QUICKEST LINE OUTAGE LOCALIZATION UNDER UNKNOWN MODEL

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ABSTRACT

Line outage detection and localization play pivotal roles in enhancing the overall reliability of the electricity grid. The existing line outage detection and localization techniques often rely on the assumption that the information about the grid, such as the topology and system parameters, is known perfectly. In practice, however, such information bears uncertainties due to inaccuracies in lines parameters and the historical data. This paper studies line outage detection and localization under the assumption that the line reactance values are known only partially, and aims to find the minimum number of measurements to perform outage detection and localization with target reliability. Specifically, based on the *nominal* values of line reactance it proposes a stochastic graphical framework that capitalizes on the correlation among the measurements generated across the grid, and designs data-adaptive data-acquisition and decision-making processes for the quickest localization of the lines in outage. The paper also analyzes the sensitivity of the proposed algorithm to the changes in the line reactance values and shows its robustness to line reactance uncertainties.

1. INTRODUCTION

Reliable delivery of electricity is one of the key goals of system operators, and a significant effort is being made to achieve it. Due to the large-scale and strong inter-connectivities in the power grid, any fault can transcend its realm and disrupt operations in other parts of the grid. Therefore, real-time monitoring of the grid is of paramount importance in securing reliable power delivery. Specifically, agile detection and localization of system failures facilitate mitigating the disruptive impacts the failure can cause to the network, prevent cascading failures in larger scales, and reduce the recovery costs.

One common type of failure is transmission line outage, which occurs due to the transmission lines being constantly exposed to various sources of disturbance, such as equipment malfunctioning and natural disasters. While the power system is designed to operate reliably under single or multiple contingencies, the monitoring task should identify those contingencies as quickly as possible to prevent overload in one section of the grid. Detecting such contingencies and localizing them accurately can expedite the process of repairing the faulty components, speed up restoration of the grid, reduce outage time, and improve power system reliability [1]. Hence, outage detection and localization have been investigated extensively in the existing literature under different settings and objectives.

Motivated by lowering the required communication and reducing the delay of decision-making and its computational complexity, the study in [2] has developed a stochastic graphical framework for modeling the bus measurements, and has devised data-adaptive data-acquisition and decision-making processes for reliably detecting and localizing the outage events with the fewest number of measurements. This framework relies on knowing the network model perfectly. However, while the introduction of phasor measurement units (PMUs) enables acquiring real-time synchronized phasor angle measurements with a satisfactory precision, accurate power grid parameters are far more difficult to obtain. For instance, reactance values of the transmission lines, which play critical roles in outage detection [2–10], are susceptible to various factors such as unstable frequency, cable quality changes, and electromagnetic fields variations, thus deviating the actual values from the nominal values. Therefore, in this paper we assume that the actual values of line reactance are unknown and different from their nominal values and we analyze the impact of such information uncertainties on the optimal line outage detection and localization.

Based on the real-time measurements used for outage detection and localization, the existing literature can be categorized into two groups. In one direction, after collecting measurements from the entire grid, the outage detection is performed by solving different formulations of the problem such as combinatorial optimization [3–5], line outage distribution factor [6, 7], graphical model learning [10], compressive sensing [8], quickest change detection [9], and joint outage detection and state estimation [11, 12]. In order to reduce the cost of data collection and processing, in the other direction outage localization is performed based on the partial observation of the grid [13–18]. These studies use pre-specified data collection rules, which, despite their effectiveness, can become inefficient in large-scale networks that are expanded over large geographic areas. The study in [2] has developed a data-adaptive data collection strategy to circumvent this issue, where it is assuming that all the parameters of the grid are known perfectly.

In this paper, we show that, interestingly, the performance of the proposed optimal sampling strategy in [2] displays outstanding resistance towards reactance fluctuations. First, we establish the resilience of the algorithm proposed in [2] against the fluctuations in line reactance values by analyzing its sensitivity. Then, through simulations we show that the degradation in the performance remains marginal as the uncertainty in the values increases.

