**Proceedings of the ASME 2021
International Mechanical Engineering Congress and Exposition
IMECE2021
November 1-5, 2021, Virtual, Online**

IMECE2021-73687

DEEP NEURAL NETWORK REAL-TIME CONTROL OF A MOTORIZED FUNCTIONAL ELECTRICAL STIMULATION CYCLE WITH AN UNCERTAIN TIME-VARYING ELECTROMECHANICAL DELAY

Hannah M. Sweatland *, **Brendon C. Allen, Max L. Greene, Warren E. Dixon**
Department of Mechanical and Aerospace Engineering
University of Florida, Gainesville, Florida 32611
Email:{hsweatland,brendoncallen,maxgreene12,wdixon}@ufl.edu

ABSTRACT

Closed-loop functional electrical stimulation (FES) control methods are developed to facilitate motor-assisted cycling as a rehabilitative strategy for individuals with neurological disorders. One challenge for this type of control design is accounting for an input delay called the electromechanical delay (EMD) that exists between stimulation and the resultant muscle force. The EMD can cause an otherwise stable system to become unstable. A real-time deep neural network (DNN)-based motor control architecture is used to estimate the nonlinear and uncertain dynamics of each leg of the cycle-rider system. The DNN estimate of the system's dynamics updates in real-time and is used as a feedforward term in the motor controller allowing the cycle crank to meet position and cadence tracking objectives. The nonsmooth Lyapunov-based stability analysis proves semi-global asymptotic tracking.

Keywords: Functional electrical stimulation (FES); input delay; switched systems; deep neural network (DNN); Lyapunov methods

INTRODUCTION[1]

Prevalent symptoms of a broad class of neurological disorders (NDs) include progressive muscle weakness and loss of voluntary coordinated limb motion. In an effort to combat the effects of NDs such as muscle atrophy and cardiovascular disease, therapeutic strategies often involve the use of functional electrical stimulation (FES) [1] and [2]. FES induces muscle contractions, which can be paired with motor assistance to facilitate stationary cycling on a recumbent cycle. While these therapies are useful, it is difficult to develop closed-loop FES and motor control schemes that maximize therapeutic benefits and minimize fatigue. These challenges are due to the inherent time-varying, nonlinear, and uncertain dynamics of the switched system. Other factors that complicate the controller design include rider asymmetries and unknown disturbances. By decoupling the two sides of the cycle, and controlling each side separately (as is done in the developed method) rider asymmetries can be compensated for. Each controller is designed such that the non-dominant leg tracks a desired cadence, and the dominant leg tracks a position that is offset 180 degrees from the non-dominant leg. This allows the non-dominant leg to set the pace to ensure it is properly exercised and prevents the dominant leg from being overworked (as is possible on a single-crank cycle). Further complicating the controller design is an unknown and time-varying input delay, called the electromechanical delay (EMD)[2], that exists between the start of stimulation and the onset of muscle contraction, as well as the end of stimulation and the end of the corresponding muscle contraction.

Input delay is not unique to FES cycle-rider systems. It exists in many engineering applications such as multi-agent systems [4], the teleoperation of robotic manipulators, and internal combustion systems [5]. Efforts have been made to design controllers to account for these delays. In the case of EMD in FES, the delay is time-varying and often increases with the onset of muscle fatigue [6] and [7]. These muscle-induced input delays impact closed-loop controller performance and can

*Address all correspondence to this author.

[1]This research is supported in part by by NSF award number 1762829. Any opinions, findings and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the sponsoring agency.

[2]In some literature, the EMD corresponds to the time latency between the onset of EMG activity and muscle force production [3].

cause an otherwise stable system to become unstable. In an effort to prevent EMD-generated instabilities, closed-loop EMD-compensating FES controllers have been designed for cycling. Other works such as [8–10] use robust control methods with a constant EMD estimate to meet cadence tracking objectives, while recent work in [11] uses a time-varying estimate of the EMD to achieve torque and cadence tracking. Unlike the developed method, the aforementioned works are not designed for a split-crank cycle, which limits their ability to compensate for rider asymmetries, and do not include adaptive terms in the controller.

The design and implementation of an adaptive controller can account for some of the problems associated with an uncertain and nonlinear cycle-rider system and allow for personalized control schemes between individuals. Deep neural networks (DNNs) are one type of function approximator that have been used in adaptive control results [12] and can be used to approximate uncertainties in the dynamics of the system. Deeper networks facilitate better function approximation; hence, they are more desirable than single-layer neural networks (NNs) [13] and [14]. Since the accuracy of the outputs of DNNs are often probabilistic, they typically lack the performance guarantees needed for use in safety-critical applications [12], such as rehabilitative cycling. Unlike updating a single hidden-layer NN in real-time (e.g., NN-based adaptive controllers in [15]), updating the inner-layer weights of a DNN involves the use of difficult-to-derive adaptive update laws because the NN weights are nested inside nonlinear activation functions. Recently, [12] developed a DNN-based controller that estimates the drift dynamics of a system. The DNNs in [12] and the developed work update in multiple times scales. The inner-layer features are iteratively (i.e., discretely) updated via online batch update training, while the output-layer weights update in real-time. The inner-layer features are estimated using data-driven function approximation methods. Periodically, the inner-layer features update, overwriting features from the previous iteration. The output-layer weights update using a gradient descent-based adaptive update law. Together, the multi-timescale approach facilitates better learning. Unlike the result in [12], the developed method applies a DNN-based multi-timescale adaptive controller to an Euler-Lagrange system with an input delay.

Leveraging the result in [12], this paper uses a DNN to estimate the dynamics of a nonlinear and delayed FES switched split-crank cycle-rider system. This results in a new control design and nonsmooth Lyapunov-based stability analysis, which yields improved FES and motor controller performance and a semi-global asymptotic tracking result. Unlike in previous works, the DNN uses the desired trajectory instead of the actual trajectory which ensures that the DNN inputs lie on a compact set. The inner-layer features can be learned *a priori* using transfer learning methods (i.e., training with healthy participants and extending those weights for use with participants with NDs). If adequate data sets are not available for pre-training, the inner-layer features can be randomized and updated after a batch update. An integral position error is developed in an effort to improve position tracking of the non-dominant leg. The developed control scheme uses a DNN, which updates in multiple timescales, as a feedforward term in the motor controller, allowing the gain conditions to be improved and the cycle crank to track a smooth desired trajectory despite uncertainties in the dynamic model.

