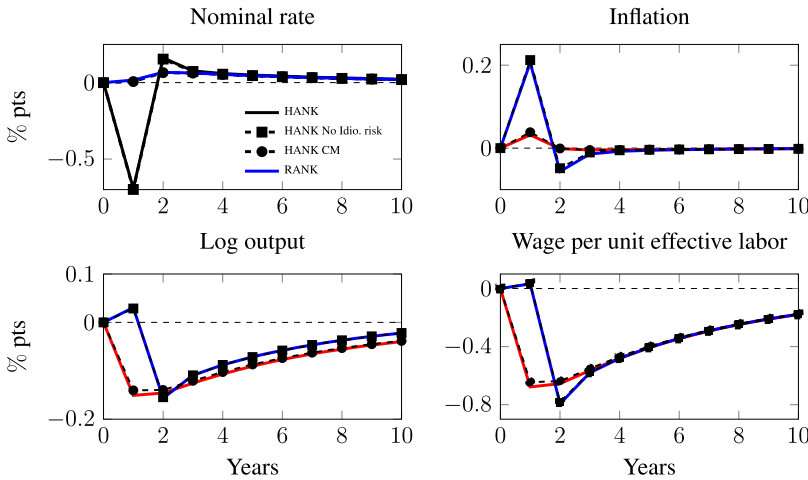


FIGURE 2.—Optimal monetary response to a markup shock. The bold lines labeled “HANK” and “RANK” are the calibrated heterogenous agent and representative agent economy responses, respectively. The dashed lines with squares and circles are responses under heterogenous agent economy with idiosyncratic shocks shut down and with complete markets, respectively.

Figure 2 shows that optimal responses in the HANK and RANK economies differ significantly. While the RANK Ramsey planner slightly *increases* nominal interest rates in response to a markup shock, the HANK planner aggressively *cuts* them. The response of inflation in HANK is an order of magnitude larger, and paths of real wages and output are temporarily above their RANK counterparts.

To illustrate how the insurance considerations drive the shapes of the impulse responses, we construct responses in intermediate economies located between HANK and RANK. We (i) start with our calibrated HANK economy (plotted as a solid line labeled as “HANK”), (ii) shut down idiosyncratic shocks (plotted as a dashed line with square markers labeled as “HANK No Idio. risk”), (iii) allow agents to trade Arrow securities contingent on aggregate shocks (plotted as a dashed line with circle markers labeled as “HANK CM”), and finally (iv) shut down heterogeneity in initial productivities and assets to obtain our RANK economy (plotted as a solid line labeled as “RANK”). This procedure allows us to isolate contributions from providing insurance against idiosyncratic shocks by comparing responses of economy (ii) with those of economy (i); contributions from providing insurance against aggregate shocks by comparing the responses of economy (iii) with those of economy (ii); and contributions from redistribution by comparing responses in economy (iv) with those of economy (iii).

Before interpreting Figure 2, note that when lump-sum transfers are available, complete and incomplete market versions of RANK are identical. Not so with HANK: The presence of heterogeneity means that the absence of complete markets puts concerns about insurance into the mind of a HANK Ramsey planner. Monetary policy cannot provide redistribution or insurance against idiosyncratic shocks beyond what taxes \tilde{Y} can do. However, monetary policy *can* provide insurance against aggregate shocks and imperfectly substitute for missing markets in Arrow securities.

The decomposition in Figure 2 shows that nearly all differences in policy responses between HANK and RANK are driven by the planner’s wish to provide insurance against aggregate shocks. To understand how motives to provide insurance account for key differences between RANK and HANK economies, it is helpful to study a one-time, fully

transient positive markup shock. The textbook effect of this shock is an increase in inflationary pressure that monetary policy can offset by depressing marginal costs. Since marginal costs are proportional to aggregate demand, a contractionary increase in nominal rates is optimal in the RANK economy. Galí (2015) called this “leaning against the wind.”

Such a one-time markup shock also changes the mix of factor payments by increasing dividends and lowering wages. When households are homogeneous as they are in the RANK economy, or when they can trade Arrow securities *ex ante*, a change in the composition of firms’ payments does not affect welfare: Agents who receive mainly wage income will hold a portfolio of Arrow securities that payoff when their wage income is low. When Arrow securities are missing, monetary policy can improve welfare by providing insurance against aggregate shocks. To offset a drop in labor income, the Ramsey planner sets interest rates to *increase* real wages. Since real wages are firms’ marginal costs, a monetary policy action that provides insurance is opposite to one that promotes price stability.

