2022 American Control Conference (ACC)
Atlanta, USA, June 8-10, 2022

Control Barrier Functions for Safe Teleoperation of a Functional Electric Stimulation Enabled Rehabilitation System

Kimberly J. Stubbs*, Axton Isaly*, and Warren E. Dixon*

***Abstract*—There is a growing need for telerehabilitation. For people with neuromuscular disorders, Functional Electrical Stimulation (FES) cycling is a commonly used rehabilitation technique. Studies have shown that by coordinating movement between the lower and upper limbs can lead to faster restoration of walking in many cases. The author's previous development of a strongly coupled bilateral telerobotic system met two separate goals: enabling a therapist to direct rehabilitative efforts remotely, and extend the benefit of in-home FES rehabilitation sessions by coordinating limb movement. However, the previously developed system did not restrict the trajectories of the leader-cycle system, thus increasing the possibility that the system might operate outside of beneficial rehabilitative cadence ranges. In this work, a control barrier function (CBF) is designed to ensure safety of the leader-cycle system (i.e., constrain the cadence within a desired operating range). Once safety of the leader is guaranteed, Lyapunov-based analysis is used to show that the follower-cycle FES/motor actuated system produces global exponential tracking of the leader-cycle position and cadence.**

***Index Terms*—Control Barrier Function, Functional Electrical Stimulation (FES), Rehabilitation Robotics, Teleoperation**

I. Introduction

Due to the ongoing global pandemic, the need for telemedicine solutions has become a pressing issue. While several remotely provided options have been developed for general medical care, people who require ongoing physical rehabilitation have, in many cases, been overlooked [1]. For people with neuromuscular disorders (NDs), functional electrical stimulation (FES) is a beneficial rehabilitation technique [2], [3], shown to produce marked improvement in both physiological and psychological health [4]–[6]. Therefore, we are motivated to develop FES telerehabilitation solutions for people with NDs.

Recent work to improve in FES rehabilitative cycling, in general, has featured the use of Lyapunov-based, nonlinear control to adapt for unknown parameters inherent to human/machine systems [7]. These developments have also allowed for switching between motor assistance and FES-induced muscle effort, ensuring that stimulation is only applied to selected muscle groups when within ideal regions of the crank cycle to produce positive torque production, leading to extended duration of rehabilitation sessions by delaying the onset of muscle fatigue and reducing uncomfortable over-stimulation [8]. General rehabilitation studies have shown that coordinating leg and arm movements on a mechanically connected cycle can improve walking for stroke patients [9] and suggest that neural connections are likely to exist between lower and upper limbs [10]. Recent results specific to FES rehabilitation, where FES is applied to the legs and arms to enforce coordinated effort, produced a significant improvement in both walking cadence and duration [11].

Motor recovery might also be improved through planned, repetitive movement [12] designed to be enjoyable for the participant, thus improving their physical and emotional state simultaneously [13]. Rehabilitation for those with NDs is typically a long process, requiring several visits with a physical therapist, as well as regular weekly rehabilitation exercise at home [14]. However, unsupervised participants often do not find in-home rehabilitation systems enjoyable enough to continue treatment, motivating the development of game-based, teleoperative rehabilitative systems [14]–[16]. These rehabilitation systems often direct the participant's desired trajectories as determined by a remote therapist-controller (i.e., leader) teleoperation system [17], [18]. However, these systems do not provide haptic feedback (i.e., kinematic feedback to the teleoperation controller) so that the leader-cycle operator is informed of the rehabilitation participant's (i.e., follower's) current performance. Therefore, there is motivation to produce a FES telerehabilitation system to meet the needs of two separate cases: first, enabling physical therapists to provide remote assistance to people with NDs, and second, improving rehabilitative outcomes through coordinated upper and lower limb movement, while encouraging completion of recurrent in-home therapies by returning the rehabilitation participant's ability to direct the performance of the FES-actuated cycle. To this end, the author's recent work [19] developed a strongly coupled, bilateral teleoperated [20] FES rehabilitation system, where FES stimulation of the upper limbs was not required to maintain limb coordination, and where split-crank cycles were used to capture asymmetric impairments (Figure 1).

However, in returning free-will to the leader-system oper-

*Department of Mechanical and Aerospace Engineering, University of Florida, Gainesville FL 32611-6250, USA Email: {kimberlyjstubbs, axtonisaly1013, wdixon}@ufl.edu

This research is supported in part by NSF Award number 1762829, and the Assistant Secretary of Defense for Health Affairs, through the Congressionally Directed Medical Research Program under Award No. W81XWH1910330. Any opinions, findings and conclusions, or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the sponsoring agency.

978-1-6654-5196-3/$31.00 ©2022 AACC

ator, no guarantee of performance within desirable rehabilitation ranges exist. Therefore, building on recent results in [21], this work introduces leader-system safety through the use of a control barrier function (CBF), where leader-cycle trajectories are constrained within a well-defined safe set of operation (i.e., a therapist-determined desirable rehabilitative cadence range) while maintaining haptic feedback to the operator. Once safety of the leader-cycle is guaranteed, Lyapunov-based analysis of the nonlinear, switched follower-cycle dynamics is performed, showing global exponential tracking to the desired leader-cycle trajectory.

Figure 1. A teleoperated cycling testbed is shown, where A denotes the leader (hand-cycle), B denotes the FES unit and corresponding stimulation pads placed on the right quadriceps, and C denotes the follower split-crank cycle.

