A PMU-based Data-Driven Approach for Estimating the Injection Shift Factors

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***Abstract*—One effective way to estimate the impact of contingencies is to utilize linear distribution sensitivity factors, such as injection shift factor and line outage distribution factor. Compared to other impact estimation approaches, estimating the line flows with sensitivity factors is computationally less demanding, as a linearized DC power flow model is utilized. However, the accuracy of the power flow model is highly dependent on the received system information. Hence, wrong or missing system information can yield inaccurate results. Phasor Measurement Units provide measurements that can be used to estimate sensitivity factors such that the impact of wrong system model information can be minimized. This paper introduces a new methodology, based on the alternating direction method of multipliers, to leverage PMU data for estimating sensitivity factors. The developed methodology is particularly applicable to near real-time conditions, where the speed of estimation is of essence. The performance of the developed method is compared with the traditional estimation methods for multiple testbeds.**

***Index Terms*—Injection shift factor, optimization, phasor measurement unit, power system reliability**

NOMENCLATURE

ϵ	PMU measurement error
$\mathbf{A}$	Edge-to-node incidence matrix
∂P_i^{line}	Active power flow change in line *i*
∂P_j^{bus}	Active power injection change at bus *j*
ρ	Small load fluctuation
H, H_i	ISF matrix and the *i*th row of the ISF matrix
M	Number of PMU time duration data
$P_i^{line}(t)$	Active power flow in line *i* at time *t*
$P_j^{bus}(t)$	Active power injection at bus *j* at time *t*

I. INTRODUCTION

Maintaining reliable power system operation is more challenging with the ever-increasing grid uncertainties and the threat of cascading failures. These uncertainties and threats are partially introduced by the increasing penetration of renewable energy resources, more complex load patterns, and more severe weather patterns. To guarantee bulk power system reliability, single or multiple component failures that are expected to occur more often than before with more grid uncertainties should be prevented. It is thus critical to perform contingency analysis in real-time to estimate the risk of potential contingencies, and determine appropriate preventive actions to keep power systems operating within secure limits [1]. A Real-time Contingency Analysis (RTCA) utilizes the estimated system states from the state estimator to identify potential contingencies and evaluate their risks [2], [3]. Contingency analysis in real-time involves two components: contingency selection and contingency evaluation [3]. The contingency selection process enables identifying potential credible *N-k* contingencies [4]–[8], that are in turn screened to determine risky contingencies. Conventional contingency evaluation methods use sensitivity factors such as Injection Shift Factor (ISF) and Line Outage Distribution Factor (LODF), which are calculated based on offline studies, to evaluate the impacts (e.g. potential overloads) of a selected set of contingencies [2]. The sensitivity factors, however, could yield incorrect contingency analysis results upon receiving the wrong system model information [9]. The wrong information can occur due to incorrect measurements, missing data, or a fake topology change due to a cyber-attack on the state estimator topology processor. Therefore, it is necessary to estimate the sensitivity factors without the system information.

Synchronized Phasor Measurement Unit (PMU) measurements provide an alternative to using system model information to estimate the sensitivity factors. Hence, the credibility issue of traditional model-based methods for calculating sensitivity factors is addressed by using PMUs. A measurement-based method for estimating ISF using PMUs was first developed in [10]. This method was applied to security constrained economic dispatch (SCED) in [11] and was further improved by a sparse representation algorithm that increases the number of zeros in a modified ISF matrix in [12]. The approach in [12] has a distinct advantage of requiring fewer PMU measurements to estimate ISF. However, this sparse method assumes that at most two lines are out of service, which limits its application, particularly to cascading event analysis. To overcome the limitation of the sparse representation method, in this paper, a nonconvex optimization problem for estimating ISF is formulated based on the past PMU measurements that reflect the most recent system operating conditions. The optimization problem is solved by decomposing the formulated nonconvex problem into subproblems that can be solved iteratively. Therefore, without the system model information, the ISF can be estimated with the obtained past PMU measurements and can be leveraged for other purposes, such

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as identifying critical contingencies and determining proper mitigative actions.

The contributions of this work can be summarized as developing a new estimation approach to estimate linear sensitivity distribution factors, only utilizing the past PMU measurement. The developed method achieves a higher estimation accuracy than the conventional ISF estimation method. This paper is organized as follows: An overview of the conventional and measurement-based ISF estimation methods is given in Section II. The developed new ISF estimation approach is described in Section III. In Section IV, the effectiveness of the introduced approach is assessed on the IEEE 57-bus and Illinois 200-bus systems. Conclusions are presented in Section V.

