In Which Matching Markets Do Costly Compatibility Inspections Lead to a Deadlock?

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With the aim of understanding congestion in matching markets, we study a matching market with N women and $M = \alpha N$ men who want to match with each other. An agent pair must perform a costly inspection to verify compatibility prior to matching with each other, and we assume they are willing to perform the inspection only if it is "mutually desirable", i.e., they mutually rank each other as their favorite potential partner who remains under consideration. The inspection and matching process progresses iteratively in the market as matches form (in the case of successful inspections) and incompatibilities are revealed. We ask which large random markets suffer from an information deadlock, i.e., in which markets will a constant fraction of agents get stuck waiting for a mutually desirable inspection to become available. We prove, by building on the machinery of message passing and density evolution from statistical physics, that the existence of an information deadlock is governed by the men-to-women ratio α, the average degree of women (or men) and the probability that an inspection is successful. We find a phase transition between the information deadlock regime and the deadlock-free regime (where a vanishingly small fraction of agents are stuck waiting) and study the dependence of deadlock and its size on market primitives. We find, e.g., that well connected markets suffer from deadlocks, and holding the degree of women fixed there is a deadlock for α below a certain threshold.