The net effect of a markup shock on optimal monetary policy depends on relative strengths of the Ramsey planner’s motives to provide price stability and insurance. The cost of inflation is set by the price adjustment cost parameter ψ , while insurance provision motives depend on inequalities in stock ownership relative to inequalities in wage income. If stock holdings are perfectly aligned with labor earnings in the sense that the share of stocks that each person owns equals his or her share of aggregate labor compensation, then the insurance provision motive vanishes; a positive effect on dividends exactly offsets the negative effect on labor earnings. In that case, optimal responses are similar to those in RANK.²⁶ HANK responses in Figure 2 differ so much from responses that would support price stability because stock holdings are much more skewed than labor earnings in U.S. data, making insurance concerns the principal motive in our calibration.

Figure 2 shows that the most efficient way to provide insurance is to *front-load* it: virtually all differences in HANK and RANK optimal outcomes occur in policy decisions in period $t = 1$. Households can borrow and lend freely so that their utilities depend only on present values of factor payments, not their timing. But price-setting firms care about the path of factor prices. A first-order approximation of a firm’s optimality condition (16) is written as

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \text{const} \cdot \hat{w}_t + \text{const} \cdot (-\ln \Phi_t), \quad (38)$$

where π_t is log inflation, \hat{w}_t is the log deviation of the wage per unit of effective labor from its average level. Solving difference equation (38) forward shows that inflation in period t is proportional to the discounted present value of future wages. As a consequence, an increase in \hat{w} in some period k increases inflation in all $t \leq k$. Therefore, to minimize costs of inflation it is optimal to deliver all of the adjustment in the present discounted value of unit labor costs at the moment the shock arrives.

An optimal response to a negative markup shock is virtually a mirror image of an optimal response to a positive markup shock. Averaged over time, the expected net flow of resources to each agent generated by a monetary policy response is approximately zero; an outcome consistent with optimal responses being driven mainly by the planner’s insurance motive and not redistribution.

²⁶The RANK economy is a special case in which both shares are equal to one, but the same result holds in an economy with heterogeneity so long as shares of dividends and earnings are aligned. See the Appendix in the Online Supplementary Material for an illustration.

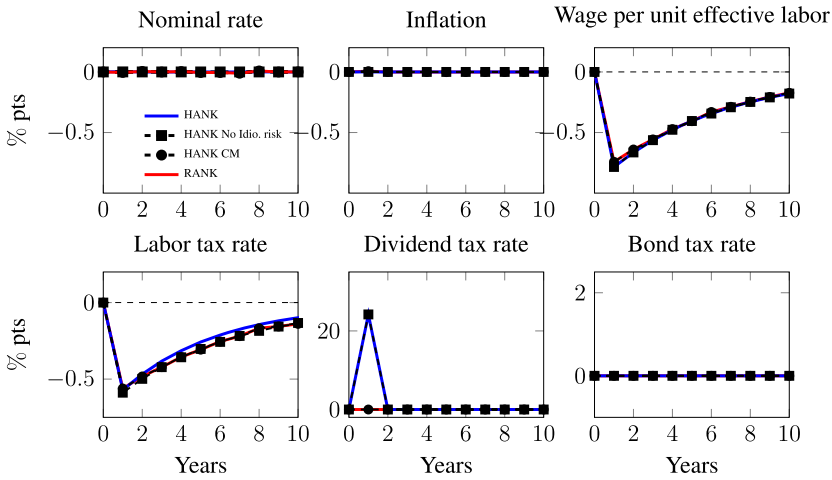


FIGURE 3.—Optimal monetary-fiscal response to a markup shock. The bold lines labeled “HANK” and “RANK” are the calibrated heterogenous agent and representative agent economy responses, respectively. The dashed lines with squares and circles are responses under heterogenous agent economy with idiosyncratic shocks shut down and with complete markets, respectively.

5.2.2. *Monetary-Fiscal Responses to a Markup Shock*

We now study a Ramsey planner who chooses monetary and fiscal policies. Figure 3 shows optimal responses to the markup shock. The planner offsets the shock by combining a labor subsidy with a dividend tax while holding the nominal interest rate unchanged.

In the RANK economy, lump-sum taxes are nondistortionary while taxes on dividend and interest income are redundant. Under the optimal monetary-fiscal policy, the planner achieves a first best by setting $Y_t^n = -1/\Phi_t$ to offset monopoly distortions, setting a path of nominal rates that delivers a constant price level and setting lump-sum taxes to satisfy the government’s budget constraint.

In the HANK economy, the burden of lump-sum taxes falls disproportionately on poor households. The RANK prescription of a proportional labor subsidy financed by reducing lump-sum transfers is therefore not optimal. Instead, a HANK Ramsey planner finances the labor income subsidy by levying a one-time tax on dividends. This tax exactly offsets the gains that stock owners receive from the higher markups, and thus the policy response provides complete insurance against the markup shock.