II. Preliminaries

To analyze the impact of system contingencies and component failures, and to perform SCED, line power flows need to be estimated as the system conditions change. Conventionally, sensitivity factors, i.e., ISF and LODF, are used to estimate the changes in line power flows. ISF determines the redistribution of the power flows for each transmission line upon system generation or load changes [10]. Conventionally, ISF is calculated based on the latest available system model, which is obtained from the state estimation solution. However, using a near-real-time system model brings about additional risks. One of the notable risks is that undetected system topology changes and inaccurate model parameters may lead to the wrong estimation of line power flows [13]. To mitigate the risks of a wrong model, the authors in [10] suggest using past PMU measurements as an alternative to the system model. Hence, leveraging the compressive sensing (CS) approach, a significant improvement in the efficiency of the ISF estimation is achieved in [12]. In this paper, the methods that utilize system model and PMU measurements are referred to as model-based and measurement-based methods, respectively.

A. Model-Based Method for Estimation of ISF

To calculate ISF based on the system model, the latest available system model information, i.e., topology and transmission line parameters, is required. Given a power system with N_{line} transmission lines and N_{bus} buses and a predefined edge direction, an edge to node incidence matrix *A*, with a dimension of $N_{line} \times N_{bus}$ is defined from the topology information, as:

$$A_{ij} = \begin{cases} 1 & \text{If bus } j \text{ is the } from \text{ bus in line } i \\ -1 & \text{If bus } j \text{ is the } to \text{ bus in line } i \\ 0 & otherwise \end{cases} \quad (1)$$

Given the transmission line parameters, a diagonal susceptance matrix *B* is constructed, where $B_{ii} = b_i$ is the susceptance of line *i*. Thus, with the susceptance matrix *B*, the incidence matrix *A*, and a known slack bus, an ISF matrix $H \in \mathbb{R}^{N_{line} \times N_{bus}}$ is estimated as [11],

$$H = \left[0_{N_{bus}} \; B\tilde{A}\tilde{B}'^{-1} \right] \quad (2)$$

where $\tilde{B}' = \tilde{A}^T B \tilde{A}$, $\tilde{A}$ is a reduced size incidence matrix of *A* where the column corresponding to the slack bus is absent, and $0_{N_{bus}}$ is an all zeros column vector to replace the slack bus elements in the *H* matrix. The H_{ij} element of the ISF matrix represents the real power flow change in line *i* given a unit increase at bus *j* and a unit decrease at the slack bus [14]. It can be observed in (2) that the accuracy of the model-based method relies highly on the accuracy of the information from the system model, such as the incidence matrix A and the line susceptance matrix B.

B. Measurement-Based Method for Estimation of ISF

To overcome the drawbacks of the model-based methods, measurement-based methods have been developed. Due to a relatively high sampling frequency of PMUs (30 to 120 messages per second) and the fact that the measurements across the system are highly synchronized, the obtained PMU measurements that reflect the present system operating conditions have been considered as a key enabling factor for measurement-based estimation of ISF [10]. The measurement-based method estimates the elements of the ISF matrix considering a linear relationship between the active power flow of the lines and the active power injection as,

$$H_{ij} = \frac{\partial P_i^{line}}{\partial P_j^{bus}} \approx \frac{\Delta P_i^{line}(t)}{\Delta P_j^{bus}(t)} \quad (3)$$

For a small $\Delta t > 0$, $\Delta P_i^{line}(t) = P_i^{line}(t + \Delta t) - P_i^{line}(t)$ and $\Delta P_j^{bus}(t) = P_j^{bus}(t + \Delta t) - P_j^{bus}(t)$. Hence, based on the approximate linear relationship in (3), the total change in the power flow of line *i* due to all bus injection changes at time *t* is estimated as,

$$\Delta P_i^{line} \approx \Delta P_1^{bus} H_{i1} + ... + \Delta P_{N_{bus}}^{bus} H_{iN_{bus}} \quad (4)$$

