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A Cross-Layer Optimal Co-Design of Control and Networking in Time-Sensitive Cyber-Physical Systems

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***Abstract*—In the design of cyber-physical systems (CPS) where multiple physical systems are coupled via a communication network, a key aspect is to study how network services are distributed. In this letter, we first describe a cross-layer model for CPS to explicitly capture the coupling between control and networking and the time-sensitive requirements of each physical system. Physical systems processes are coupled via a shared network that provides a diverse range of cost-prone and capacity-limited services with distinct latency characteristics. Service prices are given such that low latency services incur higher communication cost, and prices remain fixed over a constant period of time but will be adjusted by the network for the future time periods. Physical systems decide to use specific services over each time interval depending on the service prices and their own time sensitivity requirements. Considering the service availability, the network coordinates resource allocation such that physical systems are serviced the closest to their preferences. Performance of individual systems are measured by an expected quadratic cost and we formulate a social optimization problem subject to time-sensitive requirements of the physical systems and the network constraints. From the formulated social optimization problem, we derive the joint optimal time-sensitive control and service allocation policies.**

***Index Terms*—Cross-layer optimal design, cyber-physical systems, latency-varying services.**

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I. Introduction

Many applications of CPS such as industrial automation and autonomous vehicles include multiple controlled dynamical systems with the feedback loops closed over a shared network infrastructure [1]. This poses novel challenges for the communication and control system design to support such coupled network of systems with stringent real-time requirements and tight inter-layer dependencies [2]. Recent evolution of 5G communication technology has provided a great potential to revisit the control and networking co-design paradigm in CPS by facilitating an adaptable communication medium that can conveniently adjust its service features depending on the user demands in, e.g., latency, reliability, bandwidth and security [3]. A strictly separate design of control and network layers leads to conservative solutions and results in low quality of control as well as high cost of communication. Hence, to fulfill the tight quality of control requirements and also to exploit the flexibility of the state-of-the-art communication technology, control and networking need to be co-designed in a cross-layer fashion [4], [5].

Providing a systematic and applicable joint design framework, however, is proven to be challenging due to, first, the tight integration of the physical and cyber layers through multiple coupling sources, and second, complexity of optimal solutions that make them non-scalable and intractable to apply on real-time CPS [6], [7]. Despite the noticeable progress including [8]–[10] to develop the co-design architectures, most of the results are obtained either under oversimplification of one of the CPS layers or under the traditional average-type constraints and stationary interfaces, where the former often results in eccentric design frameworks suitable for specific CPS models [11], and the latter leads to only asymptotic averaged performance guarantees [12].

In this letter, we describe a novel cross-layer interactive ecosystem for real-time CPS wherein heterogeneous physical systems are aware of the diverse network services while their time sensitivity requirements are shared with the network for an efficient service allocation. The major novelties are, first, the model of communication network and serviceability, and second, the sampling strategy which can schedule data packets to be delivered to the controller in future time-steps. Motivated by the state-of-the-art communication technology, we assume network services provide multiple latency-varying transmission links, through which systems can close their sensor-to-controller loops subject to a given price. In a future-contract model, each system decides to pay the price for a certain network service for a known future time period. The system may change its service preference for the next future time period depending on the service price and its possibly changed communication requirements. Requests of all systems are processed by the network where some requests might be differently serviced due to service limitations. Service prices are updated for the next time periods to avoid high traffic for certain services and also to incentivize the users to select expensive services only when necessary. Performance of each physical system is measured by quadratic control cost plus the communication service price. This urges the physical systems not to always request the fastest transmission links because of higher communication prices. Service allocation is coordinated by the network such that the average sum of local performance discrepancies, resulting from network service limitations, is minimized across the physical layer over a finite time horizon. Given the described cross-layer interaction model, the joint optimal control and networking policies are derived.

Notations: In this letter, $\mathsf{E}[\,\cdot\,]$, $\mathsf{E}[\,\cdot\,|\,\cdot\,]$ and $\mathrm{tr}(\cdot)$ denote, respectively, the expectation, conditional expectation and trace operators. We denote $[x]_a^b \triangleq \max\{\min\{x, b\}, a\}$. A matrix $A \succ 0$ ($\succeq 0$) is positive definite (positive semi-definite). For time varying variables, vectors, matrices and sets, superscripts denote the corresponding system and subscripts denote the time instance, e.g., X_t^i belongs to system i and its content corresponds to time instance t. We also use $X^i_{[t_1,t_2]} \triangleq \{X^i_{t_1}, \dots, X^i_{t_2}\}$. For time-invariant matrices, we use subscript to show the belonging system. Moreover, for a general vector Y and a weight matrix Q of appropriate dimensions, we define $\|Y\|_Q^2 \triangleq Y^\top QY$.

II. Problem Statement

We consider a class of CPS consisting of N dynamical systems coupled via a common communication network. Each physical system $i \in \{1, \dots, N\}$ consists of a linear time-invariant (LTI) stochastic process $\mathcal{P}_i$, a time-sensitivity controller[1] $\mathcal{S}_i$, and a feedback controller $\mathcal{C}_i$. Let $x_k^i \in \mathbb{R}^{n_i}$, $u_k^i \in \mathbb{R}^{o_i}$ and $w_k^i \in \mathbb{R}^{n_i}$ denote, respectively, the state, control signal and exogenous disturbance for the i^{th} system at time-step k. Dynamics of the plant $\mathcal{P}_i$ is modeled as

$$x_{k+1}^i = A_i x_k^i + B_i u_k^i + w_k^i, \tag{1}$$

where $A_i \in \mathbb{R}^{n_i \times n_i}$, $B_i \in \mathbb{R}^{n_i \times o_i}$, and the process noise w_k^i is zero-mean Gaussian distributed with variance $\Sigma_{w^i} \succ 0$, and w_k^i is assumed to be independent of w_ℓ^j for all $i \neq j$ or $k \neq \ell$. Initial states x_0^i's are assumed to be randomly chosen from arbitrary i.i.d. zero-mean distributions with variance $\Sigma_{x_0^i}$, and are independent of w_k^j, $\forall j$ and k. The control cost of each physical system follows the finite horizon LQG function, i.e.,

