A Broadcast Approach to Multiple Access with Partial CSIT

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***Abstract*—A new broadcast strategy is designed for multiple access communication with *partial* channel state information at the transmitters. Specifically, a two-user multiple access channel is considered, in which the state of each channel is known only to its corresponding transmitter. In broadcast approaches, in principle, the transmitter sends multiple independent superimposed information layers, where the rate of each layer is adapted to a specific channel realization. The novel aspect of the proposed strategy is that it adapts the designed codebooks to the state of the whole network, which in contrast to the existing ones in which each transmitter adapts its transmission strategy only to the state of its direct channel to the receiver. Noting that the contribution of each user to a network-wide measure (e.g., capacity region) depends not only on the user's direct channel to the receiver, but also on the qualities of other channels, in the proposed strategy the transmitters adapt their transmissions to the combined states resulting from all users' channels. This leads to a larger achievable rate region, which is characterized and compared to two outer bounds. Furthermore, the proposed strategy is proved to achieve the sum-rate capacity asymptotically.**

I. Introduction

Wireless channels are often subject to random variations resulting from the surrounding environment, inducing uncertainties about the channel state at all transmitters and receivers in the network. While receivers can estimate the varying channel states with high fidelity, acquiring such estimates at the transmitters via feedback from the receivers, incurs additional communication and delay costs. In certain systems, it is not always feasible for the transmitters to acquire the channel state information (CSI) due to, e.g., stringent delay constraints or excessive feedback costs. Under such assumptions, the notion of outage analysis can be used for assessing the reliability of wireless networks [1] and [2]. The outage and delay-limited capacities are studied extensively for various channel models (c.f. [3]–[8] and references therein).

An effective approach to circumvent CSI uncertainty at the transmitters is a form of superposition coding, according to which each transmitter splits its data stream into a number of independently-generated coded layers with different rates. The rate of each layer is adapted to a specific channel state. The transmitter then superimposes and transmits all the generated layers and the receiver decodes as many layers as the actual quality of the channel affords.

The broadcast strategy was initially proposed for compound broadcast channels [9]. Based on that, a broadcast strategy was introduced in [10] for the slowly-fading single-user channel in which the transmitter sends the superimposed coded information layers intended to different channel states, thus creating an equivalent broadcast network. In such a network each channel state is treated as a different receiver and considered to be degraded with respect to a subset of the remaining states. Hence, each receiver is able to decode its intended information layer in addition to those adapted to all the channels with degraded states.

The information-theoretic limits of the multiple access channel (MAC) when all the transmitters and receivers have *complete* CSI are well-investigated (c.f. [1], [11], [12]). However, when the transmitters have CSI uncertainties, the performance limits are not fully known. The broadcast approach is investigated for the two-user MAC with *no* CSI at the transmitters (CSIT) in [13], [14], and [15]. In this paper, we consider the two-user MAC in which the transmitters have *partial* CSI. Specifically, each channel randomly takes one of a finite number of states, and each transmitter only knows the state of its direct channel to the receiver, while being unaware of the state of the other transmitter's channel. A similar model for the two-state MAC is considered in [16], where it adopts a broadcast approach designed for the single-user channel and directly applies it to the MAC. Specifically, in [16] each transmitter generates two coded layers, where each layer is adapted to one of the states of the channel linking the other transmitter to its receiver.

In this paper we propose a new strategy when the transmitters have partial CSI, bearing in mind two important factors. The first one is that the overall performance of the MAC channel is affected not only by the direct channel connecting each transmitter to the receiver, but also by the interference resulting from the other transmitter's channel. The second factor is that each transmitter might be able to use the available information about its channel state to adapt the number of generated information layers to avoid causing any unnecessary interference at the receiver side. Motivated by these, we propose a strategy in which the information layers are adapted to the combined state of the channels of both transmitters.