Assume the power injection change of all buses is represented as $\Delta P^{bus} = \left[\Delta P_1^{bus}, ..., \Delta P_{N_{bus}}^{bus}\right]$. Then for *M+1* available past PMU time duration data, the active power injection change at bus *k* is,

$$\Delta P_k^{bus} = \left[\Delta P_k^{bus}[1], ..., \Delta P_k^{bus}[M]\right]^T \quad (5)$$

The active power injection change at bus *k* from time m to time $m+1$ is $\Delta P_k^{bus}[m] = P_k^{bus}[(m+1)\Delta t] - P_k^{bus}[m\Delta t]$. In addition to the bus injection change, the active power flow change of line *i* is defined as,

$$\Delta P_i^{line} = \left[\Delta P_i^{line}[1], ..., \Delta P_i^{line}[M]\right]^T, \quad (6)$$

where $\Delta P_i^{line}[m] = P_i^{line}[(m+1)\Delta t] - P_i^{line}[m\Delta t]$ is the active power flow change in line *i* from time m to time $m+1$. The value of Δt is assumed to be small. Note that, PMUs are required to be present at all buses, such that line power flows and bus injections at each sampling time can be obtained. The total active power change in line *i* is,

$$\Delta P_i^{line} = \Delta P^{bus} H_i^T \quad (7)$$

where H_i is the *i*th row of the ISF matrix *H*. The dimension of ΔP_i^{line} is M and the dimension of ΔP^{bus} is $M \times N_{bus}$. If the number of PMU time duration data *M* is larger than N_{bus}, equation (7) is overdetermined and can be solved by the Least Squares method as [10],

$$H_i = (\Delta(P^{bus})^T \Delta P^{bus})^{-1} \Delta(P^{bus})^T \Delta P_i^{line} \quad (8)$$

To estimate the ISF matrix from equation (8), the Least Squares estimator requires at least $(N_{bus}+1)$ past PMU time duration data to solve equation (7), which is not desired for real-time applications. Carefully examining the ISF matrix, it is observed that many elements of ISF that correspond to a single line are close in value. This is due to the fact that the corresponding buses have similar effect on the active power flow of that line. This property is the basis of the methodology introduced in [12], referred to as Chen's difference transformation method. The difference transformation method sorts each row of the ISF matrix H_i in a descending order, referred to as $H_{i,reorder}$, and calculates the difference between each two consecutive elements. As a result, the ISF matrix is transformed to a sparse matrix, denoted as C, and thus can be estimated even when $M < N_{bus}$,

$$\begin{aligned} \underset{C_i}{\text{minimize}} \quad & \|C_i\|_0\,, \\ \text{subject to} \quad & \Delta P_i^{line} = \Delta P^{bus} S^{-1} C_i. \\ & C_i = S H_{i,reorder} \end{aligned} \tag{9}$$

where C_i is the ith row of the sparsified ISF matrix through difference transformation and $\|\cdot\|_0$ denotes the l_0 norm, $H_{i,reorder}$ is the ith row of the sorted ISF matrix based on the value of each component in H_i. The matrix *S*, aiming at calculating the difference between each two consecutive elements in $H_{i,reorder}$, is an all-zeros $M \times M$ matrix except $S_{bd} = 1$ if $b = d$ and $S_{bd} = -1$ if $b = d + 1$. Since the optimization problem in (9) is nonconvex due to the zero norm, the authors in [12] use a convex relaxation algorithm to obtain the ISF matrix, which is referred to as the baseline estimation method in this paper. The estimation accuracy of this method, however, needs to be further improved, especially when more than two lines are out of service. This required improvement is addressed in this study.

III. Estimation of ISF via Alternating Direction Method of Multipliers

To overcome the shortcomings of the previously described methods, in this paper, a measurement-based approach is developed for estimating ISF values, without making any limiting assumptions about the system conditions. This flexibility ensures that the developed method is more widely applicable to practical systems. This is achieved by deploying the Alternating Direction Method of Multipliers (ADMM) to estimate ISF. In addition to its ability to solve a nonconvex problem, ADMM has an improved convergence rate and relies on little or no assumption regarding the objective function when compared to other methods such as gradient descent [15]–[17].