$$J^i = \mathsf{E}\left[\|x_{t_f}^i\|_{Q_i^2}^2 + \sum_{k=0}^{t_f-1} \|x_k^i\|_{Q_i^1}^2 + \|u_k^i\|_{R_i}^2\right], \tag{2}$$

where, t_f represents the final time of the time horizon $[0, t_f]$, $Q_i^1 \succeq 0$, $Q_i^2 \succeq 0$ represent constant weights for the state, and $R_i \succ 0$ is the control input weight matrix. Assume that the communication network has multiple capacity-limited service opportunities, each with a distinct latency and price, that can be used by the physical systems. Let the network be comprised of $D+1$ transmission links, together providing a spectrum of network services with different latencies, denoted by $\mathcal{L} = \{s_d | d \in \mathcal{D} \triangleq \{0, \dots, D\}\}$ where d represents the link's corresponding latency. This means, if x_k^i is forwarded through the link s_d to the controller $\mathcal{C}_i$ at a time-step k, then it will be received at $\mathcal{C}_i$ at time-step $k+d$. Let the time horizon $[0, t_f-1]$ be divided into m sub-intervals. We denote the p^{th} sub-interval by T_p, $p \in \{1, \dots, m\}$, and T_p consists of $\eta_p (\eta_p \in \mathbb{N})$ time-steps. Hence, the final time-step becomes $t_f = \sum_{p=1}^m \eta_p$. For the ease of the exposition, we assume that all sub-intervals have equal lengths, i.e., $\eta_p = \eta$, $\forall p$, and thus, the time-interval becomes $T_p = [(p-1)\eta, p\eta - 1]$.

Let us denote the initial and the final time-steps of the sub-interval T_p by $\check{t}_i^p = (p-1)\eta$ and $\check{t}_f^p = p\eta - 1$, respectively. At the beginning of a sub-interval T_p, i.e., at time-step $\check{t}_i^p$, each physical system decides on its preferred service $s_d \in \mathcal{L}$ to be its sensor-to-controller communication link. The service preference remains unchanged for the entire sub-interval T_p (i.e., until $\check{t}_f^p$), and physical systems can select a different communication service only at the beginning of the next sub-interval T_{p+1}, i.e., at the time-step $\check{t}_i^{p+1}$.

During each sub-interval T_p, the service price for each transmission link s_d is denoted by λ_p^d, and is assumed to be fixed over the entire p^{th} sub-interval. They may, however, change from T_p to T_{p+1}. Prices are set such that links with lower latency are more expensive, i.e., $\lambda_p^0 \geq \dots \geq \lambda_p^D$, $\forall p$, and $\Lambda_p \triangleq [\lambda_p^0, \dots, \lambda_p^D]^\top$ represents the service price vector for the sub-interval T_p. In general, using a higher latency service results in an increase in the average control cost (2).

Let $\theta_t^i(d) \in \{0, 1\}$ denote whether system i is selected the transmission service s_d at time-step t, i.e., if $\theta_t^i(d) = 1$, then x_t^i is sent through the link s_d at time-step t to the controller $\mathcal{C}_i$ and will be delivered at time $t+d$. Since the systems may change their service preferences only at time instances $\check{t}_i^p$'s, $p \in \{1, \dots, m\}$, $\theta^i_{\check{t}_i^p}(d) = \theta^i_{\check{t}_i^p+1}(d) = \dots = \theta^i_{\check{t}_f^p}(d)$, $\forall d \in \mathcal{D}$. Hence, the decision outcome of the time-sensitivity controller $\mathcal{S}_i$, generated only at time instances $\check{t}_i^p$, is represented as

$$\theta^i_{\check{t}_i^p}(d) = \begin{cases} 1, & s_d \text{ is selected to transmit } x_t^i,\ \forall t \in T_p, \\ 0, & s_d \text{ is not selected},\ \ \forall t \in T_p. \end{cases} \tag{3}$$

We assume that each $\mathcal{S}_i$ selects one and only one of the transmission services during each sub-interval T_p, i.e.,

$$\sum_{d=0}^{D} \theta^i_{\check{t}_i^p}(d) = 1,\ \forall p = \{1, \dots, m\},\ \forall i \in \{1, \dots, N\}. \tag{4}$$

Since the decision outcome $\theta^i_{\check{t}_i^p}(d)$, $\forall d$, is fixed for the entire sub-interval T_p, with a slight abuse of notation, we define the binary-valued $\theta^i_{T_p}(d)$ as the representative for all $\theta_t^i(d)$, $t \in T_p$, and $\theta^i_{T_p} \triangleq [\theta^i_{T_p}(0), \dots, \theta^i_{T_p}(D)]^\top$. The total service cost for the physical system i over the entire horizon $[0, t_f]$ is $\sum_{p=1}^m \eta\, \theta^{i\top}_{T_p} \Lambda^p$, and the local cumulative cost for that system, that is a function of i^{th} system's local policies, becomes

$$J^i(u^i, \theta^i) = \mathsf{E}\Big[\|x_{t_f}^i\|_{Q_i^2}^2 + \sum_{k=0}^{t_f-1} \|x_k^i\|_{Q_i^1}^2 + \|u_k^i\|_{R_i}^2 + \sum_{p=1}^{m} \eta \theta^{i\top}_{T_p} \Lambda_p\Big]. \tag{5}$$

Since simultaneously minimizing the network and the control cost are conflicting objectives, the optimization problem becomes a trade-off between the two urging decision-makers

[1] Time sensitivity controller indeed determines the time-varying value of a state information n terms of its influence in reducing a cost function.