5.2.3. *Optimal Responses to Productivity Shocks*

Figure 4 shows optimal monetary response to a negative TFP shock. HANK responses differ significantly from RANK; the decomposition reveals that differences are once again driven by the absence in HANK of a market for insuring against aggregate shocks.

To understand what motivates the Ramsey planner to provide insurance, first observe that an adverse TFP shock reduces both profits and labor earnings, so firm owners and workers both suffer. A TFP shock affects agents differently when they hold different quantities of bonds: Agents who own substantial quantities of bonds suffer less than do otherwise identical agents who are debtors. Market incompleteness prevents borrowers and savers from directly hedging TFP shocks. Monetary policy fills this gap by lowering returns on debt, thereby transferring resources from savers to borrowers and smoothing relative consumption shares.

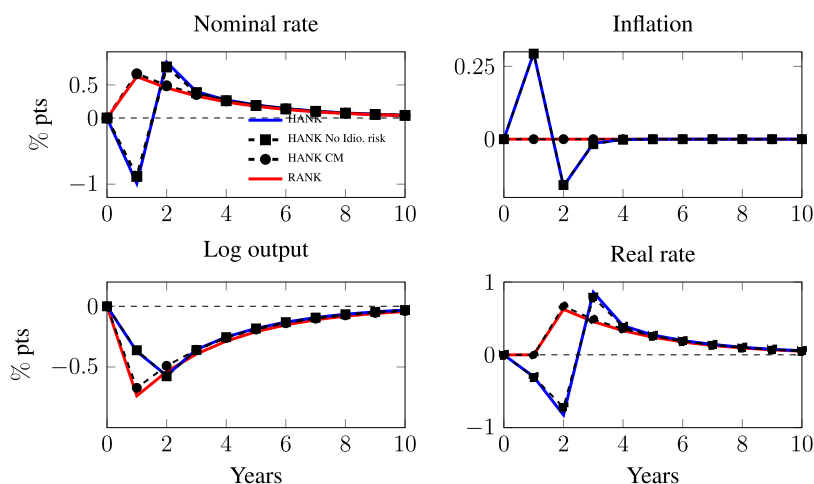


FIGURE 4.—Optimal monetary response to a TFP shock. The bold lines labeled “HANK” and “RANK” are the calibrated heterogeneous agent and representative agent economy responses, respectively. The dashed lines with squares and circles are responses under heterogeneous agent economy with idiosyncratic shocks shut down and with complete markets, respectively.

The optimal policy response evidently departs from RANK and, more generally, from the prescription that monetary policy should aim to minimize fluctuations in inflation and an “output gap.” In response to TFP shocks, by setting the interest rate to a “natural rate” that would prevail if prices were flexible, a planner can eliminate fluctuations in both inflation and the output gap. Optimal responses in the RANK economy and in the complete market version of a HANK economy both follow this prescription.²⁷ That prescription is not optimal in the incomplete markets economy because it does not provide insurance to borrowers and savers. To transfer resources from savers to borrowers, the planner lowers *ex post* real returns on debt by engineering surprise inflation and pushing the expected real rate of interest below the natural rate. A lower real rate requires temporarily higher output. A higher output triggers more inflation, as indicated by equation (38). To offset extra inflationary pressure, the Ramsey plan sets the stage for deflation at $t = 2$.

The strength of planner’s insurance motive depends on heterogeneity in holdings of nominal bonds. U.S. data indicate considerable heterogeneity in nominal asset holdings, which explains a big difference between an optimal policy in the HANK economy relative to the one in RANK.

If fiscal policy can be adjusted in response to the TFP shock, then the Ramsey plan provides optimal insurance to borrowers and savers directly via a state-contingent tax on interest income. That tax effectively completes markets and brings together optimal policy responses for HANK and RANK economies. (See Figure 5).

6. EXTENSIONS AND ROBUSTNESS

We describe several extensions to the baseline calibration and environment.

²⁷Blanchard and Galí (2007) referred to this property of New Keynesian models as a “divine coincidence.”