MODEL DYNAMICS

This paper considers a recumbent cycle with a split-crank design, where each side of the cycle can be powered independently using decoupled motors [16] and [17]. Consider a set of possible delay values $\mathbb{S} \subset \mathbb{R}_{>0}$, where $\tau : \mathbb{R}_{\geq 0} \to \mathbb{S}$ denotes the EMD, and where $t, t_0 \in \mathbb{R}_{\geq 0}$ denote the time and initial time, respectively. Throughout this work, the switching signals are piecewise right-continuous and delayed functions are defined as $f_\tau \triangleq f(t - \tau(t))$ for all $t - \tau(t) \geq t_0$, and $f_\tau \triangleq 0$, otherwise. The nonlinear uncertain switched dynamics of each leg of the motorized cycle-rider system can be modeled independently, without loss of generality, as (see [9] and [18])[3]

$$\begin{aligned} &M(q)\ddot{q} + V(q,\dot{q})\dot{q} + G(q) + P(q,\dot{q}) \\ &+b_c\dot{q} + d(t) = \underbrace{\sum_{m\in\mathcal{M}} B_m(q,\dot{q},t)\, k_m \sigma_{m,\tau} u_\tau}_{B_M^\tau(q,\dot{q},\tau,t)} \\ &\qquad + \tau_{vol} + \underbrace{B_e k_e}_{B_E} u_e(t). \end{aligned} \tag{1}$$

The measurable crank angle is denoted by $q : \mathbb{R}_{\geq 0} \to \mathcal{Q}$, where the set of all possible crank angles is denoted by $\mathcal{Q} \subseteq \mathbb{R}$. The measurable crank velocity or cadence is denoted by $\dot{q} : \mathbb{R}_{\geq 0} \to \mathbb{R}$. The unmeasurable acceleration is denoted by $\ddot{q} : \mathbb{R}_{\geq 0} \to \mathbb{R}$. The inertial, gravitational, and centripetal-Coriolis matrices are denoted by $M : \mathcal{Q} \to \mathbb{R}_{>0}$, $G : \mathcal{Q} \to \mathbb{R}$, $V : \mathcal{Q} \times \mathbb{R} \to \mathbb{R}$, and passive viscoelastic tissue forces, disturbances, and viscous damping coefficient are denoted by $P : \mathcal{Q} \times \mathbb{R} \to \mathbb{R}$, $d : \mathbb{R}_{\geq 0} \to \mathbb{R}$, and $b_c \in \mathbb{R}_{>0}$, respectively. The known lumped motor control effectiveness is $B_E \in \mathbb{R}_{>0}$, and the motor effectiveness term is $B_e \in \mathbb{R}_{>0}$. Similarly, the unknown lumped stimulation control effectiveness and individual muscle effectiveness terms are $B_M^\tau : \mathcal{Q} \times \mathbb{R} \times \mathbb{S} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ and $B_m : \mathcal{Q} \times \mathbb{R} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{>0}$, $\forall m \in \mathcal{M}$, respectively, where $\mathcal{M} \triangleq \{H, Q, Gl\}$ is the set of all admissible muscles with H, Q, and Gl indicating the hamstrings, quadriceps femoris, and gluteal muscle groups, respectively. The variables $k_e, k_m \in \mathbb{R}_{>0}$, $\forall m \in \mathcal{M}$ are user-selected constants. The delayed and implemented FES stimulation inputs $\forall m \in \mathcal{M}$ are defined as $u_{m,\tau} \triangleq k_m \sigma_{m,\tau} u_\tau$ and $u_m \triangleq k_m \sigma_m u$, respectively, where the delayed FES control input is denoted by $u_\tau : \mathbb{S} \times \mathbb{R}_{\geq 0} \to \mathbb{R}$ and the the implemented FES control input is denoted by $u : \mathbb{R}_{\geq 0} \to \mathbb{R}$. For a given $m \in \mathcal{M}$, the delayed FES switching signal, $\sigma_{m,\tau}$, indicates if muscle m receives the FES

[3]For notational brevity, all explicit dependence on time, t, within the terms $q(t)$, $\dot{q}(t)$, $\ddot{q}(t)$, and $\tau(t)$ is suppressed.

input u_τ at $t-\tau(t)$. The implemented motor control input is denoted by $u_e:\mathbb{R}_{\geq 0}\to\mathbb{R}$, and the motor current input is defined as $u_E \triangleq k_e u_e$.

The activation of the FES is determined by the implemented switching signals, $\sigma_m : \mathcal{Q}\times\mathbb{R}\to\{0,\,1\}$, defined as

$$\sigma_m(q,\dot{q}) \triangleq \begin{cases} 1, & q_\alpha(q,\dot{q})\in\mathcal{Q}_m \\ 0, & \text{otherwise}\end{cases},\forall m\in\mathcal{M}, \tag{2}$$

where $q_\alpha : \mathcal{Q}\times\mathbb{R}\to\mathbb{R}$ is a trigger condition and adjusts the activation/deactivation of the FES input. This ensures that muscle contractions occur in desired contraction regions, defined as $\mathcal{Q}_{FES}\triangleq \bigcup_{m\in\mathcal{M}}\{\mathcal{Q}_m\}$, where $\mathcal{Q}_m\subset\mathcal{Q}$, $\forall m\in\mathcal{M}$ is the desired contraction region for each muscle group. The trigger condition is designed to reduce the residual torques in kinematic dead-zones, defined as $\mathcal{Q}_{KDZ}\triangleq\mathcal{Q}\setminus\mathcal{Q}_{FES}$. To produce effective positive crank rotation, $\mathcal{Q}_m$ is defined, according to [19], as

$$\mathcal{Q}_m \triangleq \{q\in\mathcal{Q}\,|\,T_m(q)>\varepsilon_m\}\,,\forall m\in\mathcal{M},$$

where $T_m:\mathcal{Q}\to\mathbb{R}$ and $\varepsilon_m\in\mathbb{R}_{>0}$ denote a torque transfer ratio and a user-selected lower threshold.

Though the terms in (1) are unknown, the subsequently designed FES and motor controllers require known bounds on the aforementioned parameters [19].

Property: 1 The unknown terms in (1) can be bounded as $|d|\leq c_d$, $b_c\dot{q}\leq c_c|\dot{q}|$, $|P|\leq c_{P_1}+c_{P_2}|\dot{q}|$, $|G|\leq c_G$, $|V|\leq c_V|\dot{q}|$, $c_m\leq M\leq c_M$, $|\tau_{vol}|\leq c_{vol}$ respectively, where $c_d,c_c,c_{P_1},c_{P_2},c_G,c_V c_m,c_M,c_{vol}\in\mathbb{R}_{>0}$ are known constants [19].

Property: 2 The time derivative of the inertia matrix and the centripetal-Coriolis matrix are skew symmetric, $\frac{1}{2}\dot{M}(q)=V(q,\dot{q})$ [19].

Property: 3 The lumped FES (when $\sum_{m\in\mathcal{M}}\sigma_{m,\tau}>0$) control input is bounded as $c_b\leq B_M^\tau\leq c_B$, where $c_b,c_B\in\mathbb{R}_{>0}$ are known constants [8].

Property: 4 The time delay can be bounded as $\underline{\tau}\leq\tau\leq\overline{\tau}$, where $\underline{\tau},\overline{\tau}\in\mathbb{R}_{>0}$ are known constants. The delay estimate error can be bounded such that $|\tau-\hat{\tau}|\leq\overline{\overline{\tau}}$, where $\hat{\tau}\in\mathbb{R}_{\geq 0}$ is a constant estimate of the delay and $\overline{\overline{\tau}}\in\mathbb{R}_{>0}$ is a known constant [7].

