American Multi-modal Energy System: Instantiated Structural Models of California and New York

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***Abstract*—As one of the most pressing challenges of the 21st century, global climate change demands a host of changes across four critical energy infrastructures: the electric power grid, the natural gas system, the oil system, and the coal system. Unfortunately, these systems are rarely studied together. Instead, holistic multi-energy system models can serve to improve the understanding of these interdependent systems as they evolve into the future. The NSF project entitled "American Multi-Modal Energy system Synthetic & Simulated Data (AMES-3D)" seeks to fill this void with an open-source structural and behavioral model of the AMES. To that end, this paper uses a GIS-data-driven, model-based systems engineering-guided approach to develop open-source software the produces open structural models of the American Multi-modal Energy Systems. More specifically, it reports and contrasts the hetero-functional incidence tensor, the formal graph adjacency matrix and hetero-functional graph adjacency matrix statistics for the multi-energy infrastructure systems of the states of California and New York. The paper finds that the geography and the sustainable energy policies of these states are deeply reflected in the structure of their multi-energy infrastructure systems.**

***Index Terms*—Hetero-Functional Graph Theory, Model Based Systems Engineering, sustainable energy transition, American Multi-modal Energy System, Sustainability**

I. Introduction

A. Motivation

As one of the most pressing challenges of the 21st century, global climate change demands a host of changes across four critical energy infrastructures: the electric power grid, the natural gas system, the oil system, and the coal systems. These four infrastructures make up the American Multi-modal Energy System (AMES) as a system-of-systems. As the AMES undergoes a sustainable energy transition, it must not just *mitigate* climate change but it must also *adapt* to its effects with resilient architectures. These combined requirements necessitate an understanding of the AMES inter-dependencies and how they vary geographically and temporally [1].

Holistic multi-energy system models can serve to improve the understanding of these interdependent systems as they evolve into the future. Unfortunately, multi-energy system modeling remains relatively nascent [2]–[9]. A majority of the works investigating these energy systems in the past have been performed on individual energy networks [10]–[14]. More recently, work has been published analyzing only a couple systems together such as pairing the electric grid with a one of the others that compose the AMES [2]–[9].These works, however, do not include all four critical energy infrastructures and do not extend to the entire American geography. As an exception, the EIA has developed a comprehensive model called the National Energy Modeling System (NEMS) which it uses to produce the annual energy outlook [15]. Despite serving this important function and being publicly available, this software tool remains opaque and difficult to use. The EIA website itself recognize: "[The] NEMS is only used by a few organizations outside of the EIA. Most people who have requested NEMS in the past have found out that it was too difficult or rigid to use [16]". Consequently, holistic multi-energy system models of the AMES remain a present need for open citizen-based science.

The NSF project entitled "American Multi-Modal Energy system Synthetic & Simulated Data (AMES-3D)" seeks to fill this void with an open-source structural and behavioral model of the AMES. Following a Model Based Systems Engineering (MBSE) approach [17], [18], this project uses openly available datasets [19] to infer the AMES' reference architecture [20]. It uses SysML [18] to model the four interdependent energy systems, and the flows of mass and energy within and between them. These datasets are then used to instantiate the AMES' reference architecture into an instantiated architecture [20]. While the NSF project seeks to develop both a structural as well as a behavioral model of the AMES, this paper restricts its scope to the former.

The development of an AMES reference architecture provides several immediate benefits. The first is that a SysML-based reference architecture describes the system's form, function and the allocation of the latter to the former. Therefore, the reference architecture describes not just what the system is made up of but also what it does. Second, a SysML-based reference architecture can be readily translated into numerical models using Hetero-functional Graph Theory (HFGT) [21]. Thus, when this reference architecture is instantiated, its form and function are readily translated into mathematical models including both the form and function. Standard structural models include *formal graphs* that describe energy facilities and how they are interconnected. In the meantime, *hetero-functional graphs* (HFG) describe how the wide variety of functions in a system are interconnected. HFGs have been shown to provide more information than formal graphs when analyzing an evolving instantiated architecture [22], [23]. In effect, HFGT provides a means to quantitatively interpret the graphical SysML-based models from both a formal as well as a functional lens. Such an analysis has already been conducted on small electric power distribution systems [22] as well as at a large-scale for the entirety of the American electric power system [23]. This paper now builds on these electricity-only analyses to study the structure of the American Multi-modal Energy System.

