A Broadcast Approach to Multiple Access Adapted to the Multiuser Channel

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***Abstract*—A broadcast strategy for multiple access communication over slowly fading channels is introduced, in which the channel state information is known to only the receiver. In this strategy, the transmitters split their information streams into multiple independent information layers, each adapted to a specific actual channel realization. The major distinction between the proposed strategy and the existing ones is that in the existing approaches, each transmitter adapts its transmission strategy only to the fading process of its direct channel to the receiver, hence directly adopting a single-user strategy previously designed for the single-user channels. However, the contribution of each user to a network-wide measure (e.g., sum-rate capacity) depends not only on the user's direct channel to the receiver, but also on the qualities of other channels. Driven by this premise, this paper proposes an alternative broadcast strategy in which the transmitters adapt their transmissions to the combined states resulting from all users' channels. This leads to generating a larger number of information layers by each transmitter and adopting a different decoding strategy by the receiver. An achievable rate region that captures the trade-off among the rates of different information is established and is shown to subsume the existing known regions.**

***Index Terms*—Broadcast approach, fading channel, layered coding, multiple access, successive decoding.**

I. Introduction

Random fluctuations of the wireless channel states induce uncertainty about the network state at all transmitter and receiver sites [1]. Slowly varying channels can be estimated by the receivers with high fidelity, rendering the availability of the channel state information (CSI) at the receiver. Acquiring the CSI by the transmitters can be further facilitated via feedback from the receivers, which incurs additional communication and delay costs. The instantaneous and ergodic performance limits of the multiple access channel (MAC) with the CSI available to all transmitters and the receiver is well-investigated [1]–[3]. In certain communication scenarios, however, acquiring the CSI by the transmitters is not viable due to, e.g., stringent delay constraints or excessive feedback costs. In such scenarios, the notion of outage capacity evaluates the likelihood for the reliable communication for a fixed transmission rate [4]. When the actual channel realization can sustain the rate, transmission is carried out successfully, and otherwise, it fails and no message is decoded [1] and [4]. The notations of outage and delay-limited capacities are studied extensively for various networks including the multiple access channel (c.f. [5]–[10] and references therein).

Superposition coding is shown to be an effective approach for circumventing CSI uncertainty at the transmitters. The underlying motivation for this approach is that each transmitter splits its data stream into a number of independently generated coded layers with possibly different rates. These layers are superimposed and transmitted by the designated transmitter, and the receiver decodes as many layers as the quality of the channel affords. The aggregate rate of transmission, subsequently, is the sum of individual rates of the layers decoded by the receiver. Motivated by superposition coding, and following the broadcast approach to compound channels [11], the notion of broadcast strategy for slowly fading single-user channel was initially introduced for effective single-user communication [12]. In this approach, any channel realization is viewed as a broadcast receiver, rendering an equivalent network consisting of a number of receivers. Each receiver is designated to a specific channel realization and is degraded with respect to a subset of other channels. The broadcast strategy is further generalized for single-user channels with mixed delay constraints in [13], and single-user multi-antenna channels [14], where the singular values of channel matrices are leveraged to rank and order the degradedness of different channel realizations.

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The effectiveness of broadcast strategy for multiuser channels is investigated in [15] and [16] for the settings in which the transmitters have uncertainties about all channels, and in [17] for the settings in which each transmitter has uncertainties about the channels of other users. Specifically, the approaches in [15] and [16] adopt the broadcast strategy designed for single-user channels, and directly apply it to the MAC. As a result, each transmitter generates a number of information layers, each adapted to a specific realization of the direct channel linking the transmitter to the receiver. An alternative scenario in which each transmitter has the CSI of its direct channel to the receiver while being unaware of the states of other users' channels is studied in [17], where a transmission approach based on rate splitting and sequential decoding are proposed.