We start by analyzing a two-state channel, and provide inner and outer bounds on the capacity region of the network resulting from implementing the proposed broadcast approach. We also compare the resulting average achievable rate region with that of the approach in [16] to show the improvement gained from adapting the coded layers to the combined states of both transmitters' channels. Furthermore, we prove that the proposed strategy achieves the sum-rate capacity asymptotically. Finally, we discuss the generalization of the proposed

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strategy presented for the two-state channel model to the case of any arbitrary finite number of channel states.

We remark that there exists rich literature on the information-theoretic limits of the MAC under varying degrees of availability of CSIT. Representative studies on the capacity region include the impact of degraded CSIT [17], quantized and asymmetric CSIT [18], asymmetric delayed CSIT [19], non-causal asymmetric partial CSIT [20], and symmetric noisy CSIT [21]. Furthermore, bounds on the capacity region of the memoryless MAC in which the CSIT is made available to a different encoder in causal or strictly causal manners are characterized in [22]. Counterpart results are characterized for the case of common CSI at all transmitters in [23], which are also extended in [24] to address the case in which the encoder compresses previously transmitted symbols in addition to the previous states. In [25] and [26], the capacity region is characterized for a two-user MAC where the CSI and the other transmitter's message are known causally or non-causally at one of the encoders.

The remainder of the paper is organized as follows. The finite state channel model is presented in Section II. The rate-splitting and successive decoding strategies are provided in Section III for the two-state channel model. The corresponding achievable rate regions are derived and compared to two outer bounds in sections IV and V, respectively. The generalization of the proposed strategies to the finite channel model is discussed in Section VI, and Section VII concludes the paper.

II. Channel Model

Consider a two-user fading MAC in which the channel input-output relationship for one channel use is given by

$$Y = h_1 X_1 + h_2 X_2 + N \ , \tag{1}$$

where X_i is the signal of transmitter $i \in \{1,2\}$ with an average transmission power constraint P, h_i is the coefficient of the channel linking transmitter $i \in \{1,2\}$ to the receiver, Y is the received signal, and N accounts for the additive white Gaussian noise with zero mean and unit variance. The random channel coefficients independently take one of $\ell \in \mathbb{N}$ distinct values denoted by $\{\sqrt{\alpha_m} : m \in \{1,\dots,\ell\}\}$.

Transmitter $i \in \{1,2\}$ is assumed to know only the state of channel h_i, and it is unaware of the actual realization of the other channel. Also, the receiver is assumed to have access to the full CSI. Depending on the actual realization of the channel coefficients h_1 and h_2, the multiple access channel can be in one of ℓ^2 possible states. By leveraging the broadcast approach (c.f. [10] and [15]), the communication model in (1) can be equivalently presented by a broadcast network that has two inputs X_1 and X_2 and ℓ^2 outputs. Figure 1 depicts the network model for $\ell = 2$. Each output corresponds to one possible combination of channels h_1 and h_2. We denote the output corresponding to the combination $h_1 = \sqrt{\alpha_m}$ and $h_2 = \sqrt{\alpha_n}$, for $m, n \in \{1,\dots,\ell\}$ by

$$Y_{mn} = \sqrt{\alpha_m} X_1 + \sqrt{\alpha_n} X_2 + N_{mn} \ , \tag{2}$$

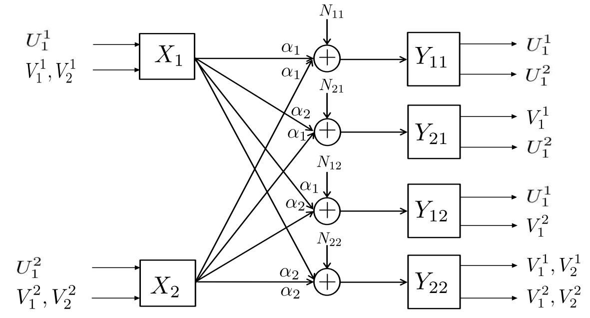

Figure 1: Equivalent broadcast channel for the two-user MAC ($\ell = 2$).

where N_{mn} is a zero-mean unit-variance Gaussian random variable. Without loss of generality, we assume that the channel gains $\{\alpha_m : m \in \{1,\dots,\ell\}\}$ are ordered as $0 < \alpha_1 < \dots < \alpha_\ell < +\infty$. We use the notation $C(x,y) \triangleq \frac{1}{2}\log_2(1 + \frac{x}{y+\frac{1}{P}})$ throughout the paper.