CONTROL DEVELOPMENT

Open-Loop Error System Development

The control objective is for the bicycle crank to track a smooth desired trajectory ($q_d:\mathbb{R}_{\geq 0}\to\mathcal{Q}$, $\dot{q}_d,\ddot{q}_d:\mathbb{R}_{\geq 0}\to\mathbb{R}$) despite an unknown input delay, discontinuous switching between different control inputs, and other uncertainties in the dynamic model. To simplify the tracking objective, the same control structure will be developed for both legs. A measurable integral position tracking error, $e_0:\mathbb{R}_{\geq 0}\to\mathbb{R}$, is designed to improve position tracking and is defined as

$$e_0 \triangleq \int_{t_0}^{t} q_d(\theta)-q(\theta)\,d\theta, \tag{3}$$

and a measurable proportional-integral-type signal, denoted by $e_1:\mathbb{R}_{\geq 0}\to\mathbb{R}$, is defined as

$$e_1 \triangleq \dot{e}_0+\alpha_0 e_0, \tag{4}$$

where $\alpha_0\in\mathbb{R}_{>0}$ is a user-defined constant. Applying Leibniz integral rule to (3) yields

$$\dot{e}_0 = q_d - q, \tag{5}$$

and taking the time derivative of (5) yields

$$\ddot{e}_0 = \dot{q}_d-\dot{q}, \tag{6}$$

where $\dot{e}_0$ and $\ddot{e}_0$ denote the measurable position and cadence errors, respectively. To add a delay-free input term into the closed-loop error system, the auxiliary error signal $e_u:\mathbb{R}_{\geq 0}\to\mathbb{R}$ is designed as

$$e_u \triangleq -\int_{t-\hat{\tau}}^{t} u(\theta)\,d\theta. \tag{7}$$

A measurable proportional-integral-derivative-based tracking error denoted by $r:\mathbb{R}_{\geq 0}\to\mathbb{R}$ is defined as

$$r \triangleq \dot{e}_1+\alpha_1 e_1+\alpha_2 e_u, \tag{8}$$

where $\alpha_1,\alpha_2\in\mathbb{R}_{>0}$ are selectable constants. The vector of error signals, $z\in\mathbb{R}^4$, is defined as

$$z \triangleq \begin{bmatrix} e_0 & e_1 & r & e_u\end{bmatrix}^T. \tag{9}$$

By taking the time derivative of (8), multiplying by M, using (1) and (3)-(8), and adding and subtracting $B_M^\tau u_{\hat{\tau}}+e_1+f_d$, the open-loop error system

$$\begin{aligned} M(q)\,\dot{r} = {} & \chi - e_1 - V(q,\dot{q})\,r + B_M^\tau(u_{\hat{\tau}}-u_\tau) - B_E u_e \\ & - M(q)\,\alpha_2 u + (M(q)\,\alpha_2 - B_M^\tau)\,u_{\hat{\tau}} + f_d, \end{aligned} \tag{10}$$

is obtained, where the auxiliary term $\chi:\mathbb{R}_{\geq 0}\to\mathbb{R}$ is defined as

$$\chi \triangleq f - f_d + d(t) - \tau_{vol}(t) + e_1 + M(q)(\alpha_0+\alpha_1)\dot{e}_1 - M(q)\alpha_0^2\dot{e}_0 + V(q,\dot{q})\left((\alpha_0+\alpha_1)e_1 - \alpha_0^2 e_0 + \alpha_2 e_u\right). \quad (11)$$

The function $f : \mathcal{Q} \times \mathbb{R}^3 \to \mathbb{R}$ is defined as

$$f \triangleq M(q)\ddot{q}_d + V(q,\dot{q})\dot{q}_d + G(q) + P(q,\dot{q}) + b_c\dot{q}, \quad (12)$$

and the function $f_d : \mathcal{Q} \times \mathbb{R}^2 \to \mathbb{R}$ is defined as

$$f_d(x_d) \triangleq M(q_d)\ddot{q}_d + V(q_d,\dot{q}_d)\dot{q}_d + G(q_d) + P(q_d,\dot{q}_d) + b_c\dot{q}_d, \quad (13)$$

where $x_d \triangleq \begin{bmatrix} q_d & \dot{q}_d & \ddot{q}_d \end{bmatrix}$.

Deep Neural Network Approximation

NNs are able to approximate smooth nonlinear functions on a compact set and can be useful in systems with unknown or uncertain dynamics such as the cycle-rider FES system in (1). NNs with more layers (i.e., more depth) have the potential to approximate functions more accurately [13], but having multiple layers complicates the selection of real-time update laws. Leveraging the result in [12], this work investigates if a DNN can be used to estimate the uncertain nonlinear dynamics of a Euler-Lagrange system with an unknown time-varying input delay. The DNN's output-layer weight matrix updates online in real-time. Simultaneously, the inner-layer feature estimates are updated iteratively (i.e., during batch updates). This iterative update of the inner-layers allows the system to collect data, update the inner-layers according to existing machine learning algorithms, and improve the quality of the estimate of $\hat{f}_{d,i}$. To ensure the DNN inputs lie on a compact set, the desired trajectory, as opposed to the actual trajectory, is used in the DNN approximation.

Since $q_d, \dot{q}_d, \ddot{q}_d \in \mathcal{L}_\infty$ by design, let $\Omega \subset \mathcal{Q} \times \mathbb{R}^2$ be a compact set such that $x_d \in \Omega$ and $f_d : \Omega \to \mathbb{R}$, where f_d is continuous. Therefore, the function $f_d(x_d)$ can be approximated by using a DNN as [20]

$$f_d(x_d) = W^{*T}\sigma^*(\Phi^*(x_d)) + \varepsilon(x_d), \quad (14)$$

where $W^* \in \mathbb{R}^{L\times 1}$ is an unknown bounded ideal output-layer weight matrix, $\sigma^* : \mathbb{R}^p \to \mathbb{R}^L$ is an unknown bounded vector of the ideal activation functions, $\Phi^* : \Omega \to \mathbb{R}^p$ is the ideal unknown DNN, $\varepsilon : \Omega \to \mathbb{R}$ is the unknown bounded function reconstruction error. The ideal unknown DNN can be expressed as $\Phi^*(x_d) = V_k\phi_k(V_{k-1}\phi_{k-1}(V_{k-2}\phi_{k-2}(\ldots x_d)))$, where $k \in \mathbb{N}$ denotes the number of inner-layers of the DNN, V_k denotes the inner-layer weights of the DNN, $\phi(\cdot)$ denotes the corresponding activation functions of the DNN.