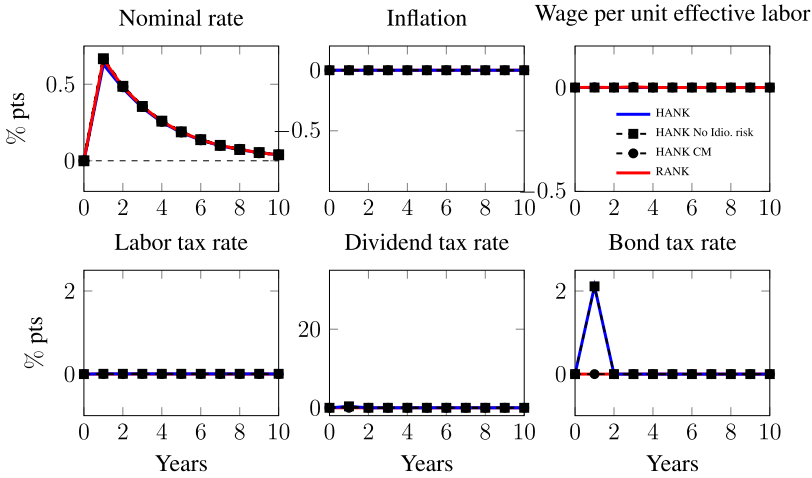


FIGURE 5.—Optimal monetary-fiscal response to a TFP shock. The bold lines labeled “HANK” and “RANK” are the calibrated heterogenous agent and representative agent economy responses, respectively. The dashed lines with squares and circles are responses under heterogenous agent economy with idiosyncratic shocks shut down and with complete markets, respectively.

6.1. Roles of Key Parameters

Optimal responses vary with parameters that shape a trade-off between price stabilization and insurance motives. The strength of a price stabilization motive is driven by the price adjustment cost parameter ψ . The strength of the planner’s insurance motive depends on relative sizes of post-tax inequalities in dividends and labor earnings. These inequalities, in turn, depend on the joint distribution of assets and labor productivities and on tax rates \bar{Y} that are pinned down by initial Pareto weights. In this section, we study how optimal monetary policy responses vary with these objects. We report main findings here and details in the Appendix of the Online Supplementary Material.

We set the value of ψ according to estimates of [Sbordone \(2002\)](#). In a staggered price adjustment model, her numbers indicate that firms change prices on average every nine months. Studies using micro evidence on price changes (see [Nakamura and Steinsson \(2013\)](#)) recover estimates that range from 6.8 to 12 months. In the Appendix of the Online Supplementary Material, we study the sensitivity of our results to variations in ψ between half to twice of our baseline value. We find on-impact changes in the nominal rates that are fairly similar to the baseline, while the peak response of inflation varies roughly linearly in ψ over this range. In addition, we set $\psi \approx 0$ to study a benchmark with fully flexible prices.

Next, we study sensitivity of outcomes to Pareto weights. In Section 4, we assigned Pareto weights using the exponential specification presented in equation (37). This specification maps a three-dimensional vector δ (loadings on the three dimensions of initial heterogeneity) to optimal tax rates $\bar{Y}(\delta)$. Our baseline calibration set $\bar{Y}(\delta) = \bar{Y}^{US}$. We explore the dependence of policy on Pareto weights by raising each component of \bar{Y} to 50%, one at a time, and computing the optimal responses for the corresponding δ . In Table IV, lines (f), (g), and (h) report the welfare decompositions for the optimal policy with higher labor, bond, and dividend taxes, respectively. In line with the benchmark model, the vast majority of welfare gains are generated by insurance, which comes at the cost of aggregate efficiency. In the Appendix of the Online Supplementary Material, we corroborate

this fact by showing that the impulse responses are quantitatively similar to the baseline responses in Figures 2 and 4 for each of these cases as well as alternative experiments where we lower the tax rates from their U.S. values.

Since inequality drifts over time in our economy, optimal responses depend on time t . In our calibrated economy, the drift is slow and therefore responses in $t = 25$ and $t = 50$ appear to be very similar to those at $t = 1$. When we initialize a Ramsey plan at an approximation to a long-run distribution of assets and productivities gleaned from simulating a competitive equilibrium for 100 periods, we find responses that are approximately the same as those in the baseline model. This outcome reflects a balance of two forces. On the one hand, the passage of time decreases the correlation between stock holdings and labor earnings, which renders inequality more misaligned. That increases the planner's gains from providing insurance. On the other hand, the correlation between shares of equities and bond holdings decreases, which diminishes gains from insuring using unanticipated inflation.

6.2. Liquidity Frictions

Thus far, we have assumed that households have unrestricted access to risk-free bond markets. Empirical work documenting large marginal propensities to consume points to presence of liquidity constrained households (see, for instance, Jappelli and Pistaferri (2014) and Johnson, Parker, and Souleles (2006)). In this section, we investigate how liquidity frictions affect optimal monetary policy. We augment our model with “hand-to-mouth” agents who own equities and fixed amounts of nominal assets. They can consume dividends, interest on their nominal bond holdings, and their labor income; but they cannot trade financial assets. Therefore, the budget constraint of a hand-to-mouth agent satisfies equation (2) subject to the restriction that the market value of nominal debt holdings $P_t Q_t b_{i,t}$ must be constant over time. Thus, we modify our baseline model to include an additional dimension of permanent heterogeneity—namely, an indicator variable $h_i \in \{0, 1\}$, where $h_i = 1$ if the agent is a hand-to-mouth type and $h_i = 0$ if the agent is not a hand-to-mouth type. All other aspects of the baseline model remain the same, including how we set initial conditions.