II. Problem Formulation

The cycling system dynamics, where the subscripts l and f denote the leader- and follower-cycling devices respectively, are modeled as [22]

$$\tau_i\left(z_i,t\right) \triangleq M_i\left(q_i\right)\ddot{q}_i + V_i\left(z_i\right)\dot{q}_i + G_i\left(q_i\right) + P_i\left(z_i\right) + b_i\dot{q}_i + d_i\left(t\right),$$

where the angular position is denoted by $q_i \in \mathcal{Q}_i \subseteq \mathbb{R}$, where $\mathcal{Q}_i$ represents the set of all measurable crank angles. Angular velocity is denoted by $\dot{q}_i \in \mathbb{R}$, and angular acceleration is denoted by $\ddot{q}_i \in \mathbb{R}$. The concatenated state vector is defined as $z_i \triangleq (q_i, \dot{q}_i)$. The collection of all input torques is denoted by $\tau_i : \mathcal{Q}_i \times \mathbb{R} \times \mathbb{R}_{>0}$. The unknown, nonlinear inertial effects, centripetal-Coriolis effects, gravitational effects, passive viscoelastic muscle forces, viscous damping effects, and system disturbances are denoted by $M_i : \mathcal{Q}_i \to \mathbb{R}$, $V_i : \mathcal{Q}_i \times \mathbb{R} \to \mathbb{R}$, $G_i : \mathcal{Q}_i \to \mathbb{R}$, $P_i : \mathcal{Q}_i \times \mathbb{R} \to \mathbb{R}$, $b_i \in \mathbb{R}_{>0}$, and $d_i : \mathbb{R}_{\geq 0} \to \mathbb{R}$, respectively. To simplify further development, an auxiliary collection of right-hand side terms is defined as $\tau_{rhs,i} \triangleq V_i\left(z_i\right)\dot{q}_i + G_i\left(q_i\right) + P_i\left(z_i\right) + b_i\dot{q}_i + d_i\left(t\right)$, such that

$$\tau_i\left(z_i,t\right) = M_i\left(q_i\right)\ddot{q}_i + \tau_{rhs,i}. \tag{1}$$

A. Follower rehabilitative FES-cycle dynamic model

An FES-enabled rehabilitative split-crank cycle, such as that used in [19] and [23], serves as the follower cycling device. The dynamic model of a single side of the cycle is modeled independently without loss of generality. The cycle-rider lower body switched dynamics for one side are modeled as [24]

$$\tau_f\left(z_f,t\right) \triangleq \tau_{e,f} + \tau_M\left(z_f\right) + \tau_{vol,f}\left(t\right). \tag{2}$$

The respective torques applied about the follower-cycle crank axis include the subsequently designed motor torque, denoted by $\tau_{e,f} \in \mathbb{R}$, the resultant torque due to application of FES stimulation, denoted by $\tau_M : \mathcal{Q}_f \times \mathbb{R} \to \mathbb{R}$, and the resultant torque due to the volitional efforts of the rehabilitation participant, denoted by $\tau_{vol,f} : \mathbb{R}_{\geq 0} \to \mathbb{R}$.

The FES induced muscle torque in (2) is modeled as the summation of all muscle forces produced by individually stimulated muscle groups, such that [25]

$$\tau_M\left(z_f\right) = \sum_{m\in\mathcal{M}} g_m\left(z_f\right)u_m, \tag{3}$$

where the subscript $m \in \mathcal{M} = \{Q, G, H\}$ indicates the quadriceps femoris (Q), gluteal (G), and hamstring (H) muscle groups. For each $m \in \mathcal{M}$, the unknown, state dependent, nonlinear muscle control effectiveness in (3) is denoted by $g_m : \mathcal{Q}_f \times \mathbb{R} \to \mathbb{R}_{\geq 0}$, and the muscle control input is denoted by $u_m : \mathcal{Q}_f \to \mathbb{R}$. Let the set of all crank angles across which each muscle group is stimulated be denoted by $\mathcal{Q}_m \subset \mathcal{Q}_f$, such that $\mathcal{Q}_m \triangleq \{q_f \in \mathcal{Q}_f \mid T_m\left(q_f\right) > \varepsilon_m\}$ [26], where $\varepsilon_m \in (0, max\left(T_m\right))$ represents the user-defined minimum torque transfer ratio required to ensure positive crank torque values for each muscle group, and $T_m\colon \mathcal{Q}_f \to \mathbb{R}$ represents the torque transfer ratio for the corresponding muscle group. Let $\mathcal{Q}_{FES} \subset \mathcal{Q}_f$ denote the region about the crank cycle where FES is applied, where $\mathcal{Q}_{FES} \triangleq \bigcup_{m\in\mathcal{M}}\{\mathcal{Q}_m\}$.

The muscle control input (i.e., stimulation intensity) applied to each muscle group $u_m\colon \mathcal{Q}_f \to \mathbb{R}$, is defined as [26]

$$u_m\left(q_f\right) \triangleq \sigma_m\left(q_f\right)k_m u_s, \tag{4}$$

for every $m \in \mathcal{M}$, where $\sigma_m : \mathcal{Q}_f \to \{0, 1\}$ denotes a switching signal such that

$$\sigma_m\left(q_f\right) \triangleq \begin{cases} 1 & \text{if } q_f \in \mathcal{Q}_m \\ 0 & \text{if } q_f \in \mathcal{Q}\backslash\mathcal{Q}_{FES}, \end{cases} \tag{5}$$

the positive constant $k_m \in \mathbb{R}_{\geq 0}$ is selected in relation to the rider's comfort level during stimulation, and $u_s \in \mathbb{R}$ represents the subsequently designed FES control input.

The electric motor torque is expressed as [25]

$$\tau_{e,f} \triangleq g_{e,f}u_{e,f}, \tag{6}$$

where the known, constant relationship between the applied electric motor current and the resulting torque about the crank axis is represented by $g_{e,f} \in \mathbb{R}_{\geq 0}$, and the subsequently designed motor control input for the follower-cycle is represented by $u_{e,f} \in \mathbb{R}$.