The DNN is trained *a priori* using data sets from previous experiments or simulations and is updated using a multiple timescale approach, meaning that output-layer training happens online in real-time, while concurrently, the inner-layer estimates update using an iterative data-driven approach. The iterative updates of the inner-layer features, in-turn, improve the real-time learning of the output-layer weights. Using (14), the function f_d can be approximated as $\hat{f}_{d,i} = \widehat{W}^T\hat{\sigma}_i\left(\hat{\Phi}_i(x_d)\right)$ where $\widehat{W} : \mathbb{R}_{\geq 0} \to \mathbb{R}^{L\times 1}$ is the estimate of the ideal output-layer weight matrix, $\hat{\sigma}_i : \mathbb{R}^p \to \mathbb{R}^L$ is the i^{th} training iteration activation function, $\widehat{\Phi}_i : \Omega \to \mathbb{R}^p$ is the i^{th} training iteration DNN estimate, and $i \in \mathbb{N}$ is the DNN estimate update index. The estimation error $\tilde{W} : \mathbb{R}_{\geq 0} \to \mathbb{R}^{L\times 1}$ is defined as

$$\tilde{W}(t) \triangleq W^* - \widehat{W}(t). \quad (15)$$

Assumption 1. Using the universal function approximation property there exists known constants $\overline{W^*}$, $\overline{\sigma^*}$, $\overline{\hat{\sigma}}$, $\bar{\varepsilon} \in \mathbb{R}_{>0}$ such that the unknown ideal weights W^*, unknown ideal activation functions $\sigma^*(\cdot)$, user-selected activation functions $\hat{\sigma}_i(\cdot)$, the unknown ideal DNN $\Phi^*(\cdot)$, and the function reconstruction error $\varepsilon(\cdot)$ can be upper bounded such that $\sup_{x_d\in\Omega}\|W^*\| \leq \overline{W^*}$, $\sup_{x_d\in\Omega}\|\sigma^*(\cdot)\| \leq \overline{\sigma^*}$, $\sup_{x_d\in\Omega,\forall i}\|\hat{\sigma}_i(\cdot)\| \leq \overline{\hat{\sigma}}$, and $\sup_{x_d\in\Omega}\|\varepsilon(\cdot)\| \leq \bar{\varepsilon}$ [20].

Closed-Loop Error System Development

Substituting (14) into (10) yields

$$M(q)\dot{r} = \chi + \varepsilon(x_d) - e_1 - V(q,\dot{q})r + B_M^\tau(u_{\hat{\tau}} - u_\tau) - B_E u_e - M(q)\alpha_2 u + (M(q)\alpha_2 - B_M^\tau)u_{\hat{\tau}} + W^{*T}\sigma^*(\Phi^*(x_d)), \quad (16)$$

where, by Property 1 and Assumption 1,

$$|\chi + \varepsilon(x_d)| \leq \Phi + \rho(\|z\|)\|z\|, \quad (17)$$

where $\Phi \in \mathbb{R}_{>0}$ is a known constant, and $\rho(\cdot)$ is a positive, strictly increasing, and radially unbounded function.

Based on (16) and the stability analysis, the FES and motor controllers are designed, respectively, as

$$u \triangleq k_s r, \quad (18)$$

$$u_e \triangleq \frac{1}{B_E}\left(\widehat{W}^T\hat{\sigma}_i\left(\widehat{\Phi}_i(x_d)\right) + k_1\operatorname{sgn}(r) + \sigma_e k_2 r\right), \quad (19)$$

respectively, where $k_s, k_1, k_2 \in \mathbb{R}_{>0}$ are selectable constants, $\mathrm{sgn}(\cdot)$ is the signum function, and $\sigma_e : \mathcal{Q} \times \mathbb{R} \to \{0, 1\}$ is the motor switching signal which is defined as

$$\sigma_e(q,\dot{q}) \triangleq \begin{cases} 1, \; q \in \mathcal{Q}_{KDZ} \\ 1, \; q \in \mathcal{Q}_{FES}, \; \sum\limits_{m\in\mathcal{M}} \sigma_m(q,\dot{q}) = 0 \\ 0, \text{ otherwise} \end{cases}. \tag{20}$$

The output-layer weight adaptation law estimate is defined as

$$\dot{\widehat{W}}(t) \triangleq \Gamma_W \hat{\sigma}_i\left(\widehat{\Phi}_i(x(t))\right) r, \tag{21}$$

where $\Gamma_W \in \mathbb{R}^{L\times L}$ is a user-defined positive definite, diagonal control gain matrix. The closed-loop error system is found by substituting (18) and (19) into (16) to yield

$$\begin{aligned} M(q)\dot{r} = {} & W^{*T}\sigma^*(\Phi^*(x_d)) - \widehat{W}^T\hat{\sigma}_i\left(\widehat{\Phi}_i(x_d)\right) \\ & +\chi + \varepsilon(x_d) - e_1 - V(q,\dot{q})r - M(q)\alpha_2 k_s r \\ & +k_s B_M^{\tau}(r_{\hat{\tau}} - r_{\tau}) - k_1 \mathrm{sgn}(r) - \sigma_e k_2 r \\ & +(M(q)\alpha_2 - B_M^{\tau})k_s r_{\hat{\tau}}. \end{aligned} \tag{22}$$

Lyapunov-Krasovskii functionals, denoted by $Q_1, Q_2 : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$, are designed to facilitate the subsequent stability analysis as

$$Q_1 \triangleq \frac{1}{2}(\varepsilon_1\omega_1 + \omega_2)k_s \int_{t-\hat{\tau}}^{t} r(\theta)^2 d\theta, \tag{23}$$

$$Q_2 \triangleq \frac{\omega_3 k_s}{\hat{\tau}} \int_{t-\hat{\tau}}^{t}\int_{s}^{t} r(\theta)^2 d\theta ds, \tag{24}$$

where ε_1, ω_1, ω_2, and ω_3 are selectable positive constants. Based on the following stability analysis, auxiliary bounding constants denoted by $\beta_1, \beta_2 \in \mathbb{R}_{>0}$ are defined as

$$\begin{aligned} \beta_1 \triangleq \min\Bigg(& \alpha_0 - \frac{1}{2\varepsilon_2}, \; \alpha_1 - \frac{\varepsilon_2}{2} - \frac{\varepsilon_3\alpha_2^2}{2}, \\ & k_s\left(\frac{c_m\alpha_2}{2} - \varepsilon_1\omega_1 - \omega_2 - \omega_3\right), \\ & \frac{\omega_3}{k_s\hat{\tau}^2} - \frac{1}{2\varepsilon_3} - \frac{k_s\omega_1}{\varepsilon_1}\Bigg), \end{aligned} \tag{25}$$

$$\begin{aligned} \beta_2 \triangleq \min\Bigg(& \alpha_0 - \frac{1}{2\varepsilon_2}, \; \alpha_1 - \frac{\varepsilon_2}{2} - \frac{\varepsilon_3\alpha_2^2}{2}, \\ & \frac{1}{2}k_2 - k_s\left(\varepsilon_1\omega_1 + \frac{1}{2}\omega_2 + \omega_3\right), \\ & \frac{\omega_3}{k_s\hat{\tau}^2} - \frac{1}{2\varepsilon_3} - \frac{k_s\omega_1}{\varepsilon_1}\Bigg), \end{aligned} \tag{26}$$

where ε_2 and ε_3 are positive constants.