To calibrate a distribution of hand-to-mouth agents, we need data on individual marginal propensities to consume (MPCs) broken down by observable characteristics that we can map to our model. Such data for the U.S. are not readily available. However, using Italian data, Jappelli and Pistaferri (2014) measured average MPCs by cash-in-hand (defined as financial wealth plus current period wage minus taxes). Their findings are broadly consistent with Kaplan, Violante, and Weidner (2014) and Kaplan, Moll, and Violante (2018), who, based on U.S. data, incorporate substantial heterogeneity among liquidity constrained agents. In view of such evidence, we incorporate liquidity-constrained agents by binning households into cash-in-hand quantile groups and, for each quantile group randomly assign a hand-to-mouth status to calibrate the model-generated MPC gradient with respect to cash-in-hand. This approach also preserves the distribution of real and nominal claims of the baseline model, which presents the same insurance motives to a Ramsey planner and, therefore, allows us to isolate effects on optimal policies that are attributable to the “liquidity frictions” that constrain hand-to-mouth agents.

Figures 6 and 7 show optimal monetary responses to markup and aggregate TFP shocks for the calibration with hand-to-mouth agents and compare them to optimal responses in the baseline calibration. Evidently, the trading frictions that give rise to heterogeneities in MPCs make paths for nominal rates, real wages, and inflation smoother than they are

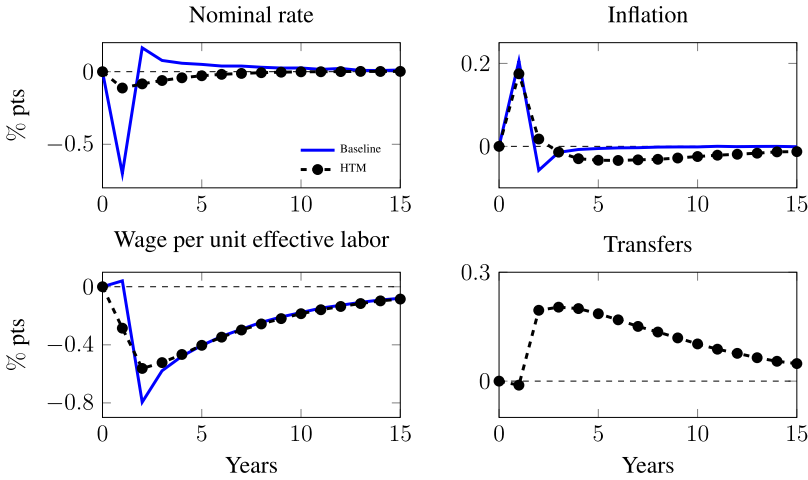


FIGURE 6.—Optimal monetary response to a markup shock with liquidity frictions. The bold lines are responses under the baseline without hand-to-mouth agents; the dashed lines with circles are responses with hand-to-mouth agents. Transfers are not plotted for the baseline because Ricardian equivalence holds and timing of transfers is indeterminate.

in the baseline model. Since Ricardian equivalence no longer holds, an optimal path of transfers is now uniquely pinned down.

We show next that deviations of optimal policy responses from those in our baseline model are driven by the inability of hand-to-mouth agents to borrow and save in order to smooth consumption over time, as well as the substantial heterogeneity within the set of liquidity constrained households. We start with optimal monetary policy responses to a markup shock in Figure 6. In the baseline calibration (solid line), the planner provides insurance against the shock by front-loading higher wages and avoiding additional future