III. Rate splitting and Decoding Schemes

In this section, we focus on the two-state channel ($\ell = 2$), and provide the proposed rate splitting, codebook assignment, and decoding schemes for the multi-terminal network depicted in Fig. 1. In this network, corresponding to receiver Y_{mn}, the state of channel $h_1 = \sqrt{\alpha_m}$ is known only to transmitter 1 and the state of channel $h_2 = \sqrt{\alpha_n}$ is known only to transmitter 2. Throughout this section, we refer to channel states α_1 and α_2 as the *weak* and *strong* channels, respectively.

A. Rate Splitting and Adapting Layers to the MAC

Due to both direct and interfering roles of each transmitter, the rates of the transmitted information streams need to be adapted to the combined state of both transmitters' channels. Furthermore, by leveraging the available partial CSIT, each transmitter can opportunistically sustain higher rates by adapting its transmission layers to the instantaneous state of its own channel.

By taking into consideration these two observations, the message of each transmitter is dynamically split into two independent codebooks, depending on the actual state of the channel known to the transmitter. Specifically, when transmitter $i \in \{1,2\}$ is in the *weak* state, it encodes its data stream into one codebook denoted by U_1^i. On the other hand, when transmitter $i \in \{1,2\}$ is in the *strong* state, it splits its data stream into two layers denoted by V_1^i and V_2^i. Based on this layering, codebook V_1^i (or V_2^i) is adapted to the state in which the other transmitter experiences a *weak* (or *strong*) channel. The details of information layering and assigning codebooks to different channel states are depicted in Fig. 2. In this figure, the cell corresponding to the combined state (α_m, α_n) specifies which codebook is assigned to that combined state.

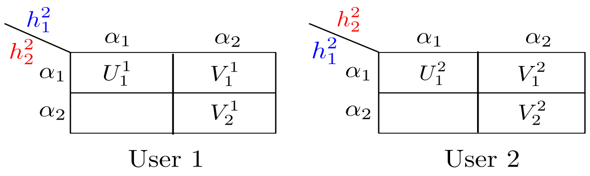

Figure 2: Layering and codebook assignments for users 1 and 2.

Table I: Successive decoding Scheme I

(h_1^2, h_2^2)	1	2
(α_1, α_1)	U_1^1, U_1^2	
(α_2, α_1)	V_1^1, U_1^2	
(α_1, α_2)	U_1^1, V_1^2	
(α_2, α_2)	V_1^1, V_1^2	V_2^1, V_2^2

B. Decoding Scheme

In this subsection, we provide two successive decoding schemes and characterize their corresponding achievable rate regions. In both schemes, based on the structure of codebook assignment presented in Fig. 2, the number of decodable codebooks for each transmitter increases as either the transmitter's channel becomes stronger, or the interfering link becomes weaker. The main idea in these decoding schemes is that two codebooks (V_2^1 and V_2^2) are reserved to be decoded *only* when both channels are strong. Except these two codebooks, whenever the channels are in any other combined state (i.e., at least one is not *strong*), the receiver decodes all the layers generated by transmitters 1 and 2 (except V_2^1 and V_2^2).

In the first decoding scheme, demonstrated in Table I, different combinations of the codebooks in different channel states are decoded as follows.