In this paper, we take a different approach based on the premise that the contribution of each user to the overall performance of the multiple access channel not only depends on the direct channel linking this user to the receiver, but also is influenced by the *relative* qualities of the other users' channels. Hence, we propose a strategy in which the information layers are generated and adapted to the combined state of the channel resulting from incorporating all individual channel states. In order to highlight the distinction with the existing approaches, consider a two-user MAC in which each channel takes one of the two possible states, referred to as *weak* and *strong* channels. The approach of [16] assigns two layers to each transmitter, one apt for the weak channel, and the second one suited to the strong channel. Each transmitter generates and transmits these layers without regard for the possible states of the other user's channel. In the proposed approach, in contrast, we leverage the fact that the two channels take a combination of four possible states. Hence, every transmitter generates four information layers, each suited to one of the four possible states. The proposed approach leads to an equivalent network with a number of receivers each corresponding to one possible combination of all channels. We show that the achievable rate region of this equivalent network is considerably larger than its counterpart presented in [16]. The proposed approach is further extended from the two-state channel to the general finite-state channels, and the corresponding achievable rate region is characterized.

The remainder of this paper is organized as follows. The finite-state channel model is presented in Section II. The encoding and decoding strategies along with an associated achievable rate region are presented in Section III. The extensions of the results to the general finite-state channels are provided in Section IV, and Section V concludes the paper.

II. Channel Model

Consider a two-user multiple access channel, in which two independent users transmit independent messages to a common receiver via a discrete-time Gaussian multiple-access fading channel. All the users are equipped with one antenna and the random channel coefficients independently take one of the $\ell \in \mathbb{N}$ distinct values, denoted by $\{\alpha_m : m \in \{1, \dots, \ell\}\}$. The fading process is assumed to remain unchanged during each transmission cycle, and can change to independent states afterwards. Channel states are *unknown* to transmitters, while the receiver is assumed to have full CSI. The users are subject to an average transmission power constraint P. By defining X_i as the signal of transmitter $i \in \{1, 2\}$ and h_i as the coefficient of the channel linking transmitter $i \in \{1, 2\}$ to the receiver, the received signal is

$$Y = h_1 X_1 + h_2 X_2 + N \ , \tag{1}$$

where N accounts for the additive white Gaussian with mean zero and variance 1. Depending on the realization of the channels h_1 and h_2, the multiple access channel can be in one of the ℓ^2 possible states.

By leveraging the broadcast approach (c.f. [12], [14], and [16]), the communication model in (1) can be equivalently presented by a broadcast network that has two inputs X_1 and X_2 and ℓ^2 outputs. Each output corresponds to one possible combinations of channels h_1 and h_2. We denote the output corresponding to the combination $h_1 = \alpha_m$ and $h_2 = \alpha_n$ by

$$Y_{mn} = \alpha_m X_1 + \alpha_n X_2 + N_{mn} \ , \tag{2}$$

where N_{mn} is a zero-mean unit-variance Gaussian random variable for all $m, n \in \{1, \dots, \ell\}$. Figure 1 depicts this network for the case of the two-state channels ($\ell = 2$). Without loss of generality and for the convenience in notations, we assume that channel coefficients take real positive values and are ordered in the ascending order, i.e.,

$$0 < \alpha_1 < \alpha_2 < \dots < \alpha_\ell \ . \tag{3}$$

We use the notation $C(x) \triangleq \frac{1}{2}\log_2(1 + x)$ throughout the paper.

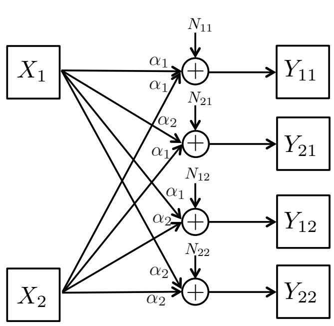

Fig. 1. Equivalent degraded broadcast channel corresponding to a two user four state multiple access channel with channel coefficients α_1 and α_2.

III. Two-state Channels ($\ell = 2$)

We start by analyzing the setting in which the channels take one of the two possible values, i.e., $\ell = 2$. This setting furnishes the context in order to highlight the differences between the proposed layering and successive decoding strategy in this paper and those investigated in [16]. By leveraging the intuition gained from the two-state setting, we generalize the codebook generation and the successive decoding strategies to accommodate a fading process with any arbitrary number of finite channel states in Section IV. Throughout the rest of this section, we refer to channels α_1 and α_2 as the *weak* and *strong* channels, respectively.