2. PRELIMINARIES

2.1. Statistical Model of Bus Measurements

Consider a power grid consisting of N buses collected in set $\mathcal{B} \triangleq \{1, \dots, N\}$ and L transmission lines denoted by set $\mathcal{E} \subseteq \mathcal{B} \times \mathcal{B}$, where $(i, j) \in \mathcal{E}$ if buses $i, j \in \mathcal{B}$ are directly connected by a line. We define θ_i and p_i as the voltage phasor angle and the injected active power at bus $i \in \mathcal{B}$, and x_{ij} as the reactance of the line connecting buses i and j. From the DC power flow model we have [19]:

$$p_i = \sum_{j \in \mathcal{N}_i} \left(\frac{\theta_i - \theta_j}{x_{ij}} \right) , \qquad (1)$$

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where $\mathcal{N}_i \triangleq \{j \in \mathcal{B} : (i,j) \in \mathcal{E}\}$ is the set of neighbors of bus i. It is assumed that line reactance values are not fully known and there exist some level of uncertainty in their values. Specifically, by denoting the set of line reactance values by $\mathcal{X} \triangleq \{x_{ij} \;:\; (i,j) \in \mathcal{E}\}$ we assume that only their nominal values, collected in set $\bar{\mathcal{X}}$, are known. By defining $\boldsymbol{p} \triangleq [p_1, \dots, p_N]^T$ and $\boldsymbol{\theta} \triangleq [\theta_1, \dots, \theta_N]^T$, from (1) we have $\boldsymbol{p} = \boldsymbol{H}\boldsymbol{\theta}$, where $\boldsymbol{H} \in \mathbb{R}^{N\times N}$ is the weighted Laplacian matrix of the connectivity graph defined as

$$\boldsymbol{H}[ij] \triangleq \begin{cases} \sum_{(i,\ell)\in\mathcal{E}} \frac{1}{x_{i\ell}} & \text{if } i = j \\ -\frac{1}{x_{ij}} & \text{if } (i,j) \in \mathcal{E} \\ 0 & \text{otherwise} \end{cases} . \tag{2}$$

Furthermore, from (1) and by accounting for the random disturbances in the system and the uncertainties of load profiles, the aggregate injected power at different buses can be modeled as independent random variables [20, 21]. Hence, we can show that the statistical relationship among the measurements collected from different buses across the grid can be modeled effectively as [10, 22, 23]

$$\boldsymbol{\theta}|\mathcal{X} \sim \mathcal{N}\big(\bar{\boldsymbol{\theta}}, (\boldsymbol{I} - \boldsymbol{R})^{-1}\big) , \tag{3}$$

where $\bar{\boldsymbol{\theta}}$ is the mean vector for $\boldsymbol{\theta}$, and $\boldsymbol{R}$ is a matrix whose (i,j)-th entry is r_{ij} which is defined as

$$r_{ij} \;\triangleq\; \frac{\beta_i}{x_{ij}} , \quad \text{where} \quad \beta_i \;\triangleq\; \left(\sum_{j\in\mathcal{N}_i} \frac{1}{x_{ij}}\right)^{-1} . \tag{4}$$

Distribution of $\boldsymbol{\theta}$ given in (3) indicates that, given the line reactance values, the bus measurements form a Gauss-Markov random field (GMRF) with the same dependency graph as the grid topology, i.e., $\mathcal{G}(\mathcal{B}, \mathcal{E})$ [10]. We note that, by construction, matrix $\boldsymbol{H}$ is rank-deficient. By removing the row and column corresponding to one reference bus, the remaining $(N-1)\times(N-1)$ matrix has full rank. In the remainder of this paper, when referring to the Laplacian matrix of the network, we always mean the modified full-rank one.