- Network state (α_1, α_1): Both transmitters are in the *weak* state and are aware of their channels. They generate codebooks $\{U_1^1, U_1^2\}$ according to Fig. 2. In this state, the baseline layers U_1^1 and U_1^2 are jointly decoded.
- Network state (α_2, α_1): When only the channel of transmitter 1 is *strong*, three codebooks are generated including $\{V_1^1, V_2^1, U_1^2\}$. As shown in Table I, codebooks $\{V_1^1, U_1^2\}$ adapted to the weak channel state (α_2, α_1) are decoded, while reserving codebook V_2^1 for state (α_2, α_2).
- Network state (α_1, α_2): This state is similar to state (α_2, α_1), except that the roles of transmitters 1 and 2 are swapped.
- Network state (α_2, α_2): Finally, when both transmitters have *strong* channels, all the codebooks transmitted by both transmitters are decoded in the order depicted in the last row of Table I. It starts by decoding the baseline layers, and at the end the layers reserved exclusively for the channel combination (α_2, α_2) are decoded.

We remark that compared with a similar network with no CSIT, presented in [15], the major distinction is due to the fact that each transmitter does not have a pre-fixed layering strategy, and each transmitter selects its layering approach dynamically, and based on the known instantaneous channel realization. Furthermore, the major distinction with a similar network with partial CSIT, presented in [16], is due to the fact that we have reserved the two codebooks V_2^1 and V_2^2 to be decoded only when both channels are *strong*, while the approach of [16] might decode them even if one of the channels is *weak*. For instance, when the combined channel state is (α_1, α_2), the approach of [16] decodes four codebooks, while in our proposed approach, we only decode two codebooks $\{U_1^1, V_1^2\}$. This leads to a larger achievable rate region, which subsumes that of [16]. As shown later via numerical evaluations, as the number of channel states increases, the gap becomes even more significant.

The second successive decoding scheme is presented in Table II. In this scheme, the set of codebooks decoded in each channel state is precisely similar to those of the first decoding scheme in Table I, except that all the codebooks are decoded successively.

IV. Achievable Rate Regions

In this section, we delineate the achievable rate regions for the codebook assignment and successive decoding schemes presented in Section III. These achievable rate regions encompass the convex combinations of all simultaneously achievable rates $R_{u_1}^i$ and $R_{v_j}^i$, which denotes the rates of information streams U_1^i and V_j^i, respectively, for $i, j \in \{1, 2\}$. For characterizing these regions, we define $\beta_{v_j}^i \in [0, 1]$ as the fraction of the power total power P assigned to information layer V_j^i, where for $i \in \{1, 2\}$ we have $\sum_{j=1}^2 \beta_{v_j}^i = 1$.

In Theorem 1, we present the achievable rate region for the the decoding scheme presented in Table I.

Theorem 1. *For the successive decoding scheme in Table I, the achievable rate region $(R_{u_1}^i, R_{v_1}^i, R_{v_2}^i)$ for user $i \in \{1, 2\}$ is the set of all rates that satisfy*

$$R_{u_1}^1 \leq \min\{d_1, d_2\}, \qquad R_{u_1}^2 \leq \min\{d_9, d_{10}\}, \qquad (3)$$
$$R_{v_1}^1 \leq \min\{d_3, d_4\}, \qquad R_{v_1}^2 \leq \min\{d_{11}, d_{12}\}, \qquad (4)$$
$$R_{v_2}^1 \leq d_5, \qquad R_{v_2}^2 \leq d_{13}, \qquad (5)$$
$$R_{u_1}^1 + R_{u_1}^2 \leq d_6, \qquad R_{v_1}^1 + R_{v_1}^2 \leq d_{14}, \qquad (6)$$
$$R_{u_1}^1 + R_{v_1}^2 \leq d_7, \qquad R_{v_1}^1 + R_{u_1}^2 \leq d_{15}, \qquad (7)$$
$$R_{v_2}^1 + R_{v_2}^2 \leq d_8, \qquad (8)$$

where constants $\{d_1, \ldots, d_{15}\}$ are defined in Appendix A.

Similarly, the achievable rate region corresponding to the successive decoding scheme presented in Table II is characterized in Theorem 2.