Substituting (3), (4), and (6) into (1) and rearranging yields the open-loop follower-cycle dynamic equation

$$g_M(z_f)\, u_s + g_{e,f} u_{e_f} = M_f(q_f)\, \ddot{q}_f + \tau_{rhs,f} - \tau_{vol,f}(t), \quad (7)$$

where the summation of muscle torque efficiencies across all muscle groups $g_M : \mathcal{Q}_f \times \mathbb{R} \to \mathbb{R}$ is represented by $g_M(z_f) \triangleq \sum_{m \in \mathcal{M}} g_m(z_f)\, \sigma_m(q_f)\, k_m$ [25].

B. Leader-cycle dynamic model

The generalized dynamic model for the leader-cycling system, following the same development as the follower FES split-crank rehabilitation cycle, is modeled as

$$g_{e,l} u_{e,l} = M_l(q_l)\, \ddot{q}_l + \tau_{rhs,l} - \tau_{vol,l}(t), \quad (8)$$

where the only torque produced by the leader-cycle operator is purely volitional. Therefore, the muscle control input has been eliminated. The known, constant relationship between the applied electric motor current and the resulting torque about the leader-cycle crank axis is represented by $g_{e,l} \in \mathbb{R}_{\geq 0}$, the subsequently designed motor control input for the leader-cycle is represented by $u_{e,l} \in \mathbb{R}$, and the torque due to the volitional efforts of the leader-system operator is denoted by $\tau_{vol,l} : \mathbb{R}_{\geq 0} \to \mathbb{R}$.

C. Universal model properties

The follower-cycle/rider dynamics in (7) and the leader-cycle/operator dynamics in (8) have the following properties and assumptions for each i, where $i \in \{l, f\}$ [23].

Property 1. $\dot{M}_i(q_i) = 2V_i(z_i)$.

Property 2. The unknown inertia term can be bounded by $c_{m,i} \leq M_i(q_i) \leq c_{M,i}$, where $c_{m,i}$ and $c_{M,i} \in \mathbb{R}_{>0}$ are known positive constants.

Property 3. The unknown centripetal-Coriolis term can be bounded by $|V_i(z_i)| \leq c_{V,i} |\dot{q}_i|$, where $c_{V,i} \in \mathbb{R}_{>0}$ is a known positive constant.

Property 4. The unknown gravitational torques can be bounded by $|G_i(q_i)| \leq C_{G,i}$, where $c_{G,i} \in \mathbb{R}_{>0}$ is a known positive constants.

Property 5. The unknown passive viscoelastic tissue torques can be bounded as $|P_i(z_i)| \leq c_{P1,i} + c_{P2,i} |\dot{q}_i|$, where $c_{P1,i}$ and $c_{P2,i} \in \mathbb{R}_{>0}$ are known positive constants.

Property 6. The unknown viscous friction term can be bounded as $|b_i| \leq c_{b,i}$, where $c_{b,i} \in \mathbb{R}_{>0}$ is a known positive constant.

Property 7. The unknown muscle stimulation efficiency term in (7) can be bounded below by $g_{\underline{M}} \leq g_M(z_f)$, for all $q_f \in \mathcal{Q}_{FES}$ and $\dot{q}_f \in \mathbb{R}$, where $g_{\underline{M}} \in \mathbb{R}_{>0}$ is a known positive constant [25].

Assumption 1. Volitional torques can be combined with unknown disturbance torques and bounded as $|\tau_{vol,i}| + |d_i| \leq c_{d,i}$, where $c_{d,i} \in \mathbb{R}_{>0}$ is a known positive constant.

III. Control Development

A. Leader-cycle Controller

The primary control objective of the leader-cycle system is to restrict the cadence of the follower-cycle within a safe operating range selected for optimal rehabilitation while maintaining the operator's ability to dictate the desired cadence of the follower-cycle within the selected range. Let $\dot{q}_d \in \mathbb{R}_{>0}$ represent the midpoint of the desired rehabilitative cadence range. The secondary objective is to apply haptic feedback proportional to the subsequently defined follower-cycle trajectory error signal $e_1 \in \mathbb{R}$, such that the leader-cycle operator is aware of the magnitude of the mismatch between the actual and desired cadence.

To quantify the primary objective, a tracking error signal $e_0 \in \mathbb{R}$ is defined as

$$e_0 \triangleq \dot{q}_d - \dot{q}_l, \quad (9)$$

where $\dot{q}_d$ represents the operator-selected set point of the desired cadence range. Using the methods developed in [21], the barrier function candidate $B\colon \mathcal{Q}_l \times \mathbb{R} \to \mathbb{R}$ is defined as

$$B(z_l) \triangleq \tfrac{1}{2} M_l(q_l) \left[e_0^2 - \bar{e}^2 \right], \quad (10)$$

where $\bar{e} \in \mathbb{R}_{>0}$ represents a user-selected constant that defines the desired cadence range about the set point $\dot{q}_d$ such that the CBF safe set is

$$\begin{aligned} \mathbb{S} &= \{ z_l \in \mathcal{Q}_l \times \mathbb{R} : |e_0| \leq \bar{e} \}, \\ &= \mathbb{R} \times [\dot{q}_d - \bar{e}, \dot{q}_d + \bar{e}]. \end{aligned} \quad (11)$$