2.2. Outage Events

We only consider the events that keep the underlying post-event graph connected and define $\mathcal{R} = \{R_1, \dots, R_J\}$ as the set of such events, where $R_k \subseteq \mathcal{E}$ contains the lines experiencing outage under event $k \in \{1, \dots, J\}$. Additionally, event R_0 is reserved to signify the no-outage event. When an outage occurs, the connectivity profile of the grid changes. We denote the connectivity graph of the grid under event R_k by $\mathcal{G}_k(\mathcal{B}, \mathcal{E}_k)$, corresponding to which we define the weighted Laplacian matrix $\boldsymbol{H}_k$ similar to (2) except for replacing $\mathcal{E}$ with $\mathcal{E}_k$. Hence, detecting and localizing outage events can be cast as the following multi-hypothesis testing problem:

$$\mathsf{H}_k : \quad \boldsymbol{\theta} = \boldsymbol{B}_k \cdot \boldsymbol{p} , \quad \text{for } k \in \{0, \dots, J\} , \tag{5}$$

where we have defined $\boldsymbol{B}_k \triangleq \boldsymbol{H}_k^{-1}$. Under each outage event, $\boldsymbol{\theta}$ follows a distinct correlation structure governed by the associated topology of the network, which is imposed by matrix $\boldsymbol{B}_k$. Due to the massive scale of power networks, collecting measurements from all the buses incurs prohibitive sensing and processing costs. Also, uncertainty in line reactance values implies that matrix $\boldsymbol{H}_k$ is not completely known. In this paper, we devise a data-adaptive decision-making framework based on the nominal ractance values that can form arbitrarily reliable decisions about the state of the grid with the *minimal* number of measurements.

3. QUICKEST OUTAGE LOCALIZATION

In this section, by capitalizing on the discrepancies among the level of information provided by different buses under different outage events, we formalize a sequential data-acquisition and decision-making process to collect measurements of voltage phasor angles, based on which we localize the outage event, when one is deemed to exist, with the *fewest* number of measurements. The data-acquisition process sequentially collects ℓ measurements at-a-time from ℓ different buses. The process continues until time $\tau \in \mathbb{N}$, as the stopping time of the process, at which point it terminates and a decision about the underlying event is formed. For modeling the dynamic decisions about the buses to be observed at time t we define the selection function $\psi(t) \in \mathcal{B}^\ell$ as the set of indices of ℓ buses to be measured at time t, and denote the vector of measurements collected at time t by $\boldsymbol{\theta}(t) \in [0, 2\pi]^\ell$. Accordingly, we denote the vector of observed buses and their corresponding measurements up to time t by ψ_t and $\boldsymbol{\theta}_t$, respectively, i.e.,

$$\psi_t \triangleq [\psi(1), \dots, \psi(t)]^T , \quad \text{and} \quad \boldsymbol{\theta}_t \triangleq [\boldsymbol{\theta}(1), \dots, \boldsymbol{\theta}(t)]^T . \tag{6}$$

Finally, we define $\delta \in \mathcal{R}$ as the decision rule at the stopping time. The quality of decision at the stopping time is captured by the decision error probability, where by denoting the true event by $\mathsf{T} \in \mathcal{R}$ is defined as $\mathsf{P_e} \triangleq \mathbb{P}(\delta \neq \mathsf{T})$. Hence, the optimal sampling strategy is obtained as the solution to the following optimization problem:

$$\underset{\tau\,,\,\delta\,,\,\psi_\tau}{\text{minimize}} \;\; \mathbb{E}\{\tau\} \quad \text{subject to} \quad \mathsf{P_e} \leq \beta , \tag{7}$$

where $\beta \in (0,1)$ controls the reliability of the decision.

4. OPTIMAL DECISION RULES

4.1. Bus Selection Rule

For identifying the buses that should be measured we assign a metric to each bus $i \in \mathcal{B}$ as follows:

$$M(i) \;\triangleq\; \max_{\mathcal{U}\subseteq\mathcal{N}_i} \frac{1}{|\mathcal{U}|} \sum_{j\in\mathcal{U}} \log \frac{1}{1 - \bar{r}_{ij}^2} , \tag{8}$$

where $\bar{r}_{ij}$ is calculated based on the nominal reactance values $\bar{x}_{ij}$. Then, at each time t we focus on the buses that are already observed and identify the ones with the largest $|\theta_i - \bar{\theta}_i|$, which provides an estimate of the location of the underlying outage event. Among the neighbors of those buses we select those with the largest metrics $M(i)$. Also, at $t = 1$, data collection is initialized by selecting ℓ buses with the most number of neighbors. The steps of bus selection rule are presented in Algorithm 1.