A. Background: Adapting Layers to the Single-user Channels

In order to motivate the proposed approach, we start by reviewing the broadcast strategy concept for a single-user channel introduced in [12], and its generalization for the two-user multiple access channel investigated in [16]. When facing a two-state channel, the single-user strategy of [12] splits the information stream of the transmitter into two layers, each corresponding to one fading state, and encodes them independently. The two encoded information layers are subsequently superimposed and transmitted over the channel. One of the streams, denoted by W_1, is always decoded by the receiver, while the second stream, denoted by W_2, is decoded only when the channel is *strong*. The successive decoding order adopted in this approach is presented in Table I.

TABLE I
Successive decoding order of [14]

h	Decoding stage 1	Decoding stage 2
α_1	W_1	
α_2	W_1	W_2

This strategy is adopted and directly applied to the multiple access channel in [16]. Specifically, it generates two coded information layers per transmitter, where the layers of user $i \in \{1, 2\}$ are denoted by $\{W_1^i, W_2^i\}$. Based on the actual realizations of the channels, a combination of these layers are successively decoded by the receiver. In the first stage, the baseline streams W_1^1 and W_1^2, which constitute the minimum amount of guaranteed information, are decoded. Additionally, when the channel between transmitter i and the receiver, i.e., h_i is strong, in the second stage information stream W_2^i is also decoded. Table II depicts the decoding sequence corresponding to each of the four possible channel combinations.

TABLE II
Successive decoding order of [16]

(h_1, h_2)	Decoding stage 1	Decoding stage 2
(α_1, α_1)	W_1^1, W_1^2	
(α_2, α_1)	W_1^1, W_1^2	W_2^1
(α_1, α_2)	W_1^1, W_1^2	W_2^2
(α_2, α_2)	W_1^1, W_1^2	W_2^1, W_2^2

B. Adapting Layers to the MAC

Contribution of user i to a network-wide performance metric (e.g., sum-rate capacity) depends not only on the quality of the channel h_i, but also on the quality of the channel of the other user. This motivates assigning more information layers to user i and adapting them to the *combined* effect of *both* channels, instead of adapting them only to channel h_i. Designing and assigning more than two information layers to each transmitter facilitates a finer resolution in successive decoding, which in turn expands the capacity region characterized in [16].

We assume that each transmitter splits its message into *four* layers corresponding to the four possible combinations of the two channels. These codebooks for transmitter $i \in \{1, 2\}$ are denoted by $\{W_{11}^i, W_{12}^i, W_{21}^i, W_{22}^i\}$, where the information layer W_{uv}^i is associated with the channel realization in which the channel of user i is α_v, and the channel of the other user is α_u. These layer assignments are demonstrated in Fig. 2.

The first initial layers $\{W_{11}^1, W_{11}^2\}$ account for the minimum amount of guaranteed information, which are adapted to the channel combination $(h_1, h_2) = (\alpha_1, \alpha_1)$ and can be decoded by all four possible channel combinations. When at least one of the channels is strong, the remaining codebooks are grouped and adapted to different channel realizations according to the assignments described in Fig. 2. Specifically:

- The second group of the layers $\{W_{12}^1, W_{21}^2\}$ are reserved to be decoded in addition to $\{W_{11}^1, W_{11}^2\}$ when h_1 is strong, while h_2 is still weak.
- Alternatively, when h_1 is weak and h_2 is strong, instead the third group of layers, i.e., $\{W_{21}^1, W_{12}^2\}$, are decoded.
- Finally, when both channels are strong, in addition to all the previous layers, the fourth group $\{W_{22}^1, W_{22}^2\}$ is also decoded.

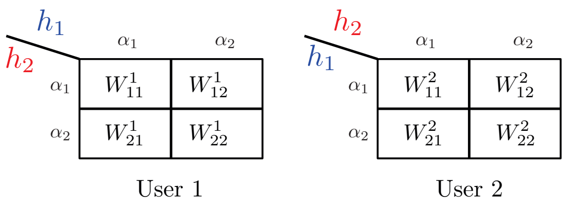

Fig. 2. Layering and codebook assignments by user 1 and user 2.