To render the set $\mathbb{S}$ uniformly globally asymptotically stable (UGAS), we constrain the leader cycle control input to ensure that $B(z_l)$ acts as a Lyapunov function outside the set $\mathbb{S}$. The leader-cycle control input is constrained to be selected from the mapping [21]

$$\mathcal{U}(z) \triangleq \left\{ u_{e,l} \in \mathbb{R} : \nabla B^T(z_l)\, f(z_l, u_{e,l}) \leq -\gamma \right\}, \quad (12)$$

where $f(z_l, u_{e,l}) \triangleq \begin{bmatrix} \dot{z}_{l,1}, & \dot{z}_{l,2} \end{bmatrix}^T = [\dot{q}_l, \;\; M_l^{-1}(q_l) [g_{e,l} u_{e,l} + \tau_{vol,l} - \tau_{rhs,l}(z_l, t)]]^T$, and γ is any function designed to be positive outside of $\mathbb{S}$. The uncertainties of the leader-cycle dynamics prevent computation of the constraint defining $\mathcal{U}$. Therefore, a worst-case upper bound of the product $\nabla B^T(z_l)\, f(z_l, u_{e,l})$ is determined for controller development and Lyapunov-based stability analysis. Using Property 1 and canceling like terms yields $\nabla B^T(z_l)\, f(z_l, u_{e,l}) = -V_l \bar{e}^2 - e_0 (g_{e,l} u_{e,l} + \tau_{vol,l} - \tau_{rhs,l}(z_l, t))$ which, using Properties 3-6 and rearranging terms, can be upper bounded by

$$\nabla B^T(z_l)\, f(z_l, u_{e,l}) \leq -e_0 g_{e,l} u_{e,l} + C_1 + C_2 |e_0| + C_3 e_0^2, \quad (13)$$

where $C_1,\ C_2,\ C_3 \in \mathbb{R}_{>0}$ are known constants.

Choosing the operator selectable control gains $K_1,\ K_2,\ K_3\ \in\ \mathbb{R}_{>0}$ such that $K_i\ \geq\ C_i$, for every $i \in \{1,\ 2,\ 3\}$, the inequality in (13) can be rewritten as

$$\nabla B^T(z_l)\, f(z_l, u_{e,l}) \leq -e_0 g_{e,l} u_{e,l} + K_u(e_0), \qquad (14)$$

where $K_u(e_0) \triangleq K_1 + K_2|e_0| + K_3 e_0^2$. We define the constraining function $\gamma : \mathbb{R} \to \mathbb{R}$ as $\gamma(e_0) \triangleq K_M\left(e_0^2 - \bar{e}^2\right)$, which is always positive outside of $\mathbb{S}$. It is now possible to define a calculable control input constraint by substituting (14) into (12), which yields

$$\mathcal{U}(e_0) \triangleq \{u_{e,l} \in \mathbb{R} : -e_0 g_{e,l} u_{e,l} + K_u(e_0) \leq -K_M\left(e_0^2 - \bar{e}^2\right)\}, \qquad (15)$$

From the resulting constrained set, the implementable leader-cycle control input is defined as

$$u_{e,l}^*(e_0) \triangleq \underset{u_{e,l}\in\mathcal{U}}{\arg\min}\ |u_{e,l} - u_{nom}|^2 \qquad (16)$$
$$s.t. \quad -e_0 g_{e,l} u_{e,l} + K_u(e_0) + \gamma(e_0) \leq 0,$$

where $u_{e,l}^* \in \mathbb{R}$ represents the minimum allowable follower-cycle control input and u_{nom} is any locally Lipschitz nominal controller [21]. From (16) the solution for the minimum follower-cycle control input $u_{e,l}^*$ can be expressed as in [21]. To satisfy the secondary objective of providing haptic feedback of follower-cycle performance to the leader-cycle operator, the nominal controller $u_{nom} : \mathbb{R} \to \mathbb{R}$ is defined as $u_{nom}(e_1) \triangleq -k_{fb} e_1$, where $e_1 {:} \mathcal{Q}_l \times \mathcal{Q}_f \to \mathbb{R}$ denotes the subsequently defined follower-cycle trajectory error signal and $k_{fb} \in \mathbb{R}_{>0}$ is an operator-selected positive constant, yielding the leader-cycle motor control input

$$u_{e,l}^*(e_0, e_1) = \begin{cases} -\frac{b(e_0)}{a(e_0)} & b(e_0) - a(e_0)\, k_{fb} e_1 > 0 \\ -k_{fb} e_1 & \text{otherwise}, \end{cases} \qquad (17)$$

where $a, b : \mathbb{R} \to \mathbb{R}$ are defined from the condition in (16) as $a(e_0) \triangleq -e_0 g_{e,l}$ and $b(e_0) \triangleq K_u(e_0) + \gamma(e_0)$, respectively. According to [21, Lem. 1], the controller in (16) is feasible and there is no division by zero in (17) if, whenever $a(e_0) = 0$, the following condition holds: $b(e_0) < 0$. In the developed application, $a(e_0) = 0$ only if $e_0 = 0$. Therefore, the user-selected parameters must be designed to ensure that

$$b(0) = K_1 - K_M \bar{e}^2 < 0. \qquad (18)$$

B. Follower-cycle Controller

The control objective of the follower-cycle system is to develop a strongly coupled telerobotic system [20], where the desired trajectory of the FES/motor actuated lower-body cycling system is defined by the angular position of the leader-cycle system. To quantify the objective, the trajectory error signal e_1 and an auxiliary error signal $e_2 : \mathbb{R} \to \mathbb{R}$ are defined as

$$e_1 \triangleq q_l - q_f, \qquad (19)$$

$$e_2 \triangleq \dot{e}_1 + \alpha e_1, \qquad (20)$$

where $\alpha \in \mathbb{R}_{\geq 0}$ is an operator-selected constant. Taking the time derivative of (20), pre-multiplying by M_f, and substituting in (19) yields

$$M_f(q_f)\, \dot{e}_2 = M_f(q_f)\left[\ddot{q}_l - \ddot{q}_f + \alpha e_1\right]. \qquad (21)$$