B. Original Contributions

This paper uses a data-driven, MBSE-guided approach to develop open-source software that produces open structural models of the American Multi-modal Energy System. More specifically, this approach uses the AMES reference architecture [20] and an asset-level GIS data called Platts Map Data Pro [19]. The instantiated structural models include, for the first time, the electric grid, the natural gas system, the oil system, the coal system and the interconnections between them as described by the AMES reference architecture. The paper organizes its initial results into a 2x2 matrix; a formal and hetero-functional graph for each of the two states of California and New York. These two states are chosen for their large size and progressive energy policies. Consequently, they have also both made strides to advance the sustainable energy transition. Despite these similarities, the two states have very different climates, relative placement with the large AMES, and trajectories in the sustainable energy transition. In 2019, CA had the most renewable energy generation out of all the states [24]. In the meantime, New York's effort to expand renewable energy capacity are balanced by its reliance on natural gas and oil pipelines to meet space heating energy demands [25]. By using MBSE and HFGT, new open data models are presented for these two states.

C. Paper Outline

The remainder of the paper proceed as follows. Section II is a description of the background literature and the data sets used to develop the instantiated architecture models. The paper then presents a comparison of the formal graphs and hetero-functional graphs network statistics for each state in Section III. A discussion of the energy resources and system capabilities is then made in Section III. The paper then presents future work for theses instantiated architectures and the AMES reference architecture. Finally the paper is brought to a conclusion in Section IV.

II. Methodology

As mentioned in the Introduction, this paper utilizes a data-driven, MBSE-guided approach to develop open structural models of the American Multi-modal Energy System. This section succinctly relays this method.

The AMES structural models are inferred from the Platts Map Data Pro (Fig. 1). This input dataset consists of Graphic

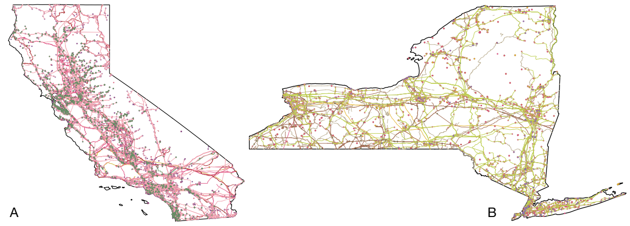

Fig. 1. GIS Layers from the Platts Map Data Pro dataset for the electric grid, natural gas system, oil system, and coal system for California (A) and New York (B).

Information System (GIS) layers for each of the four subsystems in the AMES [19]. These geo-spatial layers include attributes of the physical resources/facilities that compose the AMES infrastructure. As the Platts Map Data Pro is directed towards wholesale energy decisions, the data is limited to transmission system resources and neglects distribution level assets. Consequently, this paper's data-driven approach is similarly limited to transmission level assets. This limitation in the dataset notably excludes retail distribution of oil and gas (by truck). It also excludes distributed electric generation assets such as roof-top solar that are an integral part of the sustainable energy transition. Nevertheless, the Platts Map Data Pro is likely the best available dataset because it allows inferences of not just the AMES's form but its function too.

The first step of the data processing is to convert the Platts Map Data Pro GIS shapefile for each state (i.e. CA and NY) into an associated XML file that serves as the input for the openly-available HFGT toolbox. The GIS attribute data is cleaned and processed before being organized into an XML. When cleaning the data, all resources marked with a canceled status, closed status, or illegible attributes are removed. Additionally, geographical clustering is applied to aggregate overlapping resources, connect disjointed resources, and remove isolated resources. Once properly cleaned, the data is organized into the input XML file. Here, the AMES reference architecture plays a critical role in organizing the cleaned data into defined resources with the proper allocated functionality. Figure 2 shows the top-level context diagram of the AMES reference architecture and it is further elaborated in [20]. The AMES reference architecture provides a consistent blueprint from which to develop AMES models irrespective of the choice of region or scale. It also defines all energy resources/facilities, and the functions that they can perform. It also defines the set of operands used to track the flows of mass and energy between the AMES' many resources and between its many functions.

The second step is to run the HFGT toolbox on this newly produced XML file so as to produce the positive and negative Hetero-Functional Incidence Tensors (HFITs).