The developed ISF estimation method relies on the property that some elements of the ISF matrix are zero or close to zero. This can be explained by the fact that a change in the active power injection at a bus, that is geographically or electrically distant from a line, has little or no impact on the active power flow change of that particular line. For example, for the Illinois 200-bus system in Fig. 1, it can be observed that more than half of the lines, i.e., rows in the ISF matrix, have at least 100 elements in *H* that are smaller than 0.01. Based on the observed sparsity in the ISF matrix, the under-determined

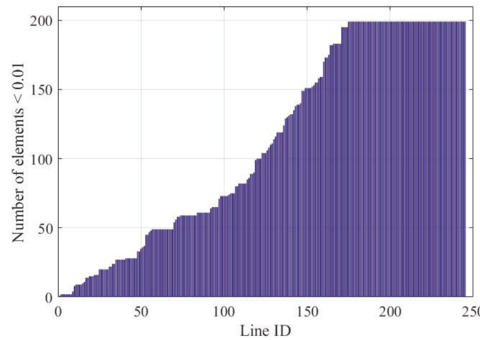

Fig. 1. The sorted number of elements in ISF that are smaller than 0.01 plotted for each line of the Illinois 200-bus (245-line) system.

equation (7) can be reformulated as an optimization problem,

$$\begin{aligned} \underset{H_i}{\text{minimize}} \quad & \left\|\Delta P_i^{line} - \Delta P^{bus} H_i^T\right\|_2^2 + \lambda \|Z\|_0\,, \\ \text{subject to} \quad & H_i - Z = \mathbb{O}. \end{aligned} \tag{10}$$

where ΔP_i^{line} is the power flow change of line *i*, ΔP^{bus} is the power injection change at all buses, and $\|\cdot\|_2$ represents the l_2 norm. The parameter λ is a positive scalar that penalizes the sparsity of H_i. A large λ leads to a more sparse estimation of H_i. Vector *Z* is an auxiliary vector that helps estimate H_i. This optimization problem is nonconvex due to the zero norm of *Z*. In this study, the ADMM method is used to solve this nonconvex problem as it enables decomposing the nonconvex problem into two subproblems. One of the sub-optimization problems can then be solved through the gradient descent method, while the other is solved analytically. The augmented Lagrangian equation of (10) is given as [15],

$$\begin{aligned} L_\rho(H_i, Z, \Lambda) = & \left\|\Delta P_i^{line} - \Delta P^{bus} H_i^T\right\|_2^2 + \lambda \|Z\|_0 \\ & + tr\left\{\Lambda^T (H_i - Z)\right\} + \frac{\rho}{2}\|H_i - Z\|_2^2 \end{aligned} \tag{11}$$

where Λ is a matrix of the Lagrangian multipliers, $tr\{\cdot\}$ denotes the trace, and ρ is a positive scalar. By using the scaled dual variable $U = \frac{\Lambda}{\rho}$, the augmented Lagrangian equation (11) is equivalent to [18],

$$\begin{aligned} L_\rho(H_i, Z, \Lambda) = & \left\|\Delta P_i^{line} - \Delta P^{bus} H_i^T\right\|_2^2 + \lambda \|Z\|_0 \\ & + \frac{\rho}{2}\|H_i - Z + U\|_2^2 - \frac{\rho}{2}\|U\|_2^2 \end{aligned} \tag{12}$$

After formulating the augmented Lagrangian equation (12) with a dual variable *U*, the ADMM estimates the minimizer of (10) by iteratively solving the following three steps outlined in (13), (14) and (15). The iterations continue until the constraints in (16) and (17) are satisfied [18],