Solving the open-loop follower-cycle dynamic system (7) for $M_f \ddot{q}_f$, substituting the result into (21), and using Property 1 to cancel like terms produces

$$\begin{aligned} M_f(q_f)\, \dot{e}_2 = &\ M_f(q_f)\, \ddot{q}_l - V_f(z_f)\, e_2 \\ & - g_M(z_f)\, u_s - g_{e,f} u_{e,f} \\ & - e_1 + \chi(z), \end{aligned} \qquad (22)$$

where $z \triangleq (z_l, z_f)$, and the auxiliary term $\chi : \mathcal{Q}_f \times \mathcal{Q}_l \times \mathbb{R}^2 \to \mathbb{R}$ is defined as

$$\begin{aligned} \chi(z) \triangleq &\ b_{c,f}\dot{q}_f + V_f(z_f)\, \dot{q}_l + V_f \alpha e_1 \\ & + G_f(q_f) + P_f(z_f) + d_f(t) \\ & + \tau_{vol,f}(t) + (M_f(q_f)\, \alpha + 1)\, e_0. \end{aligned}$$

Using Properties 2-6, Assumption 1, and the result of the subsequent stability analysis for the leader-cycle system, the upper bound on χ is defined as

$$|\chi| \leq c_1 + c_2 \alpha |e_1| + c_3 |e_2| + c_{V,f} \alpha^2 e_1^2, \qquad (23)$$

where $c_1, c_2, c_3 \in \mathbb{R}_{\geq 0}$ are known constants.

From (19), (20), (22), and the subsequent stability analysis, the FES control input $u_s : \mathbb{R} \to \mathbb{R}$ is designed as

$$u_s(e_2) = \sigma_s k_1 e_2\ , \qquad (24)$$

where $k_1 \in \mathbb{R}_{\geq 0}$ is an operator-selected constant control gain, and the discontinuous switching signal $\sigma_s : \mathcal{Q}_f \to \{0, 1\}$ indicates when the follower-cycle is within the FES stimulation regions such that

$$\sigma_s(q_f) \triangleq \begin{cases} 1 & \text{if } q_f \in \mathcal{Q}_{FES} \\ 0 & \text{if } q_f \in \mathcal{Q} \backslash \mathcal{Q}_{FES}. \end{cases} \qquad (25)$$

From (19), (20), (22), and the subsequent stability analysis, the follower-cycle motor control input $u_{e,f} : \mathbb{R}^2 \to \mathbb{R}$ is designed as

$$u_{e,f}(z) = \sigma_e k_2 e_2 + \operatorname{sgn}(e_2)\left(k_3 + k_4 |e_1| + k_5 e_1^2\right), \qquad (26)$$

where $k_2, k_3, k_4, k_5 \in \mathbb{R}_{\geq 0}$ are operator-selected constant control gains. A discontinuous switching signal $\sigma_e : \mathcal{Q}_f \to [0, 1]$, designed to allow for variable motor assistance for maximal rehabilitative benefit, is defined as [19]

$$\sigma_e(q_f) \triangleq \begin{cases} 1 & \text{if } q_f \in \mathcal{Q} \backslash \mathcal{Q}_{FES} \\ \prod_{m \in \mathcal{M}} (1 - \beta_m \sigma_m) & \text{if } q_f \in \mathcal{Q}_{FES}, \end{cases} \qquad (27)$$

where $0 < \beta_m \leq 1$, for every $m \in \mathcal{M}$, are operator-selected constants to set the proportional level of motor current applied within each FES region.

Substituting (24) and (26) into (22) yields the closed-loop follower-cycle dynamic equation

$$\begin{aligned} M_f(q_f)\dot{e}_2 &= \chi(z) + M_f(q_f)\ddot{q}_l - V_f(z_f)e_2 - e_1 \\ &\quad - g_M(z_f)\sigma_s k_1 e_2 - g_{e,f}\sigma_e k_2 e_2 \\ &\quad - g_{e,f}\left[\operatorname{sgn}(e_2)\left(k_3 + k_4|e_1| + k_5 e_1^2\right)\right]. \end{aligned} \tag{28}$$

IV. Stability Analysis

A. Leader-cycle system stability

We first analyze the safety of the leader-cycle independently. Consider the differential equation $\dot{z}_l = h_l(z_l)$, where $h_l(z_l) \triangleq (\dot{q}_l, \ddot{q}_l)$. A set of physically realistic inital conditions is defined as $\mathcal{D} \triangleq \{z_l \in \mathcal{Q}_l \times \mathbb{R} : |\dot{q}_l| \leq c_l\}$, for some $c_l \in \mathbb{R}$. Furthermore, c_l is selected large enough such that the safe set $\mathbb{S} \subset \mathcal{D}$.

Theorem 1. *For the motor actuated leader-cycle system described by the differential equation $\dot{z}_l = h_l(z_l)$, the controller $u_{e,l}^*$ is locally Lipschitz continuous and the set $\mathbb{S}$ is UGAS, provided that the feasibility condition in (18) is met such that*

$$K_M > \tfrac{K_1}{\bar{e}^2}.$$

Moreover, $\mathcal{D}$ is forward invariant, and $\ddot{q}_l$ is uniformly bounded along any solution starting in the set $\mathcal{D}$.