Table II: Successive decoding scheme II

(h_1^2, h_2^2)	1	2	3	4
(α_1, α_1)	U_1^1	U_1^2		
(α_2, α_1)	V_1^1	U_1^2		
(α_1, α_2)	U_1^1	V_1^2		
(α_2, α_2)	V_1^1	V_1^2	V_2^1	V_2^2

Theorem 2. *For the successive decoding strategy in Table II, the achievable rate region* $(R^i_{u_1}, R^i_{v_1}, R^i_{v_2})$ *for user* $i \in \{1, 2\}$ *is the set of all rates that satisfy*

$$R^1_{u_1} \leq e_1, \qquad R^2_{u_1} \leq e_4, \tag{9}$$
$$R^1_{v_1} \leq e_2, \qquad R^2_{v_1} \leq e_5, \tag{10}$$
$$R^1_{v_2} \leq e_3, \qquad R^2_{v_2} \leq e_6, \tag{11}$$

where constants $\{e_1, \dots, e_6\}$ *are defined in Appendix B.*

Corollary 1. *The maximum average rate achievable by the decoding scheme in [16] does not exceed that achieved by adopting the successive decoding scheme in Table II.*

V. Average Achievable Rate Regions

In this section, as a performance metric for the proposed strategy, we compare the average achievable rate regions characterized in theorems 1 and 2 with the capacity region of the two-user MAC channel with full CSIT, as an outer bound. Furthermore, we adapt the proposed codebook assignment strategy to the two-user MAC in which partial CSIT is available at one transmitter and full CSIT at the other, and compare the average achievable rate region of the proposed encoding strategy to the capacity region of the channel with full CSIT. Note that this region also encloses the average achievable rate regions of the codebook assignment proposed in Section III, hence serving as a second outer bound.

Outer bound 1: The first outer bound is the capacity region corresponding to the two-user MAC in which the transmitters have complete access to the CSIT.

Outer bound 2: For the second outer bound we consider a setting in which transmitter 1 is assumed to know the state of its own channel, while being unaware of the channel state of transmitter 2. Transmitter 2, on the other hand, has access to the full CSI. In this setting, transmitter 1 designs two sets of codebooks to be adopted depending on whether it is in the *weak* state or in the *strong* state. Specifically, when the channel state of transmitter 1 is *weak*, it transmits one information layer denoted by W_1^1. On the other hand, when it is *strong*, it splits its message into two information layers denoted by $\{S_1^1, S_2^1\}$, with the corresponding power allocation fractions $\beta^1_{s_1}$ and $(1 - \beta^1_{s_1})$, where $\beta^1_{s_1} \in [0, 1]$. Transmitter 2, on the other hand, is assumed to have access to the full CSI, and it adjusts its transmission rate based on the combined state of both channels. Specifically, for the combined channel state (α_2, α_2), transmitter 2 transmits two information layers, denoted by $\{S_1^2, S_2^2\}$, with power allocation fractions $\beta^2_{s_1}$ and $(1 - \beta^2_{s_1})$, where $\beta^2_{s_1} \in [0, 1]$. Corresponding to any other channel state combination, transmitter 2 transmits a single layer. Hence, W_1^2 is reserved for state (α_1, α_1), W_2^2 for state (α_1, α_2), and W_3^2 for state (α_2, α_1). It can be shown that by using this codebook allocation and performing a successive decoding strategy at the receiver, the average sum-rate capacity can be achieved for this network. Theorem 3 summarizes the average achievable rate region and identifies the area over which capacity is achieved.

