

The Multi-secretary Problem with Many Types

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The multi-secretary is one where a decision maker is presented with a sequence of T independent values representing the candidate abilities and the decision maker has a budget to hire B secretaries. After the candidate abilities are realized, the decision maker must make an irrevocable hire or reject decision. The achievable regret for this problem is defined as the expected difference between the sum of the ability of the B most capable candidates and the sum of the ability of the candidates hired by the decision maker. Recently, Arlotto and Gurvich [1] showed that, when the distribution of types is discrete, there is an algorithm whose regret is bounded uniformly for all values of number of candidates T and the hiring budget B , where the constant may scale with the reciprocal of the minimum probability mass on any type. When there are many types, e.g., a continuum of types, very little is known. The only known result on regret with many types is that of Bray [2], who considers the setting with the candidates abilities distributed as independent uniform random variables over $[0, 1]$. He showed that in this setting the best possible regret scaling is $\Theta(\log T)$.

This motivates us to ask the following questions in the present paper: *How does regret scale as a function of the underlying distribution in the presence of many types? What algorithmic approach allows to achieve optimal regret scaling with many types?*

In this work, we generalize the settings of Arlotto and Gurvich [1] and Bray [2] and study the multi-secretary problem under general distributions on $[0, 1]$ which allow for many types, ranging from a few discrete types to a continuum of types, together with gaps (intervals of absent types), and identify how the nature of the distribution affects the (best possible) scaling of regret. We refer to intervals where the probability mass is zero as *gaps*, and refer to intervals where the probability mass is positive as *mass clusters*. This generalization has both important practical and theoretical motivations. From a practical perspective, many settings which resemble the multi-secretary problem such as hiring, and closely related problems such as fulfillment problems, many types (quality levels or demand locations) are indeed present and often clustered. The algorithmic insights we develop here may be useful in such settings. From a theoretical perspective, the family of distributions we consider generalizes the distributions considered in prior work, and encompasses a broad range of distributions.

Our main contributions are as follows:

- We first establish that for any non-atomic type distribution with a gap, the certainty equivalent (CE) policy regret scales as $\Omega(\sqrt{T})$. This is in contrast to the prior finding of [2] that the regret of CE is $\Theta(\log T)$ for the uniform distribution. One or more gaps in the distribution cause the regret to increase to $\Omega(\sqrt{T})$. We note that this result applies to non-atomic distributions and hence the deterioration we prove is not due to a poor tie breaking rule (as is commonly documented in the case of a few discrete types), but to the presence of a gap in the type space. As such, this result highlights a new practical feature of distributions that could significantly affect how one should operate.
- In the presence of gaps, we establish a universal lower bound (for any policy) on the growth rate of the regret as a function of how mass accumulates around gaps. In particular, we establish that any policy must incur $\Omega(T^{1/2-1/(2(\beta+1))})$ where $\beta \geq 0$ quantifies the mass accumulation of types around gaps.

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- To operate with distributions which have gaps, we introduce a new algorithmic principle which we call *Conservativeness with respect to gaps* (CwG). We devise a new CwG policy which makes a crucial modification to the CE policy. The idea is that if at any time the CE threshold is close to the boundary of a gap, CwG instead uses the gap as the acceptance threshold to avoid incurring large regret in the future (large regret would otherwise occur in the event that the CE threshold eventually falls on the opposite side of that gap).
- We establish that for a broad class of distributions with gaps, which we refer to as (β, ϵ_0) -clustered distributions, the algorithm we devised is near-optimal, in that its worst case regret scales as $\tilde{O}(T^{1/2-1/(2(\beta+1))})$, matching the scaling of the lower bound in T up to polylogarithmic terms.

For the case of a few discrete types, our algorithm recovers bounded regret, as in Arlotto and Gurvich [1]. For the uniform distribution on $[0, 1]$, CwG is identical to CE since there are no gaps, and we recover the $O(\log T)$ result of Bray [2].

The full version of the paper can be found at <https://arxiv.org/abs/2205.09078>.

CCS Concepts: • **Theory of computation** → **Online algorithms**; **Algorithm design techniques**.

Additional Key Words and Phrases: multi-secretary, revenue management, online algorithms, regret analysis

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