Proof: The claim that $u_{e,l}^*$ is locally Lipschitz follows from [21, Lem. 1]. The proof that $\mathbb{S}$ is UGAS is identical to the proof of [27, Thm. 1]. Let $z_l : \operatorname{dom} z_l \to \mathbb{R}^2$ be a solution to $\dot{z}_l \in h_l(z_l)$ with $z_l(0) \in \mathcal{D}$. The definition of UGAS leads to the conclusion that $e_0, \dot{q}_l \in \mathcal{L}_\infty$. In particular, $\dot{q}_l(t) \in \mathcal{D}$ for all $t \in \operatorname{dom} z_l$, meaning that $\mathcal{D}$ is forward invariant. Since $u_{e,l}^*$ is continuous, $u_{e,l}^*$ is bounded along the closed-loop trajectories. Using Properties 2-6 and Assumption 1, $|\ddot{q}_l| \leq c_1 + c_2|\dot{q}_l| + c_3\dot{q}_l^2$. Thus, because $\dot{q}_l$ is bounded by a known constant on $\mathcal{D}$, so is $\ddot{q}_l$. Since $\dot{q}_l$ never leaves $\mathcal{D}$, we conclude that $\ddot{q}_l$ is uniformly bounded along any solution starting in the set $\mathcal{D}$. ■

B. Follower-cycle system stability

Consider the FES/motor actuated follower-cycle modeled in (2). The closed-loop follower-cycle dynamic equation in (28) is dependent on both the leader and follower states (i.e., the concatenated state vector z). Because the FES and motor control inputs for the follower-cycle are discontinuous by design, we use the Filippov regularization of the closed-loop dynamics to conduct a stability analysis. Using the K operator defined in [28], we analyze the differential inclusion $\dot{z} \in K[h](z)$, where $h(z) \triangleq (\dot{q}_l, \ddot{q}_l, \dot{q}_f, \ddot{q}_f)$. We consider solutions flowing in the set $\mathcal{Z} \triangleq \mathcal{D} \times \mathcal{Q}_f \times \mathbb{R}$.

A positive definite, radially unbounded, common Lyapunov function candidate for the follower-cycle, $V_L : \mathcal{Z} \to \mathbb{R}_{\geq 0}$, is defined as

$$V_L(z) = \tfrac{1}{2}e_1^2 + \tfrac{1}{2}M_f(q_f)e_2^2. \tag{29}$$

Theorem 2. *From the result in Thm. 1, for any solution starting in $\mathcal{D}$, there exists $C_l \in \mathbb{R}_{\geq 0}$ such that $|\ddot{q}_l| \leq C_l$. Therefore, for the FES and motor actuated rehabilitation follower-cycle system described by the differential inclusion $\dot{z} \in K[h](z)$, the set $\mathcal{A} \triangleq \{z \in \mathcal{Z} : e_1 = e_2 = 0\}$ is exponentially stable from $\mathcal{Z}$, and every maximal solution to the dynamics are complete, provided the following gain conditions are met*

$$g_{\underline{M}}k_1 + g_{e,f}\nu k_2 > c_3, \tag{30}$$

$$k_2 > \frac{c_3}{g_{e,f}}, \tag{31}$$

$$k_3 > \frac{c_1 + c_{M,f}C_l}{g_{e,f}}, \tag{32}$$

$$k_4 > \frac{c_2}{g_{e,f}}\alpha, \tag{33}$$

$$k_5 > \frac{c_{V,f}}{g_{e,f}}\alpha^2, \tag{34}$$

$$\alpha \in (0,1). \tag{35}$$

Proof: Using the notion of the generalized time derivative in [29, Defn. 3] and the fact that the Filippov regularization of the dynamics are identical to the original dynamics except the sgn function is replaced by the generalized SGN function, solving (20) for $\dot{e}_1$ and substituting along with (28) into the generalized time derivative of (29), using Property 1 to cancel like terms, and rearranging yields

$$\begin{aligned} \dot{\overline{V}}_L(z) =& \chi e_2 + M_f\ddot{q}_l e_2 - \alpha e_1^2 - g_M\sigma_s k_1 e_2^2 \\ & - g_{e,f}\sigma_e k_2 e_2^2 - g_{e,f}|e_2|\left(k_3 + k_4|e_1| + k_5 e_1^2\right). \end{aligned} \tag{36}$$

Using Properties 2, 6, and 7, from (36) the generalized time derivative is upper bounded for all $z \in \mathcal{Z}$ by

$$\begin{aligned} \dot{\overline{V}}_L(z) \leq& \; |e_2|\left(c_1 + c_2\alpha|e_1| + c_3|e_2| + c_{V,f}\alpha^2 e_1^2\right) \\ & + c_{M,f}C_l|e_2| - \alpha e_1^2 \\ & - \left(g_{\underline{M}}\sigma_s k_1 + g_{e,f}\sigma_e k_2\right)e_2^2 - g_{e,f}k_3|e_2| \\ & - g_{e,f}k_4|e_1||e_2| - g_{e,f}k_5 e_1^2|e_2|. \end{aligned}$$

Selecting gain values to meet the conditions in (32)-(35) yields

$$\dot{\overline{V}}_L(z) \leq c_3 e_2^2 - \alpha e_1^2 - \left(g_{\underline{M}}\sigma_s k_1 + g_{e,f}\sigma_e k_2\right)e_2^2. \tag{37}$$