The orders of successive decoding for different combinations of channel realizations are presented in Table III. Based on this successive decoding order, channel state (α_1, α_1) is degraded with respect to all other states, while (α_1, α_2) and (α_2, α_1) are degraded with respect to (α_2, α_2). Clearly, the codebook assignment and successive decoding approach presented in Table III subsumes the one proposed in [16], as presented in Table II. In particular, Table II can be recovered as a special case of Table III by setting the rate of the layers $\{W_{21}^1, W_{21}^2, W_{22}^1, W_{22}^2\}$ to zero. This implies that the proposed strategy should perform no worse than the one described in Table II. This codebook assignment and decoding order gives rise to the equivalent broadcast network with two inputs $\{X_1, X_2\}$ and four outputs $\{Y_{11}, Y_{12}, Y_{21}, Y_{22}\}$.

TABLE III
SUCCESSIVE DECODING ORDER OF THE LAYERS ADAPTED TO THE MAC

(h_1, h_2)	stage 1	stage 2	stage 3
(α_1, α_1)	W_{11}^1, W_{11}^2		
(α_2, α_1)	W_{11}^1, W_{11}^2	W_{12}^1, W_{21}^2	
(α_1, α_2)	W_{11}^1, W_{11}^2	W_{21}^1, W_{12}^2	
(α_2, α_2)	W_{11}^1, W_{11}^2	$W_{12}^1, W_{12}^2, W_{21}^1, W_{21}^2$	W_{22}^1, W_{22}^2

C. Achievable Rate Region

This subsection delineates a region of all achievable rates R_{uv}^i for $i, u, v \in \{1, 2\}$, where R_{uv}^i accounts for the rate of codebook W_{uv}^i. We define $\beta_{uv} \in [0, 1]$ as the fraction of the power that transmitter i allocates to layer W_{uv}^i for $u \in \{1, 2\}$ and $v \in \{1, 2\}$, where we clearly have $\sum_{u=1}^2 \sum_{v=1}^2 \beta_{uv} = 1$. For the convenience in notations, and in order to place the emphasis on the interplay among the rates of different information layers, we consider the case that relevant layers in different users have identical rates, i.e., rates of information layers W_{uv}^1 and W_{uv}^2, denoted by R_{uv}^1 and R_{uv}^2 respectively, are the same, and denoted by R_{uv}, i.e., $R_{uv} \triangleq R_{uv}^1 = R_{uv}^2$. The results can be readily generalized to arbitrarily different rates for different layers.

Theorem 1: The achievable rate region of the rates $(R_{11}, R_{12}, R_{21}, R_{22})$ for the channel depicted in Fig. 3 is the set of all rates satisfying

$$R_{11} \leq r_1 \triangleq \min\{a_1, \frac{a_2}{2}\} \tag{4}$$

$$R_{12} \leq r_2 \triangleq \min\{a_3, \frac{a_4}{2}\} \tag{5}$$

$$R_{21} \leq r_3 \triangleq \min\{a_5, \frac{a_6}{2}\} \tag{6}$$

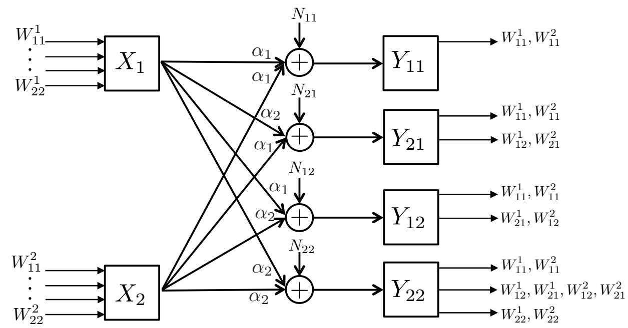

Fig. 3. Equivalent network with two inputs and four outputs.

$$R_{12} + R_{21} \leq r_4 \triangleq \min\{r_2 + r_3, a_7, \frac{a_8}{2}\} \tag{7}$$

$$2R_{12} + R_{21} \leq \min\{2r_2 + r_3, r_2 + r_4, a_9\} \tag{8}$$

$$R_{12} + 2R_{21} \leq \min\{2r_3 + r_2, r_3 + r_4, a_{10}\} \tag{9}$$

$$R_{22} \leq \frac{a_{11}}{2}, \tag{10}$$

where $\{a_i : i \in \{1, \ldots, 11\}\}$ are defined in Appendix A, over all possible power allocation factors $\beta_{uv} \in [0, 1]$ such that $\Sigma_{u=1}^2 \Sigma_{v=1}^2 \beta_{uv} = 1$.