$$H_i^{k+1} := \arg\min_{H_i} L_\rho(H_i, Z^k, \Lambda^k) \tag{13}$$

$$Z^{k+1} := \arg\min_{Z} L_\rho(H_i^{k+1}, Z, \Lambda^k) \tag{14}$$

$$U^{k+1} := U^k + H_i^{k+1} - Z^{k+1} \tag{15}$$

$$\left\|H_i^{k+1} - Z^{k+1}\right\|_2^2 \le \epsilon \tag{16}$$

$$\left\|Z^{k+1} - Z^k\right\|_2^2 \le \epsilon \tag{17}$$

The parameter k denotes an ADMM iteration, and ϵ is a small positive scalar that determines when to stop the iterations. The initial value of the dual variable U^0 is set as an all-zeros vector, and the initial value of Z can be obtained from an offline study of H to accelerate the convergence speed. The initial value of Z^0 is obtained from (2) with a full topology to improve the convergence speed of the developed ADMM-based estimation method. Although ISF changes with a change in system topology, leveraging a model-based ISF estimation to set Z^0 leads to an acceptable accuracy and is corroborated by the case studies presented in the later sections. It is also assumed that the outaged lines would not dramatically change the sparsity of the ISF matrix. If the change of ISF is sufficiently large, it is necessary to assign a different initial value to Z^0 to guarantee the convergence of the ADMM-based estimation method. However, the initial values of Z^0 can be learned off-line for different system topologies, that in turn can be used for online validation. To further increase the speed of the validation process, parallel computing can be utilized. Therefore, adjusting Z_0 will not significantly impact the performance of the developed method.

The two subproblems formulated in (13) and (14) have to be solved during each iteration. In particular, subproblem (13) can be solved through the gradient descent method, and Z^{k+1} of (14) can be obtained analytically. Hence, the optimization problem in (13) and (14) can be formulated as,

$$H_i^{k+1} := \arg\min_{H_i} \left\|\Delta P_i^{line} - \Delta P^{bus} H_i^T\right\|_2^2 + \frac{\rho}{2}\left\|H_i - Z^k + U^k\right\|_2^2 \tag{18}$$

$$Z^{k+1} := \arg\min_{Z} \lambda \left\|Z\right\|_0 + \frac{\rho}{2}\left\|H_i^{k+1} - Z + U^k\right\|_2^2 \tag{19}$$

It is observed that the sub-optimization problem in (19) is nonconvex due to l_0 norm. However, it can be solved analytically after it is decomposed into subproblems that include each component Z_q,

$$Z_q^{k+1} = \begin{cases} (H_i^{k+1} + U^k)_q & \lambda \le \frac{\rho}{2}(H_i^{k+1} + U^k)_q^2 \\ 0 & \lambda > \frac{\rho}{2}(H_i^{k+1} + U^k)_q^2 \end{cases} \tag{20}$$

where Z_q^{k+1} is the qth element of Z^{k+1} in iteration $k+1$. Hence, in this study, ADMM enables estimating H_i when $M < N_{bus}$ by solving (15), (18) and (20) iteratively until the conditions in (16) and (17) are satisfied. The developed ISF estimation algorithm is summarized in Algorithm 1. In summary, with the support of the high sampling frequency feature of PMUs, the active power flow change ΔP_i^{line} of line i and the active power injection change at all buses ΔP^{bus} can be obtained, and are utilized to estimate the ISF of line i, i.e. $\hat{H}_i$, through the developed ADMM formulations.

IV. Case Studies

The developed measurement-based ISF estimation method is studied on IEEE 57-bus [19] and Illinois 200-bus systems [20]. The performance of the ADMM-based method is evaluated and compared with the baseline method formulated in (9), with respect to the number of available PMU measurements and different system models where several lines are outaged.

Algorithm 1: Estimating H_i through ADMM method.

Input : Predefined threshold ϵ, scalar ρ and λ, $Z^0 = H_i^{off-line}$, $U^0 = \mathbb{O}$, iteration $k = 0$