When $q_f \in \mathcal{Q}_{FES}$, $\sigma_s = 1$ and $\sigma_e \in [0,1]$. Let $\nu \triangleq \min_{q_f \in \mathcal{Q}_{FES}}\{\sigma_e\}$. Selecting the gain values to meet the condition in (30) yields the negative definite inequality $\dot{\overline{V}}_L(z) \leq -\alpha e_1^2 - \lambda_1 e_2^2$, where $\lambda_1 \triangleq g_{\underline{M}}k_1 + g_{e,f}\nu k_2 - c_3$. When $q_f \in \mathcal{Q}\backslash\mathcal{Q}_{FES}$, $\sigma_s = 0$ and $\sigma_e = 1$. Selecting the gain values to meet the condition in (31) yields the negative definite inequality $\dot{\overline{V}}_L(z) \leq -\alpha e_1^2 - \lambda_2 e_2^2$, where $\lambda_2 \triangleq g_{e,f}k_2 - c_3$. Let $\lambda \triangleq \min(\lambda_1, \lambda_2)$. Then,

$$\dot{\overline{V}}_L(z) \leq -\alpha e_1^2 - \lambda e_2^2,$$

for all $z \in \mathcal{Z}$. From (29) it can be shown that $\dot{\overline{V}}_L \leq -\frac{min(\alpha,\lambda)}{\psi_2}V_L$. Using Property 1, the constraint in (34) that

$\alpha \in (0,1)$, and the fact that $\|z\|_{\mathcal{A}}^2 = \frac{1}{2}\left(e_1^2 + \dot{e}_1^2\right)$, where $\|z\|_{\mathcal{A}} := \inf_{y\in\mathcal{A}} |z-y|$, there exists $\psi_1, \psi_2 \in \mathbb{R}_{>0}$ such that

$$\psi_1 \|z\|_{\mathcal{A}}^2 \leq V_L(z) \leq \psi_2 \|z\|_{\mathcal{A}}^2$$

for all $z \in \mathcal{Z}$. By invoking [30, Thm. 1] we conclude that $\mathcal{A}$ is exponentially stable (ES) from $\mathcal{Z}$. For any maximal solution $z : \mathcal{I}_z \to \mathcal{Z}$ to $\dot{z} \in K[h](z)$, where $\mathcal{I}_z \subset [0,\infty)$, $e_1,\ e_2 \in \mathcal{L}_\infty$ using the definition of ES. Since $\dot{q}_l \in \mathcal{L}_\infty$ via Theorem 1, it follows from the definition of e_2 that $\dot{q}_f \in \mathcal{L}_\infty$. Thus, the controllers u_s and $u_{e,f}$ are bounded along the closed-loop trajectories. It follows that the dynamics $K[h](z(\mathcal{I}_z))$ are bounded, and from [31, Lem. 3.3] we conclude that $\mathcal{I}_z = [0,\infty)$, meaning that maximal solutions to the dynamic system are complete. ■

V. Conclusion

A CBF was used to produce a safe, teleoperated FES cycling system, such that the leader-cycle trajectory is constrained within a operator-defined desirable rehabilitative cadence range. Lyapunov-based analysis was used on the companion FES/motor actuated switched nonlinear follower-cycle, showing global exponential tracking of the leader-cycle trajectory. Future work includes further testing to determine system capabilities for people with NDs, as well as the development of more informative, physics-based haptic feedback (i.e., u_{nom}) to the leader-cycle operator.

References

[1] A. Bersano, M. Kraemer, E. Touze, R. Weber, A. Alamowitch, I. Sibon, and L. Pantoni, "Stroke care during the covid-19 pandemic: Experience from three large european countries," *Eur. J. Neurol.*, vol. 27, no. 9, pp. 1794–1800, 2020.

[2] T. Yan, C. W. Y. Hui-Chan, and L. S. W. Li, "Functional electrical stimulation improves motor recovery of the lower extremity and walking ability of subjects with first acute stroke: A randomized placebo-controlled trial," *Stroke*, vol. 36, no. 1, pp. 80–85, 2005.

[3] A. Kralj and T. Bajd, *Functional electrical stimulation: standing and walking after spinal cord injury*. CRC, 1989.

[4] K. J. Hunt, B. Stone, N.-O. Negård, T. Schauer, M. H. Fraser, A. J. Cathcart, C. Ferrario, S. A. Ward, and S. Grant, "Control strategies for integration of electric motor assist and functional electrical stimulation in paraplegic cycling: Utility for exercise testing and mobile cycling," *IEEE Trans. Neural Syst. Rehabil. Eng.*, vol. 12, no. 1, pp. 89–101, Mar. 2004.

[5] S. Ferrante, A. Pedrocchi, G. Ferrigno, and F. Molteni, "Cycling induced by functional electrical stimulation improves the muscular strength and the motor control of individuals with post-acute stroke," *Eur. J. Phys. Rehabil. Med.*, vol. 44, no. 2, pp. 159–167, 2008.

[6] S. P. Hooker, S. F. Figoni, M. M. Rodgers, R. M. Glaser, T. Mathews, A. G. Suryaprasad, and S. C. Gupta, "Physiologic effects of electrical stimulation leg cycle exercise training in spinal cord injured persons," *Arch. Phys. Med. Rehabil.*, vol. 73, no. 5, pp. 470–476, 1992.

[7] C. Cousin, V. Duenas, and W. E. Dixon, "FES cycling and closed-loop feedback control for rehabilitative human-robot interaction," *Robotics*, vol. 10, no. 61, pp. 1–23, 2021.

[8] M. J. Bellman, R. J. Downey, A. Parikh, and W. E. Dixon, "Automatic control of cycling induced by functional electrical stimulation with electric motor assistance," *IEEE Trans. Autom. Science Eng.*, vol. 14, no. 2, pp. 1225–1234, April 2017.

[9] T. Klarner, T. S. Bass, Y. Sun, C. Kaupp, P. M. Loadman, and E. P. Zehr, "Exploiting interlimb arm and leg connections for walking rehabilitation: A training intervention in stroke," *Neural Plast.*, 2016.

[10] D. P. Ferris, H. J. Huang, and P.-C. Kao, "Moving the arms to activate the legs," *Exercise and Sport Science Reviews*, vol. 34, no. 3, pp. 113–120, 2006.