Definition 1 – The Negative 3^{rd} Order Hetero-functional Incidence Tensor $\mathcal{M}_\rho^-$: The negative hetero-functional incidence tensor $\mathcal{M}_\rho{}^- \in \{0,1\}^{|L|\times|B_S|\times|\mathcal{E}_S|}$ is a third-order tensor whose element $\mathcal{M}_\rho^-(i,y,\psi) = 1$ when the system capability $\epsilon_\psi \in \mathcal{E}_S$ pulls operand $l_i \in L$ from buffer $b_{s_y} \in B_S$. ■

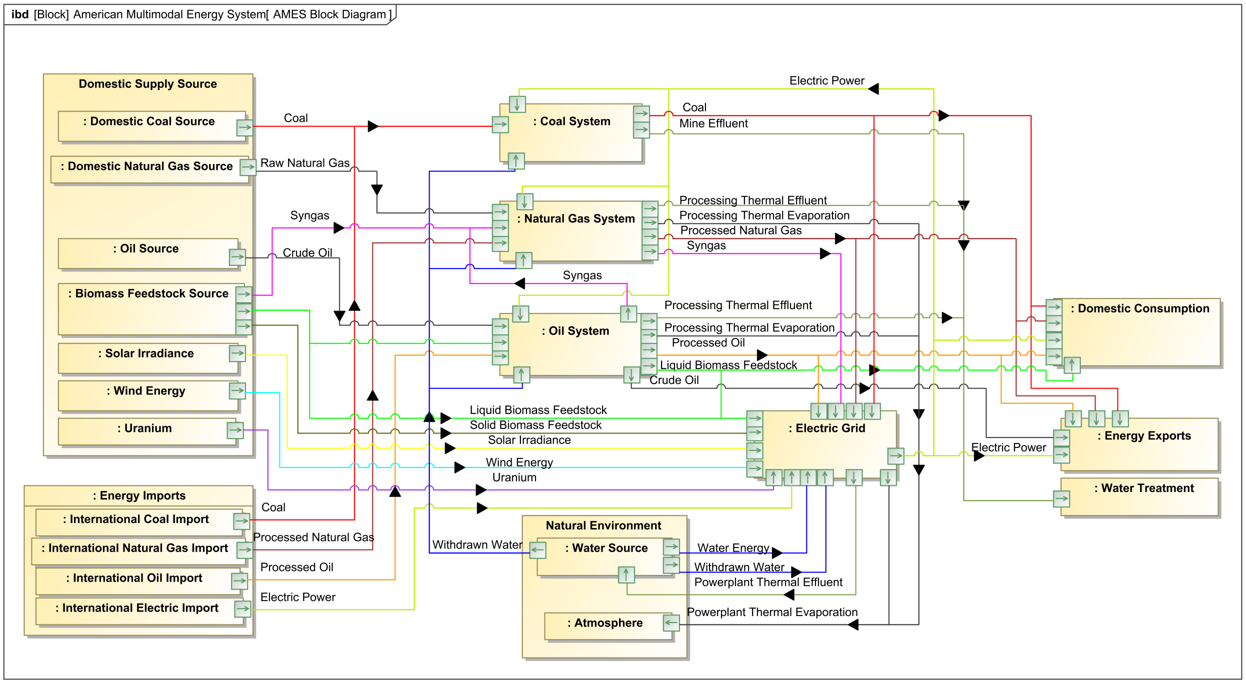

Fig. 2. The top-level context diagram for the American Multi-modal Energy Systems Reference Architecture [20]

Definition 2 – The Positive 3^{rd} Order Hetero-functional Incidence Tensor $\mathcal{M}_\rho^+$: The positive hetero-functional incidence tensor $\mathcal{M}_\rho^+ \in \{0,1\}^{|L|\times|B_S|\times|\mathcal{E}_S|}$ is a third-order tensor whose element $\mathcal{M}_\rho^+(i,y,\psi) = 1$ when the system capability $\epsilon_\psi \in \mathcal{E}_S$ injects operand $l_i \in L$ into buffer $b_{s_y} \in B_S$. ■

In the context of the AMES, the operands are flows of matter and energy like coal, oil, natural gas, and electricity. The buffers are point-facilities like electric power plants and refineries. The capabilities are "subject+verb+object" sentences like NG refinery refines raw natural gas to processed natural gas. These HFITs are important because: 1.) they include all of the information necessary to produce a formal adjacency matrix (A_{Bs}) where point-facilities are connected via edge-facilities, and 2.) they include all of the information necessary to understand how the system's capability follow one another in a hetero-functional graph (A_ρ).

