SUPPORTING GENERALIZING IN THE CLASSROOM: ONE TEACHER'S BELIEFS AND INSTRUCTIONAL PRACTICE

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The purpose of this case study is to understand how one teacher supports generalizing in her classroom by examining her beliefs about generalization and how to support generalizing in classrooms. We found that the teacher held numerous beliefs about generalization and these beliefs influenced her beliefs about how to support generalizing in the classroom. Moreover, we found that her beliefs about generalization and how to support generalizing formed a system of beliefs that were consistently evidenced in her classroom instruction. Practical implications of the findings, particularly for mathematics teacher educators, are discussed.

Keywords: Teachers' Beliefs, Instructional Practice, Generalizations

Generalization is an important aspect of learning mathematics; researchers have argued that the development of generalizations is essential to all mathematical activity (Becker & Rivera, 2006; Pierce, 1902). Researchers have investigated different types of generalizations students make (e.g., Ellis, 2007; Radford, 2006; 2008), the mental activities required to generalize (e.g., Amit & Neria, 2008; Becker & Rivera, 2006), and the types of instructional activities that support generalizing (e.g., Doerfler, 2008; Steele & Johanning, 2004). Still, studies investigating what teachers do to foster generalizations in the classroom are scant (Mata-Pereira & da Ponte, 2017) and the field lacks research that considers the *teacher's* perception of generalizations. Because understanding teachers' beliefs is an integral part of fostering substantive, lasting change to their practice (Pajares, 1992), the purpose of this case study is to examine one teacher's beliefs about generalization and how to support generalizing in classrooms as well as how these beliefs relate to her instructional practice.

Theoretical Framework and Literature Review

Generalization and How They Are Developed

Researchers investigating how instruction can foster generalization have identified a number of specific recommendations. These include techniques such as showing variation across tasks (Mason, 1996), emphasizing similarity across tasks (Radford, 2008), and ordering the structure of tasks in a progressive sequence (Ellis, 2011). Other recommendations address pedagogical moves (e.g. Amit & Neria, 2008, Koellner et al., 2008), yet research investigating teachers' efforts to foster generalizing at the classroom level is limited. Overall, however, recognizing, elucidating, and encouraging appropriate generalizations remains challenging for teachers (Callejo & Zapatera, 2017). Given these challenges, it is critical to identify teachers' perceptions of generalization and their beliefs about how to teach for generalization, in order to better support their ability to foster generalizing in the classroom.

Defining A Belief

For the purposes of this study, we use the definition provided by Rokeach (1968), who defined a belief as "any simple proposition, conscious or unconscious, inferred from what a person says or does, capable of being preceded by the phrase, 'I believe that…'" (p. 113) and added that "all beliefs are predispositions to action" (p. 113). Another construct that was influential to this study is that of a belief system. Green (1971) proposed a theory of how beliefs are held in a system, and described three characteristics of a belief system. First, individuals hold beliefs in a manner that is logical and consistent to them. The second characteristic is that beliefs can be held with varying psychological strength. Thirdly, beliefs can be held in clusters and these clusters may be isolated from other clusters.

Mathematics Teachers' Beliefs Research

The types of beliefs mathematics education researchers have most commonly investigated can be grouped into three categories: beliefs about *mathematics*, *teaching mathematics*, and *learning mathematics* (e.g. Conner et al., 2011; Thompson, 1984). Of the three categories, beliefs about mathematics may be most influential, as some researchers (e.g. Cross, 2009; Thompson, 1984) have argued that these beliefs influence a teacher's beliefs about both learning and teaching mathematics. Regarding the consistency of teachers' beliefs, a number of researchers have found teachers' beliefs to be consistent with their practice (e.g. Conner & Singletary, 2021; Cross Francis, 2015). Yet, others have claimed inconsistency between the beliefs teachers hold and their practice (e.g. Raymond, 1997). Consistent with our definition of beliefs, we agree with Leatham (2006) and Philipp (2007) who claimed that researchers should not assume that a teacher holds beliefs that are inconsistent.

Although there is extensive research on teachers' beliefs, investigations into what teachers believe about generalization and how to support generalizing in the classroom are scarce. Due to our view that beliefs have profound influence on one's actions, we believe investigations into teachers' beliefs about generalizations can yield novel insights into their instructional practices and can aid educators as they help teachers support generalizing in classrooms.

Methods

The present study was a case study (Merriam, 1998) of one teacher's beliefs about generalizations and how to support generalizing in the classroom. Ms. N, the participating teacher, was a third-year teacher who taught sixth-grade mathematics. We conducted four classroom observations during one week of instruction and recorded each observation with two cameras. For the observations, Ms. N chose lessons in which students explored various properties of ordered pairs in the coordinate plane and how to choose appropriate axes scales.

Because we believe a teacher's beliefs must be inferred from both their words and actions, we also conducted two interviews with Ms. N after the observations. The first interview was semi-structured and provided opportunities for Ms. N to discuss her beliefs about mathematics, generalizations, and how she supported generalizing in her classroom. The second interview was a videoclip interview (Speer, 2005). Two clips which Ms. N chose from one of the observations formed the basis of the interview. Both interviews were video recorded and transcribed.

The data from each observation was transcribed. We began data analysis by coding the two interviews with the broad codes of *generalization* and *classroom supports for generalizing* (CSG). We then determined emergent themes within each code and created subcodes according to these themes (Strauss & Corbin, 1998). With the new subcodes, we then analyzed Ms. N's classroom data and determined if there were any themes not captured with the current codes. After analyzing the observation data and revising our codes, we re-coded the interview data, looking for evidence for the existing codes and asking if new codes needed to be included. Once no new codes emerged from this iterative process, we began to write narratives for what Ms. N believed about generalizations and how to support generalizations in the classroom.