4.2. Stopping Time and Decision Rule

The data-acquisition process is terminated as soon as a decision can be made with the desired reliability. By denoting $\boldsymbol{C}$ as the incident matrix of the grid topology with $\boldsymbol{c}_\ell$ as its ℓ-th column that corresponds to line ℓ with reactance x_ℓ, and defining

$$\boldsymbol{s}_k[\ell] \triangleq \begin{cases} \frac{1}{x_\ell}\boldsymbol{c}_\ell^T\boldsymbol{\theta} & \text{if } \ell \in R_k \\ 0 & \text{Otherwise} \end{cases} , \tag{9}$$

at the stopping time τ we have

$$\Delta\boldsymbol{\theta}_\tau = \boldsymbol{B}_\tau \boldsymbol{C}\boldsymbol{s}_k + \boldsymbol{B}_\tau \boldsymbol{n} , \tag{10}$$

where $\boldsymbol{B}_\tau$ is the matrix constructed from $\boldsymbol{H}^{-1}$ by keeping its rows corresponding to set ψ_τ, and we have defined $\boldsymbol{n} \triangleq \boldsymbol{p} - \bar{\boldsymbol{p}}$ as the

Algorithm 1: Data-adaptive bus selection

Set $t=1$ and compute $M(i)$ for $i \in \mathcal{B}$ according to (8)
$\psi(t) \leftarrow \ell$ nodes with the largest degree
While stopping criterion is not met **do**
  Take measurements from buses in $\psi(t)$
  $S \leftarrow \psi_t\ , \quad t \leftarrow t+1\ , \quad \psi(t) \leftarrow \{\}$
  While $|\psi(t)| < \ell$ **do**
    $i \leftarrow \arg\max_{j \in S} |\theta_j - \bar{\theta}_j|$
    $\mathcal{V}_i \leftarrow$ Unobserved neighbors of i sorted by decreasing $M(\cdot)$
    If $|\mathcal{V}_i| < \ell - |\psi(t)|$ **then** $\psi(t) \leftarrow \psi(t) \cup \mathcal{V}_i$
    Else $\psi(t) \leftarrow \psi(t) \cup \{\mathcal{V}_i(1), \dots, \mathcal{V}_i(\ell - |\psi(t)|)\}$
    End if
    $S \leftarrow S \setminus i$
  End while
End while

perturbations in the power injection incurred by an outage, which can be modeled as a zero-mean uncorrelated Gaussian random vector [8]. Since the noise vector $\boldsymbol{B}_\tau \boldsymbol{n}$ is colored, we include a preprocessing whitening stage. By assuming the singular value decomposition (SVD) $\boldsymbol{B}_\tau = \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{V}^T$, and defining

$$\boldsymbol{y} \triangleq \boldsymbol{\Lambda}^{-1}\boldsymbol{U}^T \Delta\boldsymbol{\theta}_\tau\ , \quad \text{and} \quad \boldsymbol{A} \triangleq \boldsymbol{\Lambda}^{-1}\boldsymbol{U}^T \boldsymbol{B}_\tau \boldsymbol{C}\ , \tag{11}$$

from (10)–(11), corresponding to event R_k we obtain

$$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{s}_k + \tilde{\boldsymbol{n}}\ , \tag{12}$$

where $\tilde{\boldsymbol{n}}$ is a white noise vector with covariance matrix $\boldsymbol{I}$. This leads to an overcomplete representation of the sparse vector $\boldsymbol{s}_k$ by measurement vector $\boldsymbol{y}$ given in (12). Therefore, off-the-shelf tools from compressed sensing can be applied to find the non-zero elements of $\boldsymbol{s}_k$ to detect and localize the lines in outage. Specifically, we use orthogonal matching pursuit (OMP) with the modification that we stop the sampling process when the value of residual, i.e., $\boldsymbol{r} = \boldsymbol{y} - \boldsymbol{A}\boldsymbol{s}_k$, is smaller than a threshold γ that is selected such that the reliability constraint is satisfied [2].