STABILITY ANALYSIS

In the following analysis, switching times are denoted by $\{t_n^i\}$, $i \in \{m, e\}$, $n \in \{0,1,2,...\}$, which denote the instants in time when B_M^{τ} becomes nonzero $(i = m)$ and when B_M^{τ} becomes zero $(i = e)$. Open and connected sets representing the domain and initial conditions, $\mathcal{D}, S_{\mathcal{D}} \subseteq \mathbb{R}^{6+L}$, are defined as

$$\mathcal{D} \triangleq \left\{ y \in \mathbb{R}^{6+L} \,|\, \|y\| < \gamma \right\}, \tag{27}$$

$$S_{\mathcal{D}} \triangleq \left\{ y \in \mathbb{R}^{6+L} \mid \|y\| < \sqrt{\tfrac{\lambda_1}{\lambda_2}}\gamma \right\}, \tag{28}$$

where $\gamma \in \mathbb{R}_{>0}$ represents a known constant defined as[4] $\gamma \leq \inf\left\{\rho^{-1}\left(\left(\sqrt{\min(\beta_1 c_m\alpha_2 k_s, \beta_2 k_2)}, \infty\right)\right)\right\}$. A Lyapunov function candidate, denoted by $V_L : \mathcal{D} \to \mathbb{R}_{\geq 0}$, that is positive definite and continuously differentiable is defined as

$$\begin{aligned} V_L(y) = {} & \frac{1}{2}e_0^2 + \frac{1}{2}e_1^2 + \frac{1}{2}M(q)r^2 + \frac{1}{2}\omega_1 e_u^2 \\ & + \frac{1}{2}\widetilde{W}^T\Gamma_W^{-1}\widetilde{W} + Q_1 + Q_2, \end{aligned} \tag{29}$$

where $y \in \mathcal{D}$ is defined as

$$y \triangleq \left[\, z^T \;\; \sqrt{Q_1} \;\; \sqrt{Q_2} \;\; \widetilde{W}^T \,\right]^T. \tag{30}$$

Based on Property 1, (29) can be bounded as

$$\lambda_1\|y\|^2 \leq V_L \leq \lambda_2\|y\|^2, \tag{31}$$

where $\lambda_1, \lambda_2 \in \mathbb{R}_{>0}$ are known constants. To facilitate the subsequent analysis, sufficient gain conditions are defined as

$$\alpha_0 > \frac{1}{2\varepsilon_2}, \tag{32}$$

$$\alpha_1 > \frac{\varepsilon_2}{2} + \frac{\varepsilon_3\alpha_2^2}{2}, \tag{33}$$

$$\alpha_2 > \frac{2}{c_m}(\varepsilon_1\omega_1 + \omega_2 + \omega_3), \tag{34}$$

$$\omega_3 > k_s\hat{\tau}^2\left(\frac{1}{2\varepsilon_3} + \frac{k_s\omega_1}{\varepsilon_1}\right), \tag{35}$$

$$k_1 \geq \overline{W^*}\left(\overline{\sigma^*} + \overline{\hat{\sigma}}\right) + \Phi + k_s\Upsilon\max\left(c_B\bar{\bar{\tau}}, c_M\bar{\tau}\alpha_2\right), \tag{36}$$

$$k_2 > 2k_s\left(\varepsilon_1\omega_1 + \frac{1}{2}\omega_2 + \omega_3\right), \tag{37}$$

$$\omega_2 \geq \max\left(|c_M\alpha_2 - c_b|, |c_m\alpha_2 - c_B|\right). \tag{38}$$

[4]For a set A, the inverse image is defined as $\rho^{-1}(A) \triangleq \{a \mid \rho(a) \in A\}$.

Since the inner features of the DNN approximation of $\hat{f}_{d,i}$ are updated iteratively (via batch updates as in [21] and [22]), the approximation for the dynamics $\hat{f}_{d,i}$ is discontinuous. Since the estimate of the dynamics is included in the motor controller, the controller is similarly discontinuous, which introduces another discontinuous signal into the closed-loop error system. Because the time derivative of V_L is discontinuous, a nonsmooth Lyapunov-based stability analysis is used to prove the following theorem.

Theorem 1. *Consider a nonlinear system modeled by the dynamics in (1) which satisfies Assumption 1 and Properties 1-4. The control inputs in (18) and (19) and the output-layer weight adaptation law in (21) ensure the trajectory tracking error defined in (9) yields semi-global asymptotic tracking in the sense that* $\lim_{t\to\infty}\|z(t)\|\to 0$, $t \geq t_0$, *provided that* $y(t_0)\in S_{\mathcal{D}}$ *and the gain conditions (32)-(38) are satisfied.*

Proof. For $t\in[t_0,\infty)$, let $y(t)$ be a Filippov solution to the differential inclusion $\dot{y}\in K[h](y)$, where $K[\cdot]$ is defined as in [23], and let $h:\mathbb{R}^{6+L}\to\mathbb{R}^{6+L}$ be defined as $h(y)\triangleq\left[\dot{e}_0\ \dot{e}_1\ \dot{r}\ \dot{e}_u\ \sqrt{\dot{Q}_1}\ \sqrt{\dot{Q}_2}\ \dot{\tilde{W}}^T\right]^T$ [24]. For almost all $t\in[0,\infty)$, the time derivative of (29) exists almost everywhere (a.e.) due to the motor controller, B_M^τ, and σ_e being discontinuous, such that $\dot{V}_L(y)\overset{\text{a.e.}}{\in}\dot{\tilde{V}}_L(y)$, where $\dot{\tilde{V}}_L$ is a generalized time derivative of V_L along $\dot{y}=h(y)$. Taking the generalized time derivative of (29), then using (4), (8), and the calculus of $K[\cdot]$ from [25], substituting in (18), (21), and (22), applying Leibniz Rule on (7), (23), and (24) results in

$$\begin{aligned}\dot{\tilde{V}}_L\subseteq\ & e_0(e_1-\alpha_0e_0)+e_1(r-\alpha_1e_1-\alpha_2e_u)+\frac{1}{2}\dot{M}(q)r^2\\&+r\left(W^{*T}\sigma^*(\Phi^*(x_d))-\widehat{W}^TK\left[\hat{\sigma}_i\left(\widehat{\Phi}_i(x_d)\right)\right]\right)\\&+r(\chi+\varepsilon(x_d)-e_1-V(q,\dot{q})r+k_sK[B_M^\tau](r_{\hat{\tau}}-r_\tau))\\&+r(-k_1K[\mathrm{sgn}(r)]-K[\sigma_e]k_2r)\\&+r(-M(q)\alpha_2k_sr+(M(q)\alpha_2-K[B_M^\tau])k_sr_{\hat{\tau}})\\&+k_s\omega_1e_u(-r+r_{\hat{\tau}})-\widetilde{W}^TK\left[\hat{\sigma}_i\left(\widehat{\Phi}_i(x_d)\right)\right]r\\&+\frac{1}{2}(\varepsilon_1\omega_1+\omega_2)k_s\left(r^2-r_{\hat{\tau}}^2\right)\\&+\frac{\omega_3k_s}{\hat{\tau}}\left(\hat{\tau}r^2-\int_{t-\hat{\tau}}^t r(\theta)^2d\theta\right).\end{aligned}$$