Output: Estimation of line i ISF, i.e., $\hat{H}_i$

```
while (16) and (17) are not satisfied do
    Update H_i^{k+1} via (18);
    Update Z^{k+1} via (20);
    Update U^{k+1} via (15);
    k ← k + 1;
end
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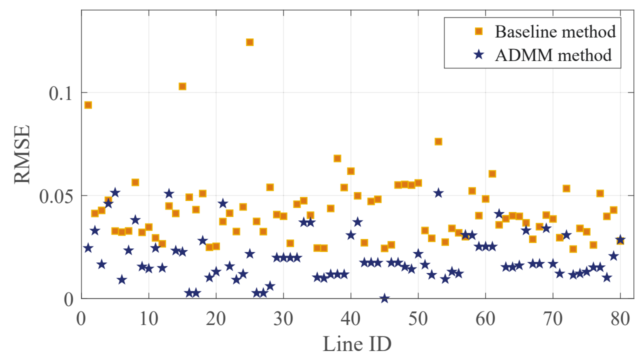

Fig. 2. Comparison of the ISF estimation accuracy in 57-bus system with no line outage.

A. PMU data Simulation

MATPOWER [21] is used to simulate PMU voltage and current time-series data. For a sample at time instant m, the load fluctuation at a bus $d \in \{\text{load buses}\}$ is simulated as,

$$L_d^{bus}[m] = (1 + \rho) L_{d,0}^{bus}[m] + \epsilon \tag{21}$$

where $L_d^{bus}[m]$ and $L_{d,0}^{bus}[m]$ are the load at bus d at time m, and the nominal value of the load, respectively. Given the load profile in (21), the bus voltages and the line currents can be obtained with an AC power flow solution. Hence, the line active power flows and bus injections are determined. In this case study, it is assumed that ρ and ϵ follow a normal distribution where the mean is zero and the standard deviation is 0.1. The choice for ρ and ϵ distributions is inspired by [10], [12]. However, other distributions can be utilized and are expected to yield similar results. It is also assumed that bad measurements have been removed by the existing bad data detection mechanisms and the sampling frequency of PMUs is 60 messages per second. The number of recorded PMU time duration data for the IEEE 57-bus system and Illinois 200-bus system are 30 and 120, i.e., 0.5-second and 2-second PMU data, respectively.

B. IEEE 57-bus system

A total of 30 PMU measurements are used to estimate the ISF vector of each transmission line with Algorithm 1 and baseline estimation method in (9). The estimation error is quantified by the root mean square error (RMSE) in Fig. 2. It is observed that, for most lines, the ADMM-based estimation method reaches better accuracy than the baseline estimation method. To illustrate the impact of the different number of measurements, the average RMSE of all lines is evaluated with

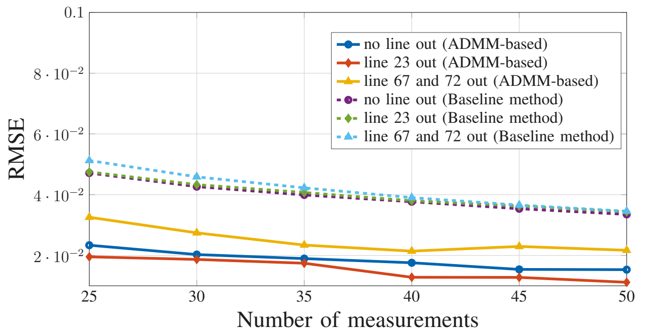

Fig. 3. Comparison of the average ISF estimation accuracy with different number of measurements and line outages in IEEE 57-bus system.

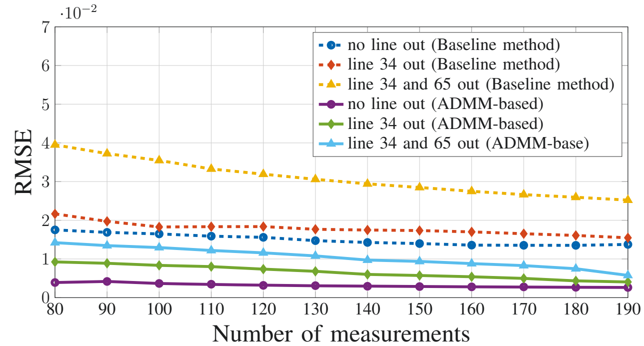

Fig. 5. The Average ISF estimation accuracy for the 200-bus system, under different number of measurements and line outages.

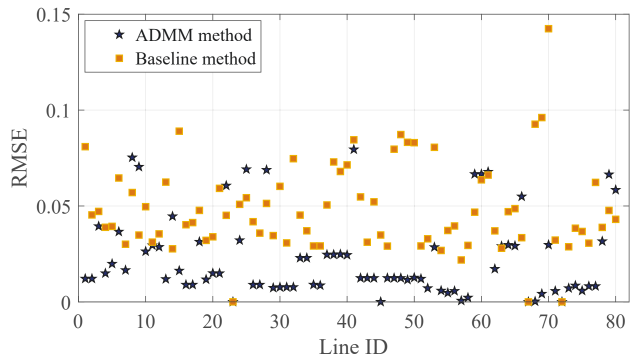

Fig. 4. Comparison of the ISF estimation accuracy for IEEE 57-bus system with lines 23, 67, 72 outaged.

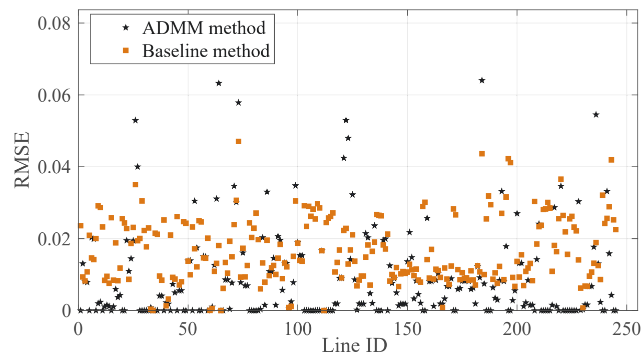

Fig. 6. ISF estimation accuracy for the Illinois 200-bus system when lines 34, 65, 112 outaged.

six different numbers of PMU measurements. Additionally, the impact of errors in the system model, where one and two outaged lines are undetected, is evaluated on the IEEE 57-bus system. The case studies in Fig. 3 show that the estimation accuracy of the ADMM-based estimation method is slightly decreased with fewer measurements and more number of undetected outaged lines. It is also observed that the ADMM-based estimation method achieves a higher estimation accuracy than the baseline method in Fig. 3.

For the baseline method in [12], at most two outaged lines can remain undetected to guarantee an estimation accuracy. To further demonstrate the benefits of the developed method, a case where three outaged lines are undetected in the 57-bus system is simulated, as shown in Fig. 4. It is observed that the ADMM-based estimation method is more accurate for most of the lines. For some lines, especially those with a small sparsity, ADMM does not reach the same accuracy as the baseline methods, while the accuracy difference between these two methods is within an acceptable range.