Each of these adjacency matrices is then calculated. The formal graph adjacency matrix A_{Bs} requires two steps. First, the two HFITs are summed along the operand dimension to produce two incidence matrices:

$$M_B^{+/-}(y,\psi) = \sum_i^{|L|} \mathcal{M}_\rho^{+/-}(i,y,\psi) \tag{1}$$

It is important to recognize that the operand heterogeneity information is lost. Then, these incidence matrices are multiplied.

$$A_{Bs} = M_B^+ M_B^{-T} \tag{2}$$

In the meantime, the hetero-functional graph adjacency matrix A_ρ is calculated without loss of information after the HFITs have been matricized (or flattened) into hetero-functional incidence matrices M_ρ^+ and M_ρ^- with dimension $|L||B_S| \times |\mathcal{E}_S|$.

$$A_\rho = M_\rho^{+T} M_\rho^- \tag{3}$$

While the formal graph adjacency matrix shows the physical connections from point facility (i.e. buffer) to another, the hetero-functional adjacency matrix shows the logical sequence of capabilities one after the other. As previous works have shown, the latter allows for more comprehensive resilience analyses; be it for small electric power distribution systems [22] or for a full scale analysis of the American electric power system [23]. The open-source HFGT toolbox [26] provides an automated means for data processing GIS shapefiles to these two structural models of the AMES.

III. Structural Model Statistics

Once created, the hetero-functional incidence tensor, the formal graph, and the hetero-functional graph for the two states of California and New York can be compared.

A. Hetero-functional Incidence Tensor Statistics

In this regard, the basic statistics of the hetero-functional incidence tensors provide a common basis for comparing the two states relative to the size of their populations and land areas (Table I). As expected by the relative size of its population

TABLE I
Hetero-functional Incidence Tensor Statistics for the Multi-modal Energy Systems in CA and NY.

	California	New York
# of Operands	14	14
# of Buffers	7853	2986
# of Capabilities	58038	17558
# of Elements in M_ρ^+.	64303	24898
# of Elements in M_ρ^-.	68870	26218
HFIT Sparsity	9.18E-15	8.76E-14
Population (millions)	39.5	19.5
Land Area (sqr miles)	155,779	47,126
Population Density(ppl/sqr mile)	253	414

and land area, California's energy infrastructure is larger than that of New York in terms of the number of point-facilities and the number of capabilities. The same holds true for the number of filled elements in the positive and negative HFIT tensors. While the two states display energy infrastructures of differing scales, they both have energy mixtures that utilize the same 14 operands. Also, despite these absolute measures, New York's population density is significantly higher than that of California. This means that, if all else is held equal, New Yorkers receive more energy infrastructure benefits and Californians must expend more in energy infrastructure costs. This same finding is reflected in the sparsity of the HFIT where New York's tensor is more than 10 times more dense.

B. Formal Graph Statistics

While it is important to assess the number of buffers (e.g. point energy facilities) in the multi-energy infrastructure of two American states, it is also important to differentiate them by type. Fig. 3 shows that 76.8% and 69.5% of the buffers in the formal graphs for California and New York respectively are electric power substations. This reflects the highly ubiquitous nature of the electric power system in both states. Furthermore, another 18.2% and 18.4% respectively are devoted to electric power generation facilities (of various types). Because coal, oil, and natural gas are very dense approximate forms of energy, their processing facilities for these types of energy have very strong economies of scale. Therefore, there is a trend towards centralization and a small number of point-facilities for energy conversion. California, notably, has a greater shift towards the the electric grid with a greater presence of substations and power plants than New York. That New York has a greater reliance on oil and gas facilities is likely a byproduct of it being located in a more colder climate. California, on the other hand, with its warm climate relies on cooling and is further along in in electrifying its energy demands and adopting renewable energy resources.

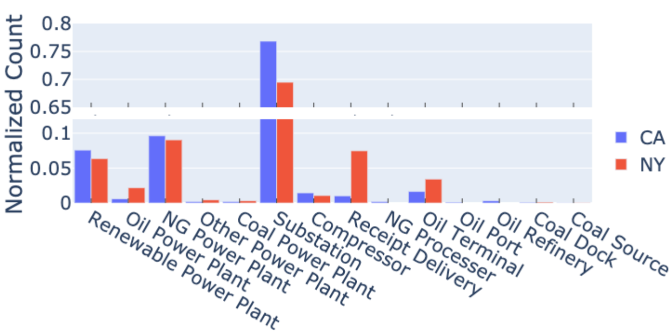

Fig. 3. The distribution of point-energy-facilities (buffers) in the formal graphs of multi-energy infrastructure systems in California and New York.