Adding and subtracting $W^{*T}K\left[\hat{\sigma}_i\left(\widehat{\Phi}_i(x_i(t))\right)\right]r$, using the estimated mismatch for the ideal output-layer weight in (15), using Property 2 (skew symmetry), and canceling common terms yields

$$\begin{aligned}\dot{\tilde{V}}_L\subseteq\ & e_0e_1-\alpha_0e_0^2-\alpha_1e_1^2-\alpha_2e_1e_u\\&+rW^{*T}\sigma^*(\Phi^*(x_d))-rW^{*T}K\left[\hat{\sigma}_i\left(\hat{\Phi}_i(x_d)\right)\right]\\&+r(\chi+\varepsilon(x_d))+k_sK[B_M^\tau]r(r_{\hat{\tau}}-r_\tau)-k_1rK[\mathrm{sgn}(r)]\\&-K[\sigma_e]k_2r^2-k_s\omega_1e_ur+k_s\omega_1e_ur_{\hat{\tau}}\\&-M(q)\alpha_2k_sr^2+(M(q)\alpha_2-K[B_M^\tau])k_srr_{\hat{\tau}}\\&+\frac{1}{2}(\varepsilon_1\omega_1+\omega_2)k_s\left(r^2-r_{\hat{\tau}}^2\right)\\&+\omega_3k_sr^2-\frac{\omega_3k_s}{\hat{\tau}}\int_{t-\hat{\tau}}^t r(\theta)^2d\theta.\end{aligned}\tag{39}$$

There are two cases to be considered overall: when $B_M^\tau>0$ and when $B_M^\tau=0$. First the case when $B_M^\tau>0$ will be considered, followed by the case when $B_M^\tau=0$.

Whenever $B_M^\tau>0$, the rider's muscles are producing forces, meaning, the FES effect is present in the system (i.e., $t\in\left[t_n^m,t_{n+1}^e\right)$). From the switching laws in (2) and (20), when $B_M^\tau>0$, there are two sub-cases: $\sigma_e=0$ or $\sigma_e>0$. The two sub-cases when muscle forces are present can be considered simultaneously by upper bounding $-K[\sigma_e]\leq 0$, using Properties 1 and 3 to bound M and $K[B_M^\tau]$, using Assumption 1 to bound W^{*T}, $\sigma^*(\cdot)$, and $\hat{\sigma}_i(\cdot)$, choosing ω_2 such that $\max(|c_M\alpha_2-c_b|,|c_m\alpha_2-c_B|)\leq\omega_2$, and using (17) and the fact that $\dot{V}_L(y)\overset{\text{a.e.}}{\in}\dot{\tilde{V}}_L(y)$, allows for (39) to be upper bounded as

$$\begin{aligned}\dot{V}_L\overset{\text{a.e.}}{\leq}\ & -\alpha_0e_0^2-\alpha_1e_1^2+|e_0e_1|+\alpha_2|e_1e_u|\\&-\left(k_1-\overline{W^*}\left(\overline{\sigma^*}+\overline{\hat{\sigma}}\right)-\Phi\right)|r|\\&+\rho(\|z\|)\|z\||r|+k_sc_B|r||r_{\hat{\tau}}-r_\tau|\\&-c_m\alpha_2k_sr^2+k_s\omega_2|rr_{\hat{\tau}}|\\&+k_s\omega_1|e_ur|+k_s\omega_1|e_ur_{\hat{\tau}}|\\&+\frac{1}{2}(\varepsilon_1\omega_1+\omega_2)k_s\left(r^2-r_{\hat{\tau}}^2\right)\\&+\omega_3k_sr^2-\frac{\omega_3k_s}{\hat{\tau}}\int_{t-\hat{\tau}}^t r(\theta)^2d\theta.\end{aligned}\tag{40}$$

Provided $y(\cdot)\in\mathcal{D}$, $\forall\cdot\in[t_0,t)$ it can be proven that $\dot{r}(\cdot)\leq\Upsilon,\forall\cdot\in[t_0,t)$, where $\mathcal{D}$ is defined in (27). Using the Mean Value Theorem on the $|r_{\hat{\tau}}-r_\tau|$ term, using Property 4, and setting k_1 such that $k_1\geq\overline{W^*}\left(\overline{\sigma^*}+\overline{\hat{\sigma}}\right)+\Phi+k_sc_B\bar{\bar{\tau}}\Upsilon$, $\dot{V}_L$ can be further bounded as

$$\begin{aligned}\dot{V}_L\overset{\text{a.e.}}{\leq}\ & -\alpha_0e_0^2-\alpha_1e_1^2+|e_0e_1|+\alpha_2|e_1e_u|\\&+\rho(\|z\|)\|z\||r|-c_m\alpha_2k_sr^2+k_s\omega_2|rr_{\hat{\tau}}|\\&+k_s\omega_1|e_ur|+k_s\omega_1|e_ur_{\hat{\tau}}|\\&+\frac{1}{2}(\varepsilon_1\omega_1+\omega_2)k_s\left(r^2-r_{\hat{\tau}}^2\right)\\&+\omega_3k_sr^2-\frac{\omega_3k_s}{\hat{\tau}}\int_{t-\hat{\tau}}^t r(\theta)^2d\theta.\end{aligned}\tag{41}$$

Next, using Young's Inequality, the following inequalities can be developed

$$|e_0e_1| \leq \frac{1}{2\varepsilon_2}e_0^2 + \frac{\varepsilon_2}{2}e_1^2, \tag{42}$$

$$|e_1e_u| \leq \frac{1}{2\varepsilon_3\alpha_2}e_u^2 + \frac{\varepsilon_3\alpha_2}{2}e_1^2, \tag{43}$$

$$|rr_{\hat{\tau}}| \leq \frac{1}{2}r^2 + \frac{1}{2}r_{\hat{\tau}}^2, \tag{44}$$

$$|e_ur| \leq \frac{1}{2\varepsilon_1}e_u^2 + \frac{\varepsilon_1}{2}r^2, \tag{45}$$

$$|e_ur_{\hat{\tau}}| \leq \frac{1}{2\varepsilon_1}e_u^2 + \frac{\varepsilon_1}{2}r_{\hat{\tau}}^2. \tag{46}$$

Substituting (42)-(46) into (41) and completing the squares on $-\frac{1}{2}c_m\alpha_2k_sr^2 + |r|\rho(\|z\|)\|z\|$ yields the following bound

$$\begin{aligned}\dot{V}_L \overset{\text{a.e.}}{\leq} & -\left(\alpha_0 - \frac{1}{2\varepsilon_2}\right)e_0^2 - \left(\alpha_1 - \frac{\varepsilon_2}{2} - \frac{\varepsilon_3\alpha_2^2}{2}\right)e_1^2 \\ & + \left(\frac{1}{2\varepsilon_3} + \frac{k_s\omega_1}{\varepsilon_1}\right)e_u^2 \\ & -k_s\left(\frac{1}{2}c_m\alpha_2 - \varepsilon_1\omega_1 - \omega_2 - \omega_3\right)r^2 \\ & + \frac{\rho^2(\|z\|)\|z\|^2}{2c_m\alpha_2k_s} - \frac{\omega_3k_s}{\hat{\tau}}\int_{t-\hat{\tau}}^{t} r(\theta)^2 d\theta. \end{aligned} \tag{47}$$