Theorem 3. *For a two-user MAC with local CSI at transmitter 1 and complete CSI at transmitter 2, the average achievable rate region is the set of all average rates enclosed by the region* $OABCDEO$ *shown in Fig. 3. The average capacity region is achieved along* AB *and* DE*, and the sum-rate capacity is achieved along* CD*. The points* $O, A, B, C, D,$ *and* E *are given by* $(0, 0)$*,* $(0, f_1)$*,* (f_2, f_1)*,* (f_3, f_4)*,* (f_5, f_6)*, and* $(f_5, 0)$ *specified in Appendix C.*

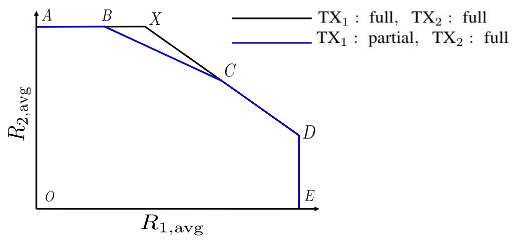

Figure 3: Outer bounds on the average achievable rate region.

In Figure 3, the region enclosed by $OAXDEO$ is the average capacity region of a two-user MAC with full CSI at each transmitter (outer bound 1), which encloses outer bound 2. Parts of the achievable rate region described in Theorem 3, i.e., AB and CDE, coincide with the average capacity region of the case of the two-user MAC with full CSIT. Specifically, along the line CD, the average sum-rate capacity is achieved for the channel even though one of the two transmitters has only local CSIT. It can be shown that if both transmitters possess local CSIT, it is possible to achieve an expected sum-rate that is close to outer bound 1, and the sum-rate capacity is achieved asymptotically for low and high signal-to-noise ratio (SNR) regions. This result is formalized in Theorem 4.

Theorem 4. *By adopting the codebook assignment presented in Section III, and setting* $\beta^1_{v_2} = \beta^2_{v_2} = \frac{\alpha_1}{\alpha_2}$*, the sum-rate capacity of a two-user MAC with full CSIT is asymptotically achievable as* $P \to 0$ *or* $P \to \infty$*.*

VI. Multi-state Channel ($\ell > 2$)

In this section, we briefly discuss the generalization of the encoding and decoding strategies proposed in Section III for the case of the two-state channel to the ℓ-state channel, where $\ell \in \mathbb{N}$. When the number of possible realizations for each transmitter's channel is ℓ, each transmitter will have ℓ different sets of codebooks each corresponding to a different

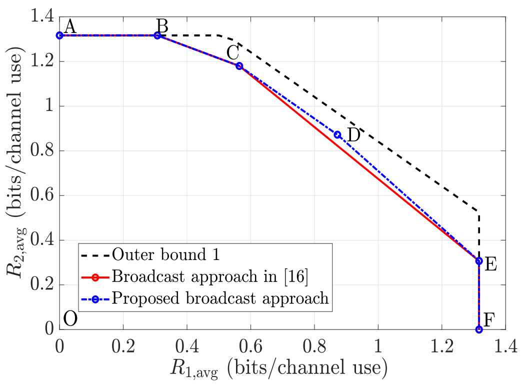

Figure 4: Achievable rate regions for two-state channel.

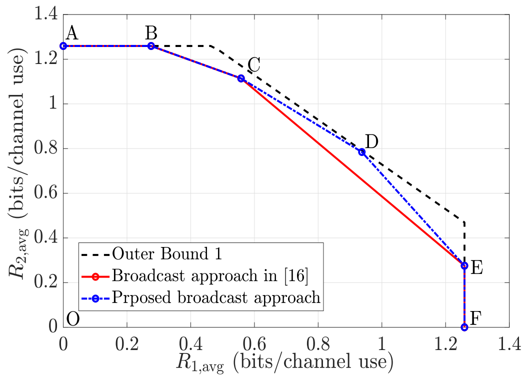

Figure 5: Achievable rate regions for three-state channel.

state. Specifically, when transmitter $i \in \{1,2\}$ knows that its channel state is α_j for $j \in \{1,\cdots,\ell\}$, it transmits the set of codebooks $\mathcal{W}_j^i$ containing j layers. The decoding order for the general case is similar the one used for the two-state channel in Table I. In particular, in channel state (α_q, α_p) the receiver successively decodes $\min\{p,q\}$ codebooks from each set of codebooks transmitted by transmitter 1 and 2. Note that the set of decoded codebooks in channel state (α_q, α_p) is related to the previously decoded set of codebooks in state (α_{q-1}, α_p) and (α_q, α_{p-1}). Similar to the two-state channel, additional codebooks can be decoded when either of the two channels becomes stronger.