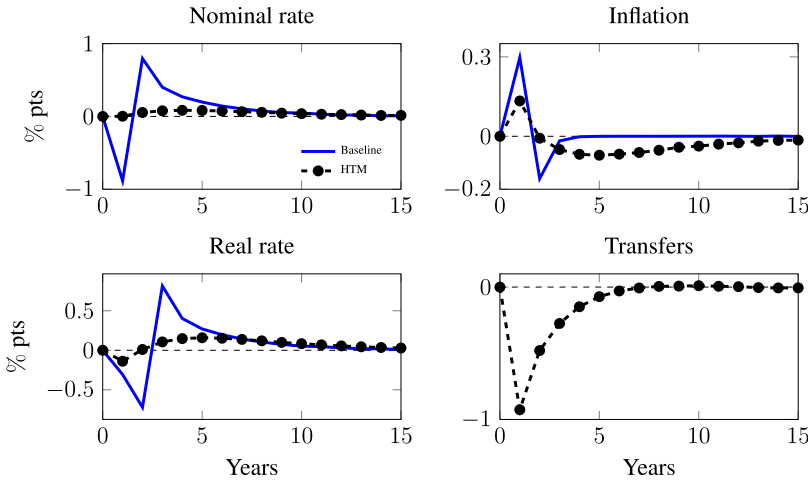


FIGURE 7.—Optimal monetary response to a TFP shock with liquidity frictions. The bold lines are responses under the baseline without hand-to-mouth agents; the dashed lines with circles are responses when hand-to-mouth agents are present. Transfers are not plotted for the baseline because Ricardian equivalence holds and the timing of transfers is indeterminate.

inflation from firms' rationally anticipating higher future marginal costs. Such a front-loading policy would be costly for hand-to-mouth agents because they would have too much income (equal to consumption) in the short run relative to the future. Thus, in addition to providing insurance, the Ramsey planner wants to smooth over time the consumption paths of hand-to-mouth agents.

A natural way to achieve insurance and also to smooth consumption would be to adjust the timing of lump-sum transfers. But hand-to-mouth agents are not homogeneous. A path of transfers that would smooth the consumption of poor hand-to-mouth agents, who rely mainly on wage income, would exacerbate the volatility of consumption of rich hand-to-mouth agents, who rely mostly on their dividend income. Thus, heterogeneity among liquidity constrained agents makes transfers a less effective tool.²⁸

In addition to timing transfers, the planner distorts allocations to induce a smoother path for the real wage (dashed line). A smoother path means a real wage above a "natural wage" that would prevail with flexible prices when markups are high and below that "natural wage" when the economy recovers. Implementing such a path for real wages requires expansionary monetary policy and associated inflation upon arrival of the shock to be followed by a contractionary monetary policy that brings persistent deflation.

Figure 7 displays optimal responses to a productivity shock. Overall, the policy actions help approximate within-agent transfers that would occur if all agents, including those who are hand-to-mouth, were free to trade assets. The TFP shock widens disparities in total income between hand-to-mouth agents who are borrowers and those who are lenders. As in the baseline, the planner inflates away debt in the short run, thereby transferring resources from lenders to borrowers during a recession. In addition, the planner engineers a persistent but small deflation. A higher price level acts like a tax on wealth by lowering obligations of hand-to-mouth borrowers and reducing assets of hand-to-mouth lenders. Since the shock is transitory, agents would want to reverse that reshuffling of resources after the shock wears off. Since they cannot trade, the planner uses a post-shock deflation to generate a smooth path of repayments from borrowers to asset holders as TFP reverts to its steady-state value. Like responses to markup shocks, these paths of prices and real interest rates require lower nominal interest rates for a few periods, followed by high nominal interest rates, later making the path of nominal rates smooth relative to outcomes in the baseline model.²⁹

The preceding discussion suggests that the persistence of individual MPC, as well as their dispersion, has important quantitative implications for optimal policy. The calibration in this section implicitly assumes that individual MPCs are permanent. The more frequently the identities of liquidity constrained agents switch, the smaller the persistence will be. Less persistence would reduce motives for the planner to smooth insurance benefits over time and bring impulse responses closer to the baseline model. Empirical work on

²⁸To highlight this point and isolate the role of wealthy hand-to-mouth consumers, in the Appendix of the Online Supplementary Material, we study optimal policy in an alternative calibration in which the bottom 15% of the cash-in-hand distribution is set to be hand-to-mouth. This example has the flavor of typical calibrations of one-asset Aiyagari models because constrained hand-to-mouth agents are more homogeneous and depend almost entirely on their labor incomes. We show that the dynamics of interest rates, inflation, output, and wages are almost identical to those of the baseline economy with no liquidity frictions. When liquidity-constrained agents are homogeneous, the planner can effectively borrow on their behalf and use transfers to smooth their consumption.

²⁹In the Appendix of the Online Supplementary Material, we report results for an extension that allows marginal propensity to consume to vary across sources of income. The general principles guiding the optimal policy are unchanged.

persistence in MPCs is still in early stages, although Auclert, Rognlie, and Straub (2018) and Kaplan, Moll, and Violante (2018) provide valuable insights into MPC dynamics.

6.3. Mutual Fund

In the baseline model, we calibrated households' portfolios to counterparts in the SCF but imposed that households could not trade claims to dividends. In this section, we follow Gornemann, Kuester, and Nakajima (2016) by having agents trade shares in a mutual fund. A competitive mutual fund sector invests in corporate equity and government bonds and remits after-tax earnings to households in proportion to their holdings of the mutual fund. Shares in the mutual fund are indirect claims to returns from a common financial portfolio and are traded by all households in a competitive market.