By using the Cauchy-Schwarz Inequality and (7), e_u^2 is bounded as

$$e_u^2 \leq \hat{\tau}k_s^2\int_{t-\hat{\tau}}^{t} r^2(\theta)\, d\theta. \tag{48}$$

Using (48) to upper bound the last term in (47) yields

$$\begin{aligned}\dot{V}_L \overset{\text{a.e.}}{\leq} & -\left(\alpha_0 - \frac{1}{2\varepsilon_2}\right)e_0^2 - \left(\alpha_1 - \frac{\varepsilon_2}{2} - \frac{\varepsilon_3\alpha_2^2}{2}\right)e_1^2 \\ & -\left(\frac{\omega_3}{k_s\hat{\tau}^2} - \frac{1}{2\varepsilon_3} - \frac{k_s\omega_1}{\varepsilon_1}\right)e_u^2 \\ & -k_s\left(\frac{1}{2}c_m\alpha_2 - \varepsilon_1\omega_1 - \omega_2 - \omega_3\right)r^2 \\ & + \frac{\rho^2(\|z\|)\|z\|^2}{2c_m\alpha_2k_s}. \end{aligned} \tag{49}$$

Using (9), (25), and the fact that $\rho^2(\|z\|) \leq \rho^2(\|y\|)$, (49) can be further bounded as

$$\dot{V}_L \overset{\text{a.e.}}{\leq} -\left(\frac{1}{2}\beta_1 - \frac{\rho^2(\|y\|)}{2c_m\alpha_2k_s}\right)\|z\|^2 - \frac{1}{2}\beta_1\|z\|^2. \tag{50}$$

Provided that $y(t) \in \mathcal{D}, \forall t \in \left[t_n^m, t_{n+1}^e\right)$, then (50) can be further bounded as

$$\dot{V}_L \leq -\frac{1}{2}\beta_1\|z\|^2, \quad t \in \left[t_n^m, t_{n+1}^e\right). \tag{51}$$

In the case where the rider's muscles are inactive, $B_M^\tau = 0$ and the FES effect is not present in the system (i.e., $t \in \left[t_n^e, t_{n+1}^m\right)$). In this case, $\sigma_e = 1$ and the system is controlled by the motor alone because of the switching laws defined in (2) and (20). During this case σ_e and B_M^τ are constant; thus, this case can be considered by setting $K[B_M^\tau] = 0$ and $K[\sigma_e] = 1$, using Property 1, Assumption 1, (17), and the fact that $\dot{V}_L(y) \overset{\text{a.e.}}{\in} \dot{\tilde{V}}_L(y)$ to upper bound (39) as

$$\begin{aligned}\dot{V}_L \overset{\text{a.e.}}{\leq} & -\alpha_0e_0^2 - \alpha_1e_1^2 + |e_0e_1| + \alpha_2|e_1e_u| \\ & -\left(k_1 - \overline{W^*}\left(\overline{\sigma^*} + \overline{\hat{\sigma}}\right) - \Phi\right)|r| + \rho(\|z\|)\|z\||r| \\ & -k_2r^2 + c_M\alpha_2k_s|r||r_{\hat{\tau}} - r| \\ & +k_s\omega_1|e_ur| + k_s\omega_1|e_ur_{\hat{\tau}}| \\ & +\frac{1}{2}(\varepsilon_1\omega_1 + \omega_2)k_s\left(r^2 - r_{\hat{\tau}}^2\right) \\ & +\omega_3k_sr^2 - \frac{\omega_3k_s}{\hat{\tau}}\int_{t-\hat{\tau}}^{t} r(\theta)^2 d\theta. \end{aligned} \tag{52}$$

Using the Mean Value Theorem on the $|r_{\hat{\tau}} - r|$ term in (52), using Property 4, and setting k_1 such that $k_1 \geq \overline{W^*}\left(\overline{\sigma^*} + \overline{\hat{\sigma}}\right) + \Phi + k_s\Upsilon c_M\bar{\tau}\alpha_2$, $\dot{V}_L$ can be further bounded as

$$\begin{aligned}\dot{V}_L \overset{\text{a.e.}}{\leq} & -\alpha_0e_0^2 - \alpha_1e_1^2 + |e_0e_1| \\ & +\alpha_2|e_1e_u| + \rho(\|z\|)\|z\||r| \\ & -k_2r^2 + k_s\omega_1|e_ur| + k_s\omega_1|e_ur_{\hat{\tau}}| \\ & +\frac{1}{2}(\varepsilon_1\omega_1 + \omega_2)k_s\left(r^2 - r_{\hat{\tau}}^2\right) \\ & +\omega_3k_sr^2 - \frac{\omega_3k_s}{\hat{\tau}}\int_{t-\hat{\tau}}^{t} r(\theta)^2 d\theta, \end{aligned} \tag{53}$$

provided $y(\cdot) \in \mathcal{D}$, $\forall \cdot \in [t_0, t)$. Following a similar development as the muscle active case, (53) can be upper bounded as

$$\dot{V}_L \overset{\text{a.e.}}{\leq} -\frac{1}{2}\beta_2\|z\|^2, \quad t \in \left[t_n^e, t_{n+1}^m\right), \tag{54}$$

where β_2 is defined in (26), provided that $y(t) \in \mathcal{D}, \forall t \in \left[t_n^e, t_{n+1}^m\right)$.

Upper bounding (51) and (54), produces the following result

$$\dot{V}_L \overset{\text{a.e.}}{\leq} -\frac{1}{2}\min(\beta_1, \beta_2)\|z\|^2, \quad \forall t \in [t_0, \infty), \tag{55}$$

provided that $y(t) \in \mathcal{D}, \forall t \in [t_0, \infty)$. It could be shown that a sufficient condition for $y(t) \in \mathcal{D}, \forall t \in [t_0, \infty)$ is that $y(t_0) \in S_{\mathcal{D}}, \forall t \in [t_0, \infty)$, where $S_{\mathcal{D}}$ is defined in (28). From (29) and (55) it can be concluded that $V_L(y) \in \mathcal{L}_\infty$ and hence $y \in \mathcal{L}_\infty$. From (9) and (30) it is clear that $e_0, e_1, r, e_u, z, \widetilde{W} \in \mathcal{L}_\infty$, and from (15), (18), (19), and Assumption 1 it can be concluded that $u, u_e, \widehat{W}, \hat{\sigma}_i(\cdot) \in \mathcal{L}_\infty$. Using the LaSalle-Yoshizawa Theorem for non-smooth systems in [24] and [26], it can be proven that $\lim_{t\to\infty} \|z(t)\| \to 0$, provided that $y(t_0) \in S_{\mathcal{D}}$ and the sufficient conditions in the theorem statement are satisfied. $\square$

CONCLUSION

This work develops the use of a real-time DNN-based adaptive controller to approximate the nonlinear dynamics of an FES cycle-rider system. This control scheme accounts for the EMD between the start and end of stimulation and the corresponding start and end of the generated muscle force. Furthermore, the developed controller uses the DNN approximation of the system dynamics as a feedforward term in the motor controller to meet the cadence and tracking objectives. The DNN updates the output-layer DNN weight online (in real-time) while using data-driven methods to update the inner-layer features iteratively. Because of the iterative switching when the inner-layer features update, a nonsmooth Lyapunov-based stability analysis is used to prove semi-global asymptotic tracking. Due to COVID-19 restrictions, sufficient experiments were not possible by the time of submission, but are now being developed.