By numerical evaluations, it can be readily verified that the average rates achieved by the proposed encoding and decoding strategies is close to outer bound 1 over a large range of SNR values. For the two-state channel model, Fig. 4 demonstrates the average rate region for SNR $= 10$ dB, channel coefficients $\sqrt{\alpha_1} = 0.5$, $\sqrt{\alpha_2} = 1$, and the channel probability model $\mathbb{P}(h_1^2 = \alpha_1) = \mathbb{P}(h_2^2 = \alpha_1) = 0.5$. Different points on the blue curve result from adopting different decoding orders of the proposed codebook allocation strategy. For example, points B and E are achieved by decoding and eliminating the first and second transmitter's messages, respectively, before decoding the other transmitter's message. Furthermore, point D is obtained by implementing the decoding strategy described in Table I, while point C can be obtained using the successive decoding order shown in Table II. Similarly, for a three-state channel model, Fig. 5 demonstrates the average rate region for channel coefficients $\sqrt{\alpha_1} = 0.5$, $\sqrt{\alpha_2} = 0.7$, $\sqrt{\alpha_3} = 1$, and the channel probability model $\mathbb{P}(h_1^2 = \alpha_1) = \mathbb{P}(h_2^2 = \alpha_1) = 0.6$ and $\mathbb{P}(h_1^2 = \alpha_2) = \mathbb{P}(h_2^2 = \alpha_2) = 0.1$. Point D in Fig. 5 shows a considerable gain achieved by the proposed joint decoding strategy compared to the sequential decoding strategy in [16]. Hence, by adapting the number of codebooks to the combined channel state as well as taking into account the degradedness of the combined channel states with respect to each other, the proposed joint successive decoding strategy achieves higher average rates. Comparing these two figures also shows that as the number of channel states increases, the gap between the achievable rate region of our proposed approach and that of [16] increases.

VII. Conclusion

In this paper, we have proposed a novel broadcast approach for the two-user multiple access channel where transmitters have access to only their local channel state information (CSI). In particular, each transmitter is assumed to know the complete state information of its own channel to the receiver, while being oblivious to the state of the other channel. Existing broadcast strategies for such a channel model adapt the number of codebooks designed at each user, as well as their rates, to the state of its individual channel. The proposed approach, in contrast, adapts the design of the information layers to the combined states of the channels resulting from all the transmitters. Achievable rate regions for the proposed approach have been characterized, demonstrating that the proposed approach and its associated achievable rate region subsume those of the existing approaches. Furthermore, it is established that the proposed strategy achieves the sum-rate capacity asymptotically.

Appendix A
Constants of Theorem 1

$$d_1 \triangleq C(\alpha_1, 0) \tag{12}$$
$$d_2 \triangleq C\left(\alpha_1, \alpha_2\beta_{v_2}^2\right) \tag{13}$$
$$d_3 \triangleq C\left(\alpha_2\beta_{v_1}^1, \alpha_2\beta_{v_2}^1\right) \tag{14}$$
$$d_4 \triangleq C\left(\alpha_2\beta_{v_1}^1, \alpha_2\beta_{v_2}^1 + \alpha_2\beta_{v_2}^2\right) \tag{15}$$
$$d_5 \triangleq C\left(\alpha_2\beta_{v_2}^1, 0\right) \tag{16}$$
$$d_6 \triangleq C(2\alpha_1, 0) \tag{17}$$
$$d_7 \triangleq C\left(\alpha_1 + \alpha_2\beta_{v_1}^2, \alpha_2\beta_{v_2}^2\right) \tag{18}$$
$$d_8 \triangleq C\left(\alpha_2\beta_{v_2}^1 + \alpha_2\beta_{v_2}^2, 0\right) \tag{19}$$
$$d_9 \triangleq C(\alpha_1, 0) \tag{20}$$
$$d_{10} \triangleq C\left(\alpha_1, \alpha_2\beta_{v_2}^1\right) \tag{21}$$
$$d_{11} \triangleq C\left(\alpha_2\beta_{v_1}^2, \alpha_2\beta_{v_2}^2\right) \tag{22}$$
$$d_{12} \triangleq C\left(\alpha_2\beta_{v_1}^2, \alpha_2\beta_{v_2}^1 + \alpha_2\beta_{v_2}^2\right) \tag{23}$$