5. SENSITIVITY ANALYSIS

The analysis in [2] shows that the bus selection rule given in Algorithm 1 combined with the stopping and the final decision rules based on OMP is optimal when $\mathcal{X} = \bar{\mathcal{X}}$ and β approaches zero. In this section we analyze the sensitivity of the proposed algorithm to the fluctuations in line reactance values, and show that this algorithm is robust against such fluctuations. To this end, we analyze the sensitivity of phasor angles and bus metrics to the line reactance x_{ij} for each line $(i,j) \in \mathcal{E}$.

5.1. Phasor Sensitivity

From the DC power flow model we have

$$\boldsymbol{H}\frac{\partial \boldsymbol{\theta}}{\partial x_{ij}} = -\frac{\partial \boldsymbol{H}}{\partial x_{ij}}\boldsymbol{\theta} = a(\boldsymbol{e}_i - \boldsymbol{e}_j)\ , \tag{13}$$

where $\boldsymbol{e}_i$ is the unit vector with i-th element being 1 and a is defined as $a \triangleq \frac{\theta_i - \theta_j}{x_{ij}^2}$. Therefore, from (13) we have

$$\begin{aligned}\frac{\partial \theta_i}{\partial x_{ij}} &= \frac{a}{\sum_{\ell \in \mathcal{N}_i} \frac{1}{x_{i\ell}}} + \frac{\sum_{\ell \in \mathcal{N}_i} \frac{1}{x_{i\ell}} \frac{\partial \theta_\ell}{\partial x_{ij}}}{\sum_{\ell \in \mathcal{N}_i} \frac{1}{x_{i\ell}}} \\ &= \frac{1}{|\mathcal{N}_i|} \cdot \frac{p_{ij}}{x_{ij}\mathsf{A}_{\mathcal{N}_i}(\frac{1}{x_{i\ell}})} + \mathsf{WA}_{\mathcal{N}_i}(\frac{\partial \theta_\ell}{\partial x_{ij}})\ ,\end{aligned} \tag{14}$$

where p_{ij} is the power flow from bus i to bus j, $\mathsf{A}_{\mathcal{N}_i}(\frac{1}{x_{i\ell}})$ is the average of the inverse reractance values of neighbors of bus i, and $\mathsf{WA}_{\mathcal{N}_i}(\frac{\partial \theta_\ell}{\partial x_{ij}})$ is the weighted average of partial derivatives of phasors of buses in $\mathcal{N}_i$ with the inverse of the reactances of the lines connected to i as the weights. The sensitivity of node j will be the same except for p_{ij} and $\mathcal{N}_i$ being replaced by $p_{ji} = -p_{ij}$ and $\mathcal{N}_j$, respectively. Furthermore, for any other bus $k \in \mathcal{B}$ (i.e., $k \neq i,j$) we have

$$\frac{\partial \theta_k}{\partial x_{ij}} = \mathsf{WA}_{\mathcal{N}_k}(\frac{\partial \theta_\ell}{\partial x_{ij}})\ . \tag{15}$$

Hence, the first term in (14) is propagated to the first neighbors of buses i and j through a weighted average, and it propagates to the second neighbors by being averaged once more. Therefore, the effect of any change in one line reactance diminishes as we go farther away from the buses connected to that line. It also explains why in the bus selection rule we have to observe the neighbors of the buses with the largest changes in their phasor angle values. When an outage occurs in line (m,n) for $(i,j) \neq (m,n)$ we have

$$\frac{\partial \Delta\theta_k}{\partial x_{ij}} = \mathsf{WA}_{\mathcal{N}_k}(\frac{\partial \theta_\ell}{\partial x_{ij}}) - \mathsf{WA}_{\bar{\mathcal{N}}_k}(\frac{\partial \bar{\theta}_\ell}{\partial x_{ij}})\ , \tag{16}$$

which is negligible since it is the difference between the average sensitivity of the neighbors of a bus, which are small values. When the outage is in line (i,j) the post-outage phasor values will be independent of the fluctuations in x_{ij} and we have

$$\frac{\partial \Delta\theta_i}{\partial x_{ij}} = -\frac{1}{|\bar{\mathcal{N}}_i|}\frac{p_{ij}}{x_{ij}\mathsf{A}_{\bar{\mathcal{N}}_i}(\frac{1}{x_{i\ell}})} - \mathsf{WA}_{\bar{\mathcal{N}}_i}(\frac{\partial \bar{\theta}_\ell}{\partial x_{ij}})\ . \tag{17}$$