Corollary 1: By setting the power allocated to layers $\{W_{21}^1, W_{21}^2, W_{22}^1, W_{22}^2\}$ to zero, the achievable rate region characterized by (4)-(10) subsumes the capacity region characterized in [16].

In order to compare the achievable rate region in Theorem 1 and the capacity region presented in [16], we group the codebooks in the way that [16] has grouped the codebooks, i.e., the codebooks adapted to the *strong* channels are grouped and their rates are aggregated, and the remaining codebooks are also grouped, and their rates are aggregated. Based on this, the region presented in Theorem 1 can be used to form the sum-rates $(R_{11}^1 + R_{12}^1 + R_{21}^1 + R_{11}^2 + R_{12}^2 + R_{21}^2)$ and $(R_{22}^1 + R_{22}^2)$. Figure 4 compares the achievable rate region based on the aforementioned grouping of the codebooks for the approached proposed in this paper with the capacity region characterized in [16] when $\alpha_1 = 0.5$, $\alpha_2 = 1$ and $\text{SNR} = 10$.

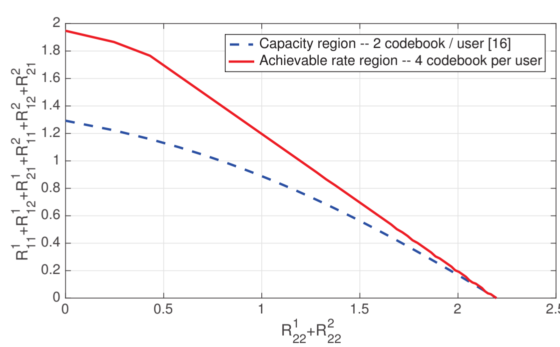

Fig. 4. Capacity region in [16] versus the achievable rate region in Theorem 1

IV. MULTI-STATE CHANNELS ($\ell \geq 2$)

A. Codebook Assignment and Decoding

In this section, we extend the proposed codebook assignment and decoding strategy designed for the two-state channel to the general multiple-state channel with $\ell \in \mathbb{N}$ states. Similar to the two-state channel, we follow the principle of assigning codebooks based on combined network state, according to which a separate layer of information is designated to each combination of the individual channel states, which necessitates ℓ^2 codebooks per user. Hence, for

TABLE IV
SUCCESSIVE DECODING ORDER FOR THE ℓ-STATE MAC.

h_2 \ h_1	α_1	α_2	..	α_q	..	α_ℓ
α_1	W^1_{11} , W^2_{11}	$\mathcal{U}_{11}$ W^1_{12} , W^2_{21}	..	.	..	$\mathcal{U}_{1(\ell-1)}$ $W^1_{1\ell}$, $W^2_{\ell 1}$
α_2	$\mathcal{U}_{11}$ W^1_{21} , W^2_{12}	$\mathcal{U}_{11}$, $\mathcal{U}_{12}$, $\mathcal{U}_{21}$ W^1_{22} , W^2_{22}	..	.	..	$\mathcal{U}_{1(\ell-1)}$, $\mathcal{U}_{2(\ell-1)}$, $\mathcal{U}_{1l}$ W^1_{2l} , W^2_{l2}
.	.	.	..	.	..	.
α_p	.	.	..	$\mathcal{U}_{(p-1)(q-1)}$,$\mathcal{U}_{p(q-1)}$,$\mathcal{U}_{(p-1)q}$, W^1_{pq} , W^2_{qp}	..	.
.	.	.	..	.	..	.
α_ℓ	$\mathcal{U}_{(\ell-1)1}$ $W^1_{\ell 1}$, $W^2_{1\ell}$	$\mathcal{U}_{(\ell-1)1}$,$\mathcal{U}_{\ell 1}$,$\mathcal{U}_{(\ell-1)2}$, $W^1_{\ell 2}$,$W^2_{2\ell}$	..	.	..	$\mathcal{U}_{(\ell-1)(\ell-1)}$, $\mathcal{U}_{\ell(\ell-1)}$, $\mathcal{U}_{(\ell-1)\ell}$ $W^1_{\ell\ell}$, $W^2_{\ell\ell}$

$i, j \in \{1, \ldots, \ell\}$, the codebook assignment strategy for the users is summarized as follows.