Results

In the following sections we discuss Ms. N's beliefs about generalization and her beliefs about supporting generalizing with examples of how she enacted those beliefs in her instruction.

Beliefs About Generalizations and How They Are Developed

The beliefs we inferred Ms. N held about generalization form a subset of her beliefs about mathematics. We identified two primary beliefs and three derivative beliefs (Green, 1971) Ms. N held about generalization. One of her primary beliefs about generalization was that a *generalization is an always true statement*. On multiple occasions she described a generalization as something that will always be true. The transcript below captures Ms. N's response when asked to contrast a generalization with a strategy. We underscore that Ms. N described generalization as the "one true thing" that she wanted students to walk away having developed.

Interviewer:	Why was this a strategy and not necessarily a generalization?
Ms. N:	Because…when I think of a generalization, I'm thinking of…one truth.
Interviewer:	Say that again, you cut up a little bit.
Ms. N:	Sorry, I'm thinking of, like, there's one truth. Like, there is one true thing that I want them to get. That's the generalization in my mind.

One of Ms. N's derivative beliefs was that *a theory is an idea that is not yet a generalization*. She described a theory as an idea that is either untested or true in some cases, but not in others. In one instance, Ms. N described a student's initial idea as a theory because it was based off one example and "it was true for that example". Implied in the way Ms. N described the student's theory was that it worked in some cases but not all cases. Ms. N also described theories as being untested. After being asked when a theory becomes a generalization, Ms. N responded by saying "Yeah. When…have you seen enough different types that you always trust it?"

Another belief Ms. N held that appeared to be derivative to her belief about generalizations was that *generalizations are tested theories*. In one instance from her classroom Ms. N claimed that she knew one student's initial theory "wasn't always true". However, after testing his theory and engaging in a discussion about the theory, the student modified his statement so that it was true for all cases. Ms. N claimed it was important for the student to refine his theory into "a more accurate statement that he could cling to." In this instance the student was able to develop a generalization as his theory went through a process of refinement.

Another notable and primary belief Ms. N held about the development of generalizations was that generalizations are actively developed rather than passively received. Throughout both interviews, Ms. N continually described generalizations as something her students would "discover", "develop", and "notice".

Beliefs About How To Support Generalizations In The Classroom

Ms. N believed it was important to engage students in examples that were "easy" and accessible for all students. The purpose of the easy examples was for students to "clearly *see* that it (the pattern she wanted students to notice) works". For instance, in a lesson in which students were to determine when two points were reflected across the *x*- or *y*-axis, Ms. N began the lesson by placing a red dart on the front board, then had a student place a yellow dart where the point would be if reflected across the *y*-axis. After discussing the coordinates of these two points and writing them on the board, Ms. N repeated this sequence for a second pair of reflected points. Rather than writing the two ordered pairs on the board and telling students that one point is the reflected image of the other, students were able to see this and presumably trust it to be true.

As alluded to in the previous paragraph, Ms. N believed it was important to leverage these easy examples to lead students to notice and discuss relationships that emerged from the initial examples. She did this by asking students a focusing question highlighting certain relationships. Ms. N also seemed to indicate that the relationships students noticed and discussed from the initial examples were not yet generalizations, but theories. Hence, Ms. N believed that accessible examples could be leveraged in a way that students could notice patterns and develop theories which then could be cultivated into generalizations.

After noticing and discussing the patterns, or theories, that emerged from the initial examples, Ms. N believed that engaging in additional examples helped students refine or reinforce their theories. For instance, after discussing the relationships between the coordinates of a point with the coordinates of its reflection, Ms. N gave additional examples for students to work in small groups. Ms. N claimed such examples were important because if "(you) do practice with them, and then they can do it independently on their own, they've like, got it (the intended generalization)".

Ms. N's System of Beliefs

Together, Ms. N's beliefs about generalization and how to support generalizing in the classroom form a system of beliefs (Green, 1971) that appear to be internally consistent and consistent with her practice. Specifically, we infer her beliefs that generalizations are developed through refining or reinforcing a theory to be consistent with and related to her beliefs about what constitutes a generalization and a theory.

Ms. N's beliefs about generalizations and how they are developed also appear to influence her beliefs regarding how to support generalizing in the classroom. Her beliefs about refining or revising theories to develop generalizations appear to influence her beliefs that students need to engage in easy, accessible examples first, then discuss patterns salient across those examples, then do additional examples so that students develop the intended generalization. As described in the previous section, Ms. N's beliefs about how to support generalizing in the classroom seem to be consistent with her instructional practice, and these beliefs also appear to be consistent with her beliefs about generalizations and their development.

Discussion

This study reveals one teacher's beliefs about generalization and how to support generalizing in the classroom and how those beliefs relate to her instructional practice. Most notable is that her primary belief that a generalization is "an always true statement" appears to heavily influence her beliefs concerning how generalizations develop and, as an extension, how to support students' in generalizing in the classroom. As we continue to work with teachers, in our own on-going project, their beliefs about generalization and how they are developed have been a paramount consideration in our interactions with each teacher. Moreover, the teachers' beliefs about generalization have been critical as we plan professional development aimed to co-construct a vision of productive generalizing in classrooms and generate instructional strategies that foster this type generalizing.

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