5.2. Metric Sensitivity

For analyzing metric sensitivity we follow the same line of argument, and from (8) we obtain

$$\frac{\partial M(m)}{\partial x_{ij}} = \max_{\mathcal{U} \subseteq \mathcal{N}_m} \frac{1}{|\mathcal{U}|}\sum_{n \in \mathcal{U}} \frac{-2r_{mn}}{1 - r_{mn}^2}\frac{\partial r_{mn}}{\partial x_{ij}}\ . \tag{18}$$

By denoting the set of neighbors of bus m that maximizes its metric by $\mathcal{U}_{\max}$, and replacing the sensitivity of r_{mn} to x_{ij} given in (18) we obtain

$$\frac{\partial M(m)}{\partial x_{ij}} = \begin{cases} \frac{1}{|\mathcal{U}_{\max}|}\left(\frac{2r_{ij}^2}{(1+r_{ij})x_{ij}} - \sum_{\substack{n \in \mathcal{U}_{\max} \\ n \neq j}} \frac{2r_{mn}r_{ij}^2}{(1-r_{mn}^2)x_{mn}}\right) & \text{if } \begin{array}{l} i = m \\ j \in \mathcal{U}_{\max} \end{array} \\ \frac{-1}{|\mathcal{U}_{\max}|}\sum_{n \in \mathcal{U}_{\max}} \frac{2r_{mn}r_{ij}^2}{(1-r_{mn}^2)x_{mn}} & \text{if } \begin{array}{l} i = m \\ j \notin \mathcal{U}_{\max} \end{array} \\ 0 & \text{otherwise} \end{cases}$$

By denoting the minimum line reactance value by $x_{\min}$, we have

$$\frac{\partial M(m)}{\partial x_{ij}} \leq \begin{cases} \frac{-2}{|\mathcal{U}_{\max}||\mathcal{N}_m|(|\mathcal{N}_m|^2-1)x_{\min}} & \text{if } \begin{array}{l} i = m \\ j \in \mathcal{U}_{\max} \end{array} \\ \frac{-2}{|\mathcal{N}_m|(|\mathcal{N}_m|^2-1)x_{\min}} & \text{if } \begin{array}{l} i = m \\ j \notin \mathcal{U}_{\max} \end{array} \\ 0 & \text{otherwise} \end{cases}\ ,$$

which shows that it is proportional to $|\mathcal{N}_m|^{-3}$. These analyses show that the selection rule is only loosely sensitive to the uncertainties in the line reactance values. In the next section, we verify our analyses by numerical evaluation of the changes in the voltage phasers and metrics, as well as the overall performance of the proposed localization strategy.

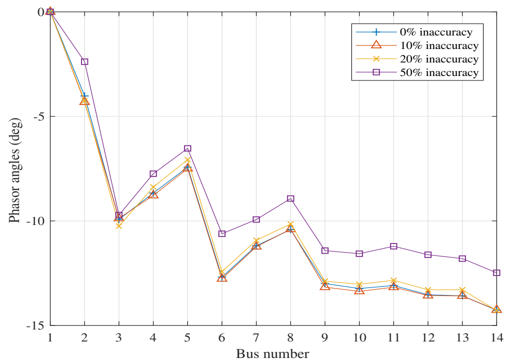

Fig. 1: Sensitivity of phasor values to reactance values

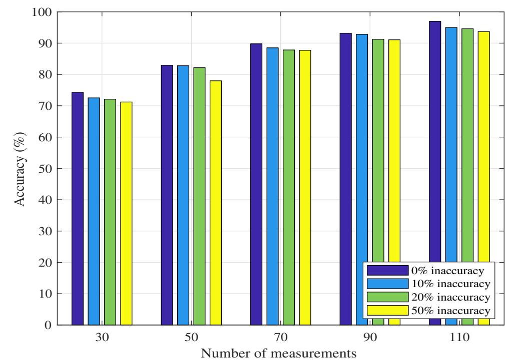

Fig. 3: Decision accuracy versus number of measurements.