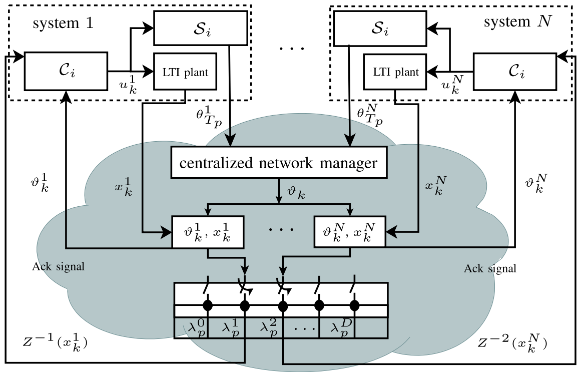

Fig. 1. Multiple LTI control systems over a service-limited network with a variety of latency-varying cost-prone transmission services over time periods of length T_p, $p=1,\ldots,m$. (Z^{-d} denotes the delay operator.)

$(\mathcal{C}_i, \mathcal{S}_i)$ to search for the best combined strategy to minimize the accumulated cost of control and communication.

Network services are assumed to have capacity limitations such that not all systems can simultaneously be serviced through one specific link. To satisfy the service capacity constraints, allocated services to the physical systems may differ from the proffered ones ($\theta^i_{T_p}$). The ultimate allocation of services is decided by a resource allocation unit in the network layer. Let $\vartheta_t^i \triangleq [\vartheta_t^i(0),\ldots,\vartheta_t^i(D)]^\top$ denote the resource allocation outcome for system i at time-step t such that $\vartheta_t^i(d)=1$ ensures that x_t^i will be forwarded to the controller $\mathcal{C}_i$ via the service link s_d and will be received by $\mathcal{C}_i$ at time-step $t+d$. Denoting the average capacity of a certain service s_d by $0<c_d<N$, the capacity constraint is

$$\frac{1}{t_f}\sum_{k=0}^{t_f-1}\sum_{i=1}^{N}\vartheta_k^i(d)\le c_d,\ \forall d\in\mathcal{D}. \tag{6}$$

The main objective of this letter is to study how each physical system optimally selects θ^i and u^i and how the network optimally reacts to the service selection θ^i's to construct appropriate ϑ^i's to satisfy the service constraints.

III. Cross-Layer Optimal Design

A. Cross-Layer Policy Makers

As depicted in Fig. 1, each system in the physical layer is steered by two local policy makers: a feedback controller $\mathcal{C}_i$ and a time-sensitivity controller $\mathcal{S}_i$. We define $\mathcal{I}_k^i$ and $\bar{\mathcal{I}}^i_{\check{t}_i^p}$ as the sets of available information for decision making for $\mathcal{C}_i$ and $\mathcal{S}_i$, respectively. We note that $\mathcal{C}_i$ generates the control input u_k^i at every time-step k, while $\mathcal{S}_i$ generates $\theta^i_{T_p}$ only at time instances $\check{t}_i^p$, $p\in\{1,\ldots,m\}$, hence, as suggested by the subscripts, $\mathcal{I}_k^i$ is updated at every k, while $\bar{\mathcal{I}}^i_{\check{t}_i^p}$ is updated at every $\check{t}_i^p$. Given the information sets $\mathcal{I}_k^i$ and $\bar{\mathcal{I}}^i_{\check{t}_i^p}$, we now introduce the causal policies $\gamma_k^i:\mathcal{I}_k^i\mapsto\mathbb{R}^{o_i}$ and $\xi^i_{\check{t}_i^p}:\bar{\mathcal{I}}^i_{\check{t}_i^p}\mapsto\{0,1\}^{D+1}$ of the system i that generate the control input at time-step k and service preferences for the sub-interval T_p, respectively. That is $u_k^i=\gamma_k^i(\mathcal{I}_k^i)$ and $\theta^i_{T_p}=\xi^i_{\check{t}_i^p}(\bar{\mathcal{I}}^i_{\check{t}_i^p})$.

We assume that a dedicated error-free acknowledgement channel exists to inform the controllers at every time-step k about the binary decision of the resource manager w.r.t. the preferred services of that system ($\theta^i_{T_p}$), i.e., ϑ_k^i are known at $\mathcal{C}_i$ at time-step k (see Fig. 1). Note that each controller uses a collocated estimator to estimate the current system state if it is not communicated. The decision on ϑ_k^i is made at every time-step k, unlike $\theta^i_{T_p}$ that is decided once for the entire sub-interval T_p. Ideally, network desires to service the dynamical systems exactly according to their preferences, i.e., $\forall k\in T_p$, $\vartheta_k^i=\theta^i_{T_p}$. If service limitations do not allow this, the allocated services are not necessarily the ones requested by some of the systems during some of the sub-intervals.

Similarly, we define $\tilde{\mathcal{I}}_k$ as the set of available information for the network to allocate resources at time-step k. We introduce $\pi_k:\tilde{\mathcal{I}}_k\mapsto\{0,1\}^{(D+1)N}$ as the causal policy for computing ϑ_k^i, i.e., $[\vartheta_k^1,\ldots,\vartheta_k^N]=\pi_k(\tilde{\mathcal{I}}_k)$.[2]

B. Information Structures of the Policy Makers

To characterize the information sets $\mathcal{I}_k^i, \bar{\mathcal{I}}^i_{\check{t}_i^p}, \tilde{\mathcal{I}}_k$, we first assume that the local decision makers $\mathcal{S}_i$ and $\mathcal{C}_i$ have the knowledge of their own constant model parameters $\mathcal{I}^i_{\text{cp}}\triangleq\{A_i,B_i,\Sigma_{w^i},Q_i^1,Q_i^2,R_i\}$. The resource allocation unit has access to $\mathcal{I}^i_{\text{cp}}$, $\forall i$. Before introducing the information interaction model, we state the following assumption.

Assumption 1: Resource allocation in the network layer is rendered independent of the local plant control inputs, i.e., none of the u_t^i, $t<k$, is incorporated in determining ϑ_k^i.