[11] R. Zhou, L. Alvarado, R. Ogilvie, S. L. Chong, O. Shaw, and V. Mushahwar, "Non-gait-specific intervention for the rehabilitation of walking after SCI: Role of the arms," *J. Neurophysiol.*, vol. 119, no. 6, pp. 2194–2211, 2018.

[12] N. Lehrer, S. Attygale, S. Wolf, and T. Rikakis, "Exploring the bases for a mixed reality stroke rehabilitation system, part I: A unified approach for representing action, quantitative evaluation, and interactive feedback," *J NeuroEng Rehabil*, vol. 8, no. 51, pp. 1–15, 2011.

[13] M. W. O'Dell, C.-C. D. Lin, and V. Harrison, "Stroke rehabilitation: Strategies to enhance motor recovery," *Annu. Rev. Med.*, vol. 60, pp. 55–68, 2009.

[14] M. Minge, E. Ivanova, K. Lorenz, G. Joost, M. Thuring, and J. Kruger, "Bemobil: Developing a user-friendly and motivating telerehabilitation system for motor relearning after stroke," in *2017 International Conference on Rehabilitation Robotics (ICORR)*, 2017, pp. 870–875.

[15] M. H. Lee, "A technology for computer-assisted stroke rehabilitation," in *23rd International Conference on Intelligent User Interfaces (IUI)*, 2018, pp. 665–666.

[16] R. Polak, A. Bistritsky, Y. Gozlan, and S. Levy-Tzedek, "Novel gamified system for post-stroke upper-limb rehabilitation using a social robot: focus groups of expert clinicians," in *2019 International Conference on Virtual Rehabilitation (ICVR)*, 2019, pp. 1–7.

[17] M. Mendoza, I. Bonilla, E. Gonzalez-Galvan, and F. Reyes, "Impedence control in a wave-based teleoperator for rehabilitation motor therapies assisted by robots." *Comput. Methods Programs Biomed.*, vol. 123, pp. 54–67, Jan. 2016.

[18] N. Norouzi-Gheidari, P. S. Archambault, and J. Fung, "Effects of robot-assisted therapy on stroke rehabilitation in upper limbs: Systematic review and meta-analysis of the literature." *J. Rehabil. Res. Dev.*, vol. 49, no. 4, pp. 479–496, 2012.

[19] K. Stubbs, B. C. Allen, and W. E. Dixon, "Teleoperated motorized functional electric stimulation actuated rehabilitative cycling," in *Proc. ASME Dyn. Syst. Control Conf.*, 2020.

[20] M. Ferre, M. Buss, R. Aracil, C. Melchiorri, and C. Balaguer, *Advances in Telerobotics*, B. Siciliano, O. Khatib, and F. Groen, Eds. Springer, 2007.

[21] A. Isaly, B. C. Allen, R. G. Sanfelice, and W. E. Dixon, "Encouraging volitional pedaling in FES-assisted cycling using barrier functions," *Front. Robot. AI*, vol. 8, pp. 1–13, 2021, special Issue on Safety in Collaborative Robotics and Autonomous Systems.

[22] M. J. Bellman, T. H. Cheng, R. J. Downey, C. J. Hass, and W. E. Dixon, "Switched control of cadence during stationary cycling induced by functional electrical stimulation," *IEEE Trans. Neural Syst. Rehabil. Eng.*, vol. 24, no. 12, pp. 1373–1383, 2016.

[23] C. Rouse, C. Cousin, B. C. Allen, and W. E. Dixon, "Split-crank cadence tracking for switched motorized FES-cycling with volitional pedaling," in *Proc. Am. Control Conf.*, 2019, pp. 4393–4398.

[24] K. J. Stubbs, B. C. Allen, and W. E. Dixon, "Admittance control of a teleoperated motorized functional electrical stimulation rehabilitation cycle," in *IFAC Conf. Cyber-Phys. Hum.-Syst.*, 2020.

[25] C. Rouse, C. Cousin, V. H. Duenas, and W. E. Dixon, "Cadence tracking for switched FES cycling combined with voluntary pedaling and motor resistance," in *Proc. Am. Control Conf.*, 2018, pp. 4558–4563.

[26] F. I. E. Estay, C. Rouse, M. Cohen, C. Cousin, and W. E. Dixon, "Cadence and position tracking for decoupled legs during switched split-crank motorized FES-cycling," in *Proc. Am. Control Conf.*, 2019, pp. 854–859.

[27] A. Isaly, B. C. Allen, R. Sanfelice, and W. E. Dixon, "Zeroing control barrier functions for safe volitional pedaling in a motorized cycle," in *IFAC Conf. Cyber-Phys. Hum.-Syst.*, 2020.

[28] N. Fischer, R. Kamalapurkar, and W. E. Dixon, "LaSalle-Yoshizawa corollaries for nonsmooth systems," *IEEE Trans. Autom. Control*, vol. 58, no. 9, pp. 2333–2338, Sep. 2013.

[29] R. Kamalapurkar, J. A. Rosenfeld, A. Parikh, A. R. Teel, and W. E. Dixon, "Invariance-like results for nonautonomous switched systems," *IEEE Trans. Autom. Control*, vol. 64, no. 2, pp. 614–627, Feb. 2019.

[30] A. R. Teel, F. Forni, and L. Zaccarian, "Lyapunov-based sufficient conditions for exponential stability in hybrid systems," *IEEE Trans. Autom. Control*, vol. 58, no. 6, pp. 1591–1596, 2012.

[31] R. Kamalapurkar, W. E. Dixon, and A. Teel, "On reduction of differential inclusions and lyapunov stability," *ESAIM: Control, Optim. Calc. of Var.*, vol. 26, no. 24, pp. 1–16, 2020.