Corresponding to the combined channel state $(h_1, h_2) = (\alpha_q, \alpha_p)$ we assign codebook W^1_{pq} to User 1 and codebook W^2_{qp} to User 2. By following the same line of analysis as in the two-state channel, the network state $(h_1, h_2) = (\alpha_1, \alpha_1)$ can be readily verified to be degraded with respect to states (α_1, α_2), (α_2, α_1), and (α_2, α_2) when $\alpha_2 > \alpha_1$. Additionally, channel combinations (α_1, α_2) and (α_2, α_1) are also degraded with respect to state (α_2, α_2). When a particular user's channel becomes stronger while the interfering channel remains constant, the user affords to decode additional codebooks. Similarly, when a user's own channel remains constant while the interfering channel becomes stronger, again the user affords to decode additional information. This can be fascilitated by decoding and removing the message of the interfering user, based on which the user experiences reduced interference. Based on these observations, for the multiple-state channels we order h_1 and h_2 in the ascending order and determine their relative degradedness by considering multiple two-state channels with α_1 and α_2 equal to any two adjacent realizations from the ordered values of h_i.

This strategy is illustrated in Table IV, in which different channel coefficients h_1 and h_2 are listed in the ascending orders. In this table $A_{p,q}$ denotes the cell in the p^{th} row and the q^{th} column, and it specifies the set of codebooks $\mathcal{U}_{pq}$ to be decoded by the combined channel state $(h_1, h_2) = (\alpha_q, \alpha_p)$. In this table, the set of codebooks to be decoded in each possible combined state is recursively related to the codebooks decoded in the weaker channels. Specifically, the state corresponding to $A_{p-1,q-1}$ is degraded with respect to states $A_{p,q-1}$ and $A_{p-1,q}$. Therefore, in the state $A_{p,q}$, the receiver decodes all layers from states $A_{p-1,q-1}$ (included in $\mathcal{U}_{p-1,q-1}$), $A_{p,q-1}$ (included in $\mathcal{U}_{p,q-1}$), and $A_{p-1,q}$ (included in $\mathcal{U}_{p-1,q}$), as well as one additional layer from each user, i.e., W^1_{pq} and W^2_{qp}. When both channel coefficients have the highest possible values, all the layers from both users will be decoded at the receiver.

B. Achievable Rate Region

In this section, we extend the achievable rate region characterized by Theorem 1 for the general multi-state channel. It can be verified that the region characterized by Theorem 1 is subsumed by this general rate region. Similarly to the two-state channel settings, we define R^i_{uv} as the rate of codebook W^i_{uv} for $i \in \{1, 2\}$ and $u, v \in \{1, \ldots, \ell\}$. We also define $\beta_{uv} \in [0, 1]$ as the fraction of the power allocated to the codebook W^i_{uv}, where $\sum_{u=1}^{\ell} \sum_{v=1}^{\ell} \beta_{uv} = 1$. Similarly, to the two-state channel setting, for the convenience in notations and for emphasizing the interplay among the rates, we consider a symmetric case in which $R_{uv} \triangleq R^1_{uv} = R^2_{uv}$.

Theorem 2: A region of simultaneously achievable rates

$$\{R_{uv} : u < v \text{ and } u, v \in \{1, \ldots, \ell\}\}$$

for an ℓ-state two-user multiple access channel is characterized as the set of all rates satisfying:

$$R_{uv} \leq \; r_1 \triangleq \min\left\{b_1(u,v), b_2(u,v), \frac{b_3(u,v)}{2}\right\} \tag{11}$$

$$R_{vu} \leq r_2 \triangleq \min\left\{\{b_4(u,v), \frac{b_5(u,v)}{2}\right\} \tag{12}$$

$$R_{uv} + R_{vu} \leq r_3 \triangleq \min\left\{r_1 + r_2, b_6(u,v), b_7(u,v), \frac{b_8(u,v)}{2}\right\} \tag{13}$$

$$2R_{uv} + R_{vu} \leq \min\{2r_1 + r_2, r_1 + r_3, b_9(u,v)\} \tag{14}$$

$$R_{uv} + 2R_{vu} \leq \min\{2r_2 + r_1, r_2 + r_3, b_{10}(u,v)\} \tag{15}$$

$$R_{uu} \leq \min\left\{b_{11}(u), \frac{b_{12}(u)}{2}\right\} \; . \tag{16}$$

where constants $\{b_i : i \in \{1, \ldots, 12\}\}$ are defined in Appendix B.