Considering the arbitrary time-step k belongs to an arbitrary sub-interval T_p, and noting the order of generating variables in one sampling cycle, $(\theta^i_{T_p}\to\vartheta_k^i\to u_k^i\to x_{k+1}^i)$, the information sets $\mathcal{I}_k^i$, $\bar{\mathcal{I}}^i_{\check{t}_i^p}$ and $\tilde{\mathcal{I}}_k$ of the three decision makers $\mathcal{C}_i$, $\mathcal{S}_i$ and the resource allocation, are as follows:

$$\mathcal{I}_k^i=\mathcal{I}^i_{\text{cp}}\cup\{\mathcal{Z}^i_{[0,k]},\theta^i_{[0,k]},\vartheta^i_{[0,k]},u^i_{[0,k-1]},\Lambda_{[1,p]}\} \tag{7}$$
$$\bar{\mathcal{I}}^i_{\check{t}_i^p}=\mathcal{I}^i_{\text{cp}}\cup\{\theta^i_{[0,\check{t}_f^{p-1}]},\vartheta^i_{[0,\check{t}_f^{p-1}]},u^i_{[0,\check{t}_f^{p-1}]},\Lambda_{[1,p]}\} \tag{8}$$
$$\tilde{\mathcal{I}}_k=\cup_{i=1}^N\{\mathcal{I}^i_{\text{cp}}\cup\{\theta^i_{[0,k]},\vartheta^i_{[0,k-1]}\}\} \tag{9}$$

and, $\mathcal{Z}_t^i=\{\vartheta_t^i(0)x_t^i,\vartheta_{t-1}^i(1)x_{t-1}^i,\ldots,\vartheta_{t-D}^i(D)x_{t-D}^i\}$. We also use $\mathcal{I}^i=\{\mathcal{I}_k^i\}_{k=0}^{t_f-1}$, $\bar{\mathcal{I}}^i=\{\bar{\mathcal{I}}^i_{\check{t}_i^p}\}_{p=1}^m$, and $\tilde{\mathcal{I}}=\{\tilde{\mathcal{I}}_k\}_{k=0}^{t_f-1}$.

Remark 1: According to (7)-(9), $u_k^i=\gamma_k^i(\mathcal{I}_k^i)$ is a function of $\vartheta^i_{[0,k]}$, but π_k does not incorporate $u^i_{[0,k]}$, $\forall i$, in computing $\vartheta_k^i=\pi_k(\tilde{\mathcal{I}}_k)$. The ultimate allocated resources to system i at a time $k\in T_p$, however, depend on $\theta^i_{[0,k]}$. Since π_k is a function of $\theta^i_{[0,k]}$ for $k\in T_p$ ($\tilde{\mathcal{I}}_k$ includes $\theta^i_{[0,k]}$, $\forall i$), control performance is indirectly considered in resource allocation as $\theta^i_{[0,k]}$ are chosen by the physical systems in order to minimize the cumulative cost (5). Moreover, it leads to a considerable complexity reduction in computing the optimal policies π_k^* and $\gamma_k^{i,*}$ (Section III-C), since the network does not need to have access to the entire control input history of all control systems, i.e., $u^i_{[0,k-1]}$, $i\in\{1,\ldots,N\}$.

C. Cross-Layer Joint Optimization Problem

Given the information sets (7) and (8), the cumulative cost function (5), for a system $i\in\{1,\ldots,N\}$, is expressed as

$$J^i(u^i,\theta^i|\mathcal{I}^i,\bar{\mathcal{I}}^i)=\mathsf{E}\Bigg[\|x^i_{t_f}\|^2_{Q_i^2}+\sum_{k=0}^{t_f-1}\|x_k^i\|^2_{Q_i^1}+\|u_k^i\|^2_{R_i}$$

[2]With slight abuse of notation, to point the resource allocation outcome for a specific system i, we will sometimes write $\vartheta_k^i=\pi_k(\tilde{\mathcal{I}}_k)$.

$$+\sum_{p=1}^{m}\eta\theta_{T_p}^{i\top}\Lambda_p\Big|\mathcal{I}_k^i,\bar{\mathcal{I}}_{\check{t}_i^p}^i\Big]. \tag{10}$$

Note that, (10) represents the local cumulative cost function without considering the resource constraint (6), thus, no resource allocation decision ϑ^i is present. The overall objective is to optimize the average performance of all systems under the constraint (6). If some of the service requests are handled differently in the network due to the constraint (6), i.e., when ϑ_k^i is applied, the corresponding control input will be changed and the cumulative control cost J^i then becomes

$$J^i(u^i,\vartheta^i|\mathcal{I}^i,\tilde{\mathcal{I}}) = \mathsf{E}\Bigg[\|x_{t_f}^i\|_{Q_i^2}^2+\sum_{k=0}^{t_f-1}\|x_k^i\|_{Q_i^1}^2+\|u_k^i\|_{R_i}^2 + \sum_{p=1}^{m}\sum_{k\in T_p}\vartheta_k^{i\top}\Lambda_p\Big|\mathcal{I}_k^i,\tilde{\mathcal{I}}_k\Bigg]. \tag{11}$$

We formulate a social cost J as the average difference between the sum of J^i's from the perspectives of the network (after resource allocation) and the physical systems, i.e.,

$$J = \frac{1}{N}\sum_{i=1}^{N}\mathsf{E}\Big[J^i(u^i,\vartheta^i|\mathcal{I}^i,\tilde{\mathcal{I}}) - \min_{u^i,\theta^i}J^i(u^i,\theta^i|\mathcal{I}^i,\bar{\mathcal{I}}^i)\Big]. \tag{12}$$

The aim is to derive the optimal policies $\gamma_k^{i,*}(\mathcal{I}_k^i)$, $\xi_{\check{t}_i^p}^{i,*}(\bar{\mathcal{I}}_{\check{t}_i^p}^i)$ and $\pi_k^*(\tilde{\mathcal{I}}_k)$ that jointly minimize J over the horizon $[0, t_f-1]$

$$\min_{\gamma^i,\xi^i,\pi} \quad J \tag{13a}$$

$$\text{s.t.}\quad u_k^i=\gamma_k^i(\mathcal{I}_k^i),\ \theta_{T_p}^i=\xi_{\check{t}_i^p}^{i,*}(\bar{\mathcal{I}}_{\check{t}_i^p}^i),\ \vartheta_k=\pi_k(\tilde{\mathcal{I}}_k) \tag{13b}$$

$$\sum_{k\in T_p}\vartheta_k^{i\top}\Lambda_p \le \eta\,\theta_{T_p}^{i\top}\Lambda_p,\ \forall i,p\in\{1,\dots,m\} \tag{13c}$$

$$\frac{1}{t_f}\sum_{k=0}^{t_f-1}\sum_{i=1}^{N}\vartheta_k^i(d)\le c_d,\ \forall d\in\mathcal{D}. \tag{13d}$$

The constraint (13b) ensures γ^i,ξ^i and π are admissible policies and measurable functions of the σ-algebras generated by their corresponding information sets, (13c) guarantees that re-allocated services impose no higher cost on the systems over the intervals T_p, and (13d) is the capacity constraint (6).