REFERENCES

[1] Ferrante, S., Pedrocchi, A., Ferrigno, G., and Molteni, F., 2008. "Cycling induced by functional electrical stimulation improves the muscular strength and the motor control of individuals with post-acute stroke". *Eur. J. Phys. Rehabil. Med.,* **44**(2), pp. 159–167.

[2] Hooker, S. P., Figoni, S. F., Rodgers, M. M., Glaser, R. M., Mathews, T., Suryaprasad, A. G., and Gupta, S. C., 1992. "Physiologic effects of electrical stimulation leg cycle exercise training in spinal cord injured persons". *Arch. Phys. Med. Rehabil.,* **73**(5), pp. 470–476.

[3] Nordez, A., Gallot, T., Catheline, S., Guèvel, A., Cornu, C., and Hug, F., 2009. "Electromechanical delay revisited using very high frame rate ultrasound". *J. Appl. Physiol.,* **106**, June, pp. 1970–1975.

[4] Olfati-Saber, R., Fax, J. A., and Murray, R. M., 2004. "Consensus problems in networks of agents with switching topology and time-delays". *IEEE Trans. Autom. Control,* **49**(9), pp. 1520–1533.

[5] Evesque, S., Annaswamy, A., Niculescu, S., and Dowling, A., 2003. "Adaptive control of a class of time delay systems". *J. Dyn. Syst. Meas. Control,* **125 (2)**, pp. 186–193.

[6] Downey, R., Kamalapurkar, R., Fischer, N., and Dixon, W. E., 2015. *Recent Results on Nonlinear Delay Control Systems: In honor of Miroslav Krstic*. Springer New York, ch. Compensating for Fatigue-Induced Time-Varying Delayed Muscle Response in Neuromuscular Electrical Stimulation Control, pp. 143–161.

[7] Allen, B. C., Stubbs, K. J., and Dixon, W. E., 2020. "Characterization of the time-varying nature of electromechanical delay during FES-cycling". *IEEE Trans. Neural Syst. Rehabil. Eng.,* **28**(10), pp. 2236–2245.

[8] Allen, B. C., Cousin, C. A., Rouse, C. A., and Dixon, W. E., to appear. "Robust cadence tracking for switched fes-cycling with an unknown time-varying input delay". *IEEE Trans. Control Syst. Tech.*

[9] Allen, B. C., Cousin, C., Rouse, C., and Dixon, W. E., 2019. "Cadence tracking for switched FES cycling with unknown input delay". In Proc. IEEE Conf. Decis. Control, pp. 60–65.

[10] Allen, B. C., Stubbs, K. J., and Dixon, W. E., 2020. "Saturated control of a switched FES-cycle with an unknown time-varying input delay". In IFAC Conf. Cyber-Phys. Hum.-Syst.

[11] Allen, B. C., Stubbs, K., and Dixon, W. E., 2020. "Robust cadence tracking for switched FES-cycling with an unknown time-varying input delay using a time-varying estimate". In IFAC World Congr.

[12] Sun, R., Greene, M., Le, D., Bell, Z., Chowdhary, G., and Dixon, W. E., 2021. "Lyapunov-based real-time and iterative adjustment of deep neural networks". *IEEE Control Syst. Lett.,* **6**, pp. 193–198.

[13] LeCun, Y., Bengio, Y., and Hinton, G., 2015. "Deep learning". *Nature,* **521**(7553), pp. 436–444.

[14] Teng, Q., and Zhanga, L., 2019. "Data driven nonlinear dynamical systems identication using multi-step CLDNN". *AIP Advances*.

[15] Lewis, F. L., 1999. "Nonlinear network structures for feedback control". *Asian J. Control,* **1**(4), pp. 205–228.

[16] Rouse, C., Cousin, C. A., Allen, B. C., and Dixon, W. E., 2021. "Shared control for switched motorized FES-cycling on a split-crank cycle accounting for muscle control input saturation". *Automatica,* **123**, pp. 1–11.

[17] Cousin, C. A., Rouse, C. A., and Dixon, W. E., to appear. "Split-crank functional electrical stimulation cycling: An adapting admitting rehabilitation robot". *IEEE Trans. Control Syst. Tech.*

[18] Allen, B., Cousin, C., Rouse, C., and Dixon, W. E., 2019. "Cadence tracking for switched FES-cycling with unknown time-varying input delay". In Proc. ASME Dyn. Syst. Control Conf.

[19] Bellman, M. J., Downey, R. J., Parikh, A., and Dixon, W. E., 2017. "Automatic control of cycling induced by functional electrical stimulation with electric motor assistance". *IEEE Trans. Autom. Science Eng.,* **14**(2), April, pp. 1225–1234.

[20] Lewis, F. L., Jagannathan, S., and Yesildirak, A., 1998. *Neural network control of robot manipulators and nonlin-*

ear systems. CRC Press, Philadelphia, PA.

[21] Chowdhary, G., Yucelen, T., Mühlegg, M., and Johnson, E. N., 2013. "Concurrent learning adaptive control of linear systems with exponentially convergent bounds". *Int. J. Adapt. Control Signal Process.,* **27**(4), pp. 280–301.

[22] Parikh, A., Kamalapurkar, R., and Dixon, W. E., 2019. "Integral concurrent learning: Adaptive control with parameter convergence using finite excitation". *Int J Adapt Control Signal Process,* **33**(12), Dec., pp. 1775–1787.

[23] Filippov, A. F., 1964. "Differential equations with discontinuous right-hand side". In *Fifteen papers on differential equations*, Vol. 42 of *American Mathematical Society Translations - Series 2*. American Mathematical Society, pp. 199–231.

[24] Fischer, N., Kamalapurkar, R., and Dixon, W. E., 2013. "LaSalle-Yoshizawa corollaries for nonsmooth systems". *IEEE Trans. Autom. Control,* **58**(9), Sep., pp. 2333–2338.

[25] Paden, B. E., and Sastry, S. S., 1987. "A calculus for computing Filippov's differential inclusion with application to the variable structure control of robot manipulators". *IEEE Trans. Circuits Syst.,* **34**(1), Jan., pp. 73–82.

[26] Kamalapurkar, R., Rosenfeld, J. A., Parikh, A., Teel, A. R., and Dixon, W. E., 2019. "Invariance-like results for nonautonomous switched systems". *IEEE Trans. Autom. Control,* **64**(2), Feb., pp. 614–627.