Beyond the number and type of point-energy-facilities, the formal graph also measures their interconnectedness. There are 15454 and 6347 edges in the formal graphs of California and New York respectively. This corresponds to a sparsity of 2.51E-4 and 7.12E-4 respectively and mimics the sparsity of the HFIT. Despite the the heterogeneity of point-energy-facilities and the sparsity of the two formal graphs, Fig. 4 shows that the formal graph degree distributions for the two states is remarkably similar. Much like the well-known results in electric power systems [2], [27]–[29], the multi-energy systems in the two states follow a decaying power law with regression coefficients of 0.99. The structural similarity of the multi-energy system in both states to their counterpart electric power systems is likely due to the fact that so much of the structural topology of both multi-energy systems consists of electrical artifacts. In the meantime, California and New York follow similar decay constants of 0.68 and 0.62 respectively. The differences in y-intercept values of 0.929 and .699 can be attributed to the states' geographic differences. The relatively dense state of NY must rely on a relatively more radial network that consists of nodes of degree 1. In contrast, California must reach its less densely populated areas with more meshed nodes.

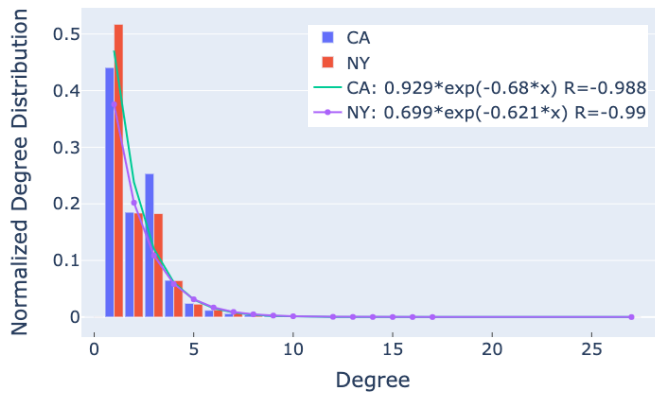

Fig. 4. The formal graph degree distributions of the multi-energy infrastructure systems of California and New York follow an exponential decay law.

C. Hetero-functional Graph Statistics

The statistics of the hetero-functional graphs for California and New York are presented similarly to those of the formal graph. In that regard, Fig. 5 differentiates the different types of capabilities found in the multi-energy infrastructure systems in California and New York. Again, both states show a predominance of facilities devoted substations that "consume electric power"[1]. Despite this common trend, CA has a greater presence of such "consume electric power" capabilities in accordance with its more electrified energy demand. The two states' relationship to natural gas also differs. CA has a greater relative presence of generating electric power from natural gas. As variable energy resources (VER) become increasingly integrated into the electric grid fast ramping power plants such as natural gas power plants become increasingly important to meet the ramping constraints placed on the electric grid. Meanwhile, in NY, there is a greater presence of capabilities that import and export natural gas and oil-based products and reflects the need for heating during the cord Northeast winters. In brief, the results found in Fig. 5 confirm those found in Fig. 3 and elaborate on them with further detail on the function of these multi-energy systems.

[1]From the perspective of the transmission system, substations serve as load buses that consume electric power when in fact they are simply routing the electric power to the distribution system for consumption by industrial, commercial, and residential end-users.