$$d_{13} \triangleq C\left(\alpha_2\beta_{v_1}^2, 0\right) \quad (24)$$

$$d_{14} \triangleq C\left(\alpha_2\beta_{v_1}^1 + \alpha_2\beta_{v_1}^2, \alpha_2\beta_{v_2}^1 + \alpha_2\beta_{v_2}^2\right) \quad (25)$$

$$d_{15} \triangleq C\left(\alpha_1 + \alpha_2\beta_{v_1}^1, \alpha_2\beta_{v_2}^1\right) \quad (26)$$

APPENDIX B
CONSTANTS OF THEOREM 2

$$e_1 \triangleq C\left(\alpha_1, \alpha_2\right) \quad (27)$$

$$e_2 \triangleq C\left(\alpha_2\beta_{v_1}^1, \alpha_2\beta_{v_2}^1 + \alpha_2\right) \quad (28)$$

$$e_3 \triangleq C\left(\alpha_2\beta_{v_2}^1, \alpha_2\beta_{v_2}^2\right) \quad (29)$$

$$e_4 \triangleq C\left(\alpha_1, \alpha_2\beta_{v_2}^1\right) \quad (30)$$

$$e_5 \triangleq C\left(\alpha_2\beta_{v_1}^2, \alpha_2\beta_{v_2}^1 + \alpha_2\beta_{v_2}^2\right) \quad (31)$$

$$e_6 \triangleq C\left(\alpha_2\beta_{v_2}^2, 0\right) \quad (32)$$

APPENDIX C
CONSTANTS OF THEOREM 3

$$f_1 \triangleq q\, C(\alpha_1, 0) + \bar{q}\, C(\alpha_2, 0) \quad (33)$$

$$f_2 \triangleq p\, C\left(\alpha_1, \alpha_2\right) + \bar{p}\, C\left(\alpha_2, \alpha_2\right) \quad (34)$$

$$\begin{aligned} f_3 &\triangleq p\, C(\alpha_1, 0) \\ &+ \bar{p}\left[C\left(\alpha_2\beta_{s_1}^1, \alpha_2(1-\beta_{s_1}^1) + \alpha_1\right) + C(\alpha_2(1-\beta_{s_1}^1), 0)\right] \end{aligned} \quad (35)$$

$$\begin{aligned} f_4 &\triangleq pq\, C(2\alpha_1, 0) + (p\bar{q} + \bar{p}q)\, C(\alpha_1 + \alpha_2, 0) \\ &+ \bar{p}\bar{q}\, C(2\alpha_2, 0) - f_3 \end{aligned} \quad (36)$$

$$f_5 \triangleq p\, C(\alpha_1, 0) + \bar{p}\, C(\alpha_2, 0) \quad (37)$$

$$\begin{aligned} f_6 &\triangleq pq\, C\left(\alpha_1, \alpha_1\right) + p\bar{q}\, C\left(\alpha_2, \alpha_1\right) \\ &+ \bar{p}q\, C\left(\alpha_1, \alpha_2\right) + \bar{p}\bar{q}\, C\left(\alpha_2, \alpha_2\right) \,, \end{aligned} \quad (38)$$

where $p \triangleq \mathbb{P}(h_1^2 = \alpha_1)$, $q \triangleq \mathbb{P}(h_2^2 = \alpha_1)$, $\bar{p} \triangleq 1 - p$, and $\bar{q} \triangleq 1 - q$.

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