Proof: Follows the same footsteps as the proof of Theorem 1. ■

V. CONCLUSIONS

We have proposed a broadcast approach for multiple access communication over a slowly fading channel. While the receiver knows the instantaneous channel states, the states are assumed to be unknown to the transmitters. The existing broadcast approaches applied to multiple access communication, directly adopt the approach designed for the single-user channel in which information layers are adapted to the state of the single-user channel. In this paper, we have proposed an encoding strategy in which the information layers are adapted to the combined states of the channels, and have presented a successive decoding strategy for decoding as many information as possible at the receiver, based on the actual channel states. We have characterized an achievable rate region, and have shown that this region subsumes the existing known capacity regions for the cases that the information layers are adapted to the single-user channels.

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Appendix A
Values of $\{a_i : i \in \{1, \ldots, 11\}\}$

$$a_1 \triangleq C\left(\frac{\alpha_1^2\beta_{11}P}{(\alpha_1^2+\alpha_2^2)(1-\beta_{11})P+1}\right),$$

$$a_2 \triangleq C\left(\frac{2\alpha_1^2\beta_{11}P}{2\alpha_1^2(1-\beta_{11})P+1}\right),$$

$$a_3 \triangleq C\left(\frac{\alpha_2^2\beta_{12}P}{\alpha_2^2(\beta_{21}+\beta_{22})P+\alpha_1^2(\beta_{12}+\beta_{22})P+1}\right),$$

$$a_4 \triangleq C\left(\frac{2\alpha_2^2\beta_{12}P}{2\alpha_2^2\beta_{22}P+1}\right),$$

$$a_5 \triangleq C\left(\frac{\alpha_1^2\beta_{21}P}{\alpha_2^2(\beta_{21}+\beta_{22})P+\alpha_1^2(\beta_{12}+\beta_{22})P+1}\right),$$

$$a_6 \triangleq C\left(\frac{2\alpha_2^2\beta_{21}P}{2\alpha_2^2\beta_{22}P+1}\right),$$

$$a_7 \triangleq C\left(\frac{\alpha_2^2\beta_{12}P+\alpha_1^2\beta_{21}P}{\alpha_2^2(\beta_{21}+\beta_{22})P+\alpha_1^2(\beta_{12}+\beta_{22})P+1}\right),$$

$$a_8 \triangleq C\left(\frac{2\alpha_2^2(\beta_{12}+\beta_{21})P}{2\alpha_2^2\beta_{22}P+1}\right),$$

$$a_9 \triangleq C\left(\frac{\alpha_2^2(2\beta_{12}+\beta_{21})P}{2\alpha_2^2\beta_{22}P+1}\right),$$

$$a_{10} \triangleq C\left(\frac{\alpha_2^2(\beta_{12}+2\beta_{21})P}{2\alpha_2^2\beta_{22}P+1}\right),$$

and

$$a_{11} \triangleq C\left(1+2\alpha_2^2\beta_{22}P\right).$$

Appendix B
Values of $\{b_i : i \in \{1, \ldots, 12\}\}$

By defining the sets

$$J_1(u,v) \triangleq \{j \in \{u,\ldots,v-1\}\},$$
$$J_2(u,v) \triangleq \{(j,k) : k \in \{u,\ldots,v-1\} \ \&\ j \in \{v+1,\ldots,\ell\}\},$$
$$J_3(u,v) \triangleq \{(j,k) : j \le k \ \&\ j,k \in \{v,\ldots,\ell\}\},$$

we have

$$b_1(u,v) \triangleq \min_{j\in J_1}\left\{C\left(\frac{\alpha_v^2\beta_{uv}P}{\alpha_j^2P(1-B_1(j,u,v))+\alpha_v^2P(1-B_2(j,u,v))+1}\right)\right\}, \tag{17}$$

$$b_2(u,v) \triangleq C\left(\frac{\alpha_v^2\beta_{uv}P}{(\alpha_v^2+\alpha_l^2)P(1-B_3(u,v))+1}\right), \tag{18}$$