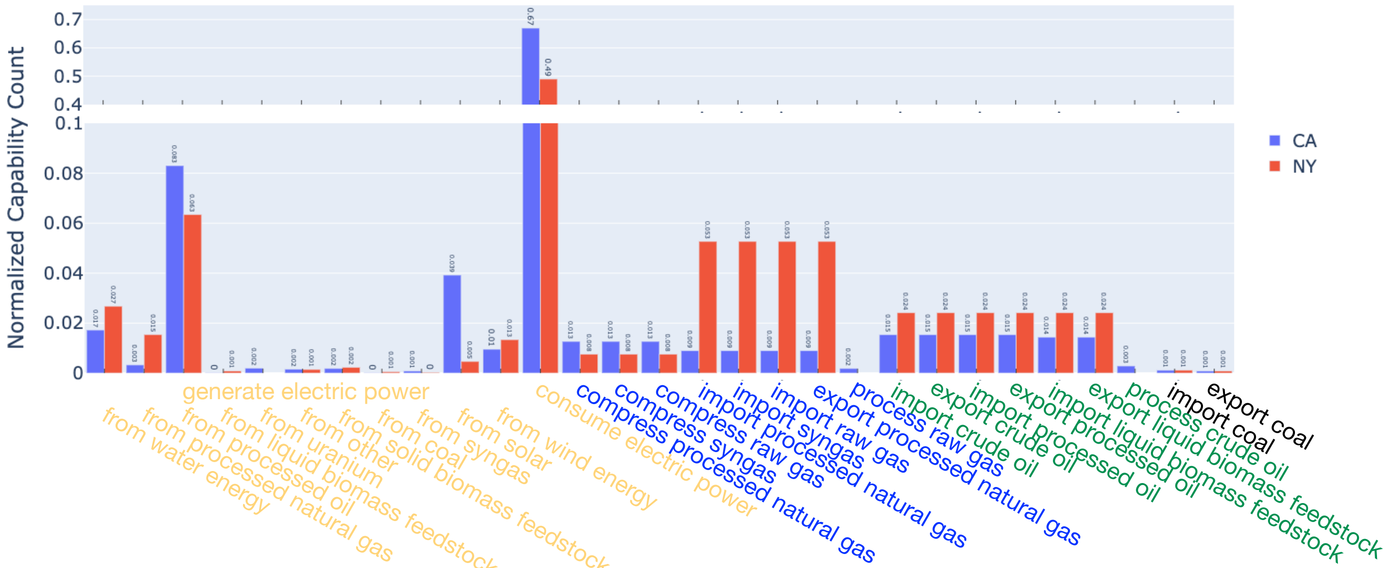

Fig. 5. The normalized distributions of the capabilities in the hetero-functional graphs of the multi-energy infrastructure systems of California and New York.

As the sustainable energy transition requires an electrification of the demands placed on the AMES, it becomes important to investigate the electric power generation mix. In this regard, when the electric power generation capabilities in Fig. 5 are weighted by their generation capacity, it produces Fig. 6 which shows the electric power generation mix for the two states. It shows that the main source of generation capacity for California comes from generating electricity from processed natural gas. This again indicates the need for quick response generation sources to support the ramping constraints placed on the California electric grid from the high penetration of variable renewable energy resources. In contrast, New York sees a relatively higher normalized generation capacity in processed oil, nuclear power, and coal than California. Finally, California's commitment to renewable energy generation comes primarily in the form of solar power. Meanwhile, NY has balanced its renewable energy between wind and solar power.

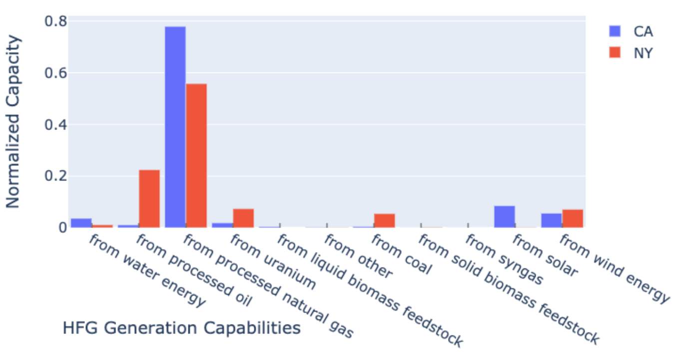

Fig. 6. The electric power generation capacity mix by fuel source for California and New York.

Finally, the degree distribution of the hetero-functional graph adjacency matrix is shown in Fig. 7. In [23], the degree distribution of the hetero-functional graph of the American electric power system was similar to that of the formal graph. From an operand point-of-view, electric power systems are relatively homogeneous, and so the formal and hetero-functional degree distributions mimic each other. In multi-energy systems, the hetero-functional graph degree distribution offers a more detailed view into the interconnectedness than a formal graph degree distribution. Similar to Fig. 4, Fig. 7 still shows an exponential decay tail. However, the exponential decay law breaks down for capabilities with a degree of five or less. This very interesting statistical phenomena is likely the result of the statistics of coal, oil, natural gas, and electric power systems being superimposed one on top of the other. In essence, the hetero-functional graph degree distribution shows "signatures" of the degree of a system's capabilities much like a power spectra shows the signatures of the frequencies of a waveform. Future work will seek to further investigate which capabilities contribute to these degree distribution peaks.