We propose a heuristic adaptive law to update the service prices for each sub-interval T_p to incentivize the systems to more evenly distribute their service requests, as follows:

$$\lambda_{p+1}^d = \left[\lambda_p^d + \alpha_d\left(\sum_{i=1}^{N}\theta_{T_p}^i(d)-c_d\right)\right]_{\lambda_{\min}^d}^{\lambda_{\max}^d}, \tag{14}$$

where, $\alpha_d\in\mathbb{R}_{\ge 0}$ is a network parameter to properly adjust the prices. The update law (14) ensures that $\lambda_p^d\in[\lambda_{\min}^d,\lambda_{\max}^d]$, where, $\lambda_{\min}^d$ and $\lambda_{\max}^d$ are known to all systems *a priori*.[3] The adaptive law (14) does not lead to an average degradation of (12) since, first, service prices are part of the local costs, and second, the prices for less-used services are decreased.

Theorem 1, for which we omit the proof due to space limitation, shows the structure of the optimal control law.

[3]Search for the α_d's to find the optimal pricing mechanism is an interesting yet challenging problem, and beyond the scope of this letter.

Theorem 1: Given the information sets $\mathcal{I}_k^i$, $\bar{\mathcal{I}}_{\check{t}_i^p}^i$ and $\tilde{\mathcal{I}}_k$ in (7)-(9) and the problem (13a)-(13d), the optimal plant control law $\gamma_k^{i,*}$, $\forall i$, is of certainty equivalence form and control inputs are obtained from linear state feedback law as

$$u_k^{i,*}=\gamma_k^{i,*}(\mathcal{I}_k^i)=-L_k^{i,*}\,\mathsf{E}[x_k^i|\mathcal{I}_k^i],\quad i\in\{1,\dots,N\} \tag{15}$$

$$L_k^{i,*}=\left(R_i+B_i^\top P_{k+1}^iB_i\right)^{-1}B_i^\top P_{k+1}^iA_i, \tag{16}$$

where, $P_T^i=Q_i^2$, and P_k^i solves the Riccati equation

$$P_k^i=Q_i^1+A_i^\top\left[P_{k+1}^i-P_{k+1}^iB_i\left(R_i+B_i^\top P_{k+1}^iB_i\right)^{-1}B_i^\top P_{k+1}^i\right]A_i.$$

Theorem 2: Consider the problem (13a)-(13d) and let $\gamma^{i,*}, i \in \{1,\dots,N\}$ follow the certainty equivalence law (15)-(16). Given $\bar{\mathcal{I}}_{\check{t}_i^p}^i$ and $\tilde{\mathcal{I}}_k$ in (8) and (9), the optimal time sensitivity control law is computed from the following constrained mixed-integer linear-programming (MILP)

$$\begin{aligned}\theta_{[k,t_f-1]}^{i,*} &= \arg\min_{\xi^i_{[\check{t}_i^p,\check{t}_i^m]}} J^i(\gamma^{i,*},\xi^i_{[\check{t}_i^p,\check{t}_i^m]}(\bar{\mathcal{I}}^i_{[\check{t}_i^p,\check{t}_i^m]}))\\ &= \arg\min_{\xi^i_{[\check{t}_i^p,\check{t}_i^m]}}\sum_{t=k}^{t_f-1}\left[\sum_{l=1}^{\tau_t^i}\sum_{j=l}^{\tau_t^i}\bar{b}_{j,t}^i\mathsf{Tr}(\tilde{P}_t^iA_i^{l-1^\top}\Sigma_{w^i}A_i^{l-1})+\theta_t^{i\top}\Lambda_{\mu(k)}\right]\\ \text{s.t.}\quad &\forall i,t\in T_p,\theta_{\check{t}_i^p}^i=\dots=\theta_t^i=\dots=\theta_{\check{t}_f^p}^i=\theta_{T_p}^i=\xi_{\check{t}_i^p}^i(\bar{\mathcal{I}}_{\check{t}_i^p}^i)\\ &\bar{b}_{0,t}^i=\theta_t^i(0),\quad \bar{b}_{j,t}^i\le\sum_{l=0}^{j}\theta_{t-j}^i(l),\ j\in\{1,\dots,\tau_t^i\},\\ &\sum_{l=0}^{D}\theta_t^i(l)=1,\ \sum_{j=0}^{\tau_t^i}\bar{b}_{j,t}^i=1,\ \sum_{j=t+2}^{D}\bar{b}_{j,t}^i=0,\ t\ge k,\\ &\theta_s^i=\vartheta_s^i,\quad\forall s<k.\end{aligned} \tag{17}$$

where, $\mu(k)=p$ for $k\in T_p$, $\tau_t^i\triangleq\min\{D,t+1\}$, and $\tilde{P}_t^i=Q_i^1+A_i^\top P_{t+1}^iA_i-P_t^i$, and $\bar{b}_{j,t}^i=[[1-\theta_t^i(0)]\prod_{d=1}^{j-1}\prod_{l=0}^{d}[1-\theta_{t-d}^i(l)]][\sum_{d=0}^{j}\theta_{t-j}^i(d)]$. For notational correctness, we use the convention $\prod_{d=d_1}^{d_2}a_d\triangleq 1$ and $\sum_{d=d_1}^{d_2}a_d\triangleq 0$, $\forall d_1>d_2$. Subsequently, the optimal resource allocation law is computed from the following constrained MILP