$$b_3(u,v) \triangleq C\left(\frac{2\alpha_v^2\beta_{uv}P}{2\alpha_v^2P(1-B_3(u,v))+1}\right), \tag{19}$$

$$b_4(u,v) \triangleq C\left(\frac{\alpha_u^2\beta_{vu}P}{\alpha_\ell^2P(1-B_4(u,v))+\alpha_u^2P(1-B_5(u,v))+1}\right), \tag{20}$$

$$b_5(u,v) \triangleq C\left(\frac{2\alpha_v^2\beta_{uv}P}{2\alpha_v^2P(1-B_3(u,v))+1}\right), \tag{21}$$

$$b_6(u,v) \triangleq \min_{(j,k)\in J_2}\left\{C\left(\frac{\alpha_k^2\beta_{vu}P+\alpha_j^2\beta_{uv}P}{\alpha_k^2P(1-B_6(k,u,v))+\alpha_j^2P(1-B_7(k,u,v))+1}\right)\right\}, \tag{22}$$

$$b_7(u,v) \triangleq C\left(\frac{\alpha_v^2(\beta_{uv}+\beta_{vu})P}{(\alpha_v^2+\alpha_l^2)P(1-B_3(u,v))+1}\right), \tag{23}$$

$$b_8(u,v) \triangleq C\left(\frac{2\alpha_v^2(\beta_{uv}+\beta_{vu})P}{2\alpha_v^2P(1-B_3(u,v))+1}\right), \tag{24}$$

$$b_9(u,v) \triangleq \min_{(j,k)\in J_3}\left\{C\left(\frac{\alpha_j^2P(\beta_{uv}+P_{vu})+\alpha_k^2P\beta_{uv}}{(\alpha_j^2+\alpha_k^2)P(1-B_3(u,v))+1}\right)\right\}, \tag{25}$$

$$b_{10}(u,v) \triangleq \min_{j,k\in J_3}\left\{C\left(\frac{\alpha_j^2P(\beta_{uv}+\beta_{vu})+\alpha_k^2P\beta_{vu}}{(\alpha_j^2+\alpha_k^2)P(1-B_3(u,v))+1}\right)\right\}, \tag{26}$$

$$b_{11}(u) \triangleq C\left(\frac{\alpha_u^2\beta_{uu}P}{(\alpha_u^2+\alpha_\ell^2)P(1-\sum_{n=1}^u\sum_{m=1}^u\beta_{mn})+1}\right), \tag{27}$$

$$b_{12}(u) \triangleq C\left(\frac{2\alpha_u^2\beta_{uu}P}{2\alpha_u^2P(1-\sum_{n=1}^u\sum_{m=1}^u\beta_{mn})+1}\right), \tag{28}$$

where were have defined

$$B_1(j,u,v) \triangleq \sum_{n=1}^{j}\sum_{m=1}^{v-1}\beta_{mn}+\sum_{n=1}^{u}\beta_{vn}, \tag{29}$$

$$B_2(j,u,v) \triangleq \sum_{n=1}^{v-1}\sum_{m=1}^{j}\beta_{mn}+\sum_{n=1}^{u}\beta_{nv}, \tag{30}$$

$$B_3(u,v) \triangleq \sum_{n=1}^{v-1}\sum_{m=1}^{v-1}\beta_{mn}+\sum_{n=1}^{u}\beta_{vn}+\sum_{n=1}^{u}\beta_{nv}, \tag{31}$$

$$B_4(u,v) \triangleq \sum_{n=1}^{v-1}\sum_{m=1}^{u}\beta_{mn}+\sum_{n=1}^{u}\beta_{nv}, \tag{32}$$

$$B_5(u,v) \triangleq \sum_{n=1}^{u}\sum_{m=1}^{v-1}\beta_{mn}+\sum_{n=1}^{u}\beta_{vn}, \tag{33}$$

$$B_6(k,u,v) \triangleq \sum_{n=1}^{k}\sum_{m=1}^{v-1}\beta_{mn}+\sum_{n=1}^{u}\beta_{vn}, \tag{34}$$

and

$$B_7(k,u,v) \triangleq \sum_{n=1}^{k}\sum_{m=1}^{v-1}\beta_{nm}+\sum_{n=1}^{u}\beta_{nv}. \tag{35}$$