C. Illinois 200-bus system

A larger system, i.e., the Illinois 200-bus system, is used to validate the efficiency of the developed ADMM-based method. With a different number of PMU measurements and different model errors, the performance of the ADMM-based algorithm and the baseline method is compared in Fig. 5. The ADMM-based estimation accuracy is increased with more PMU time duration data and fewer undetected outaged lines. Similar to the 57-bus system, the ADMM-based method enables a better estimation of the ISF matrix than the baseline method. To further demonstrate the benefits achieved, the case where three lines are out of service is studied with the aforementioned two ISF estimation methods. The results presented in Fig. 6 show that the developed method achieves a better estimation accuracy than the baseline estimation method for most lines. Although the RMSE of a few lines is higher than the baseline method when using the ADMM-based method, the magnitude of the RMSE is 0.06 in the worst case. Hence, the effectiveness of the ADMM-based method is not significantly impacted.

TABLE I
COMPUTATION TIME FOR ESTIMATION OF ISF WITH MEASUREMENTS

	IEEE 57-bus system	Illinois 200-bus system
ADMM-based ISF estimation	3.66 s	9.28 s
Baseline estimation method	0.61 s	2.25 s

D. Efficiency analysis

The average ISF estimation time for a single line are presented in Table I for comparison. The ADMM-based method takes 3.66 seconds and 9.29 seconds for IEEE 57-bus and Illinois 200-bus systems, while the baseline approach requires around 0.61 seconds and 2.25 seconds to estimate ISF. Due to the iterative nature of Algorithm 1, the developed ADMM-based approach takes a longer time to estimate the ISF matrix but achieves higher accuracy, especially when three lines are out of service. Hence, the longer computation time is a trade-off for better accuracy, particularly when more lines are out of service. Moreover, it should be noted that the ISF values do not have to be updated unless the system operating conditions change significantly. Hence, the execution time is still acceptable. The calculations are performed on a computer with an i7-8700 processor and a 3.2GHz CPU. The computation efficiency of the developed method can be further improved with more advanced computers and parallel computing techniques.

V. Conclusions

In this paper, a measurement-based estimation method was developed to enable using PMU measurements for a more accurate estimation of ISF. Specifically, an ADMM-based approach estimates the ISF matrix and is found to be more accurate than the state-of-the-art difference transformation method. By decomposing the nonconvex ISF estimation problem into subproblems that can be solved iteratively, the ADMM-based approach achieves higher accuracy. Without the knowledge of the system model information, the developed estimation method enables using PMU measurements to correctly estimate the power flows and is thus robust to undetected topology changes or wrong model parameters. Case studies performed on the IEEE 57-bus and Illinois 200-bus systems have demonstrated the promising performance of the ADMM-based estimation method. The developed estimation method can be further extended to various power system analyses such as identifying proper control actions and finding critical contingencies.

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