$$\begin{aligned}\vartheta_{[k,t_f-1]}^{*} = \arg\min_{\pi_{[k,t_f-1]}}\sum_{i=1}^{N}\sum_{t=k}^{t_f-1}\Bigg[&\vartheta_t^{i\top}\Lambda_{\mu(k)}\\ &+\sum_{l=1}^{\tau_t^i}\sum_{j=l}^{\tau_t^i}\tilde{b}_{j,t}^i\mathsf{Tr}(\tilde{P}_t^iA_i^{l-1^\top}\Sigma_{w^i}A_i^{l-1})\Bigg]\\ \text{s.t.}\quad &\frac{1}{t_f}\sum_{t=0}^{t_f-1}\sum_{i=1}^{N}\vartheta_t^i(d)\le c_d,\ \forall d\in\mathcal{D},\\ &\sum_{t\in T_p}\vartheta_t^{i\top}\Lambda_p\le\eta\theta_{T_p}^{i\top}\Lambda_p,\ \forall i,p\in\{1,\dots,m\}\end{aligned} \tag{18}$$

where, $\tilde{b}_{j,t}^i$ is similarly defined as $\bar{b}_{j,t}^i$ with the exception that θ_t^i is replaced by ϑ_t^i for all i and t (see expression (21)).

Proof: Using the optimal control law (15)-(16), the cost-to-go $V_k^i=\|x_{t_f}^i\|_{Q_i^2}^2+\sum_{t=k}^{t_f-1}\|x_t^i\|_{Q_i^1}^2+\|u_t^i\|_{R_i}^2$ is optimally computed as (see [13, Th. 1 and Proposition 1]):

$$V_k^{i,*} = \| \mathsf{E}\big[x_k^i|\mathcal{I}_k^i\big]\|^2_{P_k^i} + \mathsf{E}\left[\|e_k^i\|^2_{P_k^i} + \sum_{t=k}^{t_f-1}\|e_t^i\|^2_{\tilde{P}_t^i}\Big|\mathcal{I}_k^i\right] + \sum_{t=k+1}^{t_f}\operatorname{tr}(P_t^i\Sigma_w^i), \tag{19}$$

where, $e_k^i \triangleq x_k^i - \mathsf{E}[x_k^i|\mathcal{I}_k^i]$, and $\tilde{P}_t^i = Q_i^1 + A_i^\top P_{t+1}^i A_i - P_t^i$. Moreover, the state estimate, at time-step k, is given as

$$\mathsf{E}\big[x_k^i|\mathcal{I}_k^i\big] = \sum_{j=0}^{\min\{D,k+1\}} \tilde{b}_{j,k}^i\,\mathsf{E}\Big[x_k^i|x_{k-j}^i, u_0^i,\dots,u_{k-1}^i\Big], \tag{20}$$

and, for all $j \in \mathcal{D}$, and $k \geq j$, we have

$$\tilde{b}_{j,k}^i = \prod_{d=0}^{j-1}\prod_{l=0}^{d}[1-\vartheta_{k-d}^i(l)][\sum_{d=0}^{j}\vartheta_{k-j}^i(d)]. \tag{21}$$

For, $k < j$, $b_{0,k}^i,\dots,b_{k,k}^i$'s are defined as in (21), $b_{k+1,k}^i = \prod_{d=0}^{k}\prod_{l=0}^{d}[1-\vartheta_{k-d}^i(l)]$, and $b_{k+2,k}^i = \dots = b_{D,k}^i = 0$.

Having (19), with $k \in T_p$, the optimal time sensitivity control law $\xi^{i,*}_{[\tilde{t}_i^p,\tilde{t}_i^m]}$ is obtained by minimizing the cumulative cost $J^i(u^{i,*},\theta^i|\mathcal{I}^i,\bar{\mathcal{I}}^i)$, i.e., $\forall k \in [0, t_f-1]$ and $k \in T_p$

$$\theta^{i,*}_{[k,t_f-1]} = \arg\min_{\xi^i_{[\tilde{t}_i^p,\tilde{t}_i^m]}} \mathsf{E}\left[V_k^{i,*}(\gamma^{i,*},\xi^i) + \sum_{t=k}^{t_f-1}\theta_t^{i^\top}\Lambda_{\mu(k)}\big|\bar{\mathcal{I}}^i_{\tilde{t}_i^p}\right].$$

Since $\bar{\mathcal{I}}^i_{\tilde{t}_i^p} \subseteq \mathcal{I}_k^i$, $\forall k \in T_p$, and employing (20), one can compute $\mathsf{E}[\,\mathsf{E}[e_k^i e_k^{i^\top}|\mathcal{I}_k^i]|\bar{\mathcal{I}}^i_{\tilde{t}_i^p}] = \mathsf{E}[e_k^i e_k^{i^\top}|\bar{\mathcal{I}}^i_{\tilde{t}_i^p}]$, at $\mathcal{S}_i$ side, to be:

$$\begin{aligned}\mathsf{E}[e_k^i e_k^{i^\top}|\bar{\mathcal{I}}^i_{\tilde{t}_i^p}] &= \sum_{l=1}^{\tau_k^i}\sum_{j=l}^{\tau_k^i}\bar{b}_{j,k}^i\,\mathsf{E}[A_i^{l-1}w_{k-l}^i w_{k-l}^{i^\top}A_i^{l-1^\top}]\\ &= \sum_{l=1}^{\tau_k^i}\sum_{j=l}^{\tau_k^i}\bar{b}_{j,k}^i A_i^{l-1}\Sigma_{k-l}^i A_i^{l-1^\top},\end{aligned}$$

where, $\Sigma_{k-l}^i = \Sigma_{x_0^i}$, $k < l$, and $\Sigma_{k-l}^i = \Sigma_{w^i}$, $k \geq l$. Having this with $\bar{\mathcal{I}}^i_{\tilde{t}_i^0} = \mathcal{I}^i_{\text{cp}}$, we rewrite $\mathsf{E}[V_0^{i,*}(\gamma^{i,*},\xi^i)|\bar{\mathcal{I}}^i_{\tilde{t}_i^0}]$ as follows