The mutual fund solves

$$\max_{B_t} \mathbb{E}_0 \sum_t S_t^{mf} D_{a,t}$$

$$Q_t B_t + D_{a,t} = \frac{(1 - Y_t^b) B_{t-1}}{1 + \Pi_t} + (1 - Y_t^d) D_t,$$

where we follow Gornemann, Kuester, and Nakajima (2016) and set $\frac{S_{t+1}^{mf}}{S_t^{mf}}$ to be an asset-weighted average of intertemporal marginal rates of substitutions across households. Households' budget constraint (2) becomes

$$c_{i,t} + P_{a,t} a_{i,t} = (1 - Y_t^n) W_t \epsilon_{i,t} n_{i,t} + T_t + (D_{a,t} + P_{a,t}) a_{i,t-1}, \quad (39)$$

where $a_{i,t}$ are household i 's holdings of the mutual fund and $\int a_{i,t} di = 1$. Households freely trade $a_{i,t}$. The production side of the model is unchanged.

We study optimal responses to markup and TFP shocks and contrast them with our baseline model. Our calibration in this mutual fund setting closely follows Section 4. We initialize the distribution of mutual fund holdings by using the distribution of financial wealth from the SCF 2007 formed by summing claims to all bonds and stocks; we again set Pareto weights to rationalize observed U.S. average tax rates. Other parameters are those in the baseline model. When we study optimal monetary policy, we impose $B_t = B_0$.³⁰

Figure 8 plots optimal monetary policy responses to a markup shock. Responses in the baseline model and those for the model with a mutual fund are very close because optimal policy is driven largely by insurance motives largely drive optimal policy, as discussed extensively in Section 5. Magnitudes of optimal policy responses are determined by cross-sectional heterogeneity in exposures of labor and nonlabor incomes to aggregate shocks. Even after we sum bond and stock claims, total financial wealth remains quite skewed relative to labor earnings. As was the case before, the planner provides insurance in response to markup shocks by boosting the present discounted value of wages. In the Appendix of

³⁰This is mainly done to assure comparability of results. Different from the baseline, in which we had Ricardian equivalence, the planner in the mutual fund economy is motivated to vary the level of debt and change the riskiness of returns on the mutual fund. Setting the debt level to a constant imposes parity between the baseline monetary planner and the mutual fund monetary planner in their abilities to affect returns. We relax this restriction when we study optimal monetary-fiscal policies in a mutual fund economy in which the planner can use taxes on bonds income or dividends to directly affect returns. See the Appendix of the Online Supplementary Material.

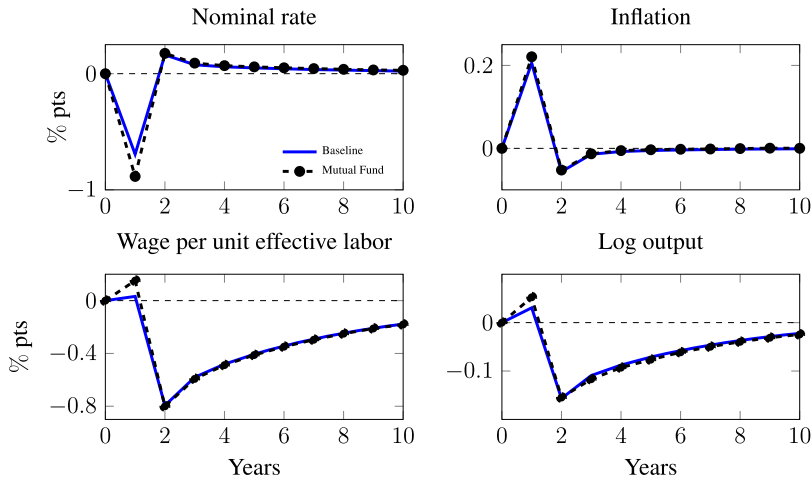


FIGURE 8.—Optimal monetary responses to a markup shock with mutual fund. The bold lines are responses under the baseline and the dashed lines with circles are responses under the mutual fund setting.

the Online Supplementary Material, we show that responses to the TFP shock are also similar to those in the baseline. In the row labeled “Mutual fund” in Table IV, we show that insurance considerations drive most of the welfare gains.