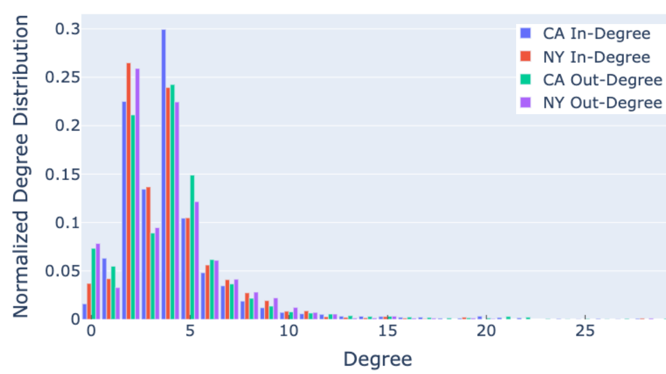

Fig. 7. The hetero-functional graph degree distributions for the multi-energy infrastructure systems of California and New York.

IV. Conclusions

This paper uses a data-driven, MBSE-guided approach to develop open-source software that produces open structural models of the American Multi-modal Energy System. It is part of a larger NSF project entitled "American Multi-Modal Energy system Synthetic & Simulated Data (AMES-3D)" which seeks to produce open-source structural and behavioral models of the American Multi-modal Energy System. The creation of open-source software and open-data models of the AMES fills an important need in the open citizen-based science in America's sustainable energy transition. It also provides one of the few multi-energy system datasets on which to advance fundamental methods. The AMES structural models are inferred from the Platts Map Data Pro GIS dataset and is complemented by the previously developed American Multi-modal Energy System Reference Architecture [20]. Together, these two data sources serve as the basis for an XML-based input data file for the open-source hetero-functional graph theory toolbox.

This paper specifically reports the hetero-functional incidence tensor, the formal graph adjacency matrix and hetero-functional graph adjacency matrix statistics for the multi-energy infrastructure systems of the states of California and New York. Here, the application of hetero-functional graph theory facilitates a nuanced analysis that respects the heterogeneity in this highly interdependent systems-of-systems. The paper finds that the geography and sustainable energy policies of the states are deeply reflected in the structure of their multi-energy infrastructure. Because New York's cold Northeastern climate drives heating demand it has a multi-energy system with greater emphasis on oil and gas. In the meantime, California's warm climate is reflected in a multi-energy system with greater emphasis on electric power system assets. Along these lines, California's natural gas infrastructure is geared toward electric power generation to support its growing reliance on variable renewable energy resources.

This paper presents multiple avenues for future open-science research. First, this analysis can be readily extended to the entire United States geograph. Second, behavioral data can be incorporated so as to develop physically-informed machined learning behavioral models of the AMES. Finally, the AMES can be studied rigorously for its sustainable and resilience properties using novel methods rooted in hetero-funtional graph theory.

V. Acknowledgement

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References

[1] G. Bridge, S. Bouzarovski, M. Bradshaw, and N. Eyre, "Geographies of energy transition: Space, place and the low-carbon economy," *Energy policy*, vol. 53, pp. 331–340, 2013.

[2] R. Albert, I. Albert, and G. L. Nakarado, "Structural vulnerability of the north american power grid," *Physical review E*, vol. 69, no. 2, p. 025103, 2004.

[3] G. Dong, R. Du, L. Tian, and R. Liu, "Robustness of network of networks with interdependent and interconnected links," *Physica A: Statistical Mechanics and its Applications*, vol. 424, pp. 11–18, 2015.

[4] P. Jean-Baptiste and R. Ducroux, "Energy policy and climate change," *Energy Policy*, vol. 31, no. 2, pp. 155–166, Jan 2003.

[5] E. Kriegler, G. Luderer, N. Bauer, L. Baumstark, S. Fujimori, A. Popp, J. Rogelj, J. Strefler, and D. P. Van Vuuren, "Pathways limiting warming to 1.5c: a tale of turning around in no time?" *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, vol. 376, no. 2119, p. 20160457, 2018.

[6] J. E. L. C. T. W. M. B. E. D. B. T. Robert Lempert, Benjamin L. Preston, "Pathways to 2050 alternative scenarios for decarbonizing the u.s. economy," Center for Climate and Energy Solutions, Climate Innovation 2050, 2019.