$$\begin{aligned}\mathsf{E}[V_0^{i,*}(\gamma^{i,*},\xi^i)|\bar{\mathcal{I}}^i_{\tilde{t}_i^0}] &= \|\mathsf{E}\big[x_0^i\big]\|^2_{P_k^i} + \sum_{t=k+1}^{t_f}\operatorname{tr}(P_t^i\Sigma_{w^i})\\ &+ \operatorname{tr}(P_0^i\sum_{l=1}^{\tau_0^i}\sum_{j=l}^{\tau_0^i}\bar{b}_{j,0}^i A_i^{l-1^\top}\Sigma_{x_0^i}A_i^{l-1})\\ &+ \sum_{t=0}^{t_f-1}\operatorname{tr}(\tilde{P}_t^i\sum_{l=1}^{\tau_t^i}\sum_{j=l}^{\tau_t^i}\bar{b}_{j,t}^i A_i^{l-1^\top}\Sigma_{t-l}^i A_i^{l-1}).\end{aligned}$$

As the only term in the last expression that is dependent on $\theta^i_{[k,t_f-1]}$ is the last term, we have for all $k \in T_p$

$$\begin{aligned}\theta^{i,*}_{[k,t_f-1]} &= \arg\min_{\xi^i_{[\tilde{t}_i^p,\tilde{t}_i^m]}} \mathsf{E}\left[V_k^{i,*}(\gamma^{i,*},\xi^i) + \sum_{t=k}^{t_f-1}\theta_t^{i^\top}\Lambda_{\mu(k)}\big|\bar{\mathcal{I}}^i_{\tilde{t}_i^p}\right]\\ &= \arg\min_{\xi^i_{[\tilde{t}_i^p,\tilde{t}_i^m]}}\sum_{t=k}^{t_f-1}\left[\operatorname{tr}(\tilde{P}_t^i\sum_{l=1}^{\tau_t^i}\sum_{j=l}^{\tau_t^i}\bar{b}_{j,t}^i A_i^{l-1^\top}\Sigma_{t-l}^i A_i^{l-1}) + \theta_t^{i^\top}\Lambda_{\mu(k)}\right]\end{aligned}$$

Note that, Λ_p is known for $\mathcal{S}_i$ assuming $k \in T_p$ (k is the current time). The optimization problem is, however, solved from k to the final time t_f over which the prices may change from T_p to T_{p+1} while future price changes are not disclosed for $\mathcal{S}_i$'s at time $k \in T_p$. Hence, the system solves the local optimization problem considering the current prices, i.e., Λ_p, for the whole horizon $[k, t_f]$. At the beginning of the next sub-interval T_{p+1} when $\mathcal{S}_i$ updates $\theta^i_{T_{p+1}}$, the adjusted price Λ_{p+1}, is considered until t_f. The constraints of the problem (17) are all linear and θ_k^i is a binary variable, hence the problem is an MILP that is solved m times over the horizon $[0, t_f]$, once per each sub-interval T_p, $p = \{1,\dots,m\}$. The constraint $\sum_{l=0}^{D}\theta_t^i(l) = 1$ ensures that only one transmission link is selected per-time, while the last two constraints are essential for correct indexes in the parameter $\bar{b}_{j,k}^i$ for $k \geq D$ and $k < D$.

To find π^*, we take similar steps to compute $\vartheta_k^{i,*}$ given the information set $\tilde{\mathcal{I}}_k$. We compute $\mathsf{E}[V_k^{i,*}(\gamma^{i,*},\pi)|\tilde{\mathcal{I}}_k]$ that results in a similar expression with the exception being $\bar{b}_{j,t}^i$ is replaced by $\tilde{b}_{j,t}^i$ in (21). Hence, considering the price and resource constraints (13c)-(13d), we derive the optimal resource allocation from the following MILP, with $k \in T_p$

$$\begin{aligned}\vartheta^*_{[k,t_f-1]} &= \arg\min_{\pi_{[k,t_f-1]}}\sum_{i=1}^{N}\mathsf{E}\left[V_k^{i,*}(\gamma^{i,*},\pi^i) + \sum_{t=k}^{t_f-1}\vartheta_t^{i^\top}\Lambda_{\mu(k)}\big|\tilde{\mathcal{I}}_k\right]\\ &= \arg\min_{\pi_{[k,t_f-1]}}\sum_{i=1}^{N}\sum_{t=k}^{t_f-1}\left[\vartheta_t^{i^\top}\Lambda_{\mu(k)} + \sum_{l=0}^{\tau_t^i}\sum_{j=l}^{\tau_t^i}\tilde{b}_{j,t}^i\operatorname{Tr}(\tilde{P}_t^i A_i^{l-1^\top}\Sigma_{w^i}A_i^{l-1})\right].\end{aligned}$$

■

Theorems 1 and 2 show that under the assumption that π_k is independent of $\gamma^i_{[0,k-1]}$'s, we can decompose the problem (13a)-(13d) and solve it for the plant control policy separately, while the resource allocation and time-sensitivity control remain coupled through the adaptive service prices and capacity constraints. Note that, the complexity of MILPs (17) and (18) to compute the mentioned policies are of orders $\mathcal{O}(NDm^2)$ and $\mathcal{O}(NDt_f^2)$, respectively, which suggests computationally feasibility for medium size CPS over finite horizons.