6.4. Heterogeneous Labor Income Exposures

In our baseline calibration, percentage falls in labor income during recessions are the same across workers. Using administrative data on W2 forms over the period 1978–2010, [Güvenen, Ozkan, and Song \(2014\)](#) documented that relative to a typical worker, individuals who have either low past incomes or very high past incomes face larger drops in earnings in recessions. In this section, we compute optimal monetary and fiscal responses under a richer stochastic process for idiosyncratic risk that captures the [Güvenen, Ozkan, and Song \(2014\)](#) patterns.

We modify equation (12) to

$$\ln \epsilon_{i,t} = (1 + f(\theta_{i,t-1})) \ln \Theta_t + \ln \theta_{i,t} + \varepsilon_{\epsilon,i,t}, \quad (40)$$

and set $f(\theta)$ so that the aggregate productivity shock has different loadings for agents with different earning histories. We assume a quadratic function $f(\theta) = f_0 + f_1\theta + f_2\theta^2$ and normalize f_0 so that an agent with median productivity faces a drop similar to the drop in aggregate TFP. We then simulate a competitive equilibrium for 30 periods and extract “recessions” as consecutive periods in which the growth rate of output falls one standard deviation below zero. Following the empirical procedure in [Güvenen, Ozkan, and Song \(2014\)](#), we rank workers by percentiles of their average log labor earnings 5 years prior to the shock and compute the percent earnings loss for each percentile relative to the median. We set parameters f_1 and f_2 to match earnings losses of the 5th and 95th percentiles.

In Figure 9, we report the optimal monetary policy response to a TFP shock with heterogeneous exposures. Amplified inequality induced by a recession increases gains that the planner earns from providing insurance. As compared to our baseline monetary re-

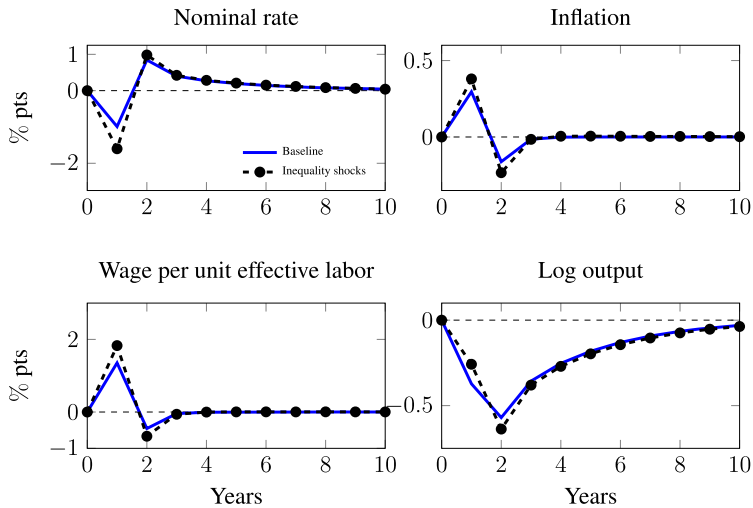


FIGURE 9.—Optimal monetary responses to a TFP shock with heterogeneity in labor income exposures. The bold lines are responses under the baseline, and the dashed lines with circles are responses when the idiosyncratic productivity process has heterogeneous exposures to aggregate TFP.

sponse, the planner further lowers the nominal rate and thereby induces higher inflation in the short run and a lower ex ante real rate.³¹

7. CONCLUDING REMARKS

We forged a method to approximate Ramsey plans in economies with heterogeneous agents and used it to reassess quantitative lessons for monetary and fiscal policy brought by earlier contributions to New Keynesian economics. Heterogeneity adds an insurance motive that quantitatively dominates the motives to stabilize nominal prices that have typically driven New Keynesian policy prescriptions. For our laboratory, we combined basic versions of the New Keynesian model and the incomplete market models. We are convinced that our method will be useful for computing optimal policies in environments that have more detailed household balance sheets, richer labor market dynamics that include realistic wage-setting frictions, and asset markets formulations capable of fitting observed returns. Adding these features is likely to affect how a Ramsey planner would deliver insurance in order to supplement missing markets and financial frictions. Nevertheless, we suspect that insurance concerns with respect to aggregate shocks and other determinants of heterogeneous income exposures will remain decisive determinants of optimal policies.

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³¹In the Appendix of the Online Supplementary Material, we plot the monetary-fiscal responses. With optimal monetary as well as fiscal policy, the planner responds by increasing labor income taxes in addition to the tax on bond income. For reasons similar to [Werning \(2007\)](#), the marginal cost of extracting resources from high-income agents is lower in times of higher inequality. Therefore, optimal labor tax rates are higher on impact and then revert as effects of inequality shocks decay to a permanently higher level.

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Co-editor Giovanni L. Violante handled this manuscript.

Manuscript received 6 June, 2018; final version accepted 20 March, 2021; available online 30 April, 2021.