[7] J. Rogers, K. Averyt, S. Clemmer, M. Davis, F. Flores-Lopez, D. Kenney, J. Macknick, N. Madden, J. Meldrum, S. Sattler, and E. Spanger-Siegfried, "Water-Smart Power: Strengthening the U.S. Electricity System in a Warming World," Union for Concerned Scientists, Cambridge, MA, Tech. Rep., 2013.

[8] P. Lara, M. Sánchez, and J. Villalobos, "Enterprise modeling and operational technologies (ot) application in the oil and gas industry," *Journal of Industrial Information Integration*, vol. 19, p. 100160, 2020. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S2452414X20300352

[9] M. Panteli, E. A. M. Ceseña, R. Moreno, and P. Mancarella, "Infrastructure planning under uncertainty: flexibility, resilience and multi-energy systems application tutorial," 2019.

[10] G. M. Masters, *Renewable and efficient electric power systems*. John Wiley & Sons, 2013.

[11] S. Mokhatab and W. A. Poe, *Handbook of natural gas transmission and processing*. Gulf professional publishing, 2012.

[12] M. V. Lurie, *Modeling of Oil Product and Gas Pipeline Transportation*. Wiley-VCH Verlag GmbH & Co. KGaA, 2009.

[13] EIA, "Coal market module of the national energy modeling system: Model documentation 2014," *Independent Statistics and Analysis, U.S. Energy Information Administration, Department of Energy*, 2014.

[14] E. Priyanka, S. Thangavel, X.-Z. Gao, and N. Sivakumar, "Digital twin for oil pipeline risk estimation using prognostic and machine learning techniques," *Journal of Industrial Information Integration*, p. 100272, 2021. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S2452414X21000704

[15] EIA, "Annual energy outlook 2020," *Independent Statistics and Analysis, U.S. Energy Information Administration, Department of Energy*, 2020.

[16] ——, "Availability of the national energy modeling system (nems) archive," United States Energy Information Administration, Tech. Rep., 2017. [Online]. Available: https://www.eia.gov/outlooks/aeo/info_nems_archive.cfm

[17] D. Dori, *Model-based systems engineering with OPM and SysML*. Springer, 2015.

[18] S. Friedenthal, A. Moore, and R. Steiner, *A Practical Guide to SysML: The Systems Modeling Language*, 2nd ed. Burlington, MA: Morgan Kaufmann, 2011.

[19] Platts, "Platts energy map data pro," S&P Global Platts, Tech. Rep., 2017. [Online]. Available: https://www.spglobal.com/platts/en/products-services/oil/map-data-pro

[20] D. J. Thompson and A. M. Farid, "A reference architecture for the american multi-modal energy system," *arXiv preprint arXiv:2012.14486*, 2020.

[21] W. C. Schoonenberg, I. S. Khayal, and A. M. Farid, *A hetero-functional graph theory for modeling interdependent smart city infrastructure*. Springer, 2019.

[22] D. J. Thompson, W. C. Schoonenberg, and A. M. Farid, "A hetero-functional graph analysis of electric power system structural resilience," in *2020 IEEE Power Energy Society Innovative Smart Grid Technologies Conference (ISGT)*, 2020, pp. 1–5.

[23] ——, "A hetero-functional graph resilience analysis of the future american electric power system," *IEEE Access*, vol. 9, pp. 68 837–68 848, 2021.

[24] EIA, "California state energy profile," *EIA U.S. States Overview*, February 2021.

[25] ——, "New york state energy profile," *EIA U.S. States Overview*, October 2021.

[26] D. Thompson, P. Hegde, W. C. H. Schoonenberg, I. Khayal, and A. M. Farid, "The hetero-functional graph theory toolbox," 2020.

[27] R. Albert, H. Jeong, and A.-L. Barabási, "Error and attack tolerance of complex networks," *Nature*, vol. 406, no. 6794, pp. 378–382, Jul 2000.

[28] L. A. N. Amaral, A. Scala, M. Barthelemy, and H. E. Stanley, "Classes of small-world networks," *Proceedings of the national academy of sciences*, vol. 97, no. 21, pp. 11 149–11 152, 2000.

[29] A.-L. Barabási and R. Albert, "Emergence of scaling in random networks," *science*, vol. 286, no. 5439, pp. 509–512, 1999.