IV. Numerical Results

We consider a set of 20 homogeneous LTI systems with $A_i = \begin{bmatrix}1.01 & 0.2\\ 0.2 & 1\end{bmatrix}$, $B_i = \begin{bmatrix}0.1 & 0\\ 0 & 0.15\end{bmatrix}$, $w_k^i \sim \mathcal{N}(\mathbf{0}, 1.5\mathbb{I}_{2\times2})$, and $Q_i^1 = Q_i^2 = R_i = \mathbb{I}_{2\times2}$, $\forall i$ and $\forall k$. We consider 6 network services with latencies $\mathcal{D} = \{0,\dots,5\}$, where for $\{s_0,\dots,s_4\}$ we assume $c_d = 4$ and $c_5 = 5$. The maximum and minimum prices for $\{s_0,\dots,s_5\}$ are $\Lambda_{\max} = [31, 19, 12, 9, 5.5, 2.5]$ and $\Lambda_{\min} = [19, 12, 9, 5.5, 2.5, 0.5]$. Each sub-interval T_p consists of 10 time-steps, and $t_f = 50$, i.e., $m = 5$. The initial service costs Λ_1 for the interval $T_1 = [0, 9]$, is $[25, 13, 11, 7, 4, 1]$, and prices are updated according to (14) with $\alpha_d = 1$, $\forall d \in \mathcal{D}$. We compare service request and allocation for the varying service costs, i.e., $\alpha_d = 1$, and constant service costs, i.e., $\Lambda_p = \Lambda_1$, $\forall p$. To capture the service usage, we define a network utilization quotient $\rho_t(d)$, $\forall t \in [0, t_f]$ and $d \in \mathcal{D}$, as follows

$$\rho_t(d) = \frac{1}{N(t+1)}\left[\sum_{k=0}^{t}\sum_{i=1}^{N}\vartheta_k^i(d)\right]. \tag{22}$$

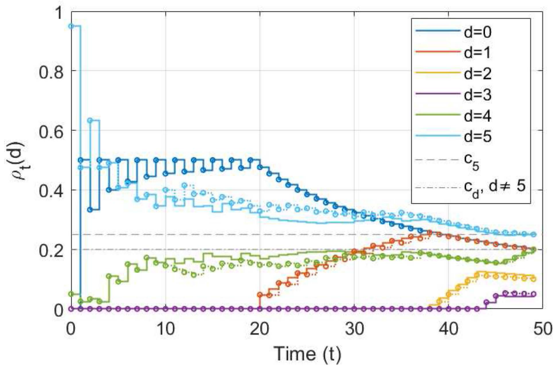

Fig. 2. Usage of different services. The solid lines (—) correspond to the time varying service costs and the dotted lines with circles (· · ◦ · ·) correspond to constant costs.

Thus, $\rho_t(d)$ shows the usage percentage of the service s_d upto time t, and from the constraint (13d), $\rho_{t_f-1}(d) \leq c_d/N$.

In Fig. 2 we plot $\rho_t(d)$ for time varying and constant service costs. In both cases, the usage for all services are the same for the first interval $[0, 9]$, as expected. Based on (14), prices for the services s_0, s_4, s_5 increase whereas the prices for the rest decrease. These cost changes incentivize the systems to choose different services (θ_t^i), and consequently, the allocation of the links (ϑ_t^i) also changes because of (13c).

In particular, during the interval $T_2 = [10, 19]$, we observe a different usage in services s_4 and s_5 between the two scenarios. The increments in the service costs, however, do not necessarily change the utilization, for example, the increased cost of s_0 did not change its usage. An interesting observation lies in the usage of services s_2 and s_3 for the final interval $T_5 = [40, 49]$. Since s_3 is not used over $T_3 = [30, 39]$, its cost is reduced for $T_4 = [40, 49]$, however we still observe a decrease in its usage, and this is because s_2 is still more efficient for many systems than s_3.

From this experiment, we notice that by adaptively changing the service costs, the utilization can be regulated, and the adaptive rule and its parameters play a significant role in regulating the usage. This is particularly a very interesting line of future research that how to *optimally* adapt the prices.

If the systems are served exactly as they request, each of them will incur a control cost of 61.1741 and a service cost of 1300. However, due to the capacity constraints, the systems do not obtain the desired service and the total control cost for the group becomes 22566.56 compared to $61.1741 \times 20 = 1223.48$ – almost a twenty-fold increase. The network would earn a total of $1300 \times 20 = 26000$ if it could serve the exact requests. However, due to the capacity constraints, the network receives a total of 9916. The total cost due to the capacity limitation becomes $22566.56 + 9916 = 32482.56$, compared to the cost of $1223.48 + 26000 = 27223.48$ with no capacity limitation.

We also studied the average deviation of the requested services from the assigned services. Let $\vartheta^{i,*}$ denote the actual service assignment to the i-th system, and $\theta^{i,*}$ denote its desired request, then the average deviation is calculated as

$$\Delta_t = \frac{\sum_{k=0}^{t} \sum_{i=1}^{N} \left| \sum_{d=0}^{D} d(\vartheta_k^{i,*}(d) - \theta_k^{i,*}(d)) \right|}{N(t+1)}, \qquad (23)$$

where in (23), $|\cdot|$ represents the absolute value. The results are plotted in Fig. 3, where we notice that Δ_t is slightly higher

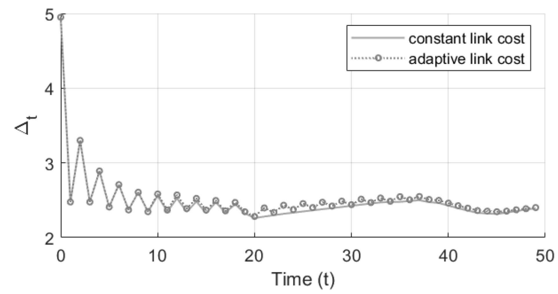

Fig. 3. Average link assignment variation.

with time varying costs as the updated costs persuade the systems to deviate further to adopt a new service.

V. Conclusion

We propose a cross-layer model of CPS wherein multiple LTI stochastic systems are coupled via a shared network that provides a range of costly and capacity-limited services with distinct latencies. Service recipients (physical systems) select certain network services for a time period for a given price. Requests are processed by the network and services are allocated taking into account the users' demands and network limitations. Service prices are adjusted for future periods with the aim of receiving more evenly distributed service requests. We formulate a social cost minimized by cross-layer decision makers, and we derive the resulting optimal policies taking into account their limitations, tolerances and constraints.

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