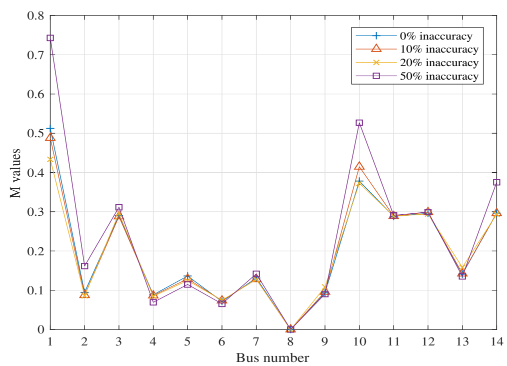

Fig. 2: Sensitivity of metric values to reactance values

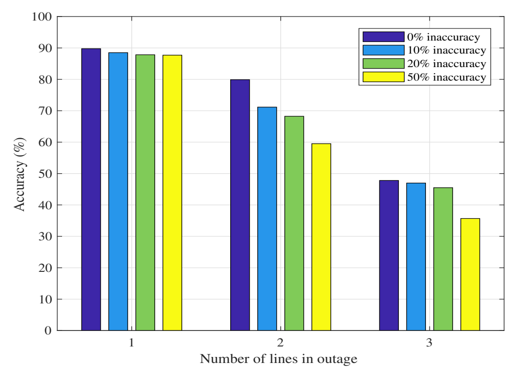

Fig. 4: Decision accuracy versus different number of line outage.

6. SIMULATION RESULTS

In this section, we evaluate the performance of the proposed algorithm and its insensitivity to the reactance fluctuations on the IEEE 118-bus standard system. The software toolbox MATPOWER is used to generate phasor angle measurements as well as the pertinent power flows under different outage events [24]. It is assumed that transmission line reactance values are uniformly distributed around their nominal value within a certain range, and all the outage events that cause network islanding are excluded.

6.1. Sensitivity of Phasor Values and Metrics

First, we assess the sensitivity of phasor angles to various ranges of line reactance fluctuations. Figure 1 shows that even with severe reactance distortion with a fluctuation level as much as 50%, the phasor angles experience limited variations. Under the same setting, we assess the reactance fluctuation resistance property of bus metrics $M(\cdot)$ in Fig. 2. It is observed that the fluctuation of the metrics is also marginal.

6.2. Sensitivity of the Localization Algorithm

In order to numerically demonstrate the inherent robustness of the proposed approach, we compare the localization accuracy of the algorithm for different levels of transmission line fluctuations in Fig. 3. It is observed that for any reactance uncertainty level, the recovery accuracy rises with the increasing number of measurements. Besides, for a fixed number of measurements, the accuracy degrades only marginally within a wide range of reactance uncertainties. The main reason is that we compare the differences between the pre-outage and post-outage phasor angles, which tend to be larger for the buses close to the outage location, regardless of the impedance values of the lines connecting those buses. In other words, the data-adaptive approach judiciously takes advantage of the information provided by phasor angle deviation, which is dominantly determined by the power grid topology rather than the fluctuations of transmission line reactance values.

Figure 4 compares the localization accuracy for multiple line outages where the lines under outage are in the same locality of the grid. Motivated by the observation made in Fig. 3, we set the number of measurements in all impedance fluctuation levels to 70. It is observed that for single and multiple line outage events, the accuracy degrades only slightly as the reactance uncertainty level increases.

7. CONCLUSION

The problem of detecting and localizing line outage events by using the minimum number of measurements under uncertainty in the line reactance values has been considered. First, by assuming that the known nominal values of line reactance are accurate, a data-adaptive information-gathering and decision-making process has been proposed. Then, through sensitivity analysis it has been shown that this strategy is robust to the fluctuations in the line reactance values. The results have been verified via numerical evaluations of the proposed rule and its performance in localizing line outage with the